MECHANICAL PROPERTIES OF MATERIALS

- Engineers are primarily concerned with the development and design of machines, structures etc.

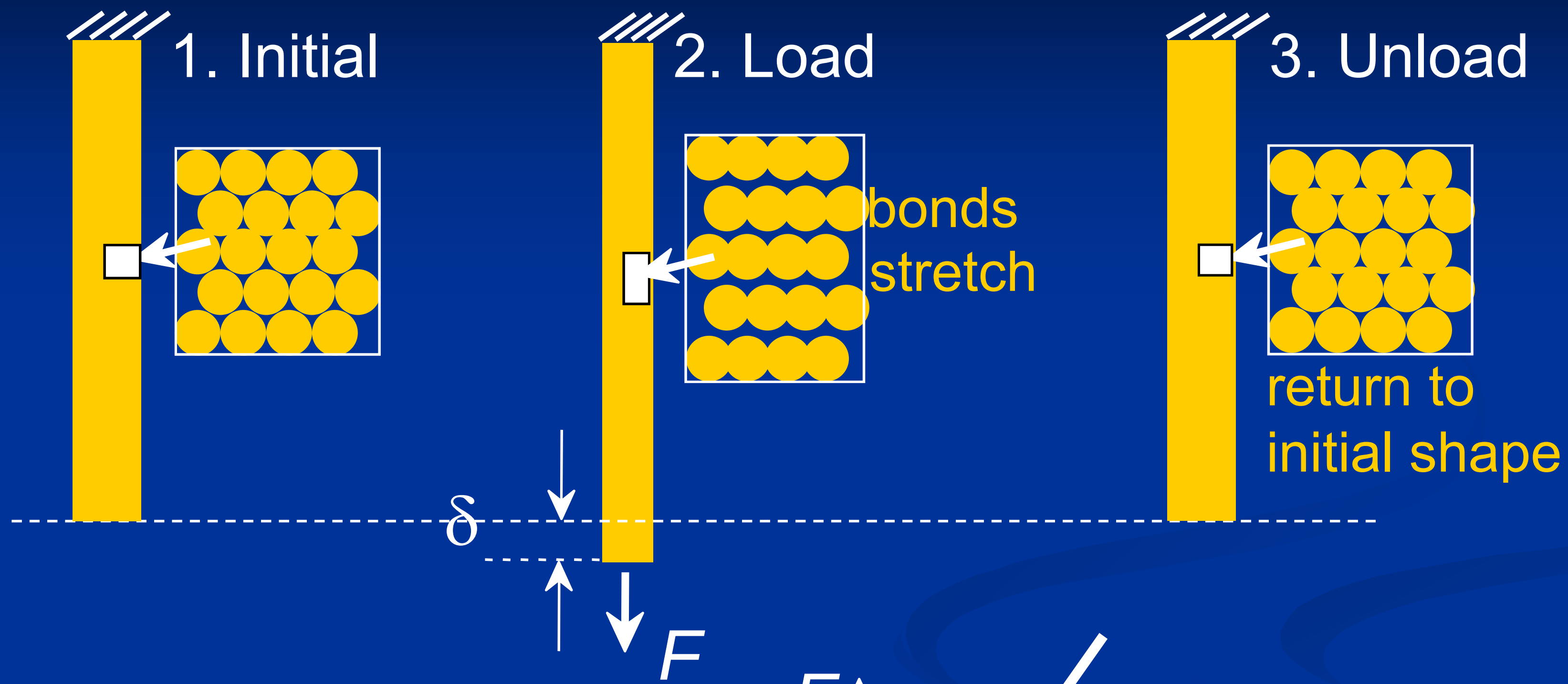
- These products are often subjected to forces/ deformations, resulting in stresses/strains, the properties of materials under the action of forces and deformations becomes an important engineering consideration.

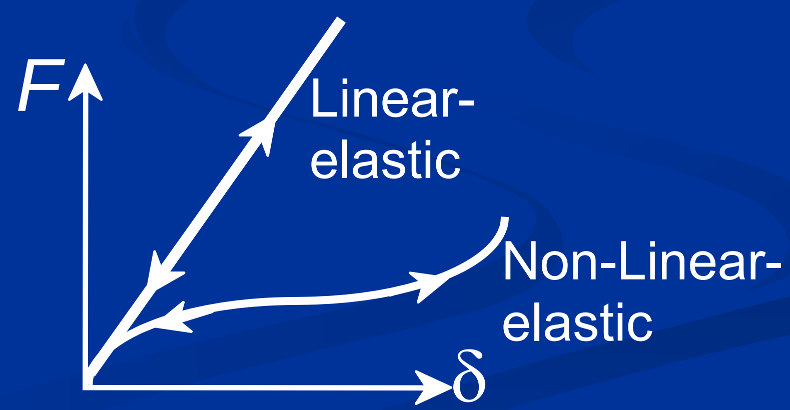
- The properties of materials when subjected to stresses and strains are called "mechanical properties". In other words the properties that determine the behavior of engineering mats under applied forces are called "mechanical properties".

- The response of a material to applied forces depends on the type and nature of the bond and the structural arrangement of atoms, molecules or ions.

- Basic deformation types for load carrying materials are:
  1. Elastic deformation (deformations are instantaneously recoverable)
  2. Plastic deformation (non-recoverable)
  3. Viscous deformation (time dependent deformation)

Elastic Deformation

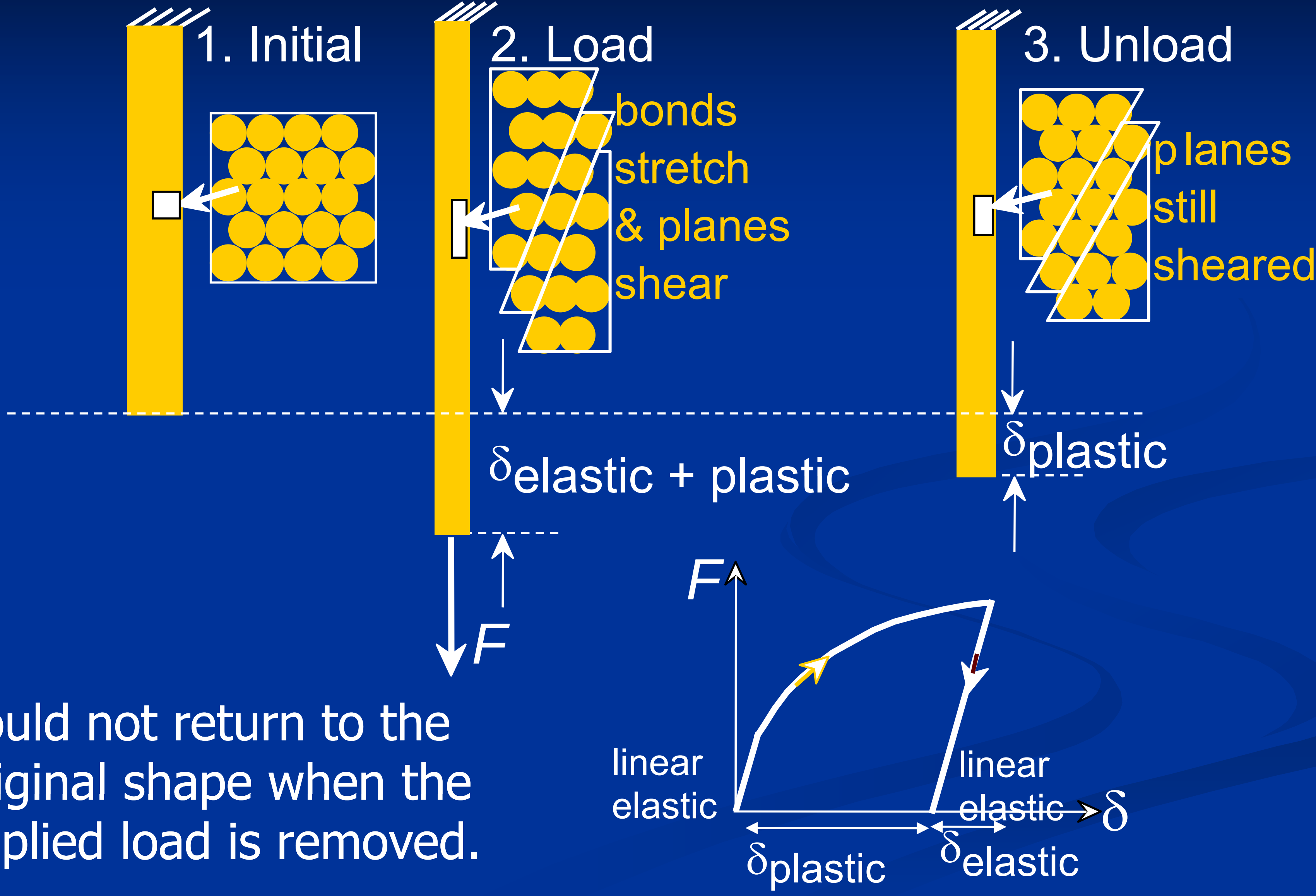




Return to the original shape when the applied load is removed.

Elastic means **reversible**!

Plastic Deformation



Could not return to the original shape when the applied load is removed.

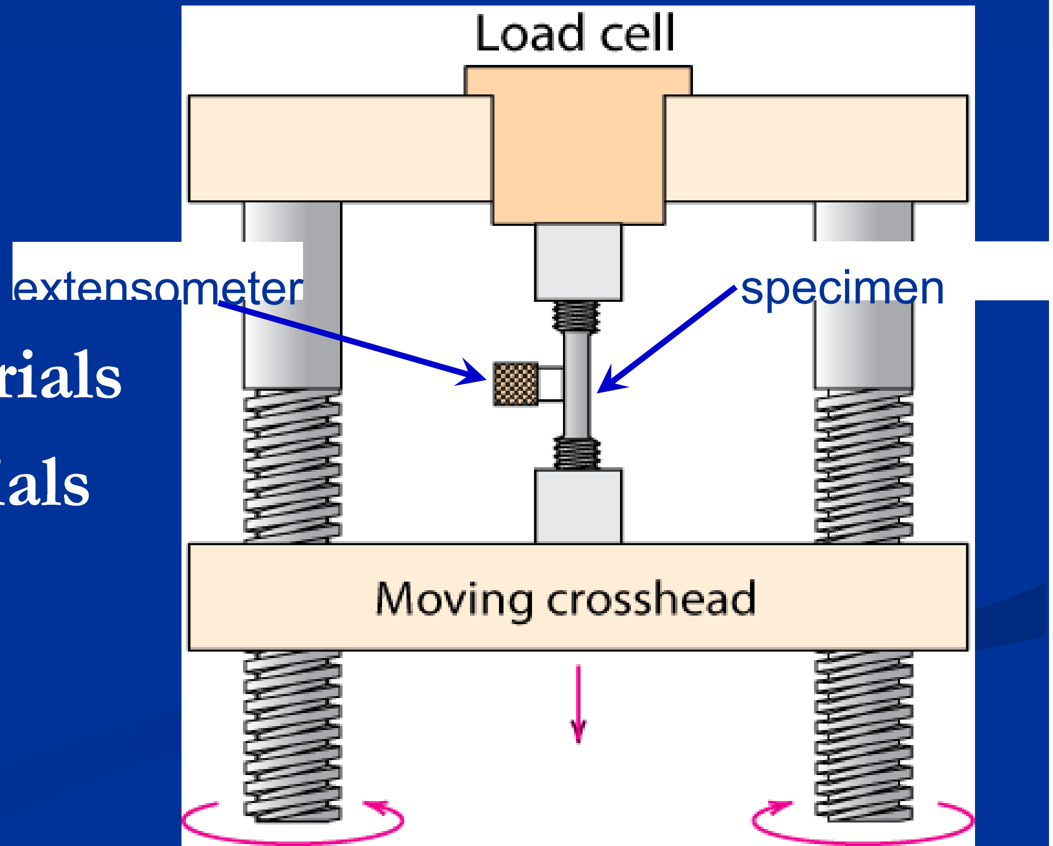
Plastic means **permanent**!

Viscous Deformation

- Plastic deformations in noncrystalline solids (as well as liquids) occurs by a viscous flow mechanism. Usually attributed to fluids. But solids may also behave like viscous materials under high temperature and pressure.
- Viscous materials deform steadily under stress.
- Deformations are time dependent.

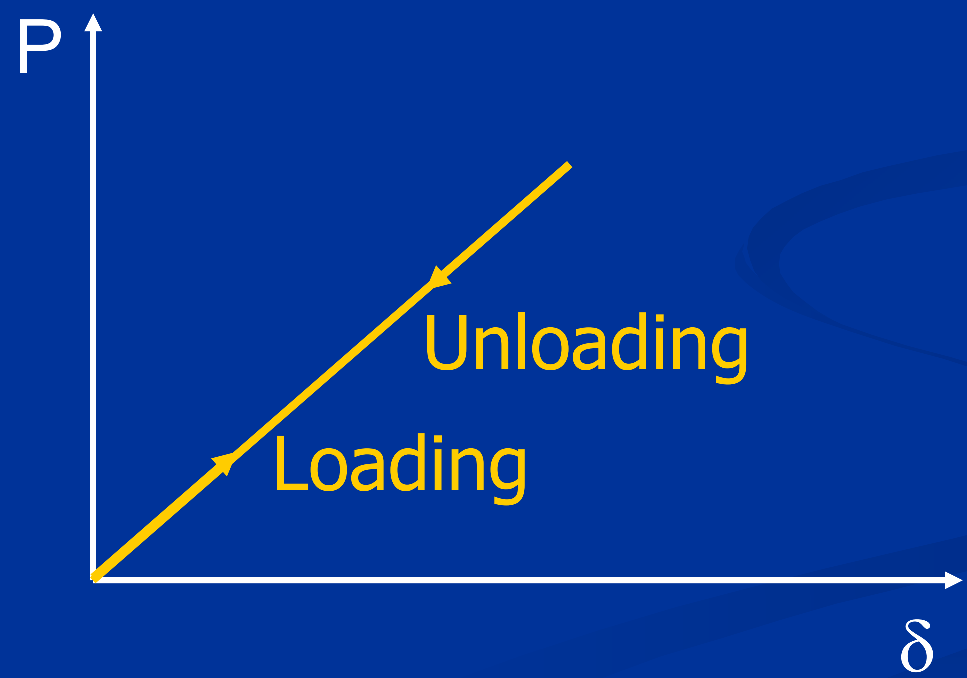
Based on the abovementioned deformation characteristics, several material idealizations could be made. Such as:

1. **Elastic Materials**
2. **Plastic Materials**
3. **Elastoplastic Materials**
4. **Viscoelastic Materials**



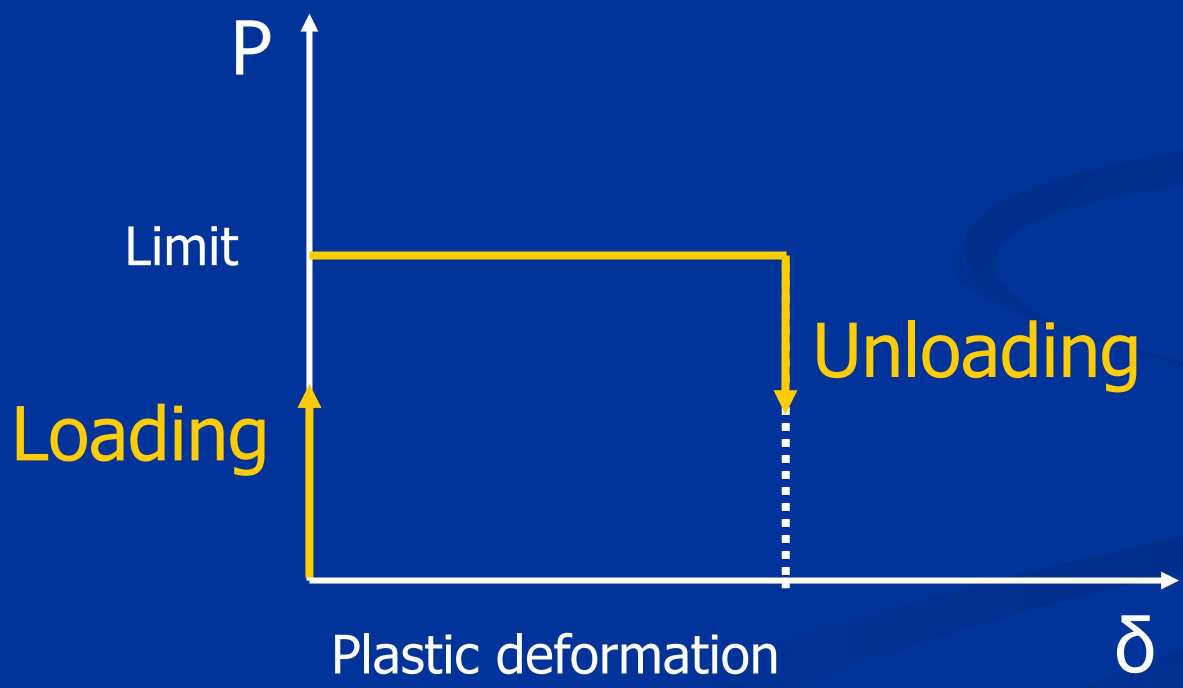
1. Elastic Materials

Return to the their original shape when the applied load is removed.



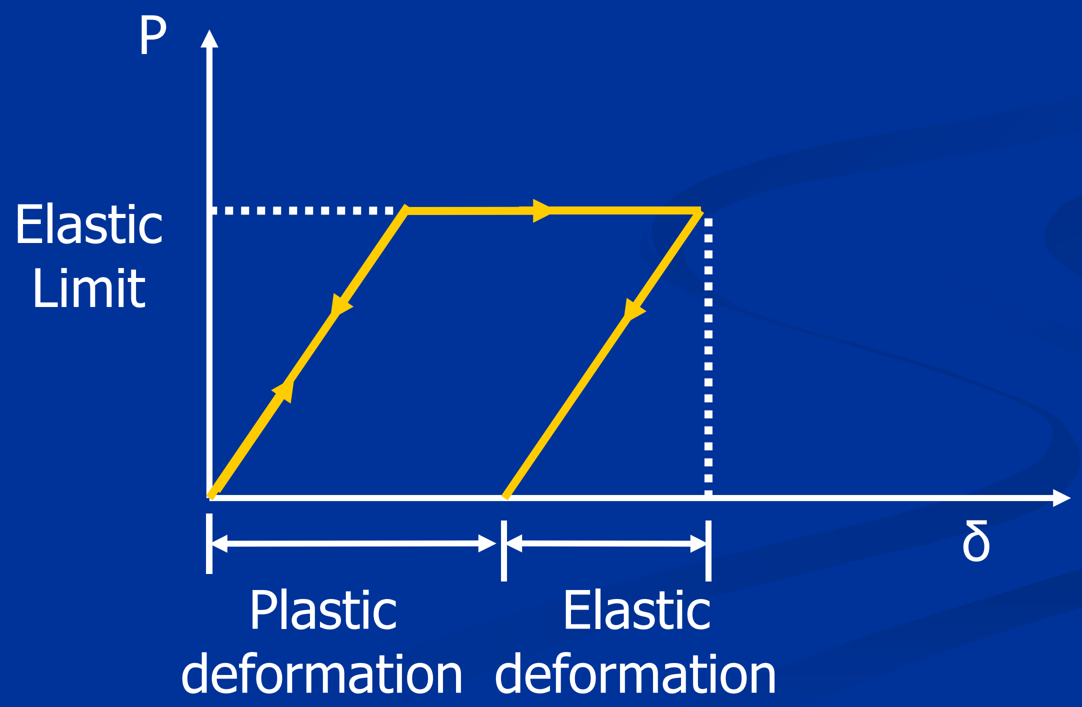
2. Plastic Materials

No deformation is observed up to a certain limit. Once the load passes this limit, permanent deformartions are observed.



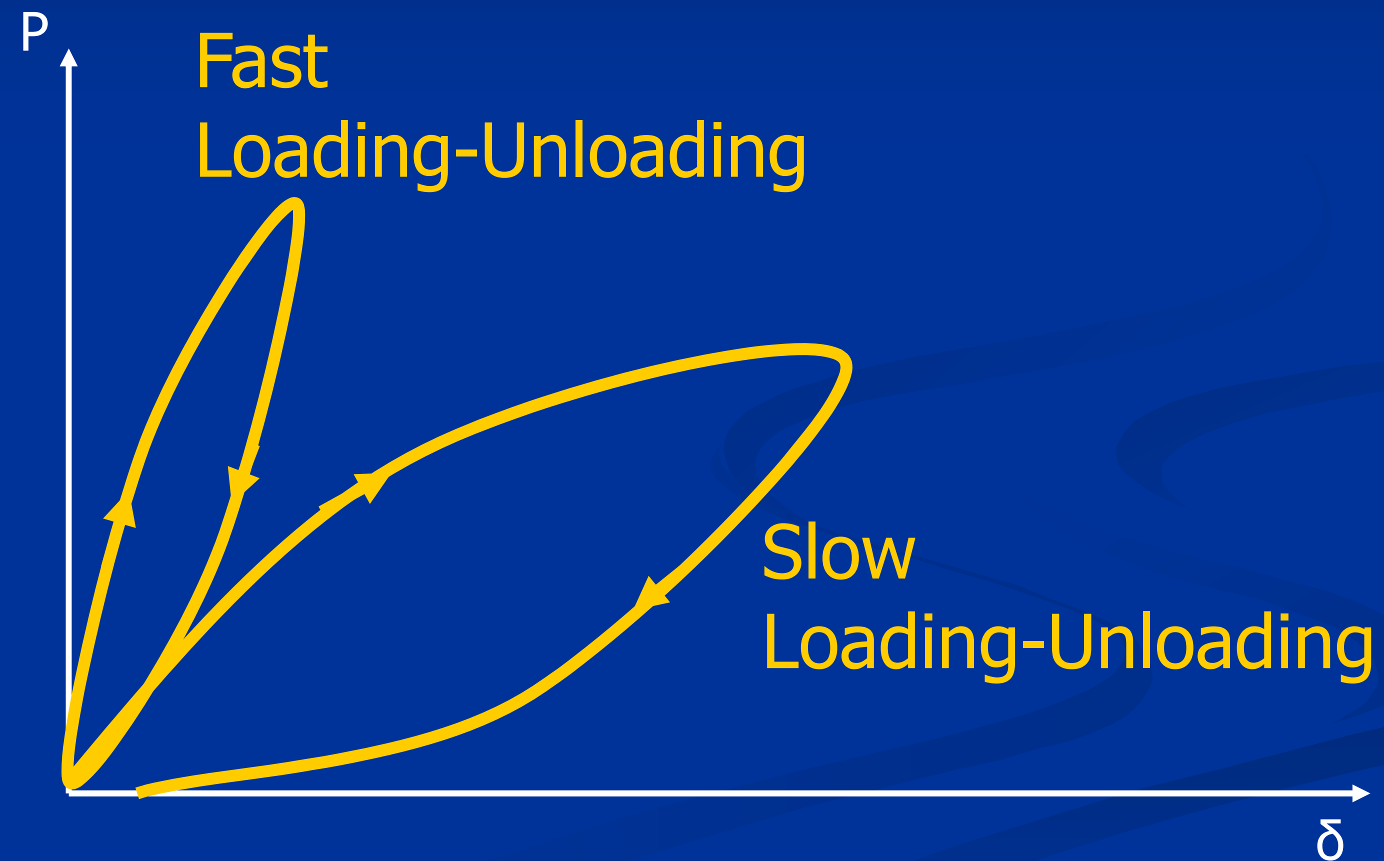
3. Elastoplastic Materials

Up to a limit shows elastic properties. Within this limit if the load is removed, returns to its original shape. If the load passes the limit, plastic deformations are observed.



4. Viscoelastic Material

Deformations are time-dependent.



ISOTROPIC and ANISTROPIC Materials

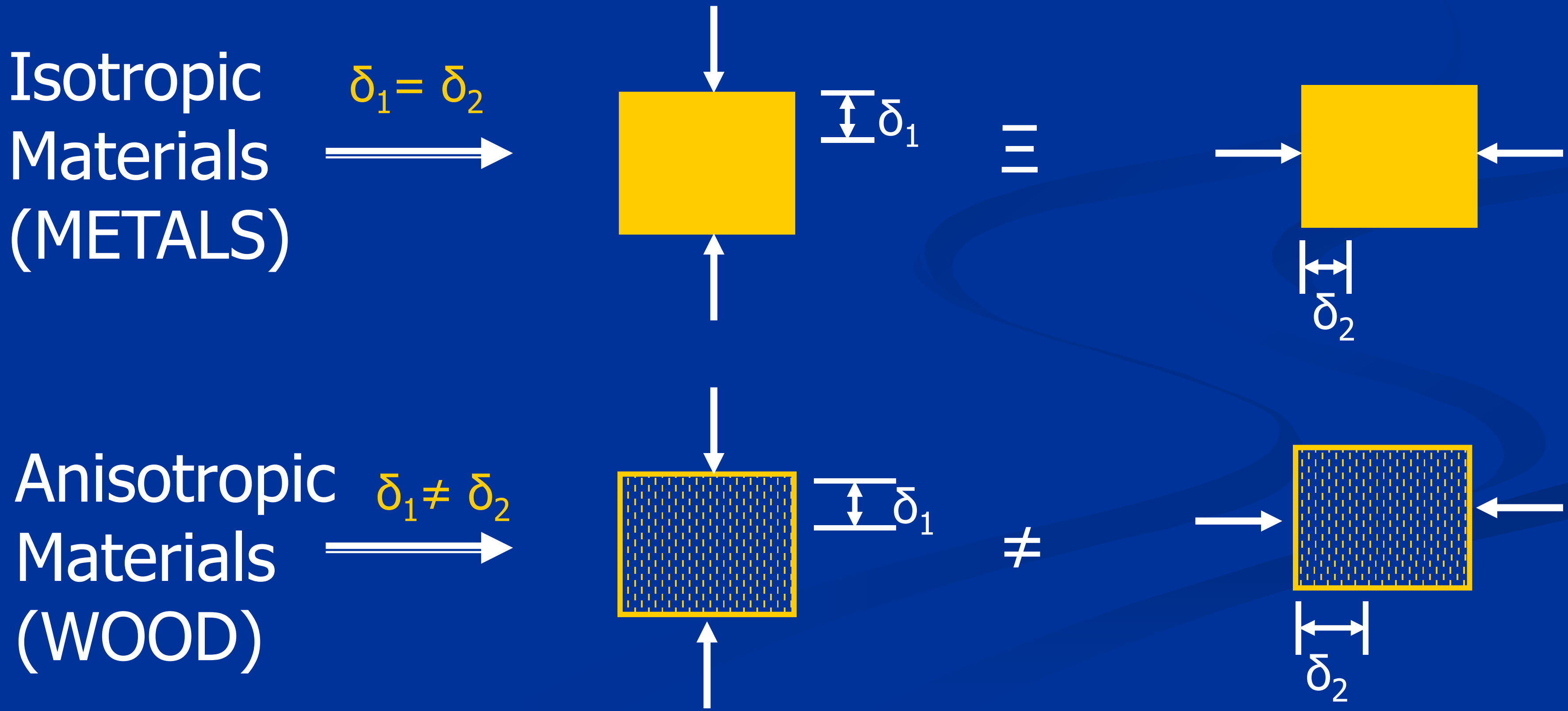
- The physical properties of some substances depend on the crystallographic direction in which the measurements are taken.
- For example, the elastic modulus, the electrical conductivity, and index of refraction may have different values in the [100] and [111] directions.
- This directionality of properties is termed as anisotropy, and it is associated with the variance of atomic or ionic spacing with crystallographic direction.
- Substances in which the measured properties are independent of the direction of measurement are called isotropic.

Table 3.3 Modulus of Elasticity Values for Several Metals at Various Crystallographic Orientations

	Modulus of Elasticity (GPa)		
Metal	[*100*]	[*110*]	[*111*]
Aluminum	63.7	72.6	76.1
Copper	66.7	130.3	191.1
Iron	125.0	210.5	272.7
Tungsten	384.6	384.6	384.6

Source: R. W. Hertzberg, *Deformation and Fracture Mechanics of Engineering Materials*, 3rd edition.

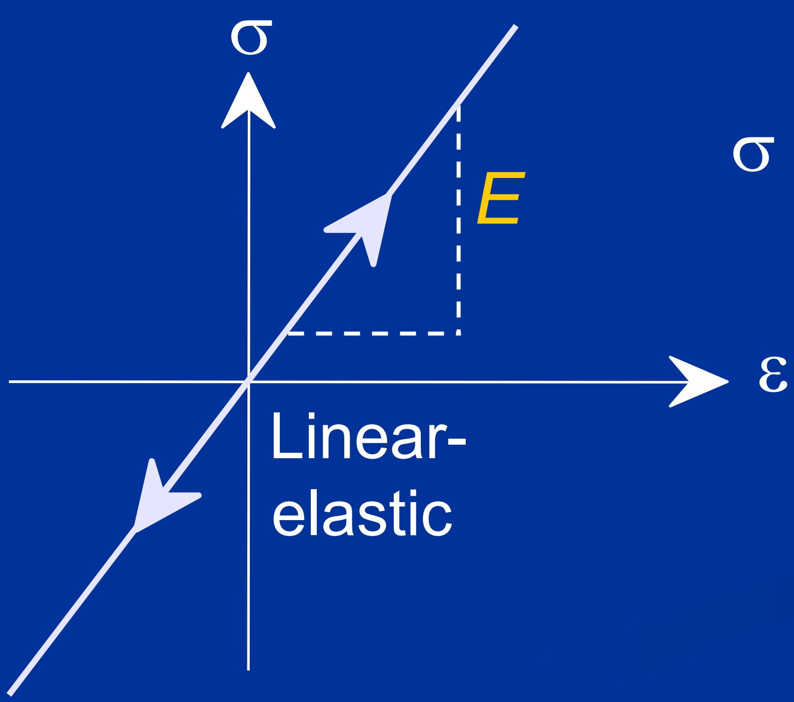
- Isotropic materials have the same mechanical properties in all directions.
- Anisotropic materials show different behavior in different directions.



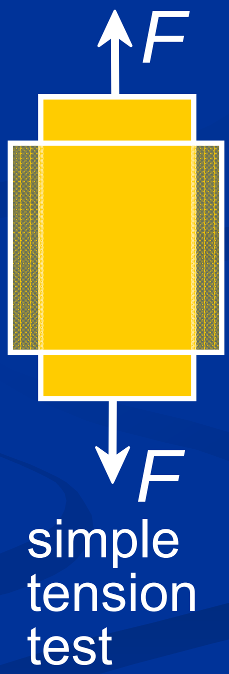
Hooke's Law

- **Hooke's Law:** For elastic materials, stress is linearly proportional to strain and is independent of time.

- **Modulus of Elasticity, *E*:**



$$\sigma = E\,\varepsilon$$



- For an anisotropic material, the linear equations between stress and strain components will be given by the following six equations known as the "Generalized Form of Hooke's law".

$$\sigma_{xx} = C_{11}\varepsilon_{xx}+C_{12}\varepsilon_{yy}+C_{13}\varepsilon_{zz}+C_{14}\gamma_{xy}+C_{15}\gamma_{xz}+C_{16}\gamma_{yz}$$

$$\sigma_{yy} = C_{21}\varepsilon_{xx} +C_{22}\varepsilon_{yy}+C_{33}\varepsilon_{zz}+\ldots\ldots\ldots\ldots$$

$$\sigma_{zz} = C_{31}\varepsilon_{xx} + C_{32}\varepsilon_{yy}+\ldots\ldots\ldots\ldots$$

$$\tau_{xy} = C_{41}\varepsilon_{xx} +\ldots\ldots\ldots\ldots$$

$$\tau_{xz} = C_{51}\varepsilon_{xx} +\ldots\ldots\ldots\ldots$$

$$\tau_{yz} = C_{61}\varepsilon_{xx} +\ldots\ldots\ldots\ldots$$

❖The six equations of Generalized Hooke's Law can be written in matrix form:

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{yz} \\ \sigma_{zx} \\ \sigma_{xy} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{41} & C_{42} & C_{43} & C_{44} & C_{45} & C_{46} \\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & C_{56} \\ C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \varepsilon_{yz} \\ \varepsilon_{zx} \\ \varepsilon_{xy} \end{bmatrix}$$

Stresses

Elastic constants

Strains

- Stress-strain relationships such as these are known as **constitutive relations**.

- It can be shown that $C_{12}=C_{21}$, $C_{31}=C_{13}$… Therefore, the number of elastic constants reduce to 21 for an anisotropic material.

- The number of independent elastic constants reduce to 2 for isotropic materials. In fact, there are 4 constants (E, ν, K, G) 2 of which are independent.

Elastic constants

- For an isotropic material the Generalized Hooke's Law yields:

$$\varepsilon_{xx} = [\sigma_{xx} - \nu(\sigma_{yy} + \sigma_{zz})]/E$$

$$\varepsilon_{yy} = [\sigma_{yy} - \nu(\sigma_{zz} + \sigma_{xx})]/E$$

$$\varepsilon_{zz} = [\sigma_{zz} - \nu(\sigma_{xx} + \sigma_{yy})]/E$$

$$\gamma_{xy} = \tau_{xy}/G$$

$$\gamma_{yz} = \tau_{yz}/G$$

$$\gamma_{zx} = \tau_{zx}/G$$

E, ν and G are known as elastic constants.

ISOTROPIC MATERIAL in UNIAXIAL TENSION

$$\begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \varepsilon_{yz} \\ \varepsilon_{zx} \\ \varepsilon_{xy} \end{bmatrix} = \frac{1}{E} \begin{bmatrix} 1 & -\nu & -\nu & 0 & 0 & 0 \\ -\nu & 1 & -\nu & 0 & 0 & 0 \\ -\nu & -\nu & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1+\nu & 0 & 0 \\ 0 & 0 & 0 & 0 & 1+\nu & 0 \\ 0 & 0 & 0 & 0 & 0 & 1+\nu \end{bmatrix} \begin{bmatrix} \sigma_{xx} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

ISOTROPIC MATERIAL in PURE SHEAR

$$\begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \varepsilon_{yz} \\ \varepsilon_{zx} \\ \varepsilon_{xy} \end{bmatrix} = \frac{1}{E} \begin{bmatrix} 1 & -\nu & -\nu & 0 & 0 & 0 \\ -\nu & 1 & -\nu & 0 & 0 & 0 \\ -\nu & -\nu & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1+\nu & 0 & 0 \\ 0 & 0 & 0 & 0 & 1+\nu & 0 \\ 0 & 0 & 0 & 0 & 0 & 1+\nu \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \tau_{xy} \end{bmatrix}$$