Production Systems:

Forward and Backward Chaining
Motivations

• A production system consists of (1) a set of production rules, (2) the working memory and (3) the control structure.

• Previously, you saw exhaustive control structure. Check all rules, one by one.

• This lecture considers forward chaining and backward chaining for control structures.
Objectives

1. 8-puzzle as production system
2. Forward chaining
3. Backward chaining
4. Comparison
5. Prolog implementation of the farmer, wolf, goat and cabbage problem
3 components of the production system

1. A set of production rules
   - knowledge base
   - condition/action pairs
   - $p(x) \rightarrow q(x)$

2. Working memory
   - utility routines
   - data structures, queues, stacks

3. Control structure
   - recursive pattern-driven search
8-puzzle as a production system

1. Production set:
   
<table>
<thead>
<tr>
<th>Condition</th>
<th>Action</th>
</tr>
</thead>
<tbody>
<tr>
<td>goal state in working memory</td>
<td>→ halt</td>
</tr>
<tr>
<td>blank is not on the left edge</td>
<td>→ move the blank left</td>
</tr>
<tr>
<td>blank is not on the top edge</td>
<td>→ move the blank up</td>
</tr>
<tr>
<td>blank is not on the right edge</td>
<td>→ move the blank right</td>
</tr>
<tr>
<td>blank is not on the bottom edge</td>
<td>→ move the blank down</td>
</tr>
</tbody>
</table>

2. Working memory is the present board state and goal state.

3. Control regime:
   
   1. Try each production in order.
   2. Do not allow loops.
   3. Stop when goal is found.
8-puzzle searched by a production system

Condition

- goal state in working memory
- blank is not on the left edge
- blank is not on the top edge
- blank is not on the right edge
- blank is not on the bottom edge

Action

- halt
- move the blank left
- move the blank up
- move the blank right
- move the blank down
Advantages of production systems

• Separation of Knowledge and Control
• A Natural Mapping onto State Space Search
• Pattern-Directed Control
• Opportunities for Heuristic Control of Search
• Tracing and Explanation
• Programming Language Independence
• A Plausible Model of Human Problem-Solving
Data-driven search: forward chaining

1. **Production set:**
   1. \( p \land q \rightarrow \text{goal} \)
   2. \( r \land s \rightarrow p \)
   3. \( w \land r \rightarrow q \)
   4. \( t \land u \rightarrow q \)
   5. \( v \rightarrow s \)
   6. \( \text{start} \rightarrow v \land r \land q \)

2. **Trace of execution:**

<table>
<thead>
<tr>
<th>Iteration #</th>
<th>Working memory</th>
<th>Conflict set</th>
<th>Rule fired</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>start</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>1</td>
<td>start, v, r, q</td>
<td>6, 5</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>start, v, r, q, s</td>
<td>6, 5, 2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>start, v, r, q, s, p</td>
<td>6, 5, 2, 1</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>start, v, r, q, s, p, goal</td>
<td>6, 5, 2, 1</td>
<td>halt</td>
</tr>
</tbody>
</table>

3. **Fire the last rule in the set.**

   - **Working memory contains true states.**
   - **Direction of search:** start -> goal
   - **Forward chaining**
Goal-driven search: backward chaining

Production set:
1. \( p \land q \rightarrow \text{goal} \)
2. \( r \land s \rightarrow p \)
3. \( w \land r \rightarrow p \)
4. \( t \land u \rightarrow q \)
5. \( v \rightarrow s \)
6. \( \text{start} \rightarrow v \land r \land q \)

Trace of execution:

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</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>goal</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>goal, p, q</td>
<td>1, 2, 3, 4, 5</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>goal, p, q, r, s</td>
<td>1, 2, 3, 4, 5</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>goal, p, q, r, s, w</td>
<td>1, 2, 3, 4, 5</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>goal, p, q, r, s, w, t, u</td>
<td>1, 2, 3, 4, 5</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>goal, p, q, r, s, w, t, u, v</td>
<td>1, 2, 3, 4, 5, 6</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>goal, p, q, r, s, w, t, u, v, start</td>
<td>1, 2, 3, 4, 5, 6</td>
<td>halt</td>
</tr>
</tbody>
</table>

Working memory contains goal and sub-goal states waiting to be satisfied (shown true).

Space searched by execution:

Backward chaining
Direction of search

start

goal
Forward chaining vs. backward chaining

- **Data-driven, forward chaining**
  - Starts with the initial given data and search for the goal.
  - At each iteration, new conclusion (RHS) becomes the pattern to look for next.
  - Working memory contains true sentences (RHS’s).
  - Stop when the goal is reached.
- **Goal-driven is the reverse.**
  - Starts with the goal and try to search for the initial given data.
  - At each iteration, new premise (LHS) becomes the new subgoals, the pattern to look for next.
  - Working memory contains subgoals (LHS’s) to be satisfied.
  - Stop when all the premises (subgoals) of fired productions are reached.
- **Sense of the arrow is in reality reversed.**
- **Both repeatedly pick the next rule to fire.**

<table>
<thead>
<tr>
<th>Condition → Action</th>
<th>Premise → Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Starts with</strong></td>
<td>premise</td>
</tr>
<tr>
<td><strong>Search for</strong></td>
<td>conclusion</td>
</tr>
<tr>
<td><strong>Working memory</strong></td>
<td>true statements</td>
</tr>
<tr>
<td><strong>Stopping criteria</strong></td>
<td>goal is reached</td>
</tr>
<tr>
<td></td>
<td>Initial data are reached</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Data-driven</th>
<th>Goal-driven</th>
</tr>
</thead>
</table>

**Sense of the arrow is in reality reversed.**
**Both repeatedly pick the next rule to fire.**
Combining forward- and backward-chaining

- Begin with data and search forward until the number of states becomes unmanageably large.
- Switch to goal-directed search to use subgoals to guide state selection.
Bidirectional search better than unidirectional search
Bidirectional search worse than unidirectional search

Forward chaining and backward chaining misses each other.
Blackboard architecture for language understanding

- Layered production systems
- Separate productions into modules
- Each module (PS) is an agent -- knowledge source
- A single global structure -- blackboard

Spoken acoustic waveform signal

Syllables

Word recognition

Parsed sentence (text)
Conclusion

• Data-driven, forward chaining
  – conditions first, then actions
  – working memory contains true statements describing the current environment

• Goal-driven, backward chaining
  – actions first, then conditions (subgoals)
  – working memory contains subgoals to be shown as true

• Mixed approach
  – Start with data and go forward until frontier is too big
  – Then start with goal and go backward
  – Try to connect the two in the middle of the state space

• Prolog implementation of production systems requires infinite loop detection.
• Break for 10 minutes.
• But you can start lab if you like.
Farmer, wolf, goat, and cabbage problem as a production system

- Farmer, wolf, goat, and cabbage come to the edge of a river.
- A boat at the river’s edge. Only the farmer can row. The boat can carry only two things, including the rower, at a time.
- If the wolf is ever left alone with the goat, the wolf will eat the goat.
- If the goat is left alone with the cabbage, the goat will eat the cabbage.
- Devise a sequence of crossings of the river so that all four characters arrives safely on the other side of the river.

- Representation
  - Predicate state(F, W, G, C) describes the location of Farmer, Wolf, Goat, and Cabbage.
  - Possible locations are e for east, w for west, bank for each of the 4 variables.
  - Initial state is state(w, w, w, w)
  - Goal state is state(e, e, e, e)
Sample solution
State space graph including unsafe states

(F, W, G, C)

(F, W, G, C)

(F, W, G, C)

(F, W, G, C)

(F, W, G, C)

(F, W, G, C)

(F, W, G, C)

(F, W, G, C)

(F, W, G, C)

(F, W, G, C)

(F, W, G, C)

(F, W, G, C)

(F, W, G, C)

(F, W, G, C)

(F, W, G, C)

(F, W, G, C)

(F, W, G, C)

(F, W, G, C)

(F, W, G, C)

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(F, W, G, C)

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(F, W, G, C)

(F, W, G, C)

(F, W, G, C)

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(F, W, G, C)

(F, W, G, C)

(F, W, G, C)

(F, W, G, C)
Prolog implementation

• Use list to represent a state or configuration [Farmer, Wolf, Goat, Cabbage]

• Initially, the game begins
  – everyone is on the West bank
  – [w,w,w,w]

• Farmer takes the wolf across
  – [e,e,w,w]
  – Goat eats the cabbage

• Final configuration
  – everyone is on the East bank
  – [e,e,e,e]
Make a move

• In each move, the farmer crosses the river with either the wolf, the goat, the cabbage, or nothing.
• Each move can be represented with a corresponding atom
  1. wolf
  2. goat
  3. cabbage
  4. nothing
• Each move transforms one configuration into another.
• \texttt{move( [w,w,w,w], wolf, [e,e,w,w] ).}
• \texttt{move( Config, Move, NextConfig ).}
  – Config and NextConfig are lists
  – Move is a variable whose value is one of the 4 possible atoms.
8 possible movements

- When the wolf and the farmer move, the goat and the cabbage do not change position.
  1.  move([w,w,Goat,Cabbage],wolf,[e,e,Goat,Cabbage]).

- Farmer and the wolf can go from East to West also.
  2.  move([e,e,Goat,Cabbage],wolf,[w,w,Goat,Cabbage]).

- Farmer takes goat
  3.  move([w,Wolf,w,Cabbage],goat,[e,Wolf,e,Cabbage]).
  4.  move([e,Wolf,e,Cabbage],goat,[w,Wolf,w,Cabbage]).

- Farmer takes cabbage
  5.  move([w,Wolf,Goat,w],cabbage,[e,Wolf,Goat,e]).
  6.  move([e,Wolf,Goat,e],cabbage,[w,Wolf,Goat,w]).

- Farmer takes nothing
  7.  move([w,Wolf,Goat,Cabbage],nothing,[e,Wolf,Goat,Cabbage]).
  8.  move([e,Wolf,Goat,Cabbage],nothing,[w,Wolf,Goat,Cabbage]).
Safety

• We need a predicate to decide whether a configuration is safe or not.
• safe([Man, Wolf, Goat, Cabbage])
• Today's lab
Implementation so far

• Use list to represent a configuration (state)
• Make a move
  – source configuration
  – farmer takes a belonging or nothing across the river from east to west or west to east
  – destination configuration
• Predicate safe returns safety status of a configuration

• Still, how to search for the sequence of moves for the solution?
Search for the solution recursively

- `solve( [w,w,w,w], S )`
- From initial configuration, search for a solution `S`.
- `S` is a list containing the sequence of belongings taken by the farmer.
- Trivial case
  - `solve( [e,e,e,e], [] )`.
  - If everything is on the east bank, stop recursion.
- Recursive case
  - `solve( Config, [ NextMove | OtherMoves ] ) :-`
    ```
    move(Config,NextMove,NextConfig),
    safe(NextConfig),
    solve( NextConfig, OtherMoves ).
    ```
Infinite loop

- solve([w, w, w, w], S) would cause an infinite loop
  - ERROR: Out of local stack
  - Never reached the trivial case in the recursion

Try farmer taking wolf.
Goat eats cabbage.
Unsafe.
Backtrack (Redo).
Main program

- farmer(S) :- length(S,7), solve([w,w,w,w], S), !.
- Search for a solution recursively.
- Limit the length of the solution to 7 to avoid infinite loop.
- Stop as soon as the first solution is found.
- ?- farmer(X).

- Be careful with the order of the predicates
- farmer(S) :- solve([w,w,w,w], S), length(S,7), !.
- ERROR: Out of local stack