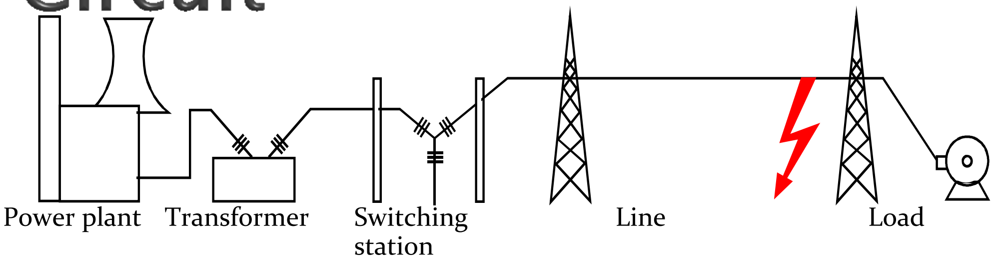
Lecture-2

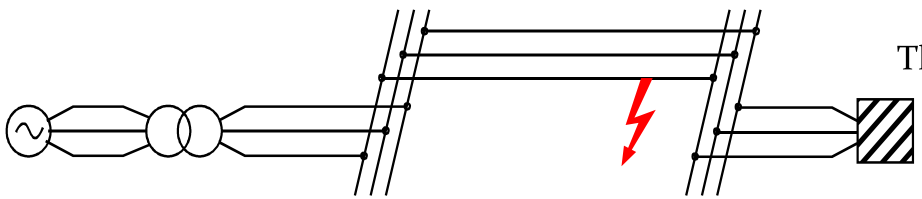
Symmetrical component transformation

Topic Covered

- Three Phase System
- Balanced and Unbalanced faults
- Symmetrical Components
- Vector Presentation
- Matrix Relations

Balanced Three-phase Systems From Network to Single-phase Equivalent Circuit

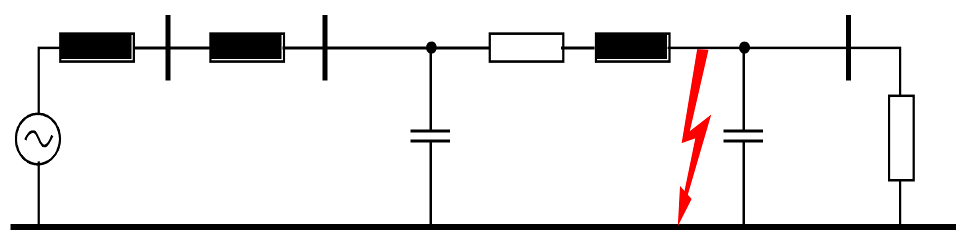




Three-phase system

Single-line diagram

Equivalent single-phase circuit



Balanced and Unbalanced faults

- Balanced Cases
  - three-phase fault
  - (symmetrical) load flow

- Unbalanced Cases
  - Single line to ground fault
  - Line to line fault
  - Double line to ground fault
  - (unsymmetrical load flow)

Analyzing unbalanced system using Fortescue's Theorem

- Unbalanced faults in power systems require a phase by phase solution method or other techniques.
- One of the most useful techniques to deal with unbalanced networks is the "symmetrical component" method, developed in 1918 by C.L. Fortescue.

Symmetrical Components

Reasons for use of symmetrical component

Unbalanced systems are difficult to handle

-> several independent balanced systems are easier to handle than one unbalanced system

Transformation of one unbalanced 3-phase system into 3 balanced 3-phase systems.

-> for each system only one phase has to be considered

Analyzing unbalanced system using Fortescue's Theorem

- Any three unbalanced set of voltages or currents can be resolved into three balanced systems of voltages or currents, referred to as the system symmetrical components, defined as follows:
  - **Positive Sequence** components: three phasors of equal magnitude displaced 120 degrees from each other following the positive sequence
  - **Negative Sequence** components: three phasors of equal magnitude displaced 120 degrees of each other following the negative sequence
  - **Zero Sequence** components: three parallel phasors having equal magnitude and angle

For a 3-ph system: 3 unbalanced phasors can be resolved into 3 balanced systems of 3 phasors each

Let Va, Vb, Vc be the Phase voltages

According to Fortescue, these can be transformed into

Positive-seq. voltages: Va1, Vb1, Vc1

Negative-seq. voltages: Va2, Vb2, Vc2

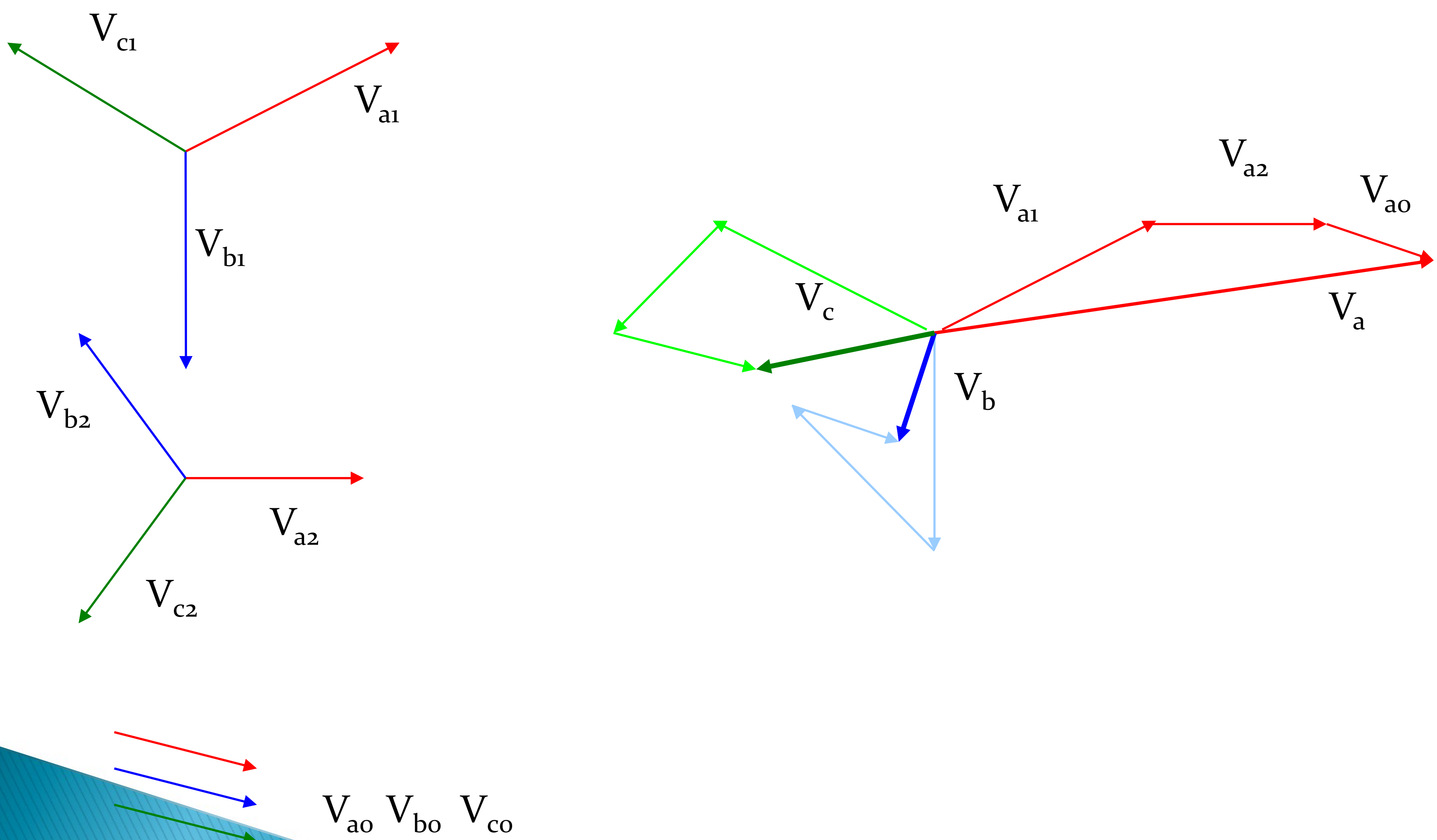
zero-sequence voltages: Va0, Vb0, Vc0

Thus, Va = Va1 + Va2 + Va0

Vb = Vb1 + Vb2 + Vb0

Vc = Vc1 + Vc2 + Vc0

Vector Presentation



The ‘a’ operator

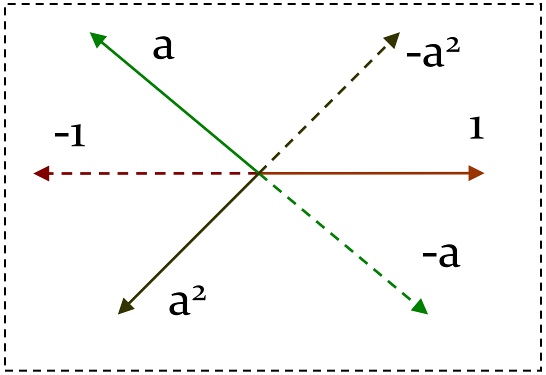
$a = 1\angle 120^0 = -0.5 + j\,0.866$

a I rotates I by 120^0

$a^2 = 1\angle 240^0 = -0.5 - j\,0.866$

$a^3 = 1\angle 360^0 = 1\angle 0^0 = 1 + j\,0$

$1 + a + a^2 = 0$



From figure previous figures

$V_{b1} = a^2 V_{a1}$ $V_{c1} = a V_{a1}$

$V_{b2} = a V_{a2}$ $V_{c2} = a^2 V_{a2}$

$V_{bo} = V_{ao}$ $V_{co} = V_{ao}$

sub. In Eq. (Slide 8) we get:

Thus, $V_a = V_{ao} + V_{a1} + V_{a2}$

$V_b = V_{ao} + a^2 V_{a1} + a V_{a2}$

$V_c = V_{ao} + a V_{a1} + a^2 V_{a2}$

Matrix Relations

Let

$$Vp = \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix}; \quad Vs = \begin{bmatrix} v_{a0} \\ v_{a1} \\ v_{a2} \end{bmatrix}; \quad A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$$

And Inverse of A is

$$A^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}$$

Matrix Relations

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} v_{a0} \\ v_{a1} \\ v_{a2} \end{bmatrix}$$

Similarly currents can be obtained using their symmetrical components

Matrix Relations

$$V_p = A\,V_s; \qquad V_s = A^{-1} V_p$$

$$V_{a0} = 1/3\,(V_a + V_b + V_c)$$

$$V_{a1} = 1/3\,(V_a + aV_b + a^2V_c)$$

$$V_{a2} = 1/3\,(V_a + a^2V_b + aV_c)$$

Matrix Relations

$$\begin{bmatrix} v_{a0} \\ v_{a1} \\ v_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix}$$