DynamicsofMechines

ThirdyearB.TechClass

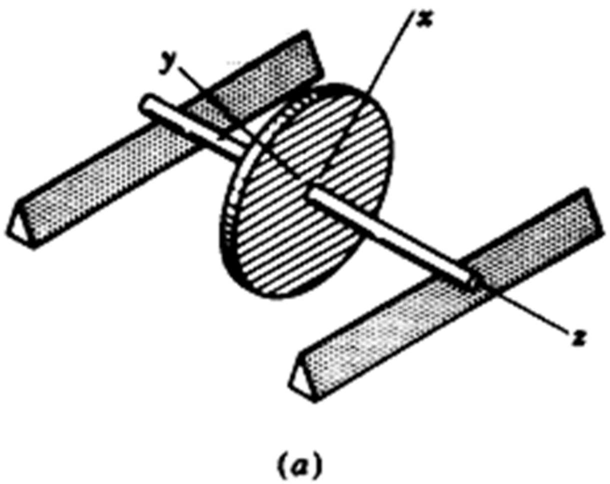
Balancingofrotatingcomponents

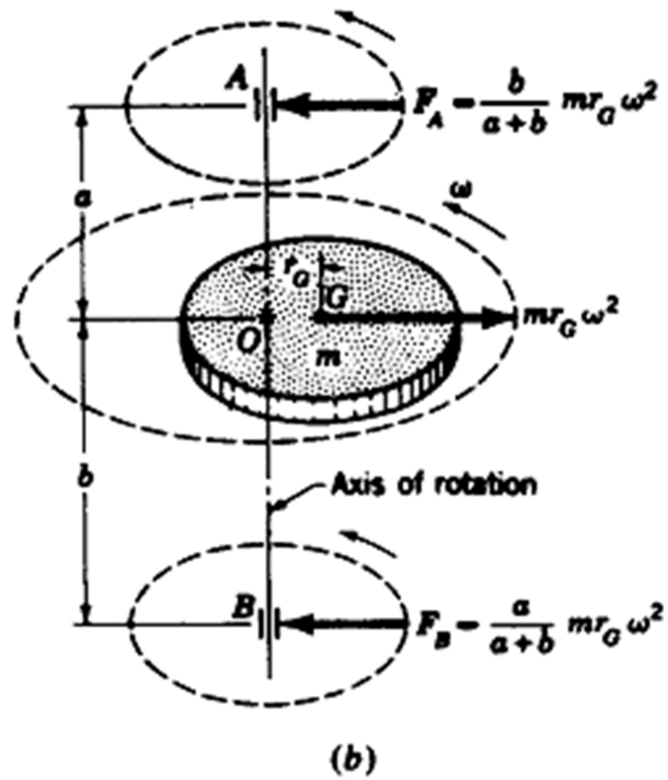
Balancing is the technique of correcting or eliminating unwanted inertia forces and moments. In previous chapters we have seen that the frame forces can vary significantly during a complete cycle of operation. Such forces can cause vibrations which at times may reach dangerous amplitudes. Even if not dangerous, vibrations increase the component stresses and subject bearings to repeated loads which cause parts to fail prematurely by fatigue. Thus it is not sufficient in the design of machinery merely to avoid operation near the critical speeds; we must also eliminate, or at least reduce, the inertia forces which produce these vibrations in the first place.

Typesofunbalance

- Staticunbalance
- Dynamicunbalance

Static unbalance





Dynamic unbalance

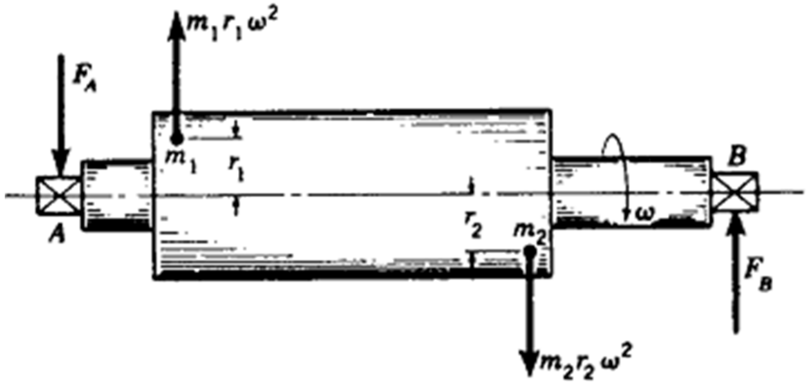


Figure 15-6 The rotor is statically balanced if $m_1 = m_2$ and $r_1 = r_2$ but dynamically unbalanced.

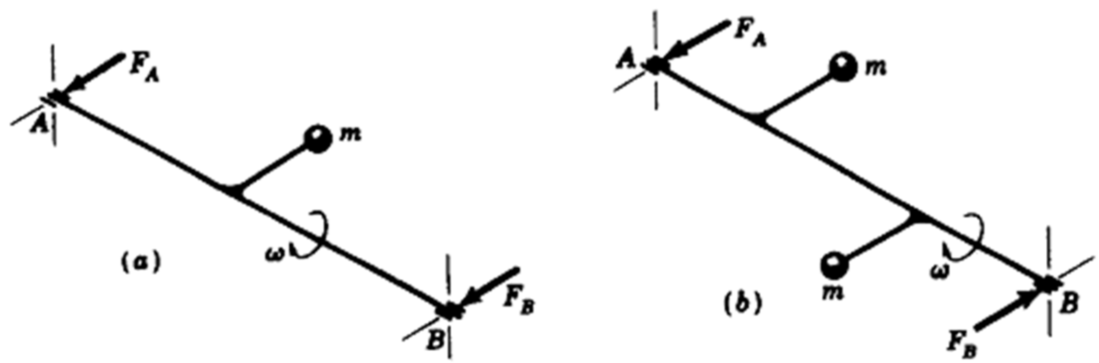


Figure 15-7 (*a*) Static unbalance; when the shaft rotates, both bearing reactions are in the same plane and in the same direction. (*b*) Dynamic unbalance; when the shaft rotates, the unbalance creates a couple tending to turn the shaft end over end. The shaft is in equilibrium because of the opposite couple formed by the bearing reactions. Note that the bearing reactions are still in the same plane but opposite in direction.

Dynamic unbalance

...contd....

If the rotor of Fig. 15-6 is placed in bearings and caused to rotate at an angular velocity ω rad/s, the centrifugal forces $m_1 r_1 \omega^2$ and $m_2 r_2 \omega^2$ act, respectively, at m_1 and m_2 on the rotor ends. These centrifugal forces produce the unequal bearing reactions $\mathbf{F}_A$ and $\mathbf{F}_B$, and the entire system of forces rotates with the rotor at the angular velocity ω. Thus a part may be statically balanced and at the same time dynamically unbalanced (Fig. 15-7).

In the general case distribution of the mass along the axis of the part depends upon the configuration of the part, but errors occur in machining and also in casting and forging. Other errors or unbalance may be caused by improper boring, by keys, and by assembly. It is the designer's responsibility to design so that a line joining all mass centers will be a straight line coinciding with the axis of rotation. However, perfect parts and perfect assembly are seldom attained, and consequently a line from one end of the part to the other, joining all mass centers, will usually be a space curve which may occasionally cross or coincide with the axis of rotation. An unbalanced part, therefore, will usually be out of balance both statically and dynamically.

Balancing machines

The purpose of a balancing machine is first to indicate whether a part is in balance. If it is out of balance, the machine must measure the unbalance by indicating its *magnitude* and *location*.

Static balancing machines are used only for parts whose axial dimensions are small, such as gears, fans, and impellers, and the machines are often called *single-plane balancers* because the mass must practically lie in a single plane.

Static balancing is essentially a weighing process in which the part is acted upon by either a gravity force or a centrifugal force. We have seen that the disk and shaft of the preceding section could be balanced by placing it on two parallel rails, rocking it, and permitting it to seek equilibrium. In this case the location of the unbalance is found through the aid of the force of gravity. Another method of balancing the disk would be to rotate it at a predetermined speed. Then the bearing reactions could be measured and their magnitudes used to indicate the amount of unbalance. Since the part is rotating while the measurements are taken, a stroboscope is used to indicate the location of the required correction.

Analysis ofunbalance

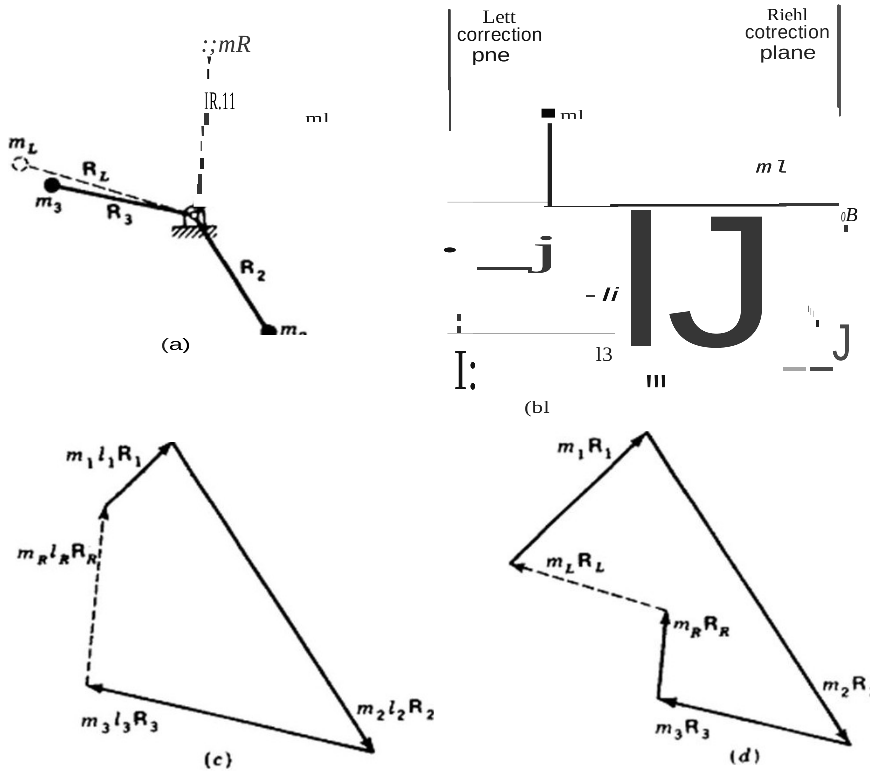
Graphical analysis The two equations

$$\sum \mathbf{F} = 0 \qquad \text{and} \qquad \sum \mathbf{M} = 0 \qquad (a)$$

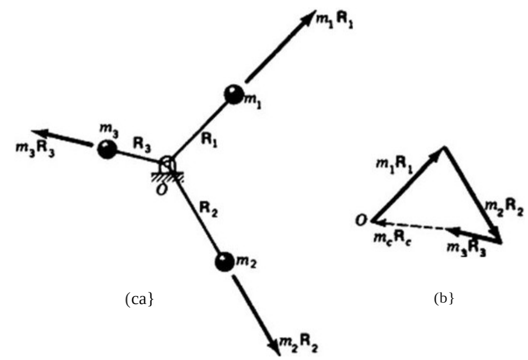
are used to determine the amount and location of the corrections. We begin by noting that the centrifugal force is proportional to the product mr of a rotating eccentric mass. Thus vector quantities, proportional to the centrifugal force of each of the three masses $m_1\mathbf{R}_1$, $m_2\mathbf{R}_2$, and $m_3\mathbf{R}_3$ of Fig. 15-8*a*, will act in radial directions as shown. The first of Eqs. (*a*) is applied by constructing a force polygon (Fig. 15-8*b*). Since this polygon requires another vector $m_c\mathbf{R}_c$ for closure, the magnitude of the correction is $m_c\mathbf{R}_c$ and its direction is parallel to $\mathbf{R}_c$. The three masses of Fig. 15-8 are assumed to rotate in a single plane, and so this is a case of static unbalance.

The first step in the solution is to take a summation of the moments of the centrifugal forces, including the corrections, about some point. We choose to take this summation about A in the left correction plane in order to eliminate the moment of the left correction mass. Thus, applying the second of Eqs. (*a*) gives

$$\sum \mathbf{M}_A = m_1 l_1 \mathbf{R}_1 + m_2 l_2 \mathbf{R}_2 + m_3 l_3 \mathbf{R}_3 + m_R l_R \mathbf{R}_R = 0 \qquad (b)$$



F1p:reJS-9 Graphicalanalysisof unbalance.



flproelM(ca)Athree-masssystemrotatina1n3siftlleplane.(blCentritupl-forcepolyfllll Jivesm,lt,astherequiredconection.

Thisisavectorequationinwhichthedi1ectionsofthevectorsareparallel,respectively,tothevectorsRNinFig.*t5-9a*.ConsequentlythemomentpolygonofFig.15-9ccanbeconstructed.Theclosingvector*mRIRRR*givesthemagnitudeanddirectionofthecorrectionrequiredfortheright-handplane.Thequantities*mR*andRRcannowbefoundbecausethemagnitudeofRRisordinarilygivenintheproblem.Thereforetheequation

$$\sum \mathbf{F} = m_1\mathbf{R}_1 + m_2\mathbf{R}_2 + m_3\mathbf{R}_3 + m_R\mathbf{R}_R + m_L\mathbf{R}_L = 0 \quad \text{(c)}$$

canbewritten.ThemagnitudeofR1.beinggiven,thisequationissolvedfor theleft-handcorrectionmi.RtbyconstructingtheforcepolygonofFig.*15-9d.*

Dynamic balancing

We have seen that static balancing is sufficient for rotating disks, wheels, gears, and the like when the mass can be assumed to exist in a single rotating plane. In the case of longer machine elements, such as turbine rotors or motor armatures, the unbalanced centrifugal forces result in couples whose effect is to tend to cause the rotor to turn end over end. The purpose of balancing is to measure the unbalanced couple and to add a new couple in the opposite direction and of the same magnitude. The new couple is introduced by the addition of masses on two preselected correction planes or by subtracting masses (drilling out) of these two planes. A rotor to be balanced will usually have both static and dynamic unbalance, and consequently the correction masses, their radial location, or both will not be the same for the two correction planes. This also means that the angular separation of the correction masses on the two planes will usually not be 180°. Thus, to balance a rotor, one must measure the magnitude and angular location of the correction mass for each of the two correction planes.

Three methods of measuring the corrections for two planes are in general use, the *pivoted-cradle*, the *nodal-point*, and the *mechanical-compensation methods*.

Balancing machines

- Pivotedcradlebalancingmachine

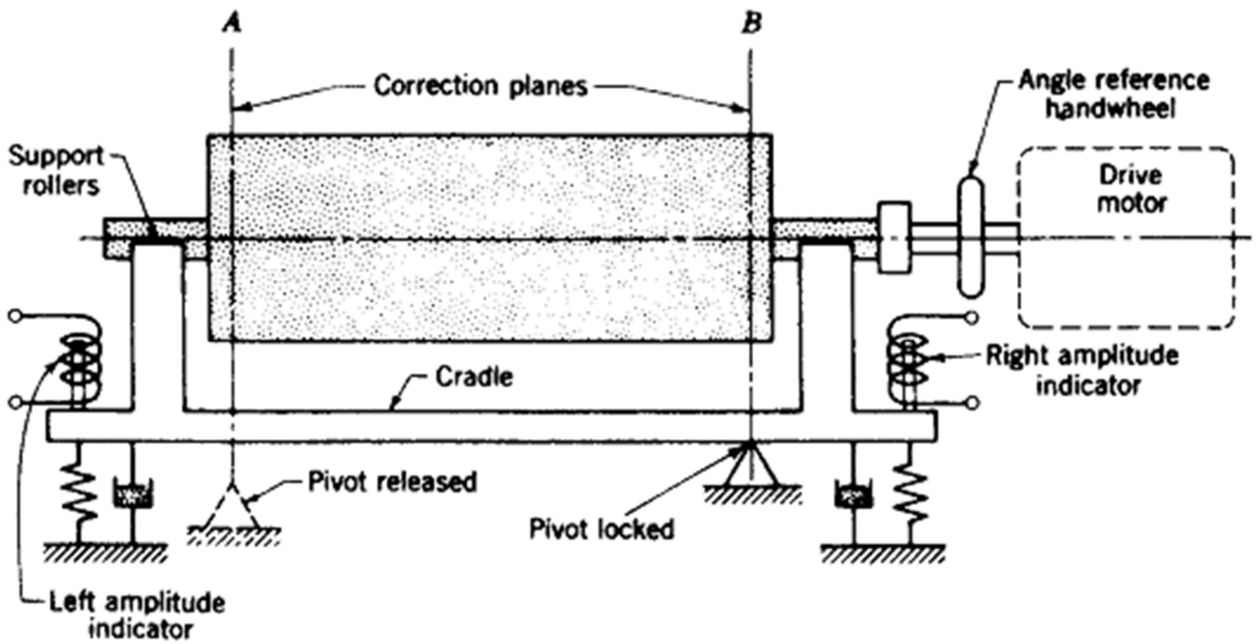


Figure 15-13 Schematic drawing of pivoted-cradle balancing machine.

Balancing machines

- Nodalpointbalancing

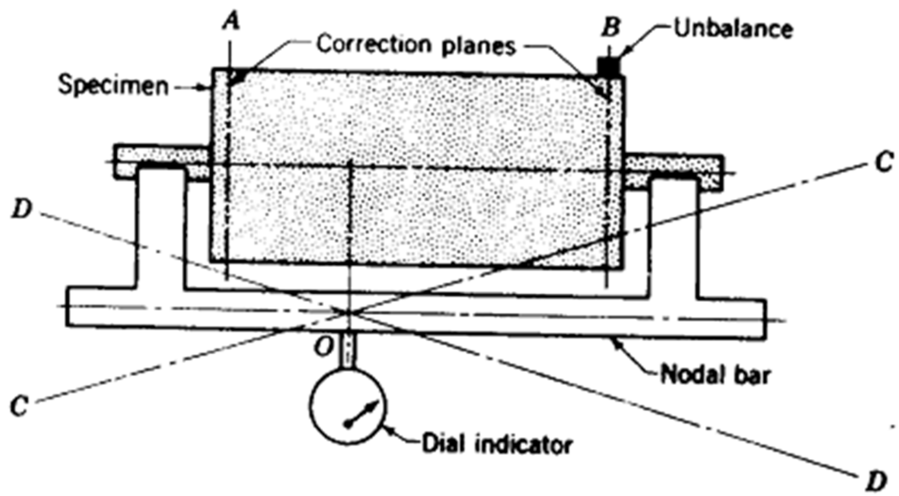
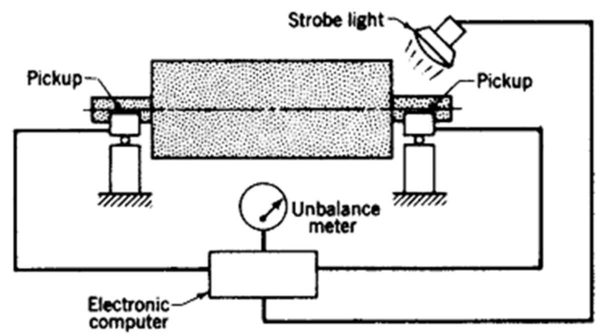


Figure 15-16 Plane separation by the nodal-point method. The nodal bar experiences the same vibration as the specimen.



Field balancing

It is possible to balance a machine in the field by balancing a single plane at a time. But cross effects and correction-plane interference often require balancing each end of a rotor two or three times to obtain satisfactory results. Some machines may require as much as an hour to bring them up to full speed, resulting in even more delays in the balancing procedure.

Field balancing is necessary for very large rotors for which balancing machines are impractical. And even though high-speed rotors are balanced in the shop during manufacture, it is frequently necessary to rebalance them in the field because of slight deformations brought on by shipping, by creep, or by high operating temperatures.

Balancingofsinglecylinderengine

The rotating masses in a single-cylinder engine can be balanced using the methods already discussed in this chapter. The reciprocating masses, however, cannot be balanced at all, and so our studies in this section are really concerned with unbalance.

Though the reciprocating masses cannot be balanced using a simple counterweight, it is possible to modify the shaking forces (see Sec. 14-9) by unbalancing the rotating masses. As an example of this let us add a counterweight opposite the crankpin whose mass exceeds the rotating mass by one-half of the reciprocating mass (from one-half to two-thirds of the reciprocating mass is usually added to the counterweight to alter the balance characteristics in single-cylinder engines).

Adding Eqs. (c) and (d) gives the resultant inertia force as

$$\mathbf{F} = \left(\frac{m_B}{2} r\omega^2 \cos \omega t + m_B r\omega^2 \frac{r}{l} \cos 2\omega t\right)\hat{\mathbf{i}} - \frac{m_B}{2} r\omega^2 \sin \omega t \; \hat{\mathbf{j}} \qquad (15\text{-}15)$$

The vector

$$\frac{m_B}{2} r\omega^2 (\cos \omega t \; \hat{\mathbf{i}} - \sin \omega t \; \hat{\mathbf{j}})$$

Balancing single cylinder engine

is called the *primary component* of Eq. (15-15). This component has a magnitude $m_B r\omega^2/2$ and can be represented as a *backward* (clockwise) rotating vector with angular velocity ω. The remaining component in Eq. (15-15) is called the *secondary component*; it is the x projection of a vector of length $m_B r\omega^2(r/l)$ rotating *forward* (counterclockwise) with an angular velocity of 2ω.

The maximum inertia force occurs when $\omega t = 0$ and from Eq. (15-15) is seen to be

$$F_{max} = m_B r\omega^2\left(\frac{r}{l} + \frac{1}{2}\right) \qquad (e)$$

because $\cos \omega t = \cos 2\omega t = 1$ when $\omega t = 0$. Before the extra counterweight was added, the maximum inertia force was

$$F_{max} = m_B r\omega^2\left(\frac{r}{l} + 1\right) \qquad (f)$$

Thus in this instance the effect of the additional counterweight is to reduce the maximum shaking force by 50 percent of the primary component and to add vertical inertia forces where formerly none existed.

Balancingofmulti-cylinderengine

To obtain a basic understanding of the balancing problem in multicylinder engines, let us consider a two-cylinder in-line engine having cranks 180° apart and rotating parts already balanced by counterweights. Such an engine is shown in Fig. 15-22. Applying the imaginary-mass approach for the first harmonics results in the diagram of Fig. 15-22*a*. This figure shows that masses +1 and +2, rotating clockwise, balance each other, as do masses −1 and −2,

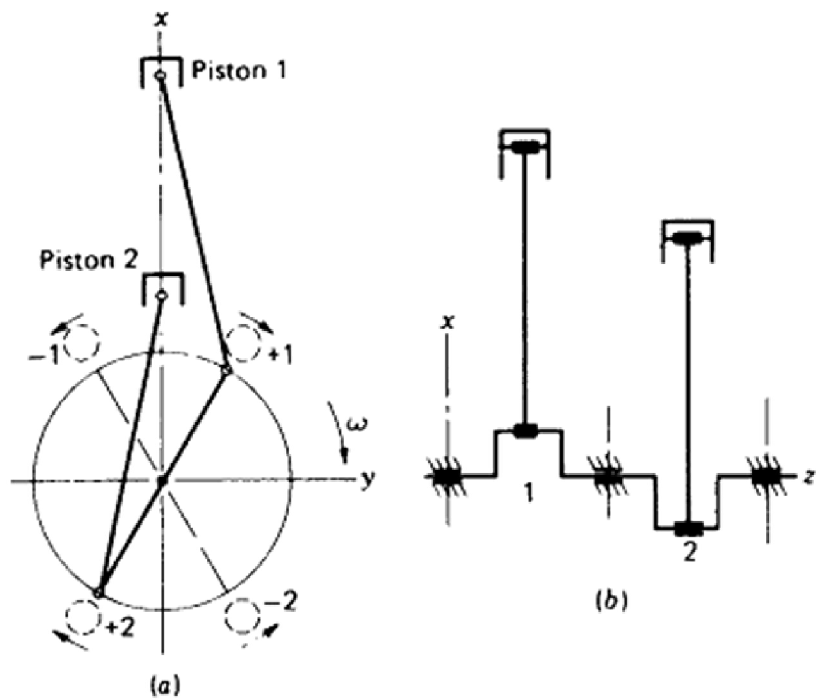


Figure 15-22 (*a*) First harmonics; (*b*) a two-throw crankshaft with three main bearings.

Balancingofmulti-cylinderengine

rotating counterclockwise. Thus, the first harmonic forces are inherently balanced for this crank arrangement. Figure 15-22*b* shows, however, that these forces are not in the same plane. For this reason unbalanced couples will be set up which tend to rotate the engine about the *y* axis. The values of these couples can be determined using the force expressions in Table 15-1 together with the coupling distance because the equations can be applied to each cylinder separately. It is possible to balance the couple due to the real rotating masses as well as the imaginary half-masses that rotate with the engine; however, the couple due to the half-mass of the first harmonic that is counterrotating cannot be balanced.

Summarymulti-cylinderenginebalancing

1. In a three-cylinder radial engine with one crank and three connecting rods having the same crankpin, the negative masses are inherently balanced for the first harmonic forces while the positive masses are always located at the crankpin. These two findings are inherently true for all radial engines. Also, since the radial engine has its cylinders in a single plane, unbalanced couples do not occur. The three-cylinder engine will have unbalanced forces in the second and higher harmonics.
2. A two-cylinder opposed-piston engine with a crank spacing of 180° is balanced for forces in the first, second, and fourth harmonics but unbalanced for couples.
3. A four-cylinder in-line engine with cranks at 90° is balanced for forces in the first harmonic but unbalanced for couples. In the second harmonic it is balanced for both forces and couples.
4. An eight-cylinder in-line engine with the cranks at 90° is inherently balanced for both forces and couples in the first and second harmonics but unbalanced in the fourth harmonic.
5. An eight-cylinder V engine with cranks at 90° is inherently balanced for forces in the first and second harmonics and for couples in the second. The unbalanced couples in the first harmonic can be balanced by counterweights that introduce an equal and opposite couple. Such an engine is unbalanced for forces in the fourth harmonic.