Control System Engineering

Section A : INTRODUCTORY CONCEPTS

Topic covered : **System/Plant model, types of models, illustrative examples of plants and their inputs and outputs, controller, servomechanism, regulating system, linear time invariant (LTI) system, time-varying system, causal system, open loop control system, closed loop control system, illustrative examples of open-loop and feedback control systems, continuous**

time and sampled data control systems. Effects of feedback on sensitivity (to parameter variations), stability, external disturbance (noise), overall gain etc. Introductory remarks about non-linear control systems.

System/Plant model

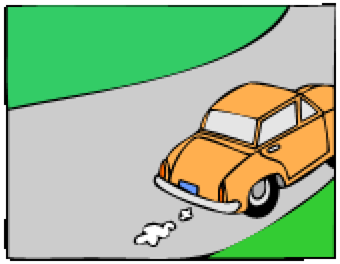
- **System** – An interconnection of elements and devices for a desired purpose.
- **Control System** – An interconnection of components forming a system configuration that will provide a desired response.
- **Process** – The device, plant, or system under control. The input and output relationship represents the cause-and-effect relationship of the process.

Process to be controlled.

Types of Models

- Servo Systems: the desired speed (set–point) changes fast. Major requirement: to follow the changing "set–point" at an acceptable speed and accuracy.
- Regulation Systems: the desired speed does not changes very fast. It may be constant. Major concern: substantial uncertainties/disturbances and high accuracy.

Car and Driver Example

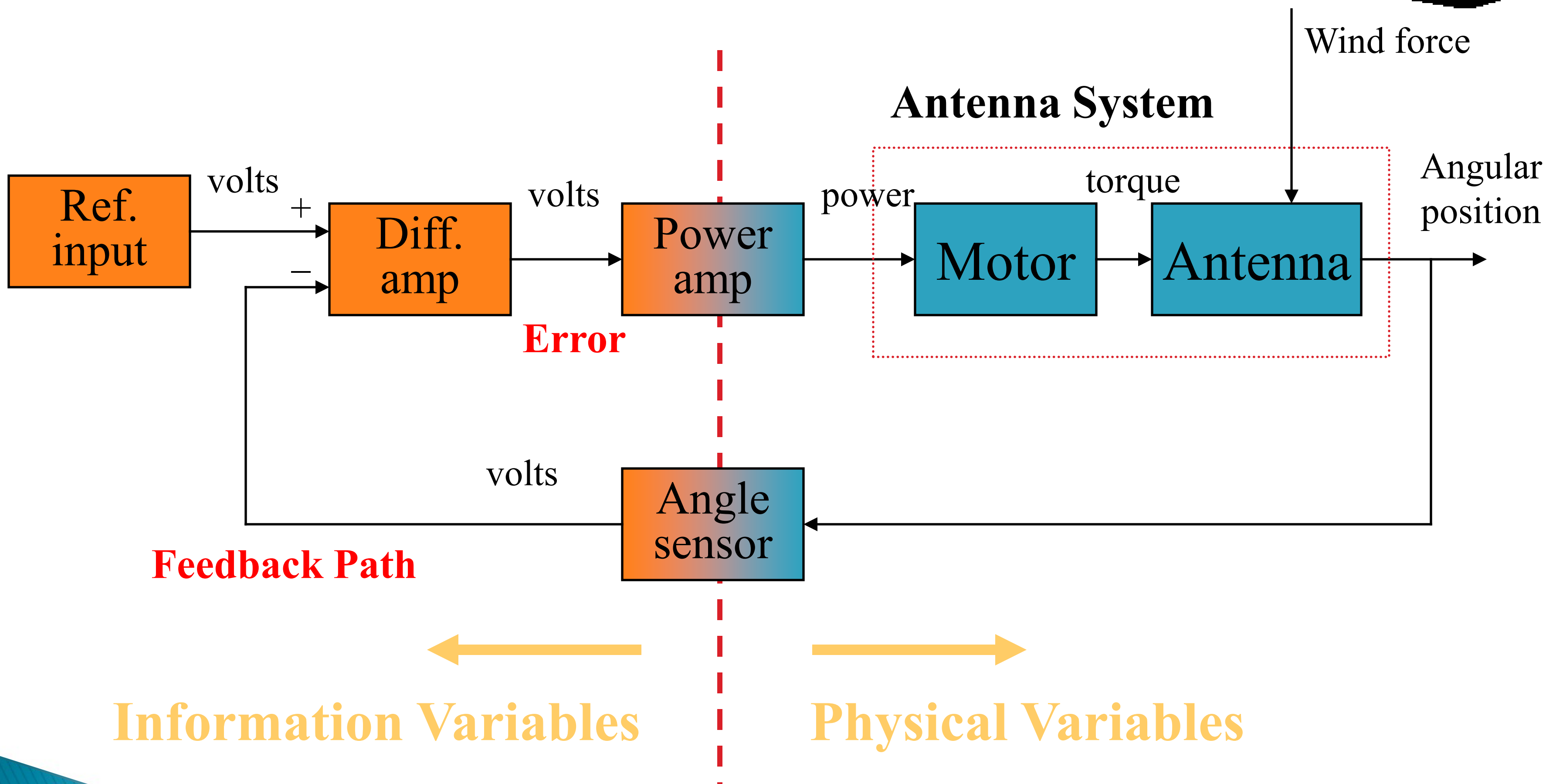


- Objective function: to control the direction and speed of the car.
- Outputs: actual direction and speed of the car
- Control inputs: road markings and speed signs
- Disturbances: road surface and grade, wind, obstacles.
- Possible subsystems: the car alone, power steering system, braking system, . . .

Antenna Positioning Control System

- Original system: the antenna with electric motor drive systems.
- Control objective: to point the antenna in a desired reference directi...
- Control inputs: drive motor voltages.
- Outputs: the elevation and azimuth of the antenna.
- Disturbances: wind, rain, snow.

Antenna Control System Functional Block Diagram



Controller

- What does a controller do? Decides how to respond to the observed difference between the measured speed and the desired speed set-point.
- How should the controller respond? Primarily based on the model, which describes the relationship between the input (voltage) and the output(speed) Robust Control: also largely based on the uncertainties
- An important Step in System Design: Find the model (system identification)
- Design: compromise between the uncertainties /disturbance and the response speed.

Servomechanism

- A **servomechanism**, sometimes shortened to **servo**, is an automatic device that uses error-sensing negative feedback to correct the performance of a mechanism and is defined by its function. It usually includes an in built encoder.
- The term correctly applies only to systems where the feedback or error-correction signals help control mechanical position, speed or other parameters.
- For example, an automotive power window control is not a servomechanism, as there is no automatic feedback that controls position—the operator does this by observation.
- By contrast a car's cruise control uses closed loop feedback, which classifies it as a servomechanism.

Linear-Time Invariant System

- Special importance for their mathematical tractability
- Most signal processing applications involve LTI systems
- LTI system can be completely characterized by their impulse response

$$\delta[n\text{-}k] \longrightarrow \boxed{T\{.\}} \longrightarrow h_k[n]$$

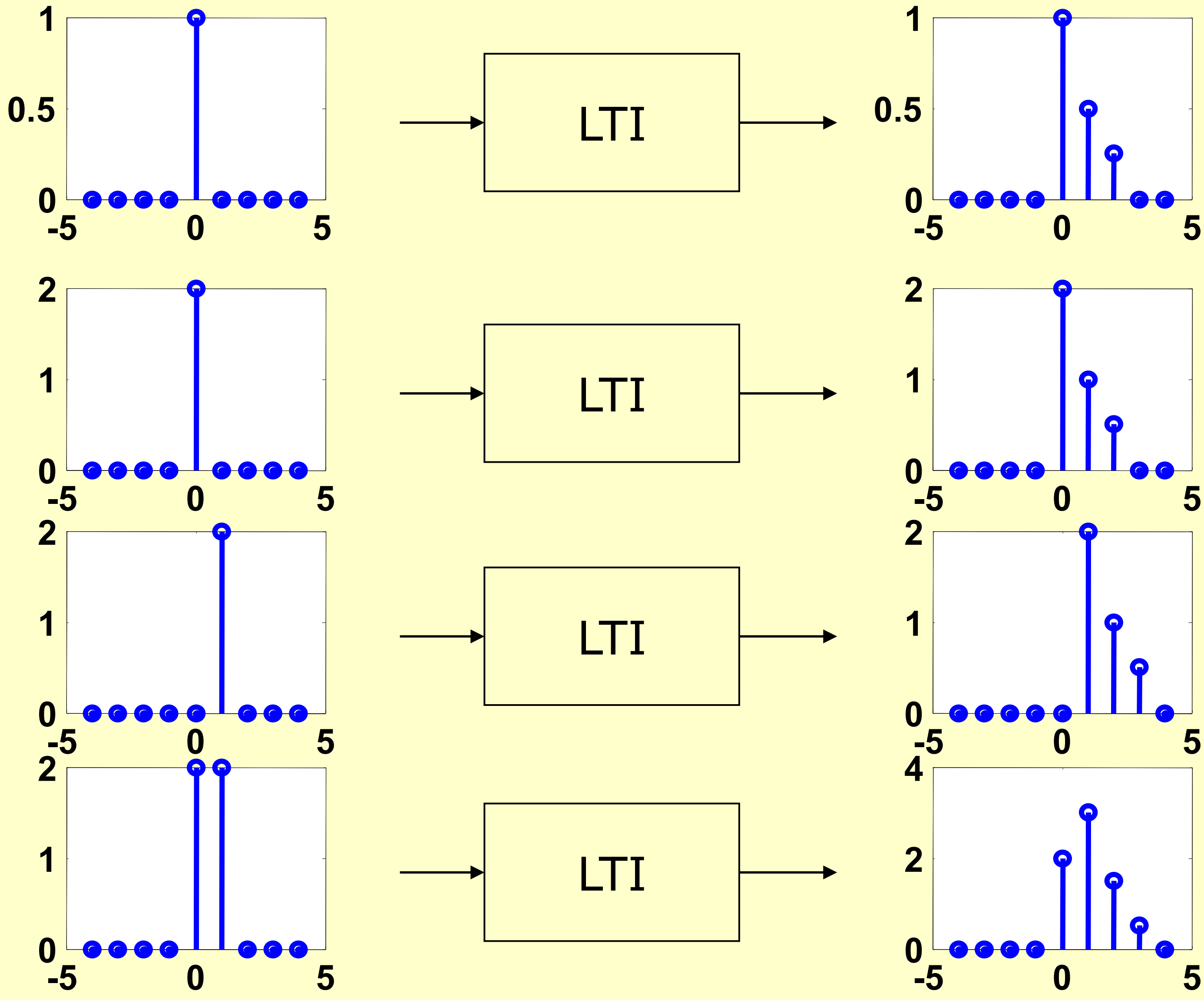
- Represent any input $x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$

$$y[n] = T\left\{\sum_{k=-\infty}^{\infty} x[k]\delta[n-k]\right\} = \sum_{k=-\infty}^{\infty} x[k]T\{\delta[n-k]\} = \sum_{k=-\infty}^{\infty} x[k]h_k[n]$$

- From time invariance we arrive at convolution

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = x[k] * h[k]$$

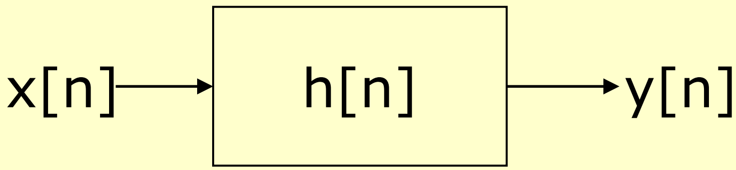
LTI System Example

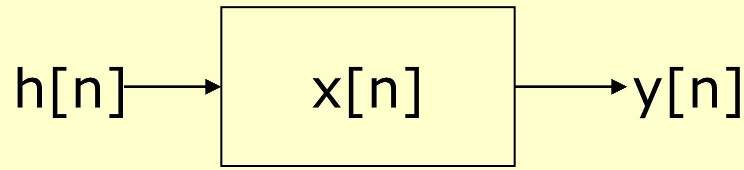


Properties of LTI Systems

- Convolution is commutative

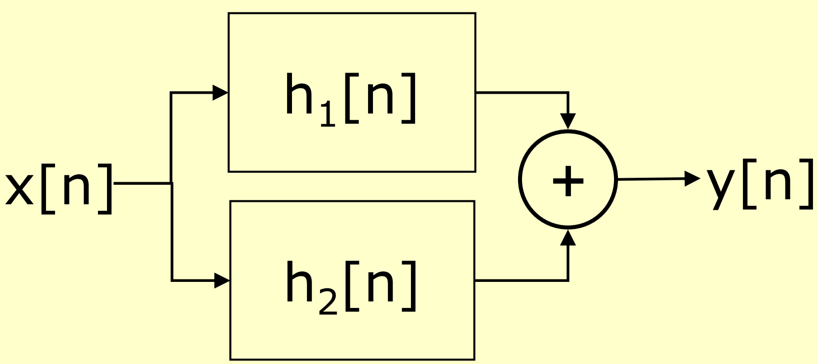
$$x[k] * h[k] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = \sum_{k=-\infty}^{\infty} h[k]x[n-k] = h[k] * x[k]$$

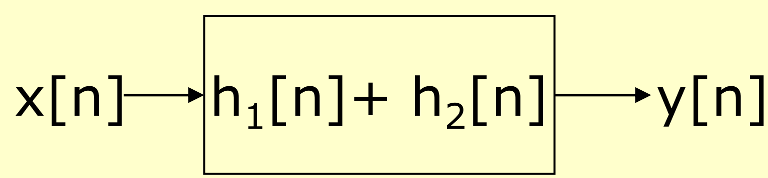




- Convolution is distributive

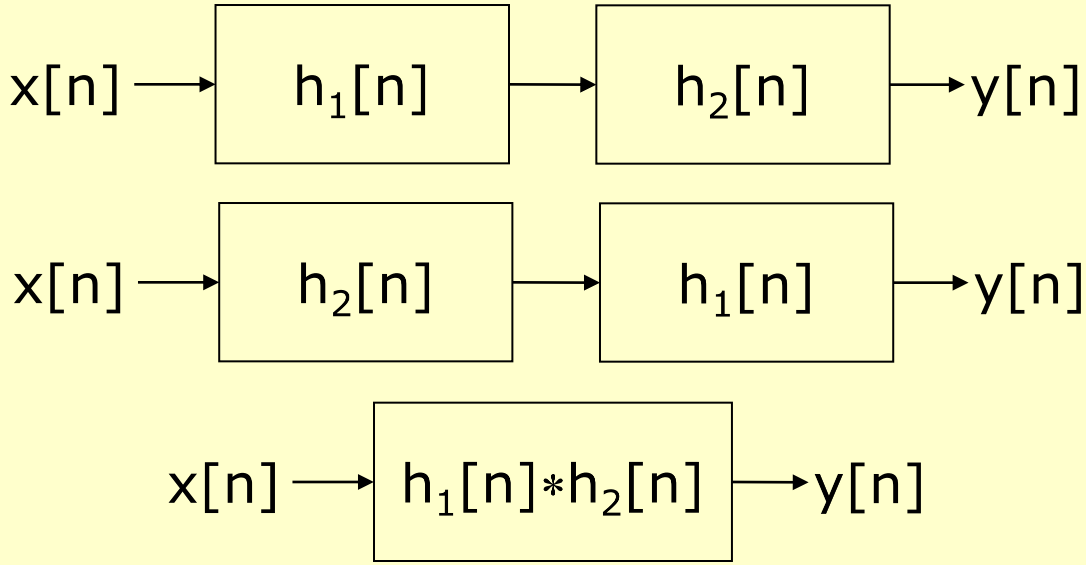
$$x[k] * \left(h_1[k] + h_2[k]\right) = x[k] * h_1[k] + x[k] * h_2[k]$$





Properties of LTI Systems

- Cascade connection of LTI systems



Stable and Causal LTI Systems

- An LTI system is (BIBO) stable if and only if
  - Impulse response is absolute summable

$$\sum_{k=-\infty}^{\infty} \left| h[k] \right| < \infty$$

  - Let's write the output of the system as

$$\left| y[n] \right| = \left| \sum_{k=-\infty}^{\infty} h[k] x[n-k] \right| \leq \sum_{k=-\infty}^{\infty} \left| h[k] \right| \left| x[n-k] \right|$$

  - If the input is bounded

$$\left| x[n] \right| \leq B_x$$

  - Then the output is bounded by

$$\left| y[n] \right| \leq B_x \sum_{k=-\infty}^{\infty} \left| h[k] \right|$$

  - The output is bounded if the absolute sum is finite
- An LTI system is causal if and only if

$$h[k] = 0 \quad \text{for } k < 0$$

Linear Constant-Coefficient Difference Equations

- An important class of LTI systems of the form

$$\sum_{k=0}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k]$$

- The output is not uniquely specified for a given input
  - The initial conditions are required
  - Linearity, time invariance, and causality depend on the initial conditions
  - If initial conditions are assumed to be zero system is linear, time invariant, and causal
- Example
  - Moving Average

$$y[n] = x[n] + x[n-1] + x[n-2] + x[n-3]$$

  - Difference Equation Representation

$$\sum_{k=0}^{0} a_k y[n-k] = \sum_{k=0}^{3} b_k x[n-k] \quad \text{where } a_k = b_k = 1$$

Eigenfunctions of LTI Systems

- Complex exponentials are eigenfunctions of LTI systems:

$$x[n] = e^{j\omega n}$$

- Let's see what happens if we feed x[n] into an LTI system:

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k] = \sum_{k=-\infty}^{\infty} h[k]e^{j\omega(n-k)}$$

$$y[n] = \left(\sum_{k=-\infty}^{\infty} h[k]e^{-j\omega k}\right)e^{j\omega n} = \underbrace{H\left(e^{j\omega}\right)}_{\text{eigenvalue}}\overbrace{e^{j\omega n}}^{\text{eigenfunction}}$$

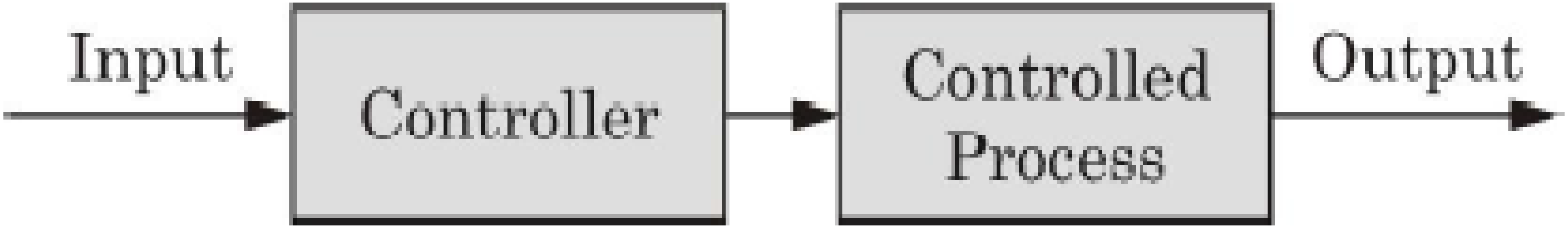
- The eigenvalue is called the frequency response of the system

$$H\left(e^{j\omega}\right) = \sum_{k=-\infty}^{\infty} h[k]e^{-j\omega k}$$

- $H(e^{j\omega})$ is a complex function of frequency
  - Specifies amplitude and phase change of the input

Open Loop Control System

- An open loop control system is one in which the control action is independent of the output.
- Non feedback control systems or control systems without feedback are known as open loop control systems.
- Open-loop control systems represent the simplest form of controlling devices.
- In open-loop systems the control action is independent of the desired output.



- The components of the open loop systems are controller and controlled process.
- The control adjustment of an open-loop system always depends on human intervention and estimation.

Examples:

1. Field control D.C. Motors.
2. Immersion rod used for heating water is another example of open loop system.

Advantages:

1. Open-loop control systems are simple.
2. Open-loop control systems are cheap and require low maintenance.
3. In open-loop control systems, calibration is not a problem.

Disadvantages:

(*i*) Open loop control systems are not reliable.

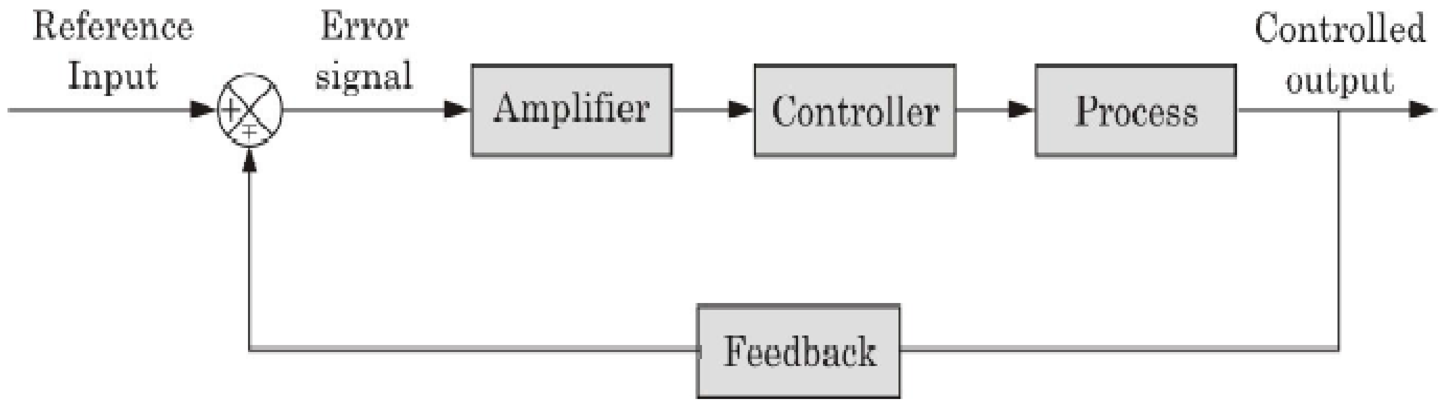
(*ii*) Open loop control systems are slow.

(*iii*) Open loop control systems are inaccurate.

(*iv*) Optimization is not possible.

Closed Loop Control System

- A closed loop control system is one in which the control action is somehow dependent on the output.
- Any system having one or more feedback paths forms a closed loop system. In the closed loop system the output is compared with the reference input and error signal is produced.



- Feedback control systems are also known as closed loop control systems. The term closed loop control always implies the use of feedback control action in order to reduce error.

Examples:

A pilot landing an aircraft is an example of a closed loop control system.

Advantages :

1. Closed loop systems are faster

2. Closed loop systems are more reliable.

3. Optimization is possible in closed loop control system.

4. Feedback element present in a closed loop system presents inherent capability to sense errors, disturbances and correct itself .

Disadvantages :

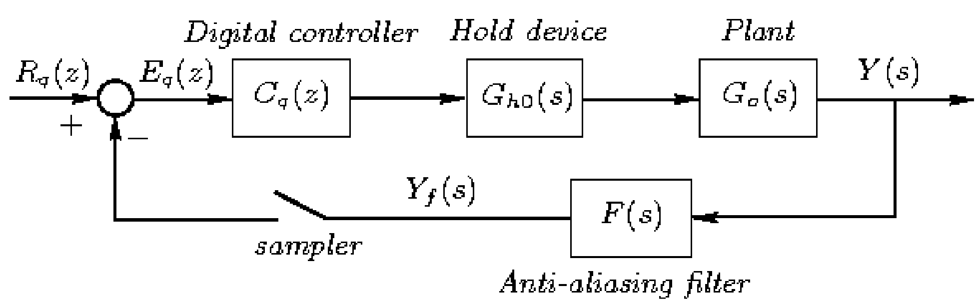
1. Closed loop systems are complex in design and costlier.

2. Maintenance of such type of systems are difficult.

3. Stability presents a major problem to the control system design of a closed loop system.

Continuous Time And Sampled Data Control Systems

A key idea is that if one is only interested in the at-sample response, these samples can be described by discrete time models in either the shift or delta operator. For example, consider the sampled data control loop shown below.

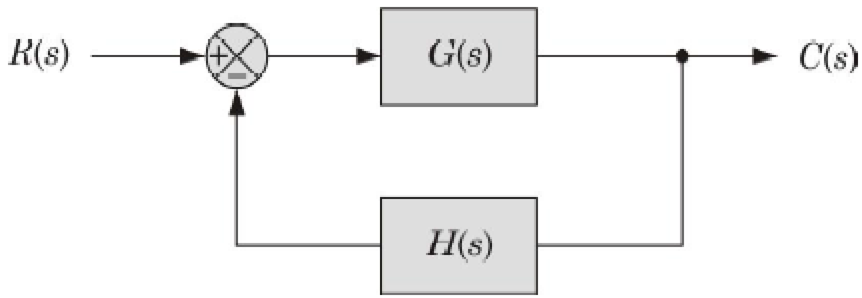


Sampled data control loop

FEEDBACK AND ITS EFFECTS

- Feedback is that characteristic of closed loop control systems which distinguishes them from open loop system.

Effect of Feedback on Overall Gain



In closed loop case, the output is

$$C(s) = \frac{G(s)}{1 + G(s)H(s)} R(s)$$

Changes to

$$C(s) + \Delta C(s) = \frac{G(s) + \Delta G(s)}{1 + G(s)H(s) + \Delta G(s)H(s)} \cdot R(S)$$

due to variation $\Delta G(s)$ in $G(s)$, the forward path transfer function. Since $|G(s)| >> |\Delta G(s)|$. Then output is

$$\Delta G(s) \approx \frac{\Delta G(s)}{1 + G(s)\, H(s)} R(S)$$

From above equation it is clear that the change in output due to feedback in $G(s)$ in closed loop system reduces by a factor of $1 + G(s)H(s)$ which is much greater than unity $(1 + G(s)\, H(s)) >> 1$.

Effect of Feedback on Sensitivity

The term system sensitivity is used to describe the relative variation in the overall transfer function $T(s) = \dfrac{C(s)}{R(s)}$ due to variation in transfer function sensitivity is defined as.

$$\text{Sensitivity} = \frac{\%\text{ change in } T(s)}{\%\text{ change in } G(s)}$$

Now, for small incremental variation in $G(s)$, the sensitivity can be written in quantitative form as

$$\delta_G^T = \frac{\partial T/T}{\partial G/G} = \frac{\partial T}{\partial G} \times \frac{G}{T}$$

Here δ_G^T denotes the sensitivity of T with respect to G.

Now according to above definition, the sensitivity of the closed-loop system is

$$\delta_G^T = \frac{\partial T}{\partial G} \times \frac{G}{T} = \frac{(1+GH) - GH}{(1+GH)^2} \times \frac{G}{G/(1+GH)}$$

or

$$\delta_G^T = \frac{1}{1+GH}$$

Effect of Feedback on Sensitivity...

Similarly, the sensitivity of the open-loop system is

$$\delta_G^T = \frac{\partial T}{\partial G} \times \frac{G}{T} = 1 \qquad \text{(in this case } T = G\text{)}$$

So, the sensitivity of a closed-loop system with respect to variation in G is reduced by a factor $(1 + GH)$ as compared to that of an open-loop system.

Now, we calculate the sensitivity of T with respect to H, is given as.

$$\delta_H^T = \frac{\partial T}{\partial H} \times \frac{H}{T} = G\left[\frac{-G}{(1+GH)^2}\right]\frac{H}{G/(1+GH)}$$

$$\Rightarrow \qquad \delta_H^T = \frac{-GH}{1+GH}$$

Above equation shows that for large values of GH, sensitivity of the feedback system with respect to H approaches unity.

The use of feedback in reducing sensitivity to parameters variations is an important advantage of feedback control system.

Effect of Feedback on Stability

Consider the open loop system with overall transfer function is given as

$$G(s) = \frac{K}{s + T}$$

The pole is located at $s = -T$

Now, suppose closed loop system with unity negative feedback, then overall transfer function is given as

$$\frac{C(s)}{R(s)} = \frac{K}{s + (T + K)}$$

Here, the pole is located at $s = -(T + K)$.

We know that the stability depends upon the location of poles. Thus, we can say that the feedback affects the stability.