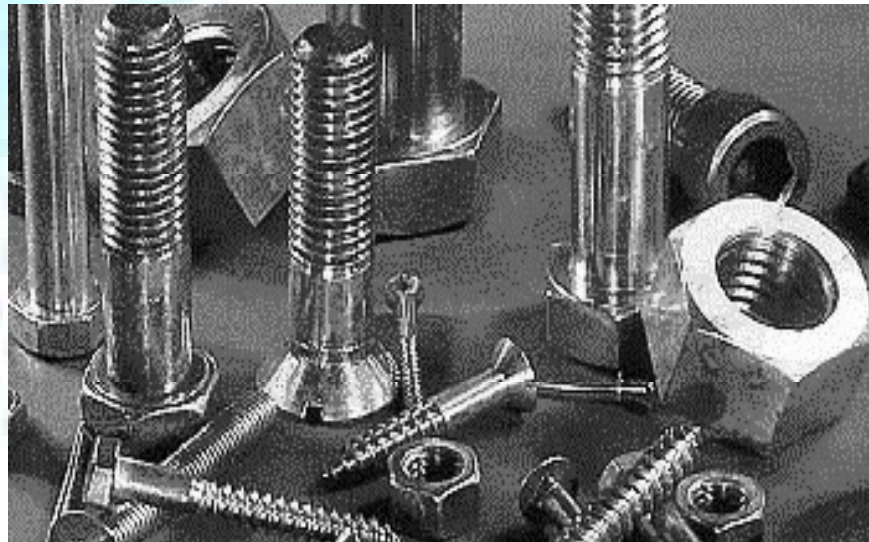
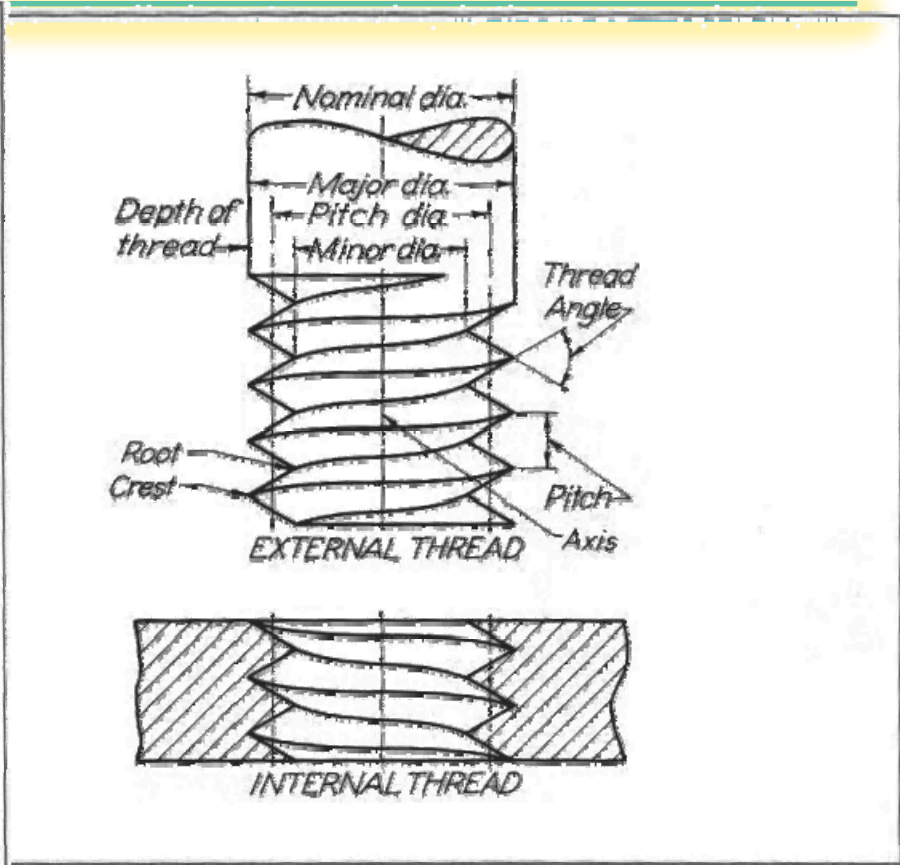
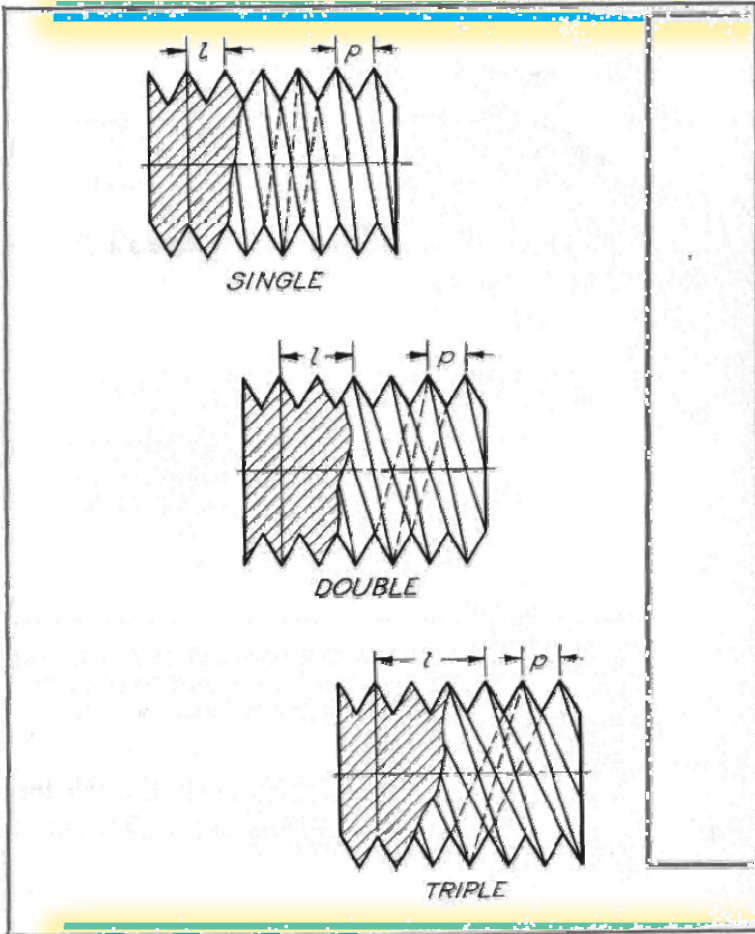
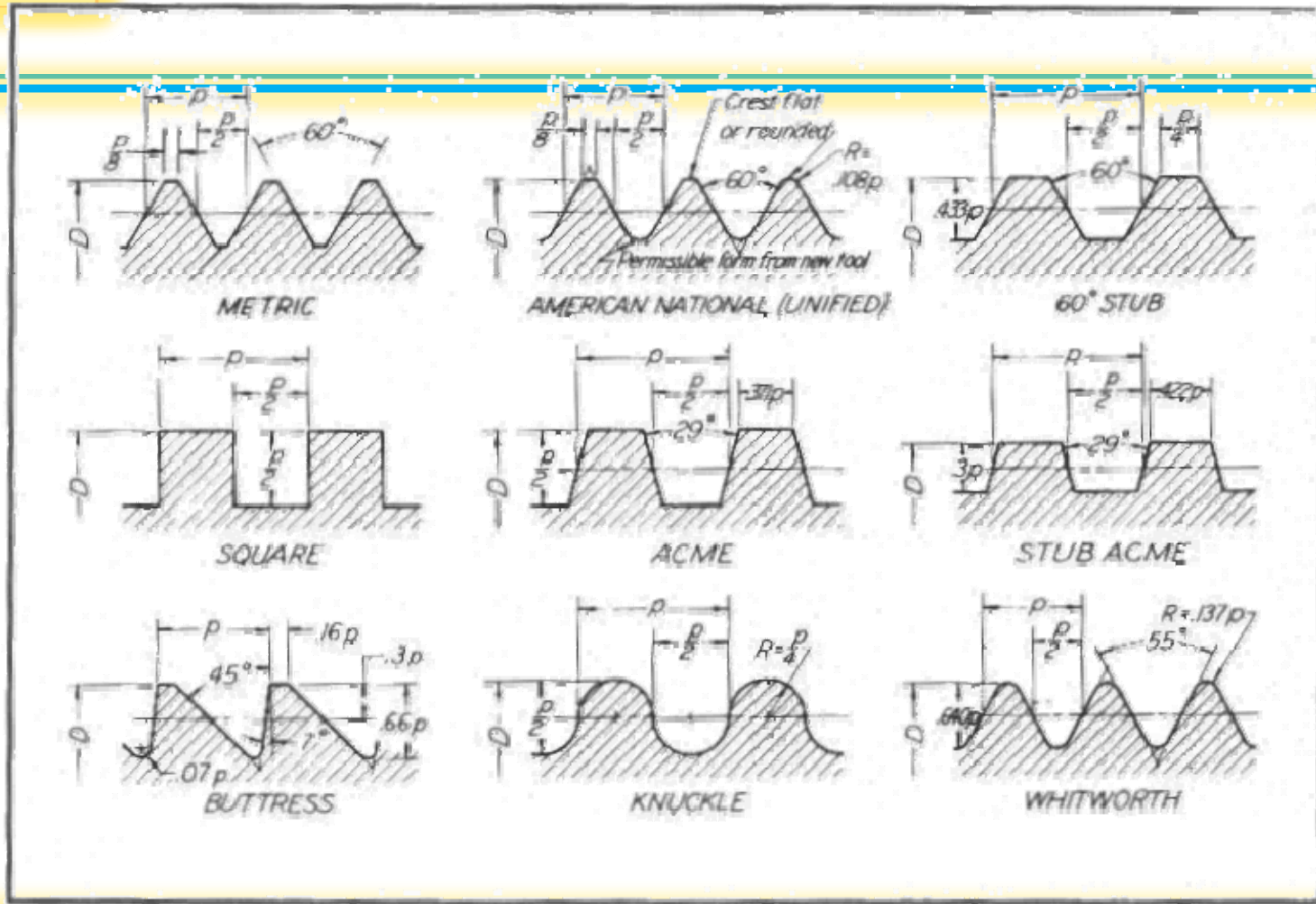
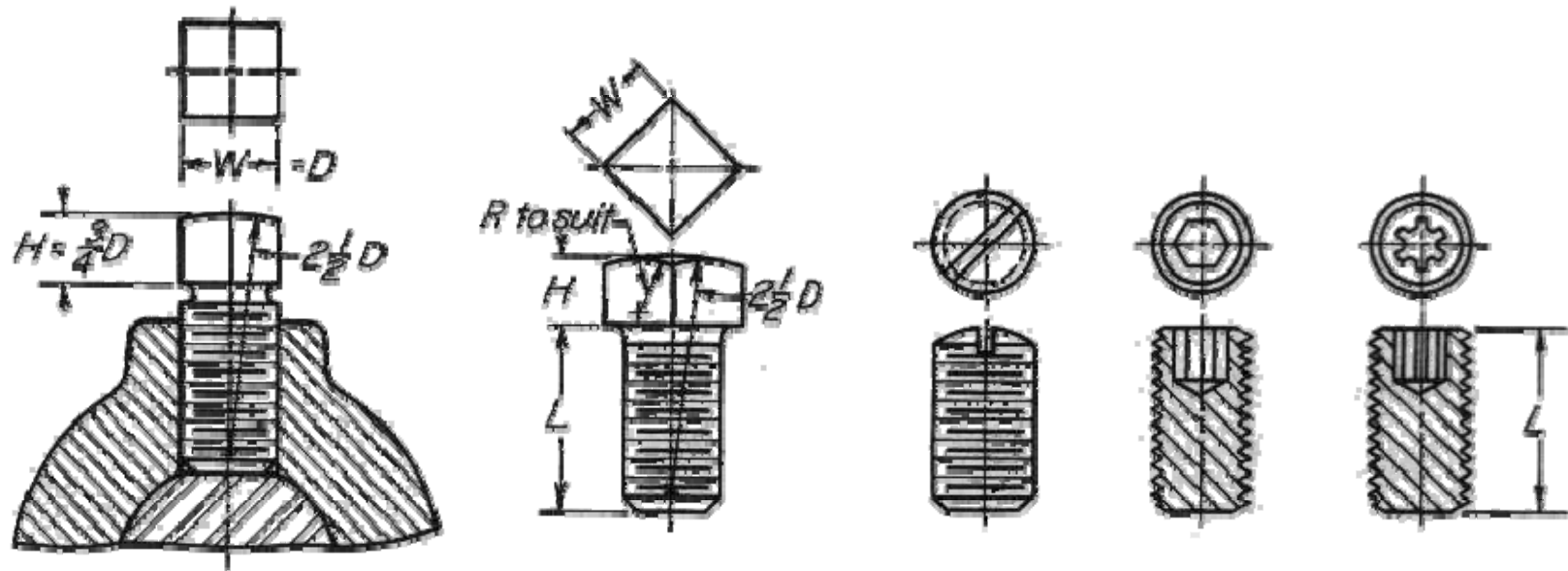


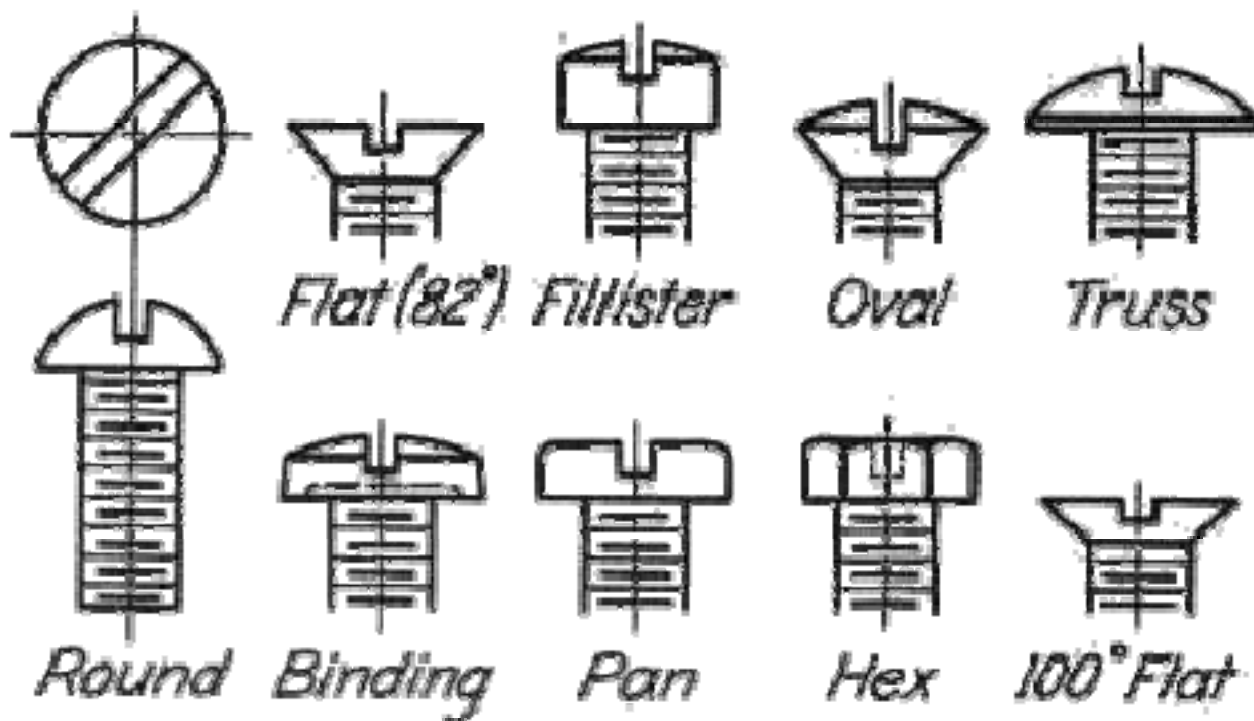
# **SCREWS, FASTENERS, AND THE DESIGN OF NONPERMANENT JOINTS**













# Chapter Outline

- 8-1 Thread Standards and Definitions
- 8-2 The Mechanics of Power Screws
- 8-3 Strength Constraints
- 8-4 Joints-Fasteners Stiffness
- 8-5 Joints-Member Stiffness
- 8-6 Bolt Strength
- 8-7 Tension Joints-The External Load
- 8-8 Relating Bolt Torque to Bolt Tension
- 8-9 Statically Loaded Tension Joint with Preload
- 8-10 Gasketed Joints
- 8-11 Fatigue Loading of Tension Joints
- 8-12 Shear Joints
- 8-13 Setscrews
- 8-14 Keys and Pins
- 8-15 Stochastic Considerations



# LECTURE 35

- 8-3 Strength Constraints
- 8-4 Joints-Fasteners Stiffness
- 8-5 Joints-Member Stiffness
- 8-6 Bolt Strength



## Example-3

A single-threaded **20 mm** power screw is **20 mm** in diameter with a **pitch** of **5 mm**. A vertical load on the screw reaches a maximum of **3 kN**. The coefficients of friction are **0.06** for the collar and **0.09** for the threads. The frictional diameter of the collar is **45 mm**. Find the overall efficiency and the torque to "raise" and "lower" the load.





## Example-3

Given

$$l = 5 \text{ mm}$$

$$d = 20 \text{ mm}$$

$$\text{Pitch, } p = 5 \text{ mm}$$

$$d_m = (d - p/2) = 20 - 2.5 = 17.5 \text{ mm}$$

$$\text{Load, } F = 3 \text{ kN}$$

$$\text{Frictional coefficient for the thread, } f = 0.09$$

$$\text{Frictional coefficient for the collar, } f_c = 0.06$$

$$\text{Frictional diameter of the collar, } D_c = 45 \text{ mm}$$



## Example-3 (Cont.'d)

### Solution

The torque required to raise the load is:

$$T_R = \frac{Fd_m}{2} \left( \frac{l + \pi fd_m}{\pi d_m - fl} \right) + \frac{Ff_c d_c}{2}$$

$$T_R = \frac{3(17.5)}{2} \left( \frac{5 + \pi(0.09)(17.5)}{\pi(17.5) - (0.09)(5)} \right) + \frac{3(0.06)(45)}{2}$$
$$= 4.78902 + 4.05 = 8.84 \text{ N.m}$$

The torque required to lower the load is:

$$T_L = \frac{Fd_m}{2} \left( \frac{\pi fd_m - l}{\pi d_m + fl} \right) + \frac{Ff_c d_c}{2}$$

$$T_L = \frac{3(17.5)}{2} \left( \frac{\pi(0.09)(17.5) - 5}{\pi(17.5) + (0.09)(5)} \right) + \frac{3(0.06)(45)}{2}$$
$$= -0.02462 + 4.05 = 4.025 \text{ N.m}$$



## Example-2 (Cont.'d)

Since  $T_L$  is positive, the thread is self-locking. The efficiency is:

$$e = \frac{T_0}{T_R} = \frac{Fl}{2\pi T_R}$$

$$e = \frac{3(5)}{2\pi(8.84)} = 0.27$$



## Power Screws Stress Analysis

The following stresses should be checked on both the nut and the screw:

1. Shearing stress in screw body.

$$\tau = \frac{16T}{\pi d_r^3} \quad (8-7)$$

2. Axial stress in screw body

$$\sigma = \frac{F}{A} = \frac{4F}{\pi d_r^2} \quad (8-8)$$



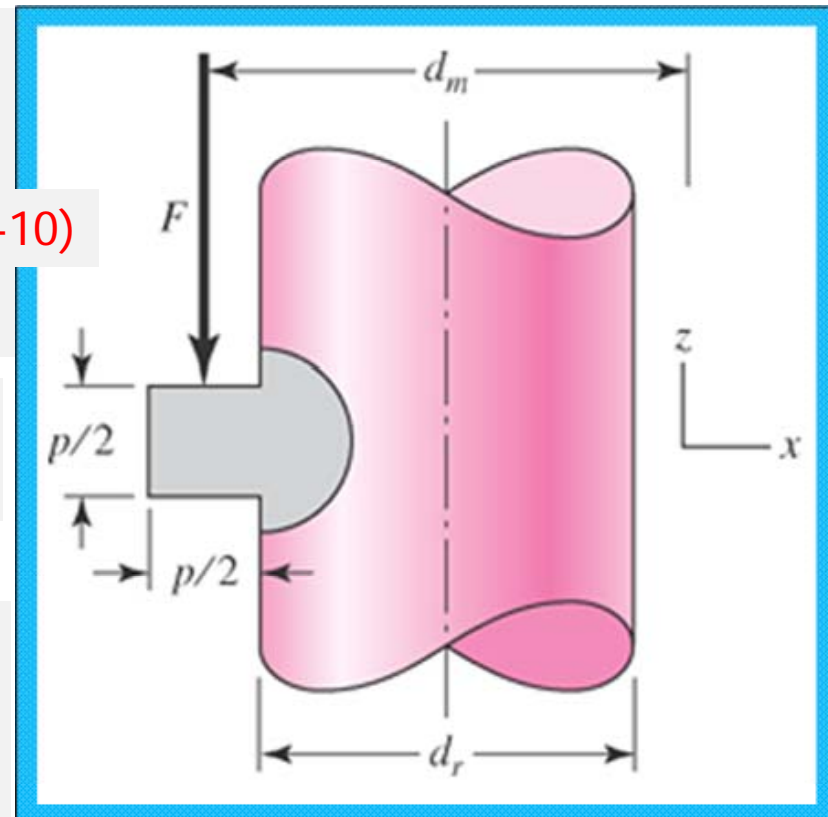
## Power Screws Stress Analysis

### 3. Thread bearing stress

$$\sigma_B = \frac{F}{\pi d_m n_t p / 2} = \frac{2F}{\pi d_m n_t p} \quad (8-10)$$

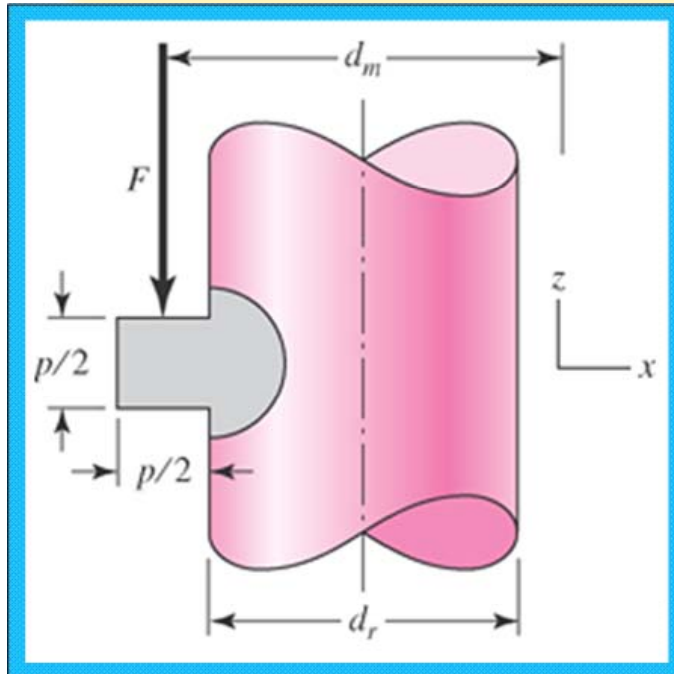
where  $n_t$  is the number of engaged threads.

Figure 8-8  
Geometry of square thread useful in finding bending and transverse shear stresses at the thread root





## Power Screws Stress Analysis

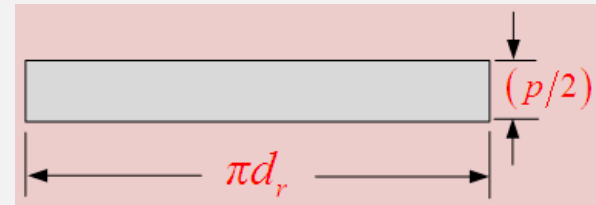


### 4. Thread bending stress

The bending stress at the root of the thread is given by

$$\frac{I}{C} = \frac{(1/12)(\pi d_r n_t)(p/2)^3}{(p/2)/2} = \frac{\pi}{24} d_r n_t p^2$$

$$M = \frac{Fp}{4}$$



$$\sigma_b = \frac{M}{I/c} = \left( \frac{Fp}{4} \right) \frac{24}{\pi d_r n_t p^2} = \frac{6F}{\pi d_r n_t p} \quad (8-11)$$



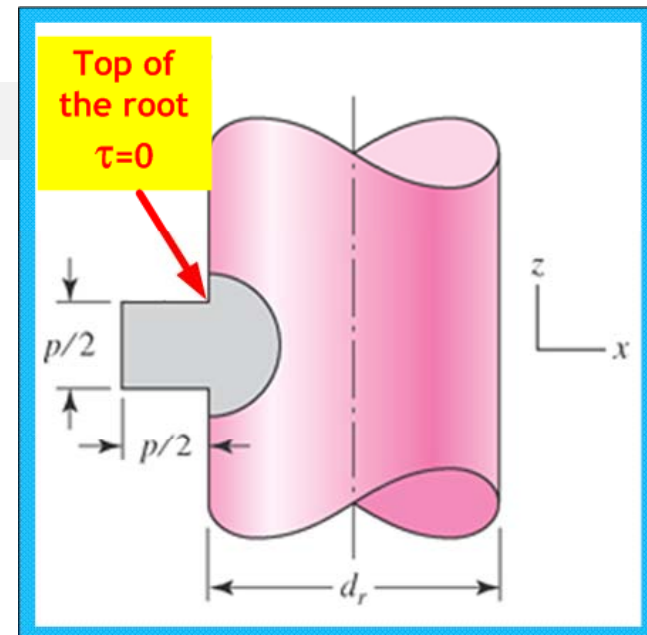
## Power Screws Stress Analysis

5. Transverse shear stress at the center of the thread root

$$\tau = \frac{3V}{2A} = \frac{3}{2} \frac{F}{\pi d_m n_t p/2} = \frac{3F}{\pi d_m n_t p} \quad (8-12)$$

Notice that the transverse shear stress at **the top of the root** is zero

$$\tau = 0$$





## Power Screws Stress Analysis

The state of stress at top of root "plane" is

$$\begin{aligned}\sigma_x &= \frac{6F}{\pi d_r n_t p} & \tau_{xy} &= 0 \\ \sigma_y &= 0 & \tau_{yz} &= \frac{16T}{\pi d_r^3} \\ \sigma_z &= -\frac{4F}{\pi d_r^2} & \tau_{zx} &= 0\end{aligned}$$

$$\sigma' = \frac{1}{\sqrt{2}} \left\{ (\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2) \right\}^{1/2}$$

- Von-Mises Stress at top of root plane is calculated using Eq. (6-14) of Sec. (6-5) and failure criteria applied (see example 8-1).





## Power Screws Stress Analysis

- ❑ The engaged threads cannot share the load equally. Some experiments show that
  - the first engaged thread carries 0.38 of the load
  - the second engaged thread carries 0.25 of the load
  - the third engaged thread carries 0.18 of the load
  - the seventh engaged thread is free of load
- ❑ In estimating thread stresses by the equations above, substituting  $n_t$  to 1 will give the largest level of stresses in the thread-nut combination



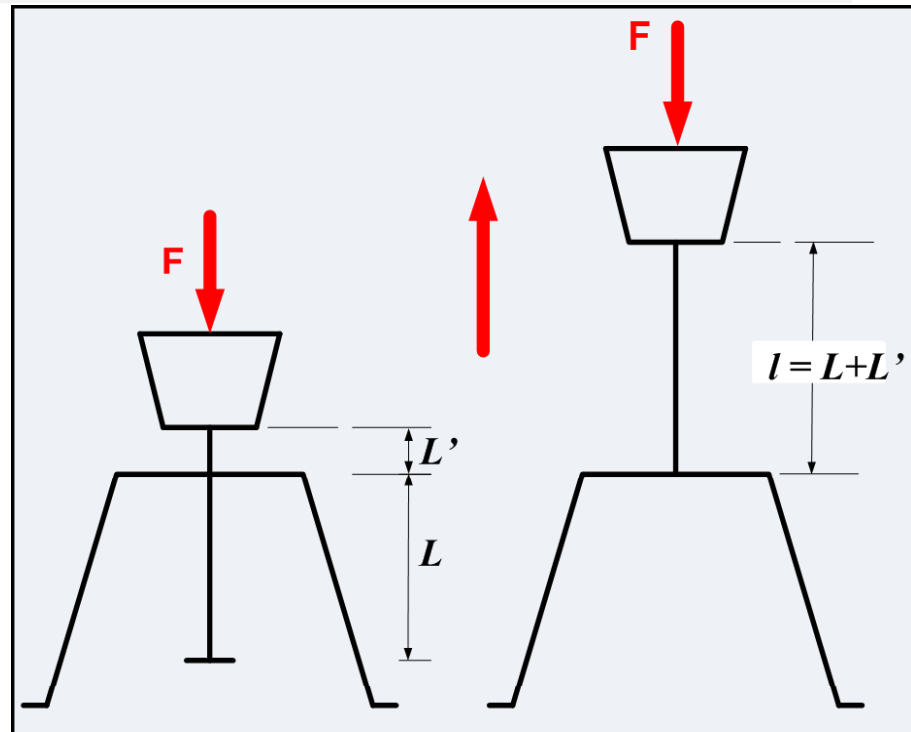
## Power Screws Buckling

Assuming that the column (screw) is a Johnson column

$$\left(\frac{F}{A}\right)_{crit} = S_y - \left(\frac{S_y l}{2\pi k}\right)^2 \frac{1}{CE}$$

where

$$l = L + L'$$





## Example-3 (Example 8-1 in Textbook)

A square-thread power screw has a major diameter of 32 mm and a pitch of 4 mm with double threads, and it is to be used in an application similar to that in Fig. 8–4. The given data include  $f = f_c = 0.08$ ,  $d_c = 40$  mm, and  $F = 6.4$  kN per screw.

- (a) Find the thread depth, thread width, pitch diameter, minor diameter, and lead.
- (b) Find the torque required to raise and lower the load.
- (c) Find the efficiency during lifting the load.
- (d) Find the body stresses, torsional and compressive.
- (e) Find the bearing stress.
- (f) Find the thread stresses bending at the root, shear at the root, and von Mises stress and maximum shear stress at the same location.



(a) From Fig. 8-3a the thread depth and width are the same and equal to half the pitch, or 2 mm. Also

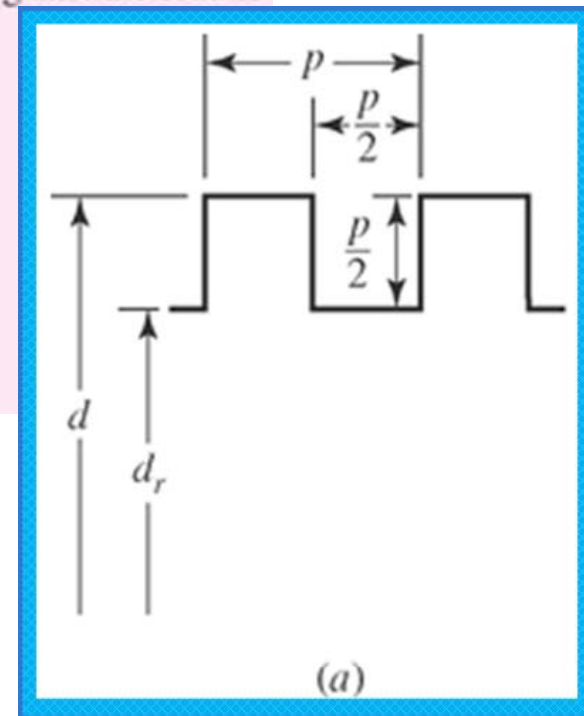
$$d_m = d - p/2 = 32 - 4/2 = 30 \text{ mm}$$

$$d_r = d - p = 32 - 4 = 28 \text{ mm}$$

$$l = np = 2(4) = 8 \text{ mm}$$

(b) Using Eqs. (8-1) and (8-6), the torque required to turn the screw against the load is

$$\begin{aligned} T_R &= \frac{F d_m}{2} \left( \frac{l + \pi f d_m}{\pi d_m - fl} \right) + \frac{F f_c d_c}{2} \\ &= \frac{6.4(30)}{2} \left[ \frac{8 + \pi(0.08)(30)}{\pi(30) - 0.08(8)} \right] + \frac{6.4(0.08)40}{2} \\ &= 15.94 + 10.24 = 26.18 \text{ N} \cdot \text{m} \end{aligned}$$





## Example-3 (Example 8-1 in Textbook)

Using Eqs. (8-2) and (8-6), we find the load-lowering torque is

$$\begin{aligned} T_L &= \frac{F d_m}{2} \left( \frac{\pi f d_m - l}{\pi d_m + f l} \right) + \frac{F f_c d_c}{2} \\ &= \frac{6.4(30)}{2} \left[ \frac{\pi(0.08)30 - 8}{\pi(30) + 0.08(8)} \right] + \frac{6.4(0.08)(40)}{2} \\ &= -0.466 + 10.24 = 9.77 \text{ N} \cdot \text{m} \end{aligned}$$

The minus sign in the first term indicates that the screw alone is not self-locking and would rotate under the action of the load except for the fact that the collar friction is present and must be overcome, too. Thus the torque required to rotate the screw "with" the load is less than is necessary to overcome collar friction alone.

(c) The overall efficiency in raising the load is

$$e = \frac{F l}{2\pi T_R} = \frac{6.4(8)}{2\pi(26.18)} = 0.311$$



## Example-3 (Example 8-1 in Textbook)

(d) The body shear stress  $\tau$  due to torsional moment  $T_R$  at the outside of the screw body is

$$\tau = \frac{16T_R}{\pi d_r^3} = \frac{16(26.18)(10^3)}{\pi(28^3)} = 6.07 \text{ MPa}$$

The axial nominal normal stress  $\sigma$  is

$$\sigma = -\frac{4F}{\pi d_r^2} = -\frac{4(6.4)10^3}{\pi(28^2)} = -10.39 \text{ MPa}$$

(e) The bearing stress  $\sigma_B$  is, with one thread carrying  $0.38F$ ,

$$\sigma_B = -\frac{2(0.38F)}{\pi d_m(1)p} = -\frac{2(0.38)(6.4)10^3}{\pi(30)(1)(4)} = -12.9 \text{ MPa}$$

(f) The thread-root bending stress  $\sigma_b$  with one thread carrying  $0.38F$  is

$$\sigma_b = \frac{6(0.38F)}{\pi d_r(1)p} = \frac{6(0.38)(6.4)10^3}{\pi(28)(1)4} = 41.5 \text{ MPa}$$



## Example-3 (Example 8-1 in Textbook)

The transverse shear at the extreme of the root cross section due to bending is zero. However, there is a circumferential shear stress at the extreme of the root cross section of the thread as shown in part (d) of 6.07 MPa. The three-dimensional stresses, after Fig. 8-8, noting the  $y$  coordinate is into the page, are

$$\begin{aligned}\sigma_x &= 41.5 \text{ MPa} & \tau_{xy} &= 0 \\ \sigma_y &= 0 & \tau_{yz} &= 6.07 \text{ MPa} \\ \sigma_z &= -10.39 \text{ MPa} & \tau_{zx} &= 0\end{aligned}$$

Equation (5-14) of Sec. 5-5 can be written as

$$\begin{aligned}\sigma' &= \frac{1}{\sqrt{2}} [(41.5 - 0)^2 + [0 - (-10.39)]^2 + (-10.39 - 41.5)^2 + 6(6.07)^2]^{1/2} \\ &= 48.7 \text{ MPa}\end{aligned}$$



Alternatively, you can determine the principal stresses and then use Eq. (5-12) to find the von Mises stress. This would prove helpful in evaluating  $\tau_{\max}$  as well. The principal stresses can be found from Eq. (3-15); however, sketch the stress element and note that there are no shear stresses on the  $x$  face. This means that  $\sigma_x$  is a principal stress. The remaining stresses can be transformed by using the plane stress equation, Eq. (3-13). Thus, the remaining principal stresses are

$$\frac{-10.39}{2} \pm \sqrt{\left(\frac{-10.39}{2}\right)^2 + 6.07^2} = 2.79, -13.18 \text{ MPa}$$

Ordering the principal stresses gives  $\sigma_1, \sigma_2, \sigma_3 = 41.5, 2.79, -13.18$  MPa. Substituting these into Eq. (5-12) yields

$$\begin{aligned}\sigma' &= \left\{ \frac{[41.5 - 2.79]^2 + [2.79 - (-13.18)]^2 + [-13.18 - 41.5]^2}{2} \right\}^{1/2} \\ &= 48.7 \text{ MPa}\end{aligned}$$

The maximum shear stress is given by Eq. (3-16), where  $\tau_{\max} = \tau_{1/3}$ , giving

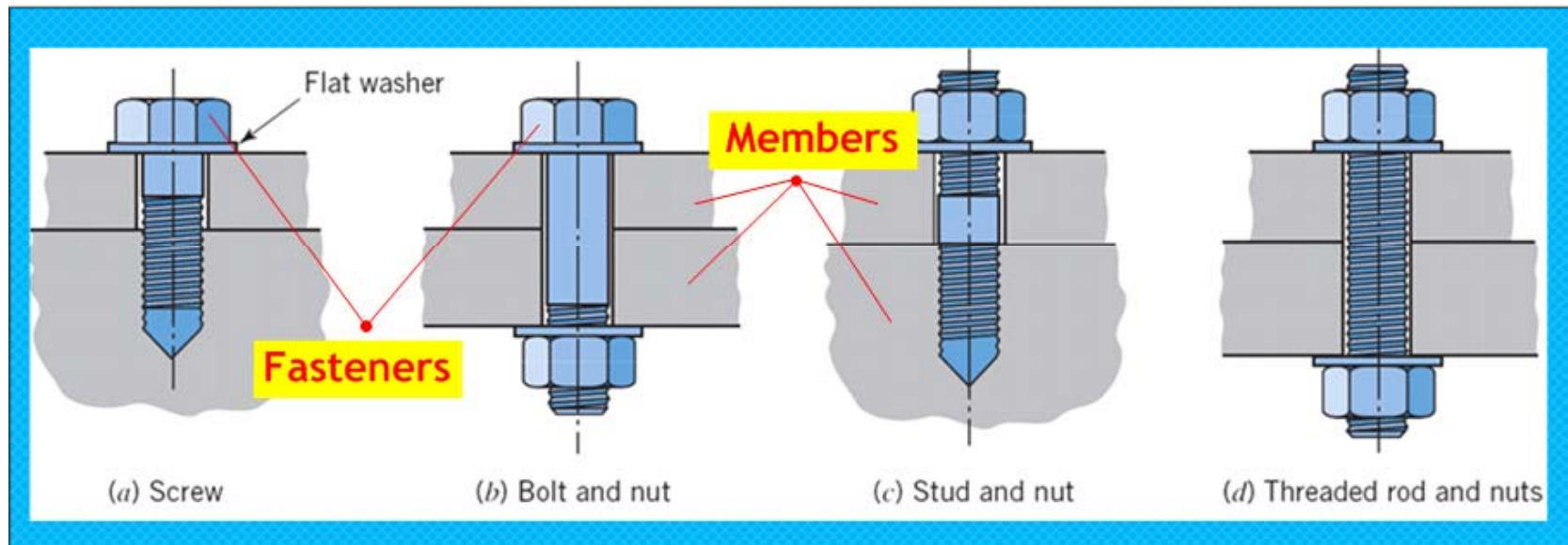
$$\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{41.5 - (-13.18)}{2} = 27.3 \text{ MPa}$$





# Types of fasteners

## Nonpermanent Joints



Three types of threaded fastener: (a) Screw (b) Bolt and nut; (c) Stud and nut, (d) Threaded rod and nuts



# Threaded Fasteners

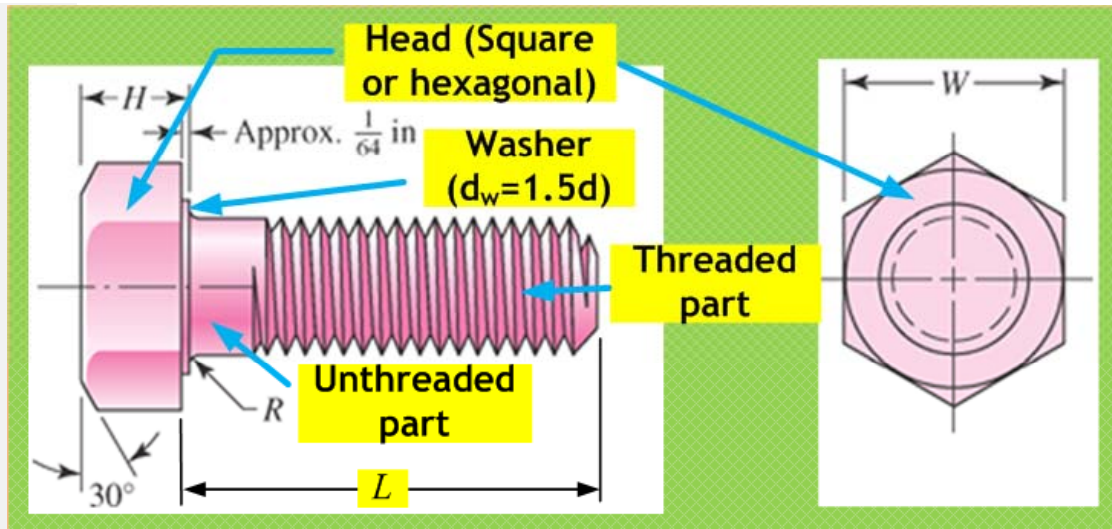
## A- BOLTS:

### Purpose:

to clamp two or more members together.

### Parts:

- (1) Head (Square or Hexagonal)
- (2) Washer ( $d_w = 1.5d$ )
- (3) Threaded part
- (4) Unthreaded part



Dimensions of square and hexagonal bolts are given in TABLE A-29



# Threaded Fasteners

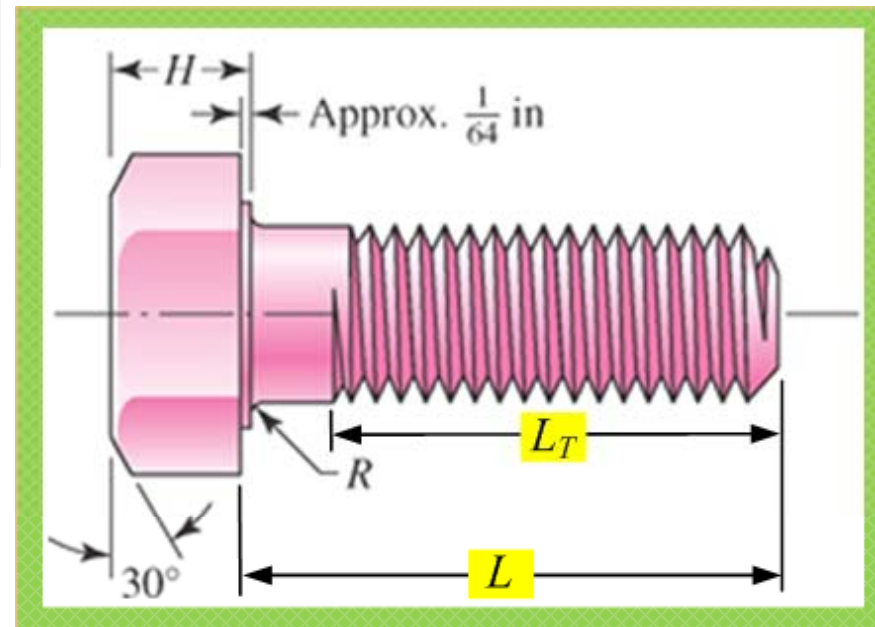
The diameter of the washer face is the same as the width across the flats of the hexagon. The thread length ( $L_T$ ) is :

English

$$L_T = \begin{cases} 2D + \frac{1}{4} \text{ in} & L \leq 6 \text{ in} \\ 2D + \frac{1}{2} \text{ in} & L > 6 \text{ in} \end{cases}$$

Metric (in mm)

$$L_T = \begin{cases} 2D + 6 & L \leq 125 \quad D \leq 48 \\ 2D + 12 & 125 < L \leq 200 \\ 2D + 25 & L > 200 \end{cases}$$

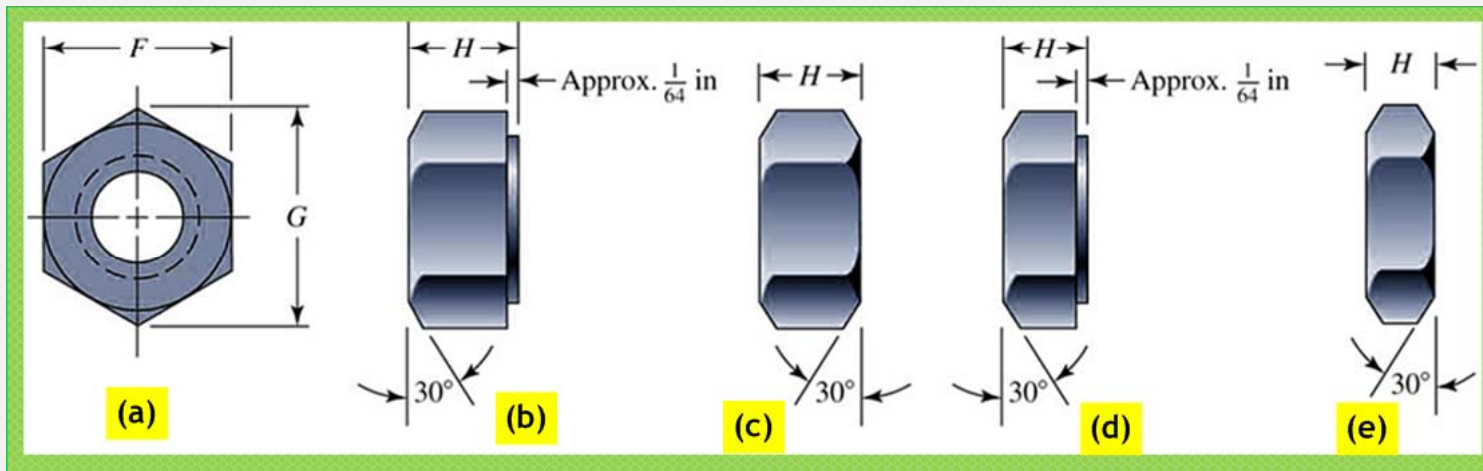


$D$  is the nominal diameter



# Threaded Fasteners

**B- NUTS:** Same material as that of a screw  
Table A-31 gives dimensions of Hexagonal nuts



## Good Practice:

1. Never re-use nuts;
2. Tightening should be done such that 1 or 2 threads come out of the nut;
3. Washers should always be used under bolt head to prevent burr stress concentration.



# Threaded Fasteners

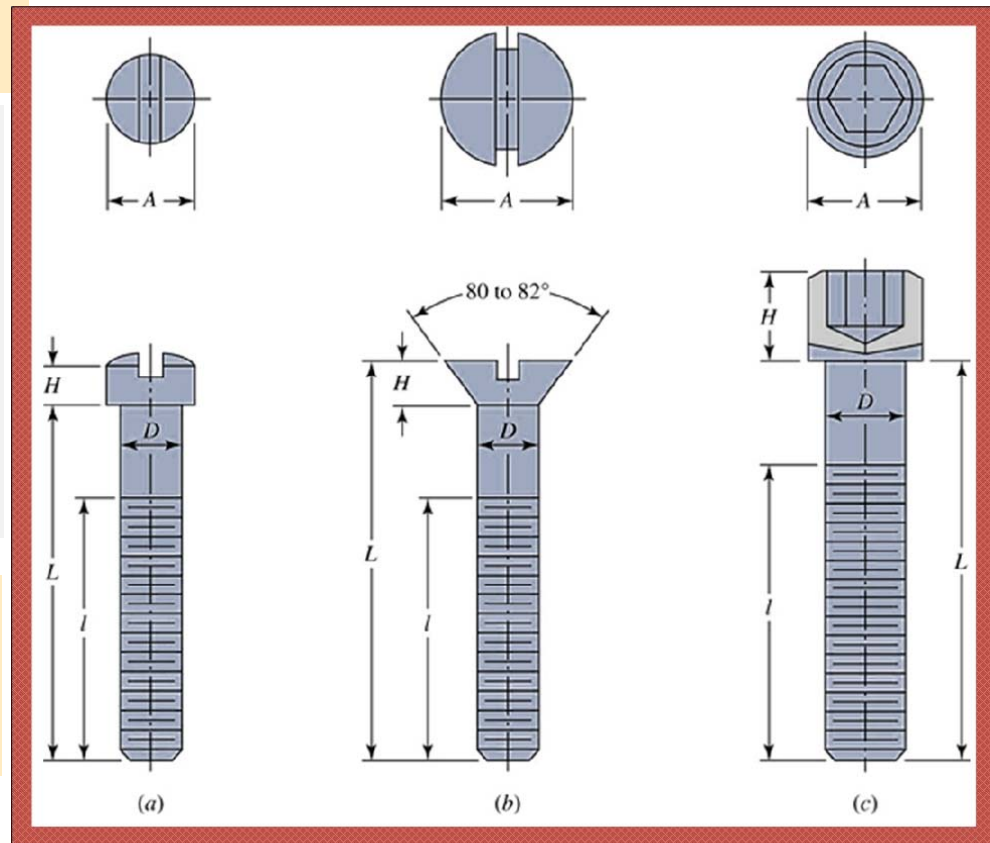
## Common Cap screws

Used for clamping members same as bolt except that 1 member should be threaded.

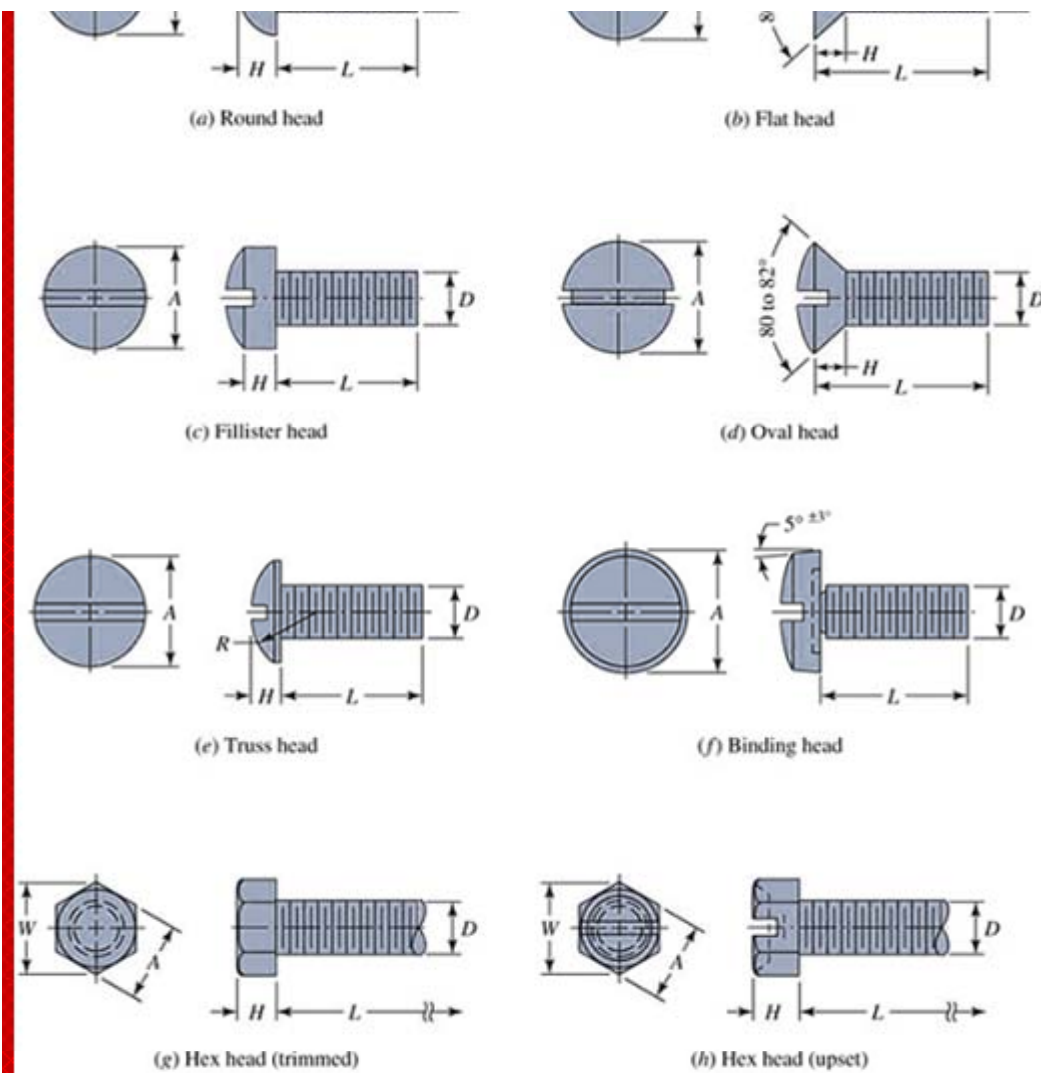
The head of a hexagon-head cap screw  $H_{cap}$  is slightly thinner than that of a hexagonal head bolt  $H_{bolt}$ .

Figure 8-10

Typical cap-screw heads: (a) fillister head; (b) flat head; (c) hexagonal socket head



# Threaded Fasteners



**Figure 8.11:**  
Types of heads used in machine screws



# Joints: Fastener Stiffness

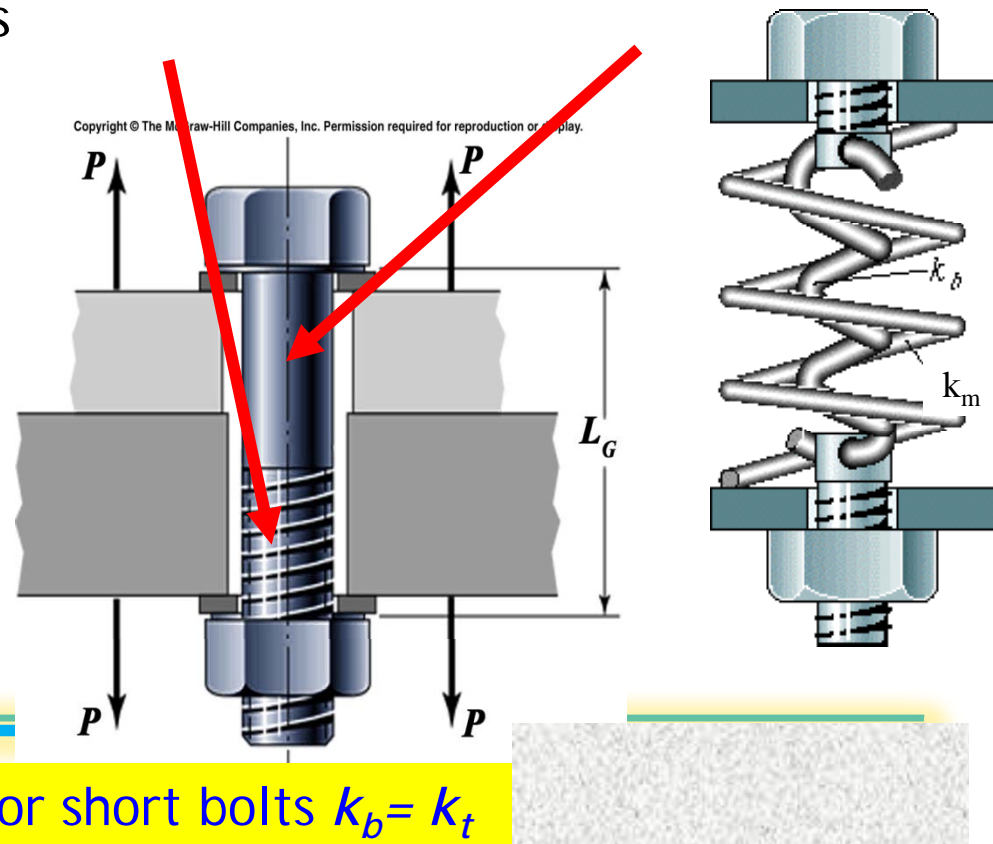
In joint under tension the members are under compression and the bolt under tension:  $k_b$  = equivalent spring constant of bolt composed of threaded ( $k_t$ ) and unthreaded ( $k_d$ ) parts acting as springs in series

$$\frac{1}{k_b} = \frac{1}{k_d} + \frac{1}{k_t}$$

$$k_b = \frac{k_d k_t}{k_d + k_t}$$

$$k_d = \frac{A_d E}{l_d} \quad k_t = \frac{A_t E}{l_t}$$

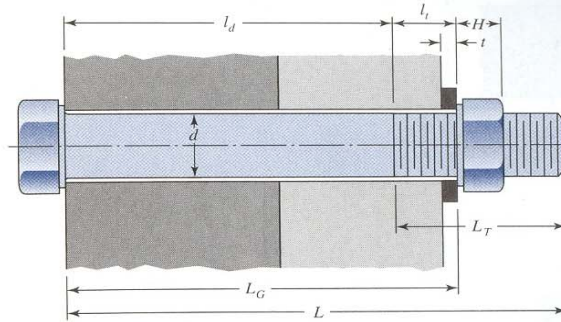
$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d}$$



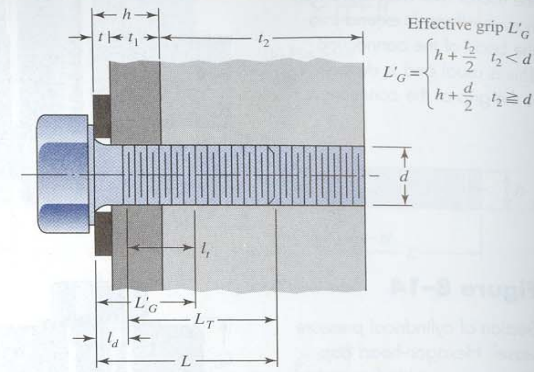


## Joints: Fastener Stiffness

To find different  
parameters use  
table 8-7



(a)



Effective grip

$$L'_G = \begin{cases} h + t_2/2, & t_2 < d \\ h + d/2, & t_2 \geq d \end{cases}$$

Given fastener diameter  $d$   
and pitch  $p$  or number of threads

Grip is thickness  $L_G$

Washer thickness from  
Table A-32 or A-33  
Threaded length  $L_T$   
Inch series:

$$L_T = \begin{cases} 2D + \frac{1}{4} \text{ in}, & L \leq 6 \text{ in} \\ 2D + \frac{1}{2} \text{ in}, & L > 6 \text{ in} \end{cases}$$

Metric series:

$$L_T = \begin{cases} 2D + 6 \text{ mm}, & L \leq 125, D \leq 48 \text{ mm} \\ 2D + 12 \text{ mm}, & 125 < L \leq 200 \text{ mm} \\ 2D + 25 \text{ mm}, & L > 200 \text{ mm} \end{cases}$$

Fastener length:  $L > L_G + H$

Length of useful unthreaded  
portion:  $l_d = L - L_T$   
Length of useful threaded  
portion:  $l_t = L_G - l_d$

Round up using Table A-17\*

Fastener length:  $L > h + 1.5$

Length of useful unthreaded  
portion:  $l_d = L - L_T$   
Length of useful threaded  
portion:  $l_t = L_G - l_d$

Area of unthreaded portion:

$$A_d = \pi d^2 / 4$$

Area of threaded portion:

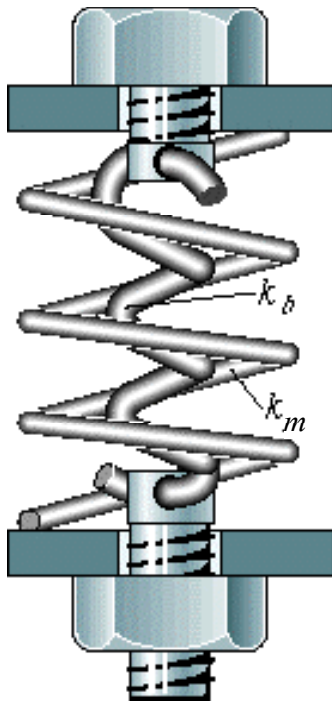
$A_t$ , Table 8-1 or 8-2

Fastener stiffness:

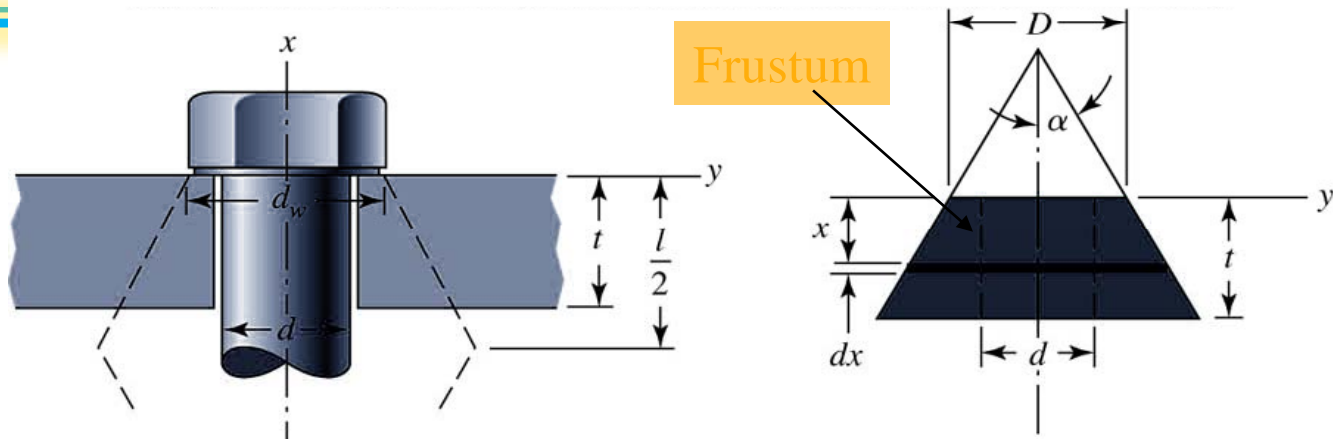
$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d}$$



# Joints: Member Stiffness



Members act as springs under compression



Compression stress distribution from experimental data

Equivalent spring constant  $k_m$

$$\frac{1}{k_m} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + \dots + \frac{1}{k_i}$$

$$d\delta = \frac{Pdx}{EA}$$

Integrating from 0 to l

$$\delta = \frac{P}{\pi E d \tan \alpha} \ln \frac{(2t \tan \alpha + D - d)(D + d)}{(2t \tan \alpha + D + d)(D - d)}$$

$$k = \frac{P}{\delta} = \frac{\pi E d \tan \alpha}{\ln \frac{(2t \tan \alpha + D - d)(D + d)}{(2t \tan \alpha + D + d)(D - d)}}$$

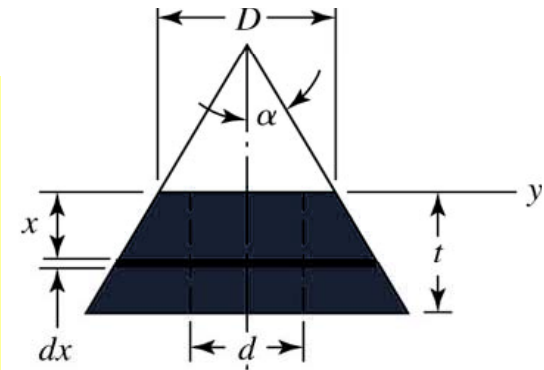




# Joints: Member Stiffness

For Members made of Aluminum, hardened steel and cast iron  $25 < a < 33$   
For  $a = 30$

$$k = \frac{0.5774 \pi E d}{\ln \frac{(1.155 t + D - d)(D + d)}{(1.155 t + D + d)(D - d)}} \quad (8 - 20)$$



If Members have same E with symmetrical frusta ( $l = 2t$ ) they act as 2 identical springs  $k_m = k/2$  For  $a = 30^\circ$  and  $D = d_w = 1.5d$

$$k_m = \frac{0.5774 \pi E d}{2 \ln \left( 5 \frac{0.5774 l + 0.5 d}{0.5774 l + 2.5 d} \right)} \quad (8 - 22)$$

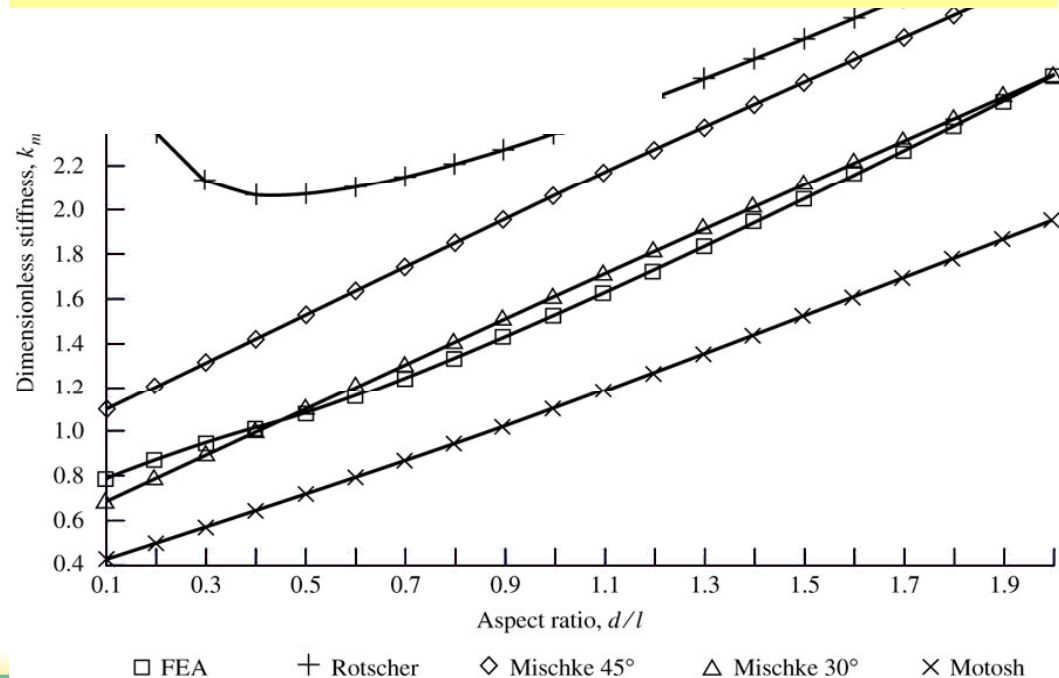


# Joints: Member Stiffness

## Other equations

From Finite element analysis results, A and B from table 8.8 for standard washer Faces and members Of same material

$$\frac{k_m}{Ed} = A \exp( Bd / l) \quad (8 - 23)$$





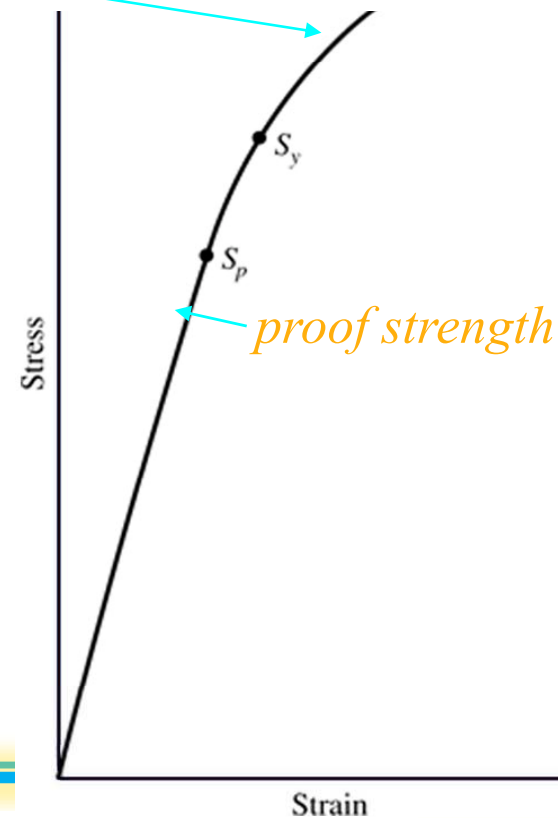
# Bolt Strength

Bolt strength is specified by *minimum proof strength*  $S_p$  or *minimum proof load*,  $F_p$  and *minimum tensile strength*,  $S_{ut}$

The SAE specifications are given in Table 8-9 bolt grades are numbered according to *minimum tensile strength*

The ASTM Specs for steel bolts (structural) are in Table 8-10. Metric Specs are in table 8-11.

If  $S_p$  not available use  $S_p = 0.85 S_y$   
 $F_p = A_t S_p$

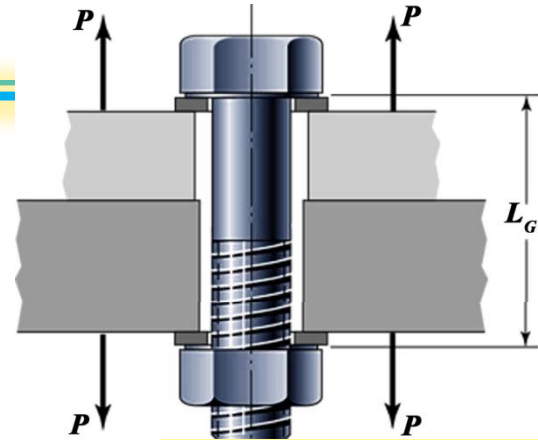


# Tension Joints

## Static Analysis

### a) External Load

*External Load  $P$  is shared by bolt and members*



Equilibrium

$$P = P_b + P_m \quad (1)$$

Compatibility

$$\delta = \delta_b = \delta_m \quad (2)$$

Relation P- $\delta$

$$\delta = \frac{P_m}{k_m} = \frac{P_b}{k_b} \quad \& \quad P_m = \frac{P_b k_m}{k_b} \quad (3)$$

$$P_b = \frac{k_b}{k_b + k_m} P = CP$$

$$P_m = \frac{k_m}{k_b + k_m} P = (1 - C)P$$

$$C = \frac{k_b}{k_b + k_m}$$

(4)

*$C$  is the stiffness constant of the joint,*

*For typical values of  $C$  see table 8-12*

*Most of external Load  $P$  is taken by members*



# Tension Joints

## Static Analysis

### d) Joint Safety

Failure of Joint occurs when:

1) Bolt yields

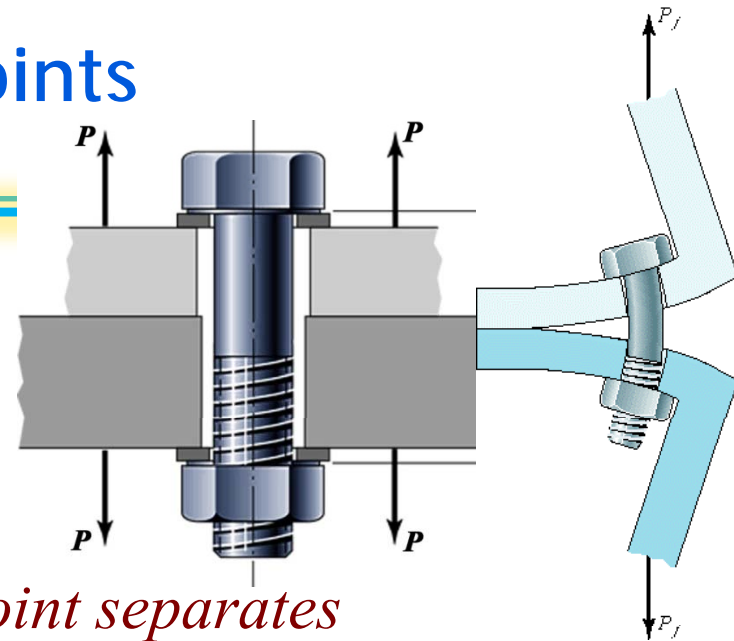
$$\sigma_b = \frac{F_b}{A_t} = \frac{CP}{A_t} + \frac{F_i}{A_t}$$

Failure starts  $\sigma_b = S_P$

$$\frac{C(nP)}{A_t} + \frac{F_i}{A_t} = S_P$$

Load factor  $n = \frac{S_P A_t - F_i}{CP}$

For N Bolts  $n = \frac{S_P A_t - F_i}{CP / N}$



or 2) Joint separates

Let  $P_0$  be external load causing separation  $F_m = 0$

$$\left. \begin{aligned} F_m &= (1 - C)P - F_i \\ F_m &= 0 \end{aligned} \right\} \Rightarrow 0 = (1 - C)P_0 - F_i$$

$$n = \frac{F_i}{P(1 - C)}$$

For n bolts  $n = \frac{F_i}{P / N (1 - C)}$

# Tension Joints

## Static Analysis

### e) Gasketed Joints

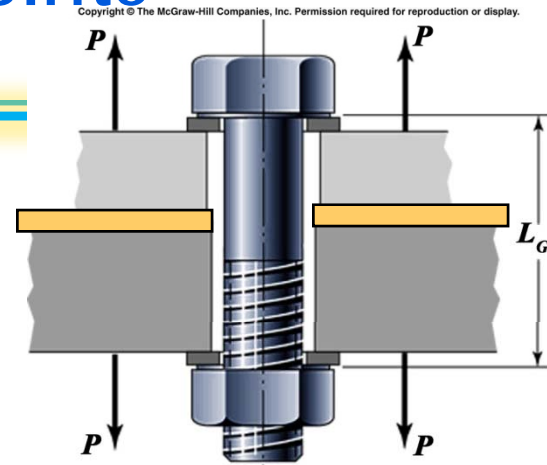
If a full gasket is present in joint  
The gasket pressure  $p$  is:

$$p = - \frac{F_m}{A_g / N}$$

With load factor  $n$

$$F_m = (1 - C)nP - F_i$$

$$p = [F_i - nP(1 - C)] \frac{N}{A_g}$$



To maintain uniformity of pressure adjacent bolts should not be placed more than 6 nominal diameters apart on bolt circle. To maintain wrench clearance bolts should be placed at least 3  $d$  apart

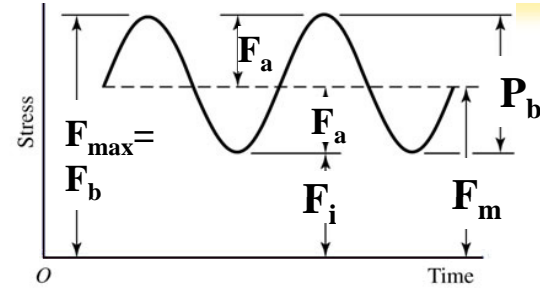
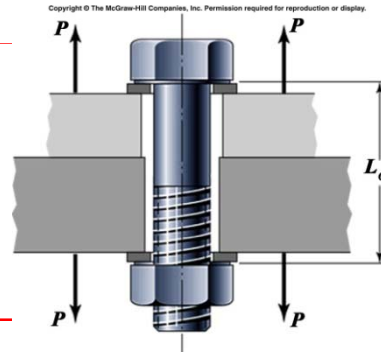
$$3 \leq \frac{\pi D_b}{Nd} \leq 6$$

$D_b$  is the diameter of the bolt circle

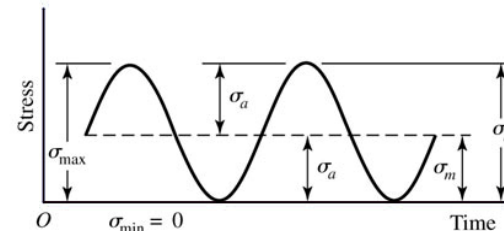


# Tension Joints

## Fatigue Analysis



(d)



*In general, bolted joints are subject to  $0-P_{max}$ , e.g. pressure vessels, flanges, pipes, ...*

$$F_{ba} = \frac{F_b - F_i}{2} \Rightarrow \sigma_{ba} = \frac{F_b - F_i}{2A_t} = \frac{(CP + F_i) - F_i}{2A_t} = \frac{CP}{2A_t}$$

$$F_{bm} = \frac{F_b + F_i}{2} \Rightarrow \sigma_{bm} = \frac{F_b + F_i}{2A_t} = \frac{(CP + F_i) + F_i}{2A_t} = \frac{CP}{2A_t} + \frac{F_i}{A_t} = \sigma_{ba} + \sigma_i$$

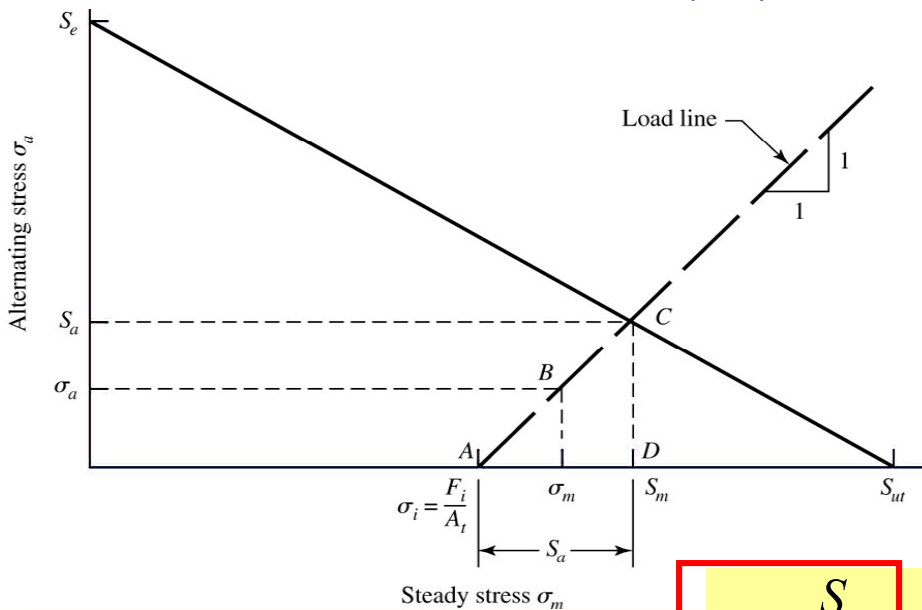
(c)

(f)

# Tension Joints

## Fatigue Analysis

*High Preload is especially important in fatigue.  $\sigma_i$  is a constant the load line at  $F_i/A_t$  has a unit slope,  $r=1.0$*



To find  $s_a$  use Goodman

$$\frac{S_a}{S_e} + \frac{S_m}{S_u} = 1 \quad \text{for conservative assessment of } n:$$

$$S_a = \frac{S_e(S_{ut} - \sigma_i)}{S_{ut} + S_e}$$

or Gerber Eq. 8.42

or ASME-elliptic. Eq. 8.43

Safety factor (using Goodman)

$$n_f = \frac{S_a}{\sigma_i} = \frac{2S_e(S_{ut}A_t - F_i)}{CP(S_{ut} + S_e)} \quad \text{with } \sigma_i = \frac{F_i}{A_t}$$

# Tension Joints

## Fatigue Analysis

*In case of **cut threads** use the method described in chapter 7 with  $K_f$  values of table 8-16.*

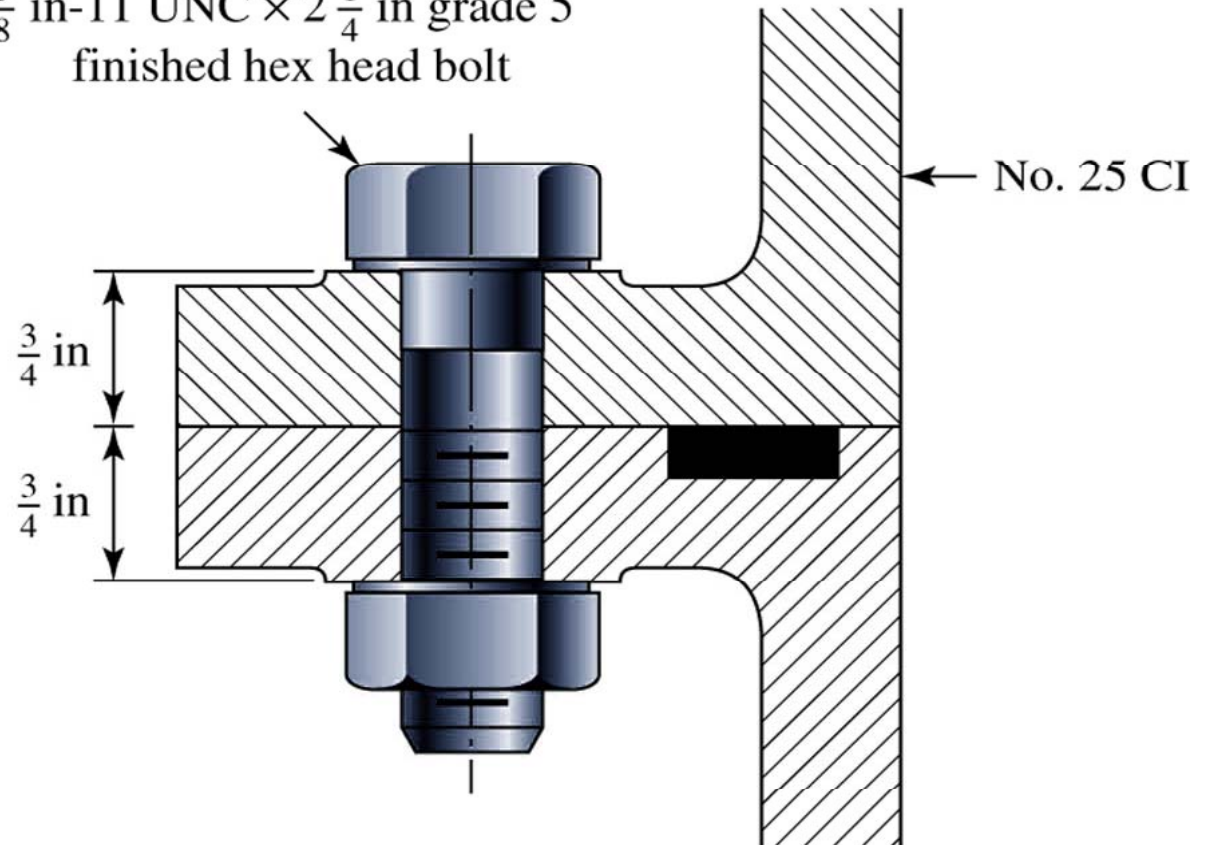
*The fully corrected endurance limit for **rolled threads** is given in table 8-17*

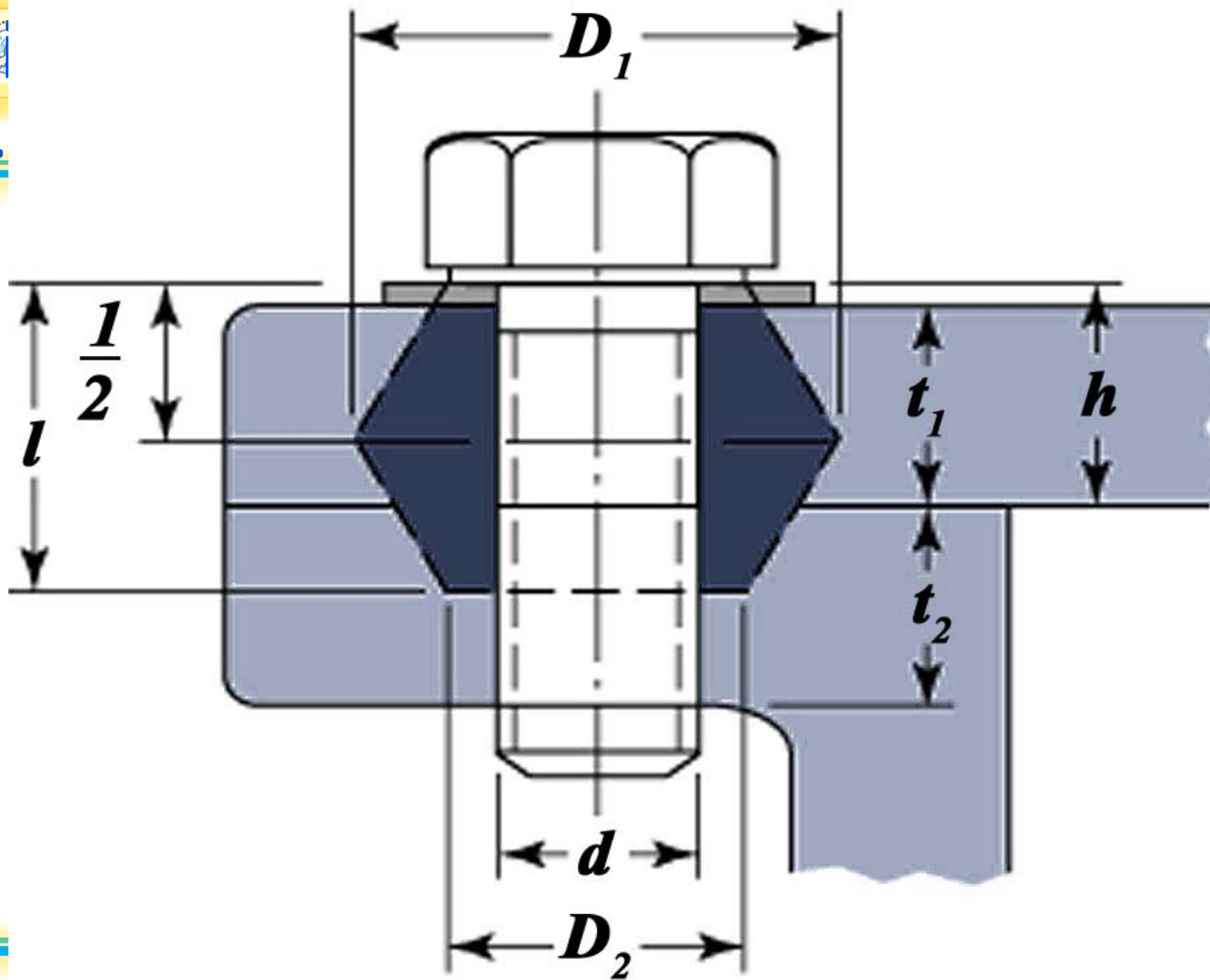


# Fig. 8.10

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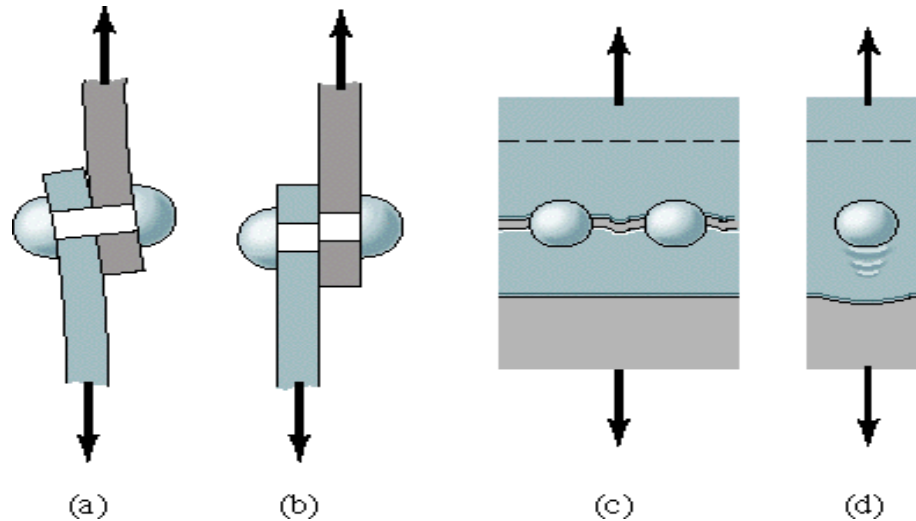
$\frac{5}{8}$  in-11 UNC  $\times$   $2\frac{1}{4}$  in grade 5  
finished hex head bolt







# Failure Modes of Riveted Fasteners Under Shear



Failure modes due to shear loading of riveted fasteners. (a) Bending of member; (b) shear of rivet; (c) tensile failure of member; (e) bearing of rivet on member or bearing of member on rivet.



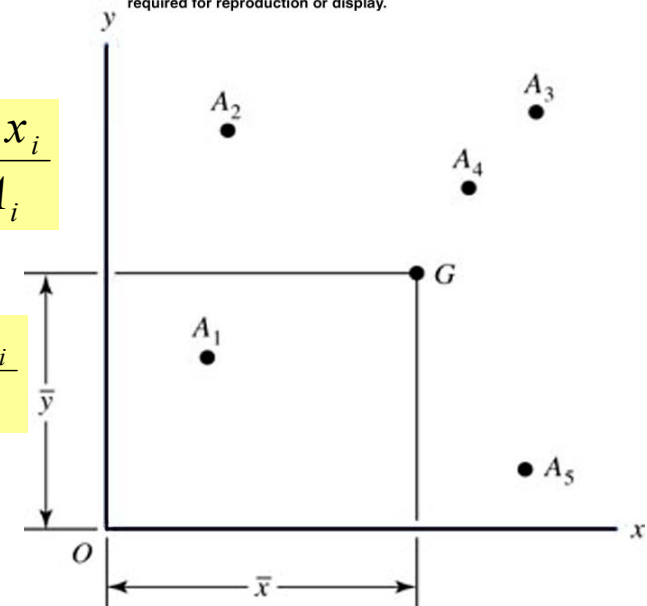
# Shear Joints

## Centroid of pins, rivets or bolts

$$x' = \frac{A_1x_1 + A_2x_2 + A_3x_3 + A_4x_4 + A_5x_5}{A_1 + A_2 + A_3 + A_4 + A_5} = \frac{\sum A_i x_i}{\sum A_i}$$

$$y' = \frac{A_1y_1 + A_2y_2 + A_3y_3 + A_4y_4 + A_5y_5}{A_1 + A_2 + A_3 + A_4 + A_5} = \frac{\sum A_i y_i}{\sum A_i}$$

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# Shear Joints

## Shear of pins, rivets bolts due to eccentric loading

V & M statically indeterminate problem

4 steps (assuming same diameter bolts, load shared equally)

1) Direct load  $F'$

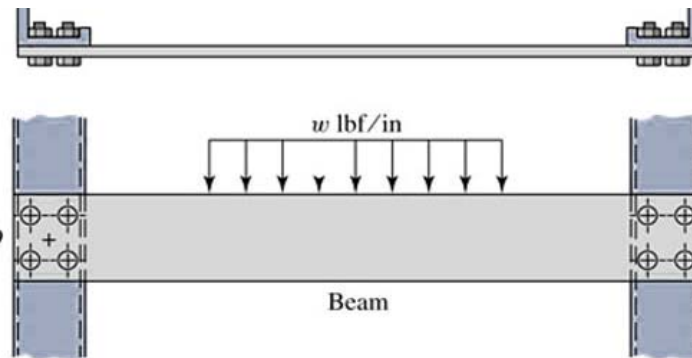
Primary Shear

$$F' = \frac{V}{N}$$

2) Centroid

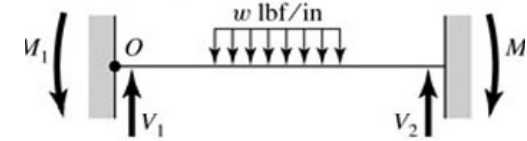
$$x' = \frac{\sum A_i x_i}{\sum A_i}$$

$$y' = \frac{\sum A_i y_i}{\sum A_i}$$

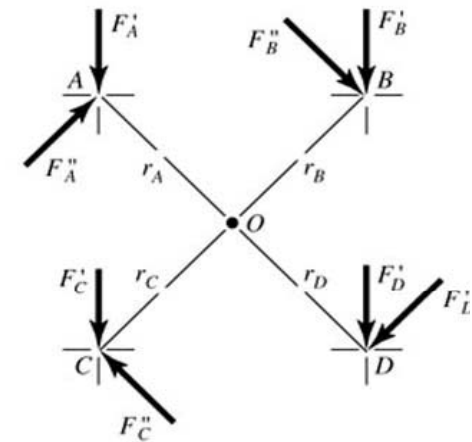


(a)

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(b)



(c)





# Shear Joints

## Shear of pins, rivets bolts due to eccentric loading

### 3) Moment load M

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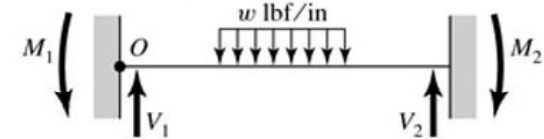
#### Secondary Load

$$M_1 = F_A'' r_A + F_B'' r_B + F_C'' r_C + F_D'' r_D$$

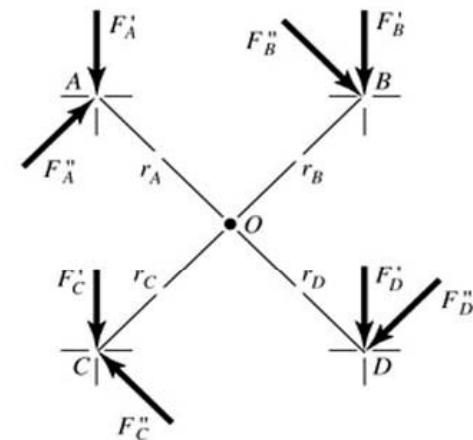
The force taken by each bolt is proportional to its radial distance from the centroid

$$\frac{F_A''}{r_A} = \frac{F_B''}{r_B} = \frac{F_C''}{r_C} = \frac{F_D''}{r_D}$$

$$F_n'' = \frac{M_1 r_n}{r_A^2 + r_B^2 + r_C^2 + \dots}$$



(b)



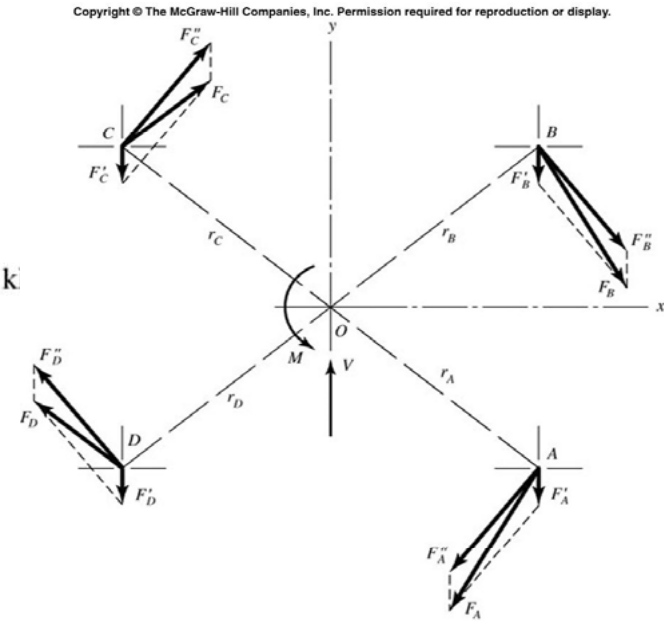
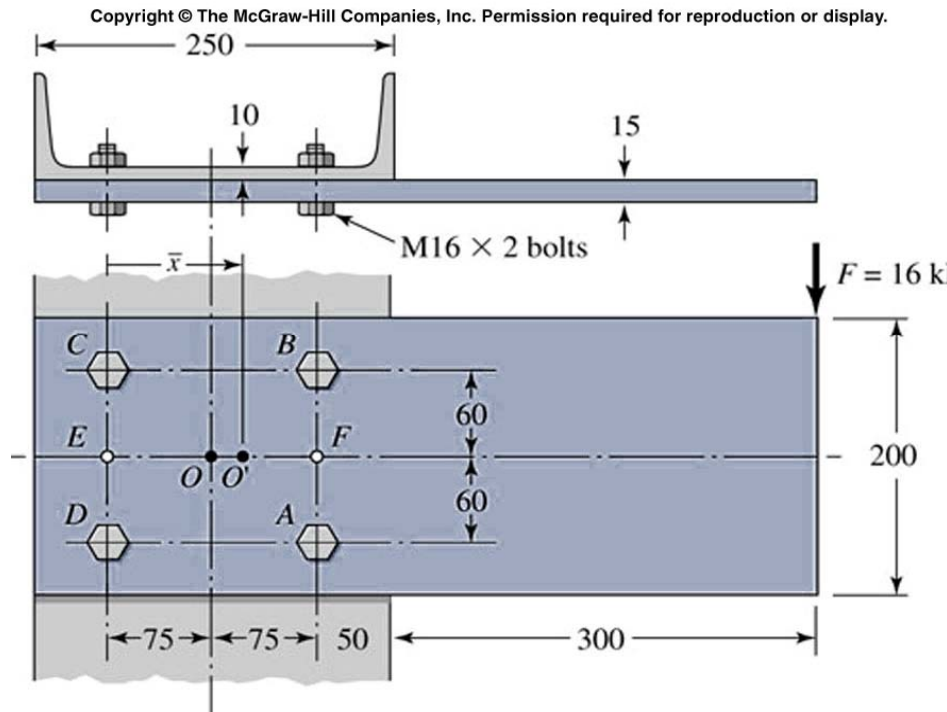
(c)

### 4) Add Vectorially the direct

and moment loads *If bolts are not same size only bolts with max. R should be considered*

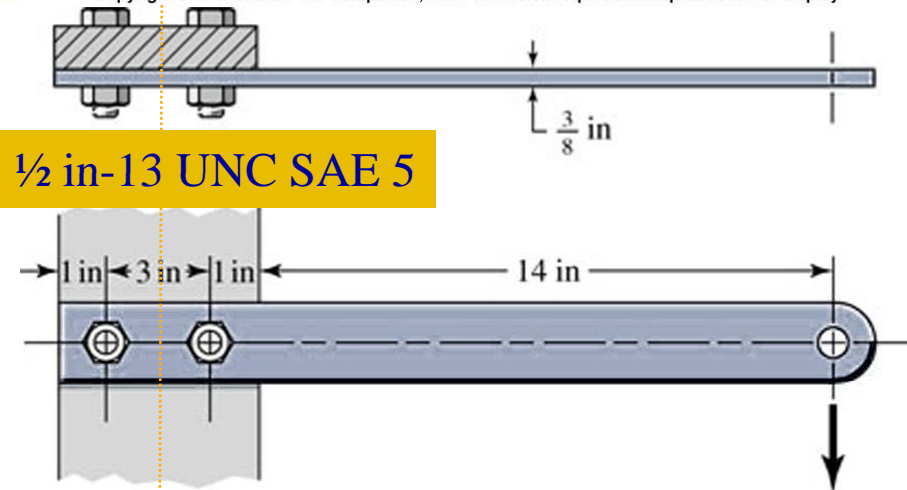


# See Examples 8.6 and 8.7



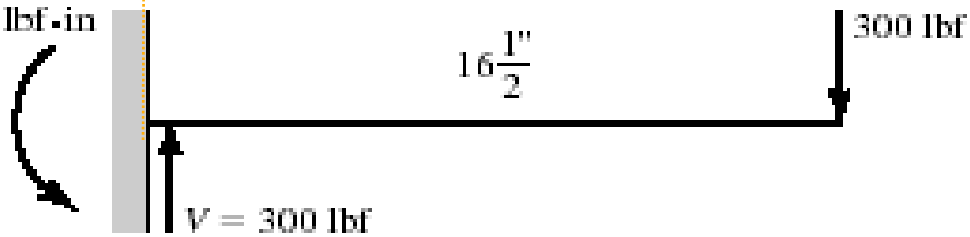
# Problem 8-50

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$\frac{1}{2}$  in-13 UNC SAE 5

$$M = 16.5(300) \\ = 4950 \text{ lbf}\cdot\text{in}$$

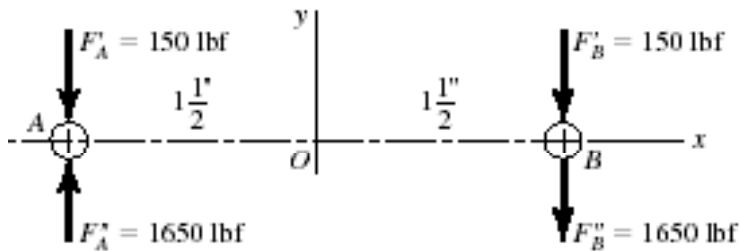




## Problem 8-50

1) Direct load  $F'$

$$F' = \frac{V}{N} = \frac{300}{2} = 150 \text{ lb}$$



2) Centroid is at O

3) Secondary Load

$$F_A'' = F_B'' = \frac{M_1 r_A}{r_A^2 + r_B^2} = \frac{4950 \times 1.5}{1.5^2 + 1.5^2} = 1650 \text{ lb}$$

4) Add Vectorially the direct and moment loads

$$F_A = 1650 - 150 = 1500 \text{ lb}$$

$$F_B = -1650 - 150 = -1800 \text{ lb}$$

Shear of bolt:

$$A_s = \frac{\pi}{4} (0.5)^2 = 0.1963 \text{ in}^2$$

$$\tau = \frac{F}{A} = \frac{1800}{0.1963} = 9170 \text{ psi}$$

$$S_{sy} = 0.577(92) = 53.08 \text{ kpsi}$$

$$n = \frac{53.08}{9.17} = 5.79 \text{ Ans.}$$

$$\text{Is } A_s = A_t? \quad L_T = 2d + 1/4 = 1.25''$$

$$H_{\text{nut}} + 2p + 3/8 = 7/16 + 2(1/13) + 3/8 = 0.97'' < 1.25''$$

So  $A_s = A_t = 0.1419 \text{ in}^2$  and  $t_p = 12.7 \text{ Kpsi}$ :

$$n = 4.2$$



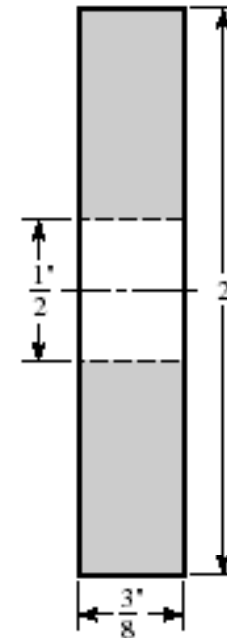
## Problem 8-50

*Bearing on bolt:*

$$A_b = \frac{1}{2} \left( \frac{3}{8} \right) = 0.1875 \text{ in}^2$$

$$\sigma = -\frac{F}{A} = -\frac{1800}{0.1875} = -9600 \text{ psi}$$

$$n = \frac{92}{9.6} = 9.58 \text{ Ans.}$$



*Bearing on members:*  $S_y = 54 \text{ kpsi}$ ,  $n = \frac{54}{9.6} = 5.63 \text{ Ans.}$



## Problem 8-50

*Strength of members:* Considering the right-hand bolt

$$M = 300(15) = 4500 \text{ lbf} \cdot \text{in}$$

$$I = \frac{0.375(2)^3}{12} - \frac{0.375(0.5)^3}{12} = 0.246 \text{ in}^4$$

$$\sigma = \frac{Mc}{I} = \frac{4500(1)}{0.246} = 18\,300 \text{ psi}$$

$$n = \frac{54(10)^3}{18\,300} = 2.95 \quad \text{Ans.}$$

