Control System Engineering

Section D: Compensation

Topic Covered : Necessity of compensation, compensation networks, application of lag and lead compensation, basic modes of feedback control, proportional, integral and derivative controllers, illustrative examples.

Compensation

- The design of a control system is concerned with the arrangement of the system structure and the selection of a suitable components and parameters.

- A compensator is an additional component or circuit that is inserted into a control system to compensate for a deficient performance.

- Types of Compensation
  - Cascade compensation
  - Feedback compensation
  - Output compensation
  - Input compensation

PID Controllers

- PID control consists of a proportional plus derivative (PD) compensator cascaded with a proportional plus integral (PI) compensator.

- The purpose of the PD compensator is to improve the transient response while maintaining the stability.

- The purpose of the PI compensator is to improve the steady state accuracy of the system without degrading the stability.

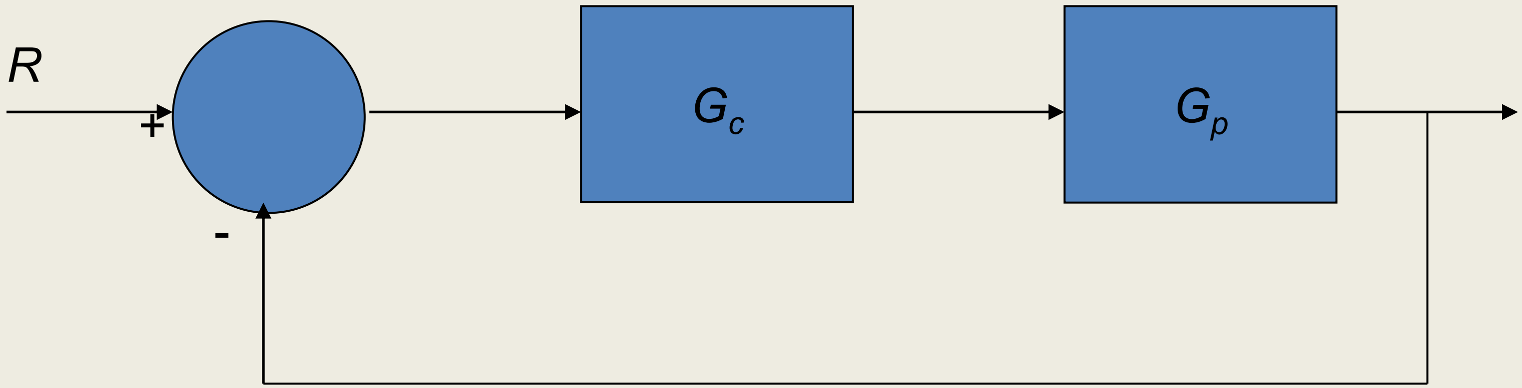
- Since speed of response, accuracy, and stability are what is needed for satisfactory response, cascading PD and PI will suffice.

The Characteristics of P, I, and D Controllers

Note that these correlations may not be exactly accurate, because *Kp*, *Ki*, and *Kd* are dependent of each other. In fact, changing one of these variables can change the effect of the other two. For this reason, the table should only be used as a reference when you are determining the values for *Ki*, *Kp* and *Kd*.

Response	Rise Time	Overshoot	Settling Time	SS Error
K_P	Decrease	Increase	Small Change	Decrease
K_I	Decrease	Increase	Increase	Eliminate
K_D	Small Change	Decrease	Decrease	Small Change

The Simplest form of compensation is gain compensation



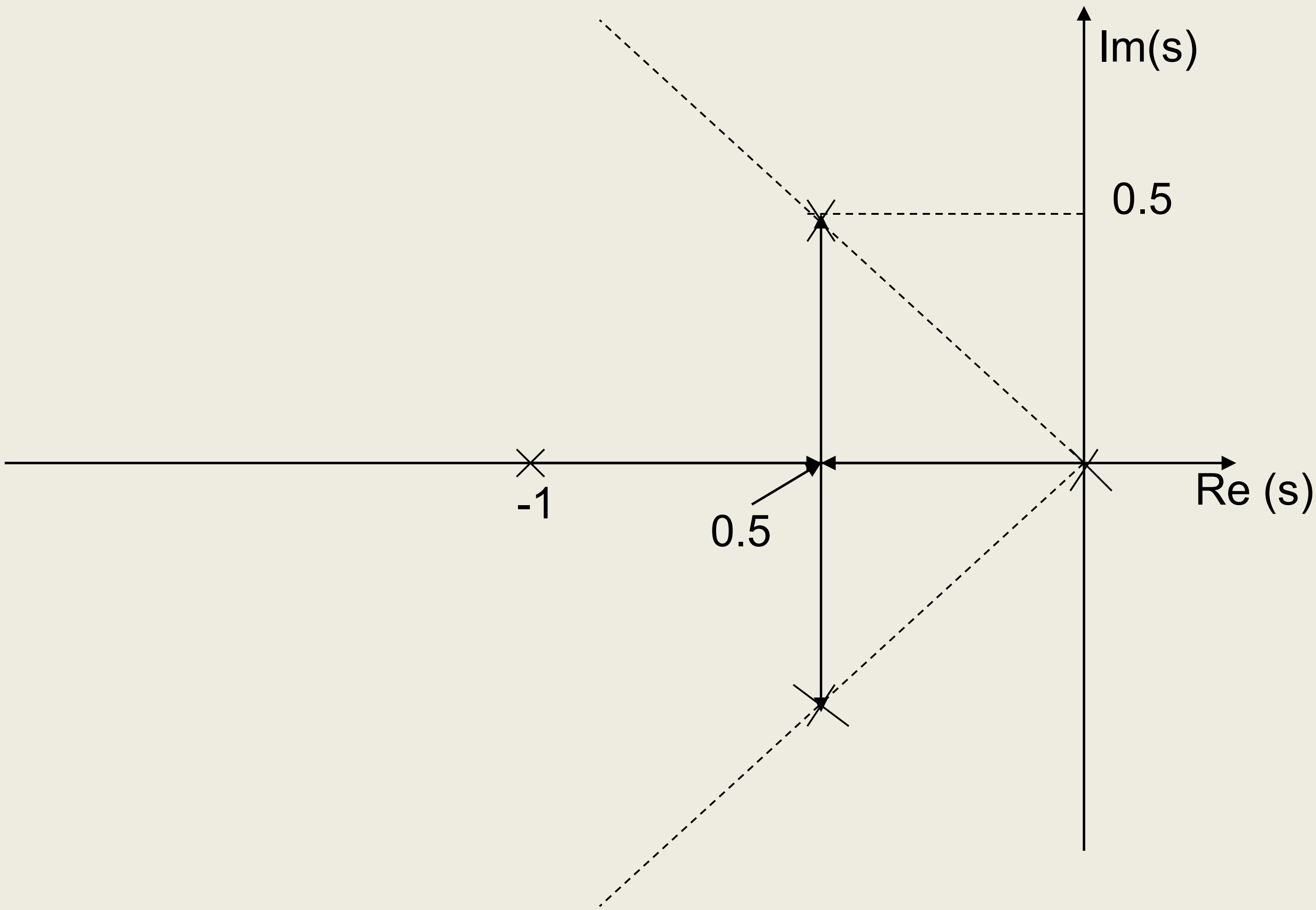
$G_p(s) = \frac{10}{s(s+1)}$ Our goal is to improve the transient response of this system

The system is type I; the closed - loop system will exhibit zero error to a step function

Design requirement : P.O 5%; Draw the root locus; Chose $\xi = 1/\sqrt{2}$. Simple geometry shows that to achieve the specified damping ratio the closed loop poles will be at

$$s = -0.5 \pm j0.5; K = \frac{1}{\left|G_p(s)\right|_{s=-0.5+j0.5}} = \frac{\left|-0.5+j0.5\right|\left|0.5+j0.5\right|}{10} = 0.05$$

Root Locus for Simple Gain Compensator



Lead/Lag Compensation

- Lead/Lag compensation is very similar to PD/PI, or PID control.
- The lead compensator plays the same role as the PD controller, reshaping the root locus to improve the transient response.
- Lag and PI compensation are similar and have the same response: to improve the steady state accuracy of the closed-loop system.
- Both PID and lead/lag compensation can be used successfully, and can be combined.

Lead Compensation Techniques Based on the Root-Locus Approach

- From the performance specifications, determine the desired location for the dominant closed-loop poles.

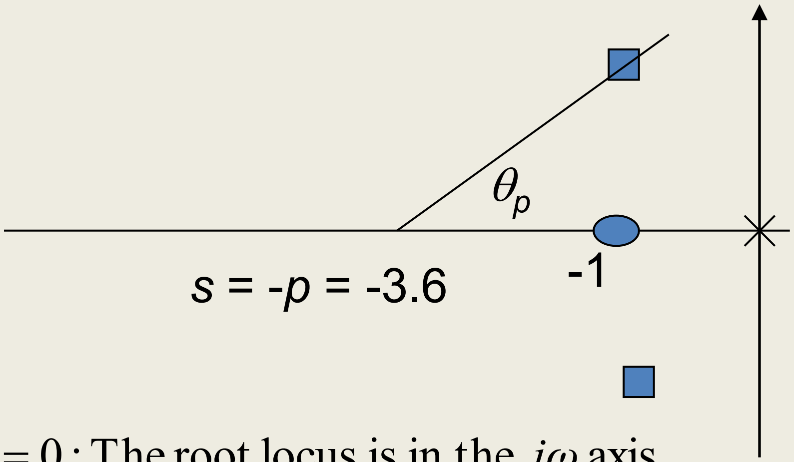
- By drawing the root-locus plot of the uncompensated system ascertain whether or not the gain adjustment alone can yield the desired closed-loop poles. If not calculate the angle deficiency. This angle must be contributed by the lead compensator.

- If the compensator is required, place the zero of the phase lead network directly below the desired root location.
- Determine the pole location so that the total angle at the desired root location is 180^o and therefore is in the compensated root locus.

- Assume the transfer function of the lead compensator.

- Determine the open-loop gain of the compensated system from the magnitude conditions.

Lead Compensator using the Root Locus



$$GH(s) = \frac{K_1}{s^2}; 1 + GH(s) = 1 + \frac{K_1}{s^2} = 0 : \text{The root locus is in the } j\omega \text{ axis}$$

$$\text{We desire to compensate this system with a network, } G_c(s) = \frac{s+z}{s+p}$$

$$Ts \le 4s; P.O \le 35\%; \xi \text{ should be} \ge 0.32; T_s = \frac{4}{\xi\omega_n} = 4; \xi\omega_n = 1$$

$$\text{We will choose a desired dominant root location as } r_1, \hat{r}_1 = -1 \pm j2$$

$$\text{We place the zero of the compensator directly below the desired location at } s = -z = -1$$

$$\phi = -2 \times 116 + 90 = -142^{\circ}; -180^{o} = -142 - \theta_p; \theta_p = 38^{o}; G_c(s) = \frac{s+1}{s+3.6}$$

$$GH(s)G_c(s) = \frac{K_1(s+1)}{s^2(s+3.6)}; K_1 = \frac{(2.23)^2(3.25)}{2} = 8.1$$

Adding Lead Compensation

The lead compensator has the same purpose as the PD compensator: to improve the transient response of the closed-loop system by reshaping the root locus. The lead compensator consists of a zero and a pole with the zero closer to the origin of the *s* plane than the pole. The zero reshapes a portion of the root locus to achieve the desired transient response. The pole is placed far enough to the left that it does not have much influence of the portion influenced by the zero.

$$\text{Consider } G_p = \frac{10}{s(s+1)}$$

$$\text{Design Specifications}: \text{P.O} \leq 20\%; t_p \leq 1.0\text{s}$$

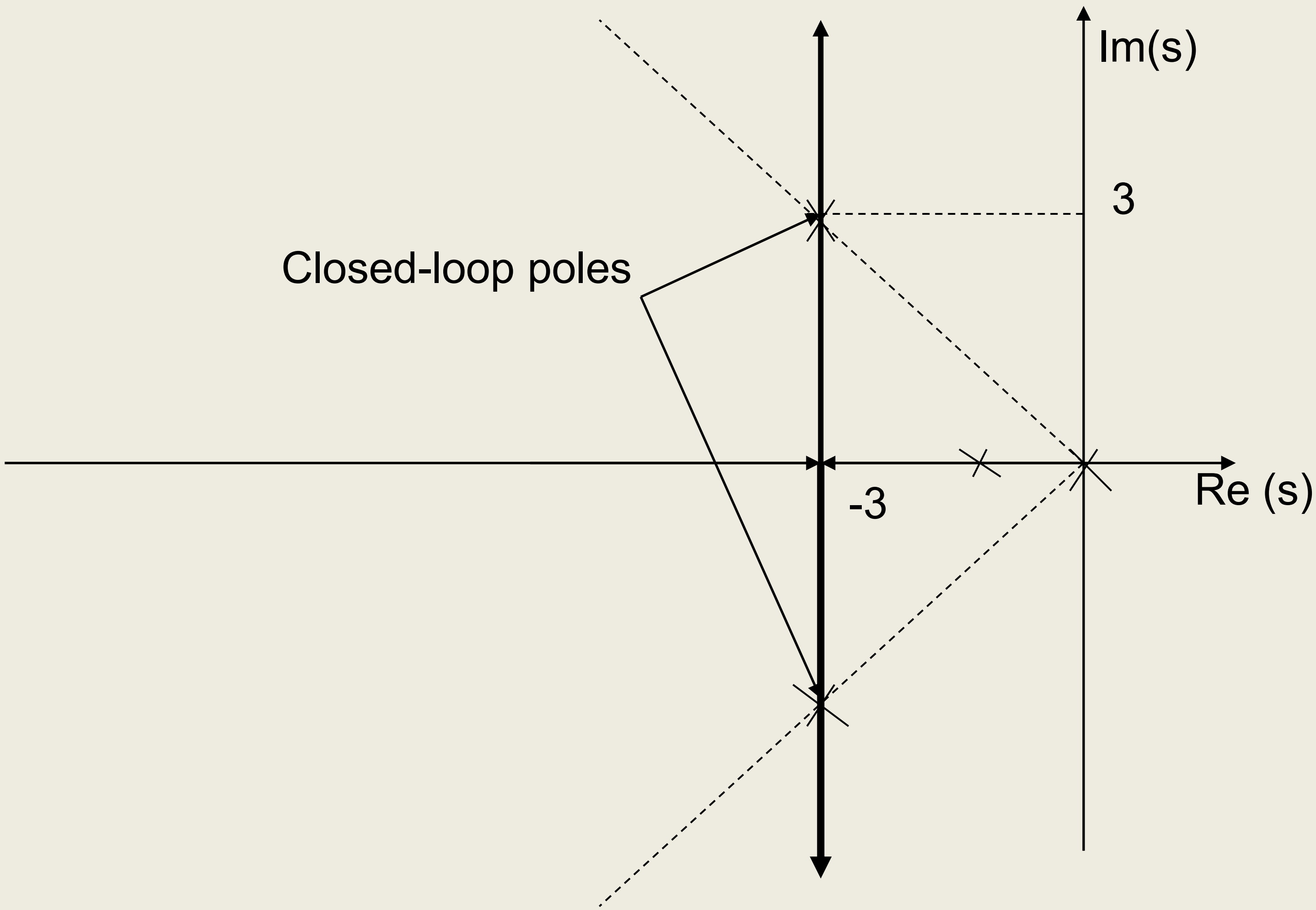
$$\text{To achieve the desired tp, we place the closed-loop poles at } s = -3 \pm j3.$$

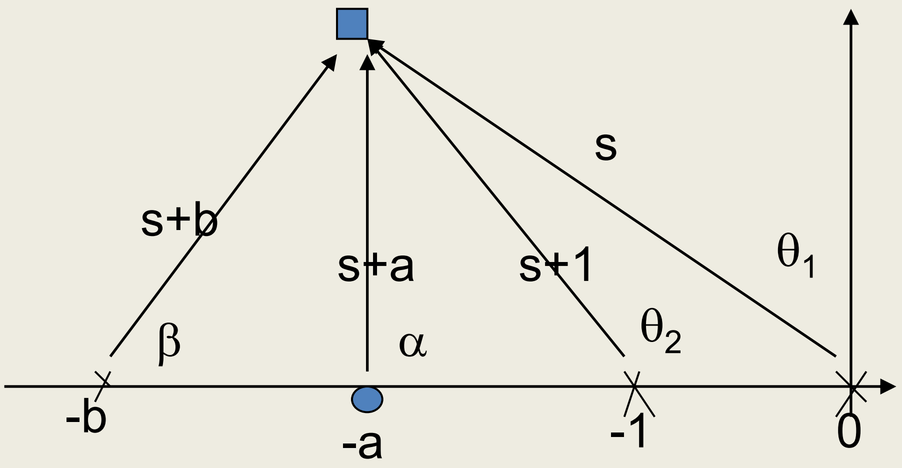
$$\xi = 1/\sqrt{2}; \text{Expect P.O to be 5\%; The general formular for the compensator is}$$

$$G_c(s) = \frac{K_c(s+a)}{s+b}; 0 \langle a \langle b$$

$$\angle G_c(s) G_p(s)_{s=-3\pm j3} = -180$$

Root Locus for Simple Gain Compensator





$$\alpha - \beta - \theta_1 - \theta_2 = 180^o; \alpha - \beta = \theta_1 + \theta_2 - 180^o = 78.7^o$$

$$\text{Fix s at -3}; \beta = 90 - 78.7^o = 11.3^o; b = 3 + \frac{3}{\tan 11.3^o} = 3 + 15 = 18$$

$$G_c(s) = \frac{K_c(s+3)}{s+18}; Kc = \left(\frac{|s||s+1||s+18|}{10|s+3|} \right)_{s=3+j3} = 7.8$$

$$G_c(s) = \frac{7.8(s+3)}{s+18}$$

Adding a Lag Controller

- A first-order lag compensator can be designed using the root locus. A lag compensator in root locus form is given by

$$G(s) = \frac{s - z_o}{s - p_o}$$

- where the magnitude of z_o is greater than the magnitude of p_o. A phase-lag compensator tends to shift the root locus to the right, which is undesirable. For this reason, the pole and zero of a lag compensator must be placed close together (usually near the origin) so they do not appreciably change the transient response or stability characteristics of the system.

How does the Lag Controller Shift the Root Locus to the Right?

- Recall finding the asymptotes of the root locus that lead to the zeros at infinity, the equation to determine the intersection of the asymptotes along the real axis is:

$$\alpha = \frac{\sum \text{poles} - \sum \text{zeros}}{\text{poles} - \text{zeros}}$$

- When a lag compensator is added to a system, the value of this intersection will be a smaller negative number than it was before. The net number of zeros and poles will be the same (one zero and one pole are added), but the added pole is a smaller negative number than the added zero. Thus, the result of a lag compensator is that the asymptotes' intersection is moved closer to the right half plane, and the entire root locus will be shifted to the right.

Control Modes

There are many ways by which a control unit can react to an error and supply an output for correcting elements.

- **The two-step mode:** The controller is just a switch which is activated by the error signal and supplies just an on-off correcting signal. Example of such mode is the bimetallic thermostat.
- **The proportional mode (P):** This produces a control action that is proportional to the error. The correcting signal thus becomes bigger the bigger the error. Therefore, the error is reduced the amount of correction is reduced and the correcting process slows down. A summing operational amplifier with an inverter can be used as a proportional controller.
- **The derivative mode:** This produces a control action that is proportional to the rate at which the error is changing. When there is a sudden change in the error signal the controller gives a large correcting signal. When there is a gradual change only a small correcting signal is produced. An operational amplifier connected as a differentiator circuit followed by another operational amplifier connected as an inverter make an electronic derivative controller circuit.

- **The integral mode (I):** This produces a control action that is proportional to the integral of the error with time. Therefore, a constant error signal will produce an increasing correcting signal. The correction continues to increase as long as the error persists.

- **Combination of modes:** Proportional plus derivative modes (PD), proportional plus integral modes (PI), proportional plus integral plus derivative modes (PID). The term three-term controller is used for PID control.

- The controller may achieve these modes by means of pneumatic circuits, analog electronics involving operational amplifiers or by the programming of a microprocessor or computer.