DynamicsofMechines

ThirdyearB.TechClass

Machine & Mechanism

If a number of bodies are assembled in such a way that the motion of one causes constrained and predictable motion to the others, it is known as a *mechanism*. A mechanism transmits and modifies a motion. A *machine* is a mechanism or a combination of mechanisms which, apart from imparting definite motions to the parts, also transmits and modifies the available mechanical energy into some kind of desired work. Thus, a mechanism is a fundamental unit and one has to start with its study.

Analysis&Synthesis

The study of a mechanism involves its analysis as well as synthesis. *Analysis* is the study of motions and forces concerning different parts of an existing mechanism, whereas *synthesis* involves the design of its different parts. In a mechanism, the various parts are so proportioned and related that the motion of one imparts requisite motions to the others and the parts are able to withstand the forces impressed upon them. However, the study of the relative motions of the parts does not depend on the strength and the actual shapes of the parts.

Definitions of terms

Kinematics It deals with the relative motions of different parts of a mechanism without taking into consideration the forces producing the motions. Thus, it is the study, from a geometric point of view, to know the displacement, velocity and acceleration of a part of a mechanism.

Dynamics It involves the calculations of forces impressed upon different parts of a mechanism. The forces can be either static or dynamic. Dynamics is further subdivided into *kinetics* and *statics*. Kinetics is the study of forces when the body is in motion whereas statics deals with forces when the body is stationary.

DegreesofFreedom

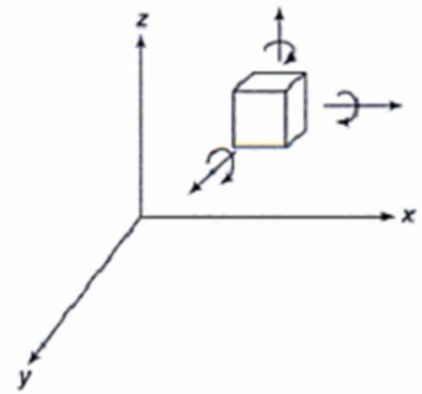
An unconstrained rigid body moving in space can describe the following independent motions (Fig. 1.10):

1. Translational motions along any three mutually perpendicular axes x, y and z
2. Rotational motions about these axes

Thus, a rigid body possesses six degrees of freedom. The connection of a link with another imposes certain constraints on their relative motion. The number of restraints can never be zero (joint is disconnected) or six (joint becomes solid).

Degrees of freedom of a pair is defined as the number of independent relative motions, both translational and rotational, a pair can have.

Degrees of freedom = 6 – Number of restraints



Analysis offorces

If a number of forces *P,Q,R* etc. are acting simultaneously on a particle, then a single force, which will produce the same effect as that of all the given forces, is known as a ***resultant force.*** The forces *P,Q,R* etc. are called ***component forces.*** The process of finding out the resultant force of the given component forces, is known as ***composition of forces.***

A resultant force may be found out analytically, graphically or by the following three laws:

1. *Parallelogram law of forces.* It states, "If two forces acting simultaneously on a particle be represented in magnitude and direction by the two adjacent sides of a parallelogram taken in order, their resultant may be represented in magnitude and direction by the diagonal of the parallelogram passing through the point."

2. *Triangle law of forces.* It states, "If two forces acting simultaneously on a particle be represented in magnitude and direction by the two sides of a triangle taken in order, their resultant may be represented in magnitude and direction by the third side of the triangle taken in opposite order."

3. *Polygon law of forces.* It states, "If a number of forces acting simultaneously on a particle be represented in magnitude and direction by the sides of a polygon taken in order, their resultant may be represented in magnitude and direction by the closing side of the polygon taken in opposite order."

Definition of terms

Force Our earliest ideas concerning forces arose because of our desire to push, lift, or pull various objects. So force is the action of one body acting on another. Our intuitive concept of force includes such ideas as *place of application*, *direction*, and *magnitude*, and these are called the *characteristics of a force.*

Matter Matter is any material or substance; if it is completely enclosed, it is called a body.

Mass Newton defined mass as *the quantity of matter of a body as measured by its volume and density*. This is not a very satisfactory definition because density *is* the mass of a unit volume. We can excuse Newton by surmising that he perhaps did not mean it to be a definition. Nevertheless, he recognized the fact that all bodies possess some inherent property that is different from weight. Thus, a moon rock has a certain constant amount of substance, even though its moon weight is different from its earth weight. This constant amount of substance, or quantity of matter, is called the *mass* of the rock.

Inertia Inertia is the property of mass that causes it to resist any effort to change its motion.

Weight Weight is the force of gravity acting upon a mass. The following quotation is pertinent:

Definition of terms

.....contd....

Particle A particle is a body whose dimensions are so small that they may be neglected.

Rigid body All bodies are either elastic or plastic and will be deformed if acted upon by forces. When the deformation of such bodies is small, they are frequently assumed to be rigid, i.e., incapable of deformation, in order to simplify the analysis.

Deformable body The rigid-body assumption cannot be used when internal stresses and strains due to the applied forces are to be analyzed. Thus we consider the body to be capable of deforming. Such analysis is frequently called *elastic-body analysis*, using the additional assumption that the body remains elastic within the range of the applied forces.

Newton's Laws

[Law 1] Every body perseveres in its state of rest or of uniform motion in a straight line, except in so far as it is compelled to change that state by impressed forces.

[Law 2] Change of motion is proportional to the moving force impressed, and takes place in the direction of the straight line in which such force is impressed.

[Law 3] Reaction is always equal and opposite to action; that is to say, the actions of two bodies upon each other are always equal and directly opposite.

Analysis ofstaticforces

- Conditionsofequilibrium

A particle is said to be in *equilibrium* if it remains at rest if originally at rest, or has a constant velocity if originally in motion. Most often, however, the term "equilibrium" or, more specifically, "static equilibrium" is used to describe an object at rest. To maintain equilibrium, it is *necessary* to satisfy Newton's first law of motion, which requires the *resultant force* acting on a particle to be equal to *zero*. This condition may be stated mathematically as

$$\Sigma \mathbf{F} = 0 \qquad (3\text{–}1)$$

where $\Sigma \mathbf{F}$ is the vector *sum of all the forces* acting on the particle.

Analysis ofdynamic forces

- Weuseelasticbodyanalysis,whenweareinterestedindeformation,deflection,extensionorthemotionsofthevariousparticlesofthebody,thenitiscalled***ElasticBody*** analysis.
- Weuse***RigidBody*** analysis,whenweareinterestedinoverallmotionofthebody. Theentirebodyisassumedtobeveryrigid&incapableofdeforminginanymanner.

Centroid & center of mass

In solving engineering problems, we frequently find that forces are distributed in some manner over a line, an area, or a volume. The resultant of these distributed forces is usually not too difficult to find. In order to have the same effect, this resultant must act at the *centroid* of the system. Thus, *the centroid of a system is a point at which a system of distributed forces may be considered concentrated with exactly the same effect.*

Instead of a system of distributed forces, we may have a distributed mass. Then, by *center of mass* we mean *the point at which the mass may be considered concentrated so that the same effect is obtained.*

Inertiaforce&D'Alembert'sprinciple

Consider a moving rigid body of mass m acted upon by any system of forces, say $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$, as shown in Fig. 13-6*a*. Designate the center of mass of the body as point G, and find the resultant of the force system from the equation

$$\sum \mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 \qquad (a)$$

In the general case the line of action of this resultant will *not* be through the mass center but will be displaced by some distance, such as the distance h, as shown in the figure. In the study of mechanics it is shown that the effect of this unbalanced force system is to produce linear and angular accelerations

Inertiaforce&D'Alembert'sprinciple

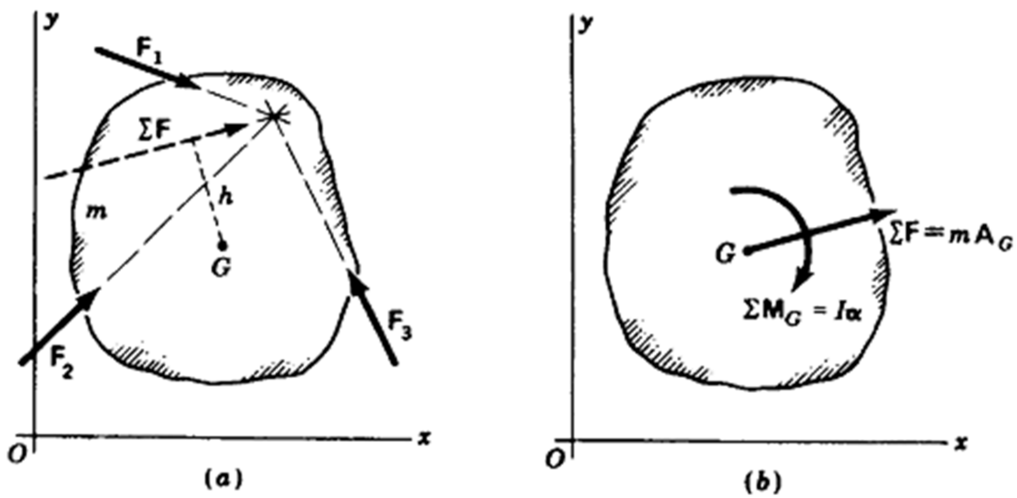


Figure 13-6

whose values are given by

$$\sum \mathbf{F} = m\mathbf{A}_G \quad (13\text{-}16)$$

$$\sum \mathbf{M}_G = I\alpha \quad (13\text{-}17)$$

in which $\mathbf{A}_G$ is the acceleration of the mass center and α is the angular acceleration of m (Fig. 13-6*b*). The quantity $\Sigma \mathbf{F}$ is the resultant of all the external forces acting upon the body, and $\Sigma \mathbf{M}_G$ is the sum of the external moments together with the moments of the external forces taken about G in the plane of motion. The mass moment of inertia is designated as I and is taken with reference to the mass center G also.

Friction, types & characteristics

1. ***Static friction.*** It is the friction, experienced by a body, when at rest.

2. ***Dynamic friction.*** It is the friction, experienced by a body, when in motion. The dynamic friction is also called ***kinetic friction*** and is less than the static friction. It is of the following three types :

(*a*) ***Sliding friction.*** It is the friction, experienced by a body, when it ***slides*** over another body.

(*b*) ***Rolling friction.*** It is the friction, experienced between the surfaces which has ***balls*** or ***rollers*** interposed between them.

(*c*) ***Pivot friction.*** It is the friction, experienced by a body, due to the ***motion of rotation*** as in case of foot step bearings.

The friction may further be classified as :

1. Friction between unlubricated surfaces, and

2. Friction between lubricated surfaces.

Limiting friction

Consider that a body A of weight W is lying on a rough horizontal body B as shown in Fig. 10.1 (*a*). In this position, the body A is in equilibrium under the action of its own weight W, and the normal reaction R_N (equal to W) of B on A. Now if a small horizontal force P_1 is applied to the body A acting through its centre of gravity as shown in Fig. 10.1 (*b*), it does not move because of the frictional force which prevents the motion. This shows that the applied force P_1 is exactly balanced by the force of friction F_1 acting in the opposite direction.

If we now increase the applied force to P_2 as shown in Fig. 10.1 (*c*), it is still found to be in equilibrium. This means that the force of friction has also increased to a value $F_2 = P_2$. Thus every time the effort is increased the force of friction also increases, so as to become exactly equal to the applied force. There is, however, a limit beyond which the force of friction cannot increase as shown in Fig. 10.1 (*d*). After this, any increase in the applied effort will not lead to any further increase in the force of friction, as shown in Fig. 10.1 (*e*), thus the body A begins to move in the direction of the applied force. This maximum value of frictional force, which comes into play, when a body just begins to slide over the surface of the other body, is known as ***limiting force of friction*** or simply ***limiting friction***. It may be noted that when the applied force is less than the limiting friction, the body remains at rest, and the friction into play is called **static friction** which may have any value between zero and limiting friction.

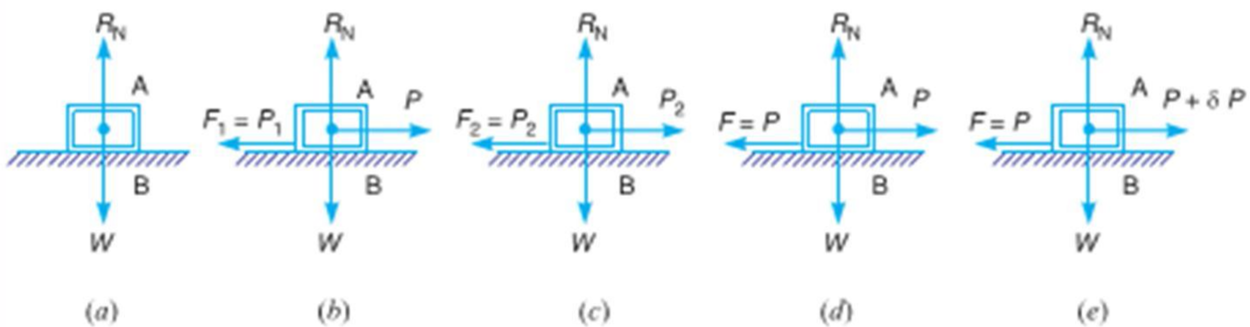


Fig. 10.1. Limiting friction.

Laws of static friction

Following are the laws of static friction :

1. The force of friction always acts in a direction, opposite to that in which the body tends to move.
2. The magnitude of the force of friction is exactly equal to the force, which tends the body to move.
3. The magnitude of the limiting friction (F) bears a constant ratio to the normal reaction (R_N) between the two surfaces. Mathematically

$$F/R_N = \text{constant}$$

4. The force of friction is independent of the area of contact, between the two surfaces.
5. The force of friction depends upon the roughness of the surfaces.

Lawsofkineticordynamicfriction

Following are the laws of kinetic or dynamic friction :

1. The force of friction always acts in a direction, opposite to that in which the body is moving.
2. The magnitude of the kinetic friction bears a constant ratio to the normal reaction between the two surfaces. But this ratio is slightly less than that in case of limiting friction.
3. For moderate speeds, the force of friction remains constant. But it decreases slightly with the increase of speed.

Coefficientoffriction

It is defined as the ratio of the limiting friction (F) to the normal reaction (R_N) between the two bodies. It is generally denoted by μ. Mathematically, coefficient of friction,

$$\mu = F/R_N$$

Limiting angle of friction

Consider that a body A of weight (W) is resting on a horizontal plane B, as shown in Fig. 10.2. If a horizontal force P is applied to the body, no relative motion will take place until the applied force P is equal to the force of friction F, acting opposite to the direction of motion. The magnitude of this force of friction is $F = \mu.W = \mu.R_N$, where R_N is the normal reaction. In the limiting case, when the motion just begins, the body will be in equilibrium under the action of the following three forces :

1. Weight of the body (W),
2. Applied horizontal force (P), and
3. Reaction (R) between the body A and the plane B.

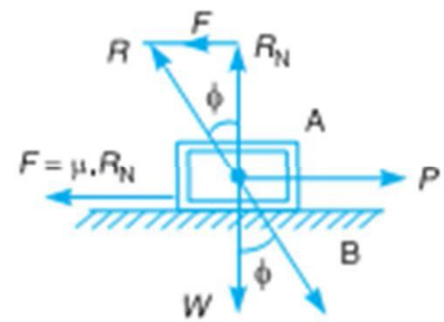


Fig. 10.2. Limiting angle of friction.

The reaction R must, therefore, be equal and opposite to the resultant of W and P and will be inclined at an angle ϕ to the normal reaction R_N. This angle ϕ is known as the ***limiting angle of friction***. It may be defined as the angle which the resultant reaction R makes with the normal reaction R_N.

From Fig. 10.2, $\tan \phi = F/R_N = \mu\ R_N / R_N = \mu$

Engine, types &characteristics

- Mainlytwotypesofprimemovers:
  - Internalcombustionengine(usedin automobile, marine, industrialapplications &veryoldaircrafts)
  - Steamengine(external combustion)(was usedforlocomotives)

Boththetypesofenginesarere ciprocatoryinnature.

Bothofthemgeneratelargeinertiaforces &requiresbalancingtechniquetocontrole ffectofinertiaforces.

Steam engine indicatordiagram

The valves are used to control the steam which drives the piston of a reciprocating steam engine. The valves have to perform the four distinct operations on the steam used on one side (*i.e.* cover end) of the piston, as shown by the indicator diagram (also known as pressure-volume diagram) in Fig. 17.1. These operations are as follows:

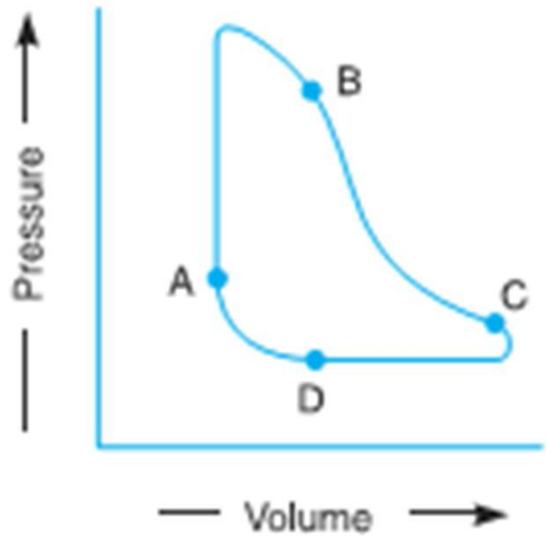


Fig. 17.1. Indicator diagram of a reciprocating steam engine.

Internalcombustionenginetypes

- Singlecylinder&multi-cylinders
  - Inline multi-cylinders, 3, 4&6cylinders aremostcommonly beingusedin automobiles.
  - V-type multi-cylinderengines,V6&V8are mostcommon configurationforlarge commercialvehicles&stationary applications.Includedanglegenerally kept120degree.

IndicatordiagramforanICengine

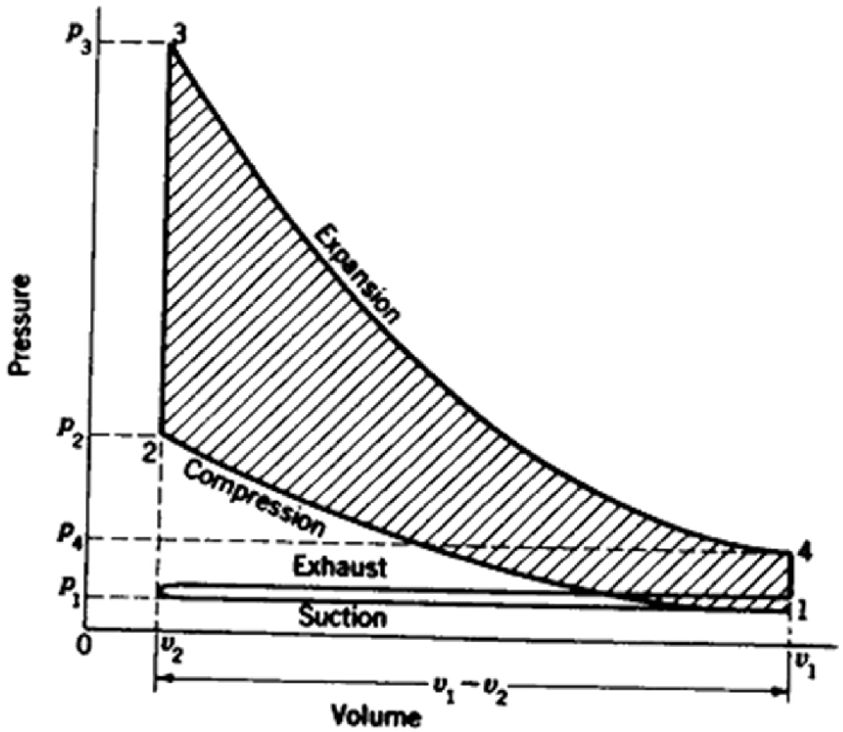


Figure 14-12 An ideal indicator diagram for a four-cycle engine.

where bhp = brake horsepower per cylinder

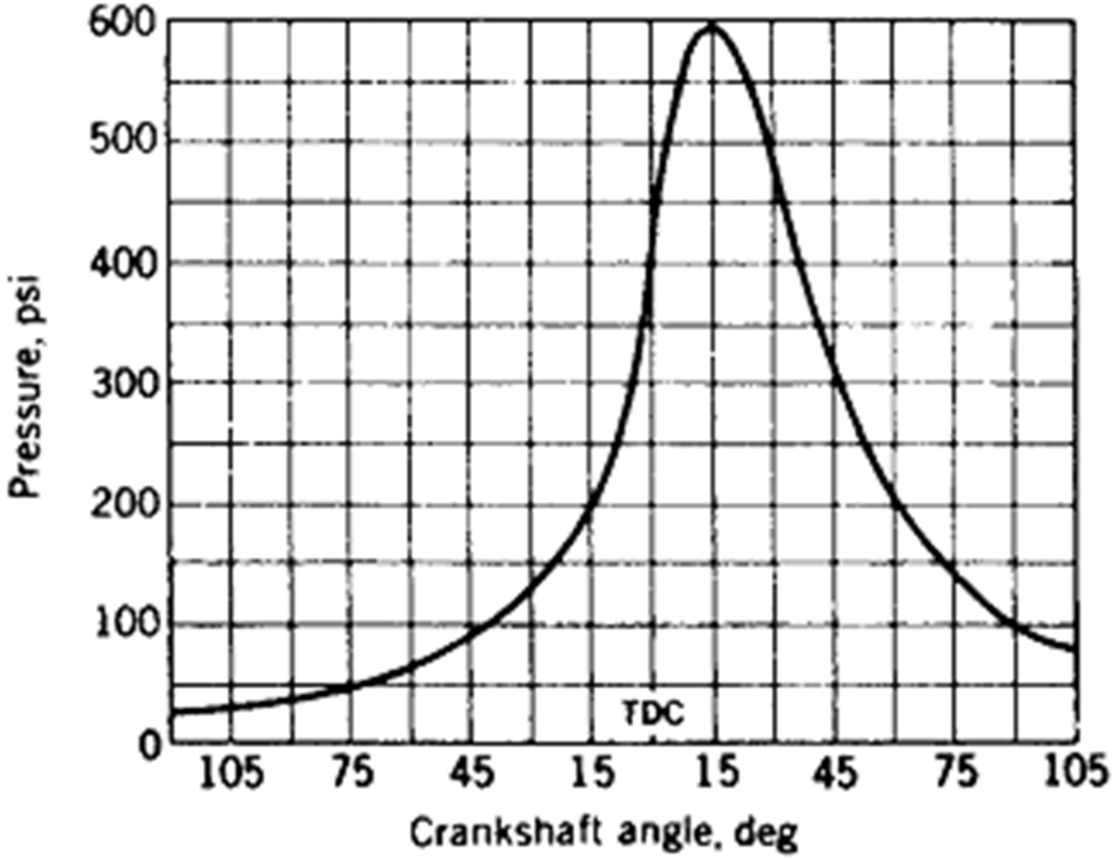
p_b = brake mean effective pressure, psi

l = length of stroke, in

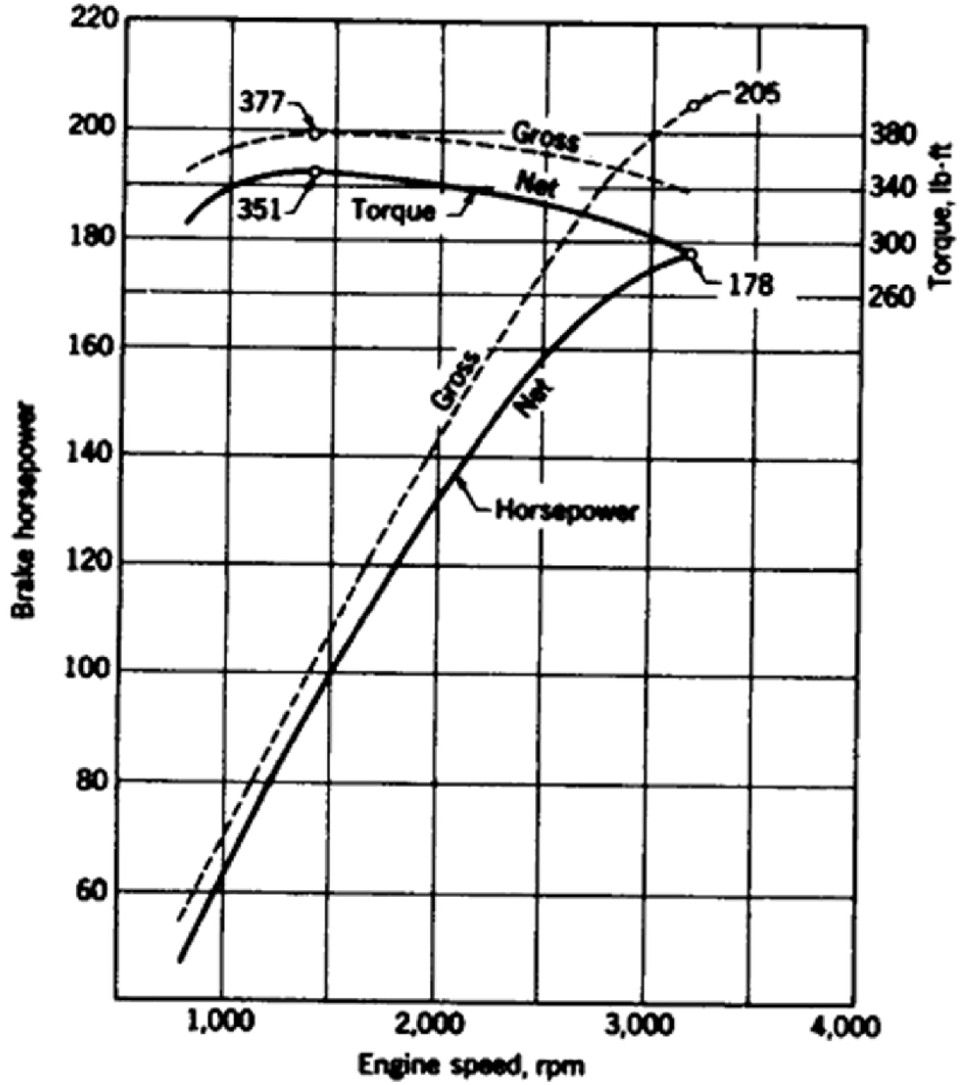
a = piston area, in^2

n = number of working strokes per minute

Pressure curve (Gas force)



Power&torque curves for an IC engine



Equivalent masses

In analyzing the inertia forces due to the connecting rod of an engine it is often convenient to concentrate a portion of the mass at the crankpin A and the remaining portion at the wrist pin B (Fig. 14-14). The reason for this is that the crankpin moves on a circle and the wrist pin on a straight line. Both of these motions are quite easy to analyze. However, the center of gravity G

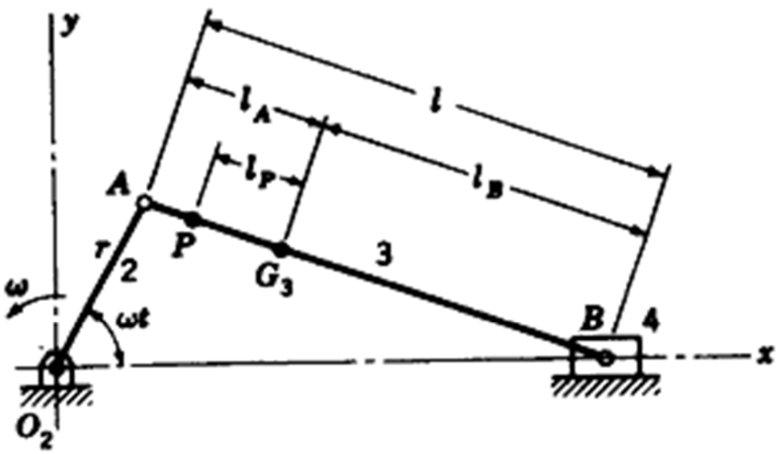


Figure 14-14

is somewhere between the crankpin and wrist pin, and its motion is more complicated and consequently more difficult to determine in algebraic form.

Equivalent masses

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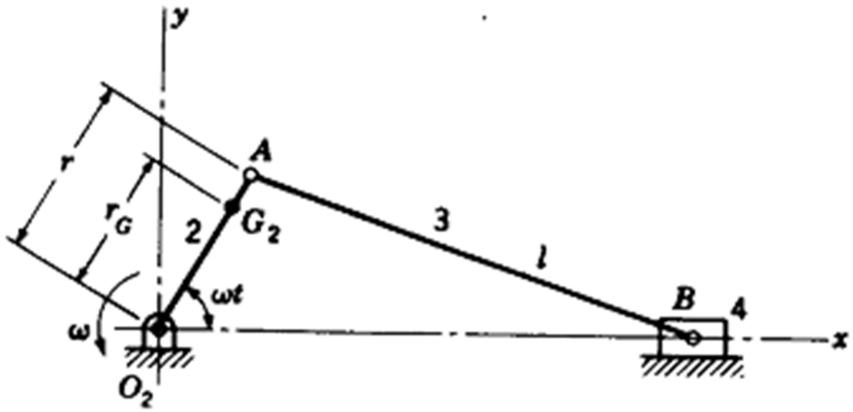

Figure 14-15

For estimating and checking purposes, about two-thirds of the mass should be concentrated at A and the remaining third at B.

Figure 14-15 illustrates an engine linkage in which the mass of the crank m_2 is not balanced, as evidenced by the fact that the center of gravity G_2 is displaced outward along the crank a distance r_G from the axis of rotation. In the inertia-force analysis, simplification is obtained by locating an equivalent mass m_{2A} at the crankpin. Thus, for equivalence

$$m_2 r_G = m_{2A} r \qquad \text{or} \qquad m_{2A} = m_2 \frac{r_G}{r} \tag{14-19}$$

Inertia forces

Using the methods of the preceding section, we begin by locating equivalent masses at the crankpin and at the wrist pin. Thus

$$m_A = m_{2A} + m_{3A} \tag{14-20}$$

$$m_B = m_{3B} + m_4 \tag{14-21}$$

Equation (14-20) states that the mass m_A located at the crankpin is made up of the equivalent masses m_{2A} of the crank and m_{3A} of part of the connecting rod. Of course, if the crank is balanced, all its mass is assumed to be located at the axis of rotation and m_{2A} is then zero. Equation (14-21) indicates that the reciprocating mass m_B located at the wrist pin is composed of the equivalent mass m_{3B} of the other part of the connecting rod and the mass m_4 of the piston assembly.

Figure 14-16 shows the slider-crank mechanism with masses m_A and m_B located at points A and B, respectively. If we designate the angular velocity of the crank as ω and the angular acceleration as α, the position vector of the

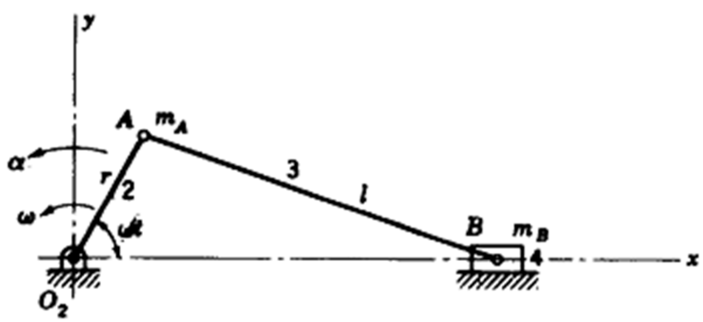

Figure 14-16

crankpin relative to the origin O_2 is

Inertia force

.....contd...

$$F^y = m_A r\omega^2 \sin \omega t \qquad (14\text{-}29)$$

It is customary to refer to the portion of the force occurring at the circular frequency ω rad/s as the *primary inertia force* and the portion occurring at 2ω rad/s as the *secondary inertia force*. We note that the vertical component has only a primary part and that it therefore varies directly with the crankshaft speed. On the other hand, the horizontal component, which is in the direction of the cylinder axis, has a primary part varying directly with the crankshaft speed and a secondary part moving at twice the crankshaft speed.

Bearing loads

The designer of a reciprocating engine must know the values of the forces acting upon the bearings and how these forces vary in a cycle of operation. This is necessary to proportion and select the bearings properly, and it is also needed for the design of other engine parts. This section is an investigation of the force exerted by the piston against the cylinder wall and the forces acting against the piston pin and against the crankpin. Main bearings forces will be investigated in a later section because they depend upon the action of all the cylinders of the engine.

The resultant bearing loads are made up of the following components:

1. The gas-force components, designated by a single prime
2. Inertia force due to the weight of the piston assembly, designated by a double prime
3. Inertia force of that part of the connecting rod assigned to the piston-pin end, triple-primed
4. Connecting-rod inertia force at the crankpin end, quadruple-primed

Crankshafttorque

The torque delivered by the crankshaft to the load is called the *crankshaft torque* and it is the negative of the moment of the couple formed by the forces $\mathbf{F}_{41}$ and $\mathbf{F}^{y}_{21}$. Therefore, it is obtained from the equation

$$\mathbf{T}_{21} = -F_{41}x\hat{\mathbf{k}} = [(m_{3B} + m_4)\ddot{x} + P]x \tan\phi\, \hat{\mathbf{k}} \qquad (14\text{-}44)$$

Engine shakingforces

The inertia force due to the reciprocating masses is shown acting in the positive direction in Fig. 14-21*a*. In Fig. 14-21*b* the forces acting upon the engine block due to these inertia forces are shown. The resultant forces are $\mathbf{F}_{21}$, the force exerted by the crankshaft on the main bearings, and a positive couple formed by the forces $\mathbf{F}_{41}$ and $\mathbf{F}_{21}^y$. The force $\mathbf{F}_{21}^x = -m_B\mathbf{A}_B$ is frequently termed a *shaking force*, and the couple $T = xF_{41}$ a *shaking couple*. As indicated by Eqs. (14-27) and (14-30), the magnitude and direction of this force and couple change with ωt; consequently, the shaking force induces linear vibration of the block in the x direction, and the shaking couple a torsional vibration of the block about the crank center.

A graphical representation of the inertia force is possible if Eq. (14-27) is rearranged as

$$F = m_B r\omega^2 \cos \omega t + m_B r\omega^2 \frac{r}{l} \cos 2\omega t \qquad (14\text{-}45)$$

Engine shakingforces

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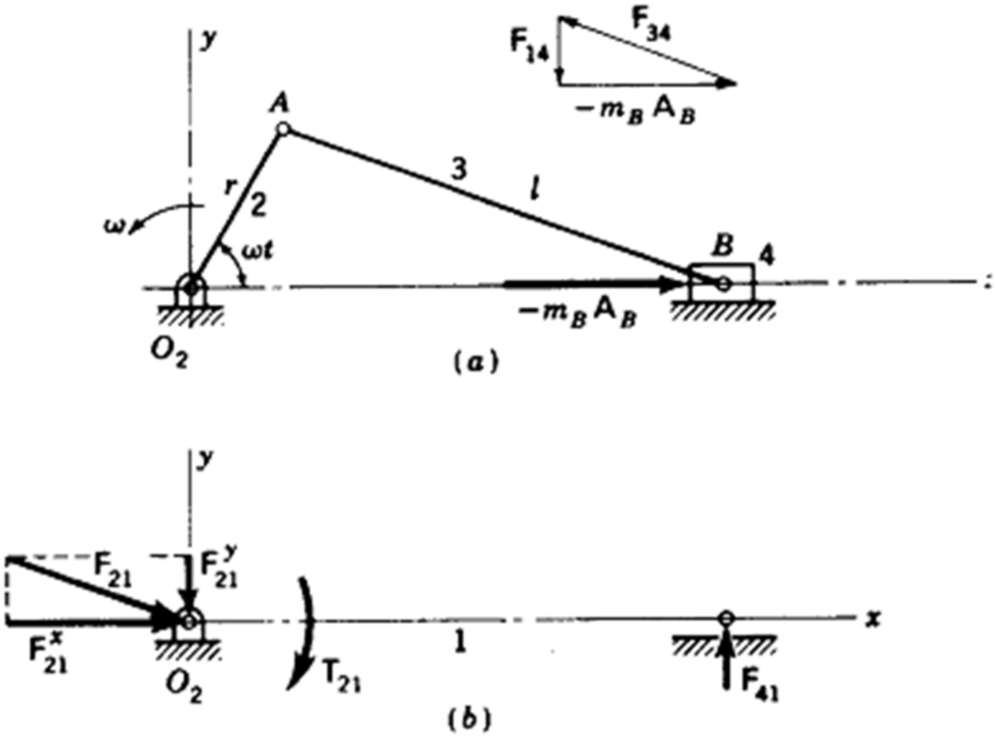

Figure 14-21 Inertia forces due to the reciprocating masses; the primes have been omitted for simplification.