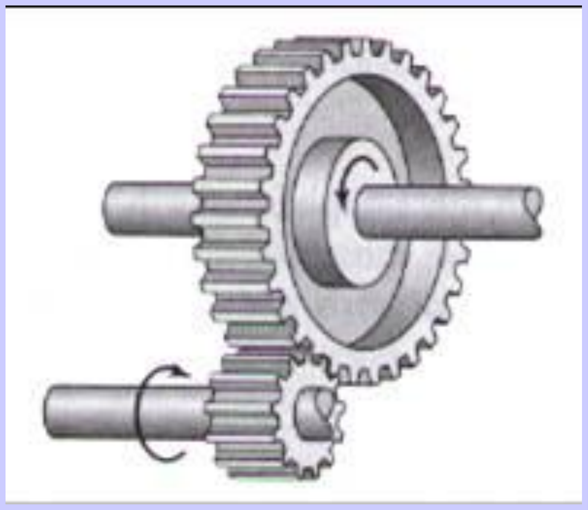
Gears

Applications of Gears

- ***Toys and Small Mechanisms*** **– small, low load, low cost**
  kinematic analysis
- ***Appliance gears*** **– long life, low noise & cost, low to moderate load**
  kinematic & some stress analysis
- ***Power transmission*** **– long life, high load and speed**
  kinematic & stress analysis
- ***Aerospace gears*** **– light weight, moderate to high load**
  kinematic & stress analysis
- ***Control gears*** **– long life, low noise, precision gears**
  kinematic & stress analysis

Types of Gears

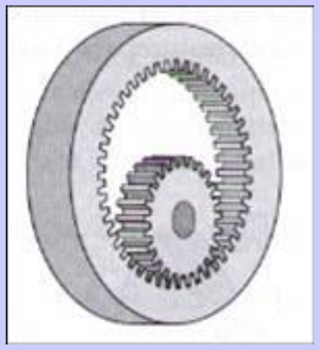
Gear (large gear)



Pinion (small gear)

Spur gears – tooth profile is parallel to the axis of rotation, transmits motion between parallel shafts.

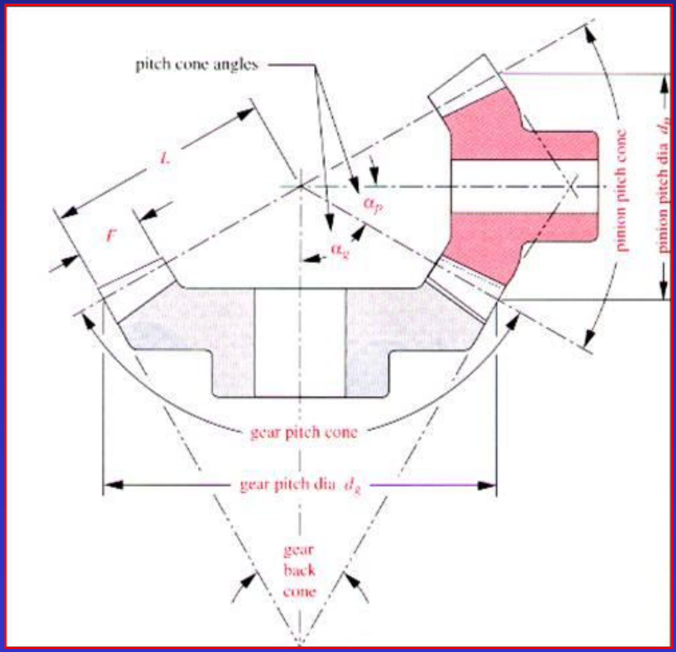
Internal gears



Helical gears – teeth are inclined to the axis of rotation, the angle provides more gradual engagement of the teeth during meshing, transmits motion between parallel shafts.

Types of Gears

Bevel gears – teeth are formed on a conical surface, used to transfer motion between non-parallel and intersecting shafts.

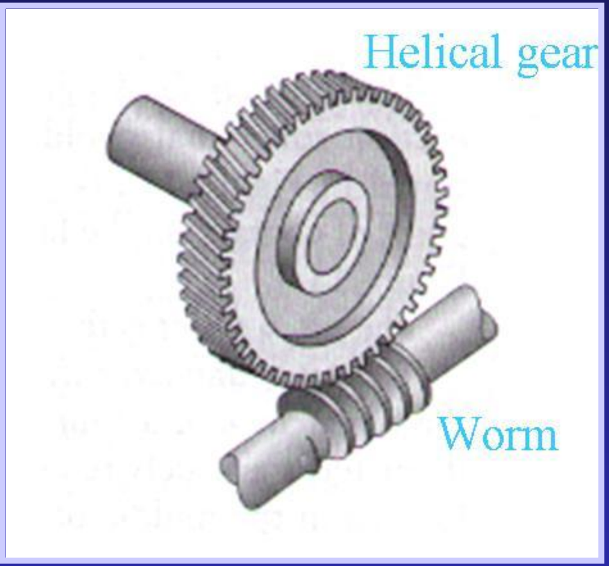


Straight bevel gear

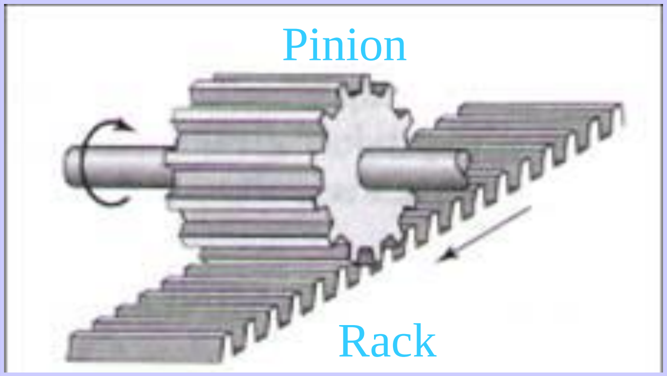
Spiral bevel gear

Types of Gears

Worm gear sets – consists of a helical gear and a power screw (worm), used to transfer motion between non-parallel and non-intersecting shafts.



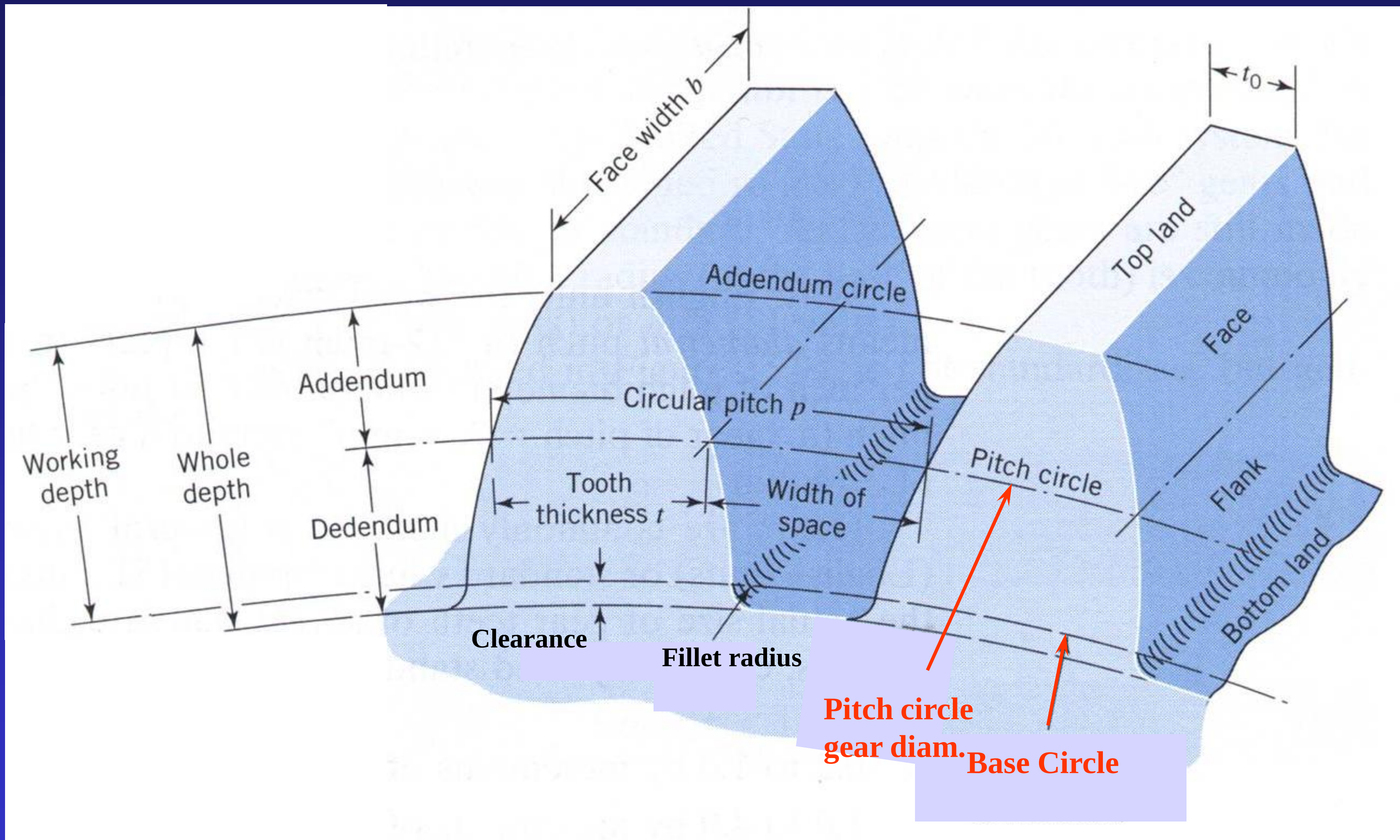
Rack and Pinion sets – a special case of spur gears with the gear having an infinitely large diameter, the teeth are laid flat.



Gear Design and Analysis

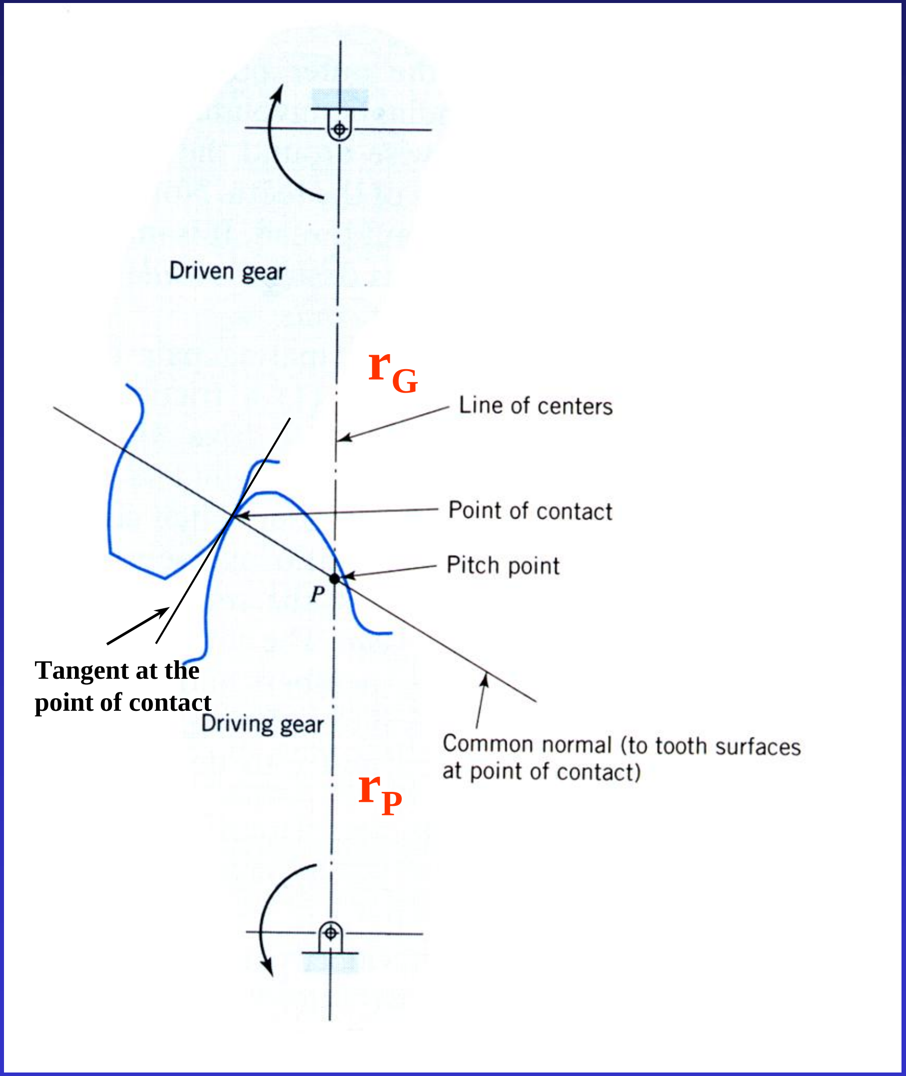
- Kinematics of gear teeth and gear trains.
- Force analysis.
- Design based on tooth ***bending*** strength.
- Design based on tooth ***surface*** strength.

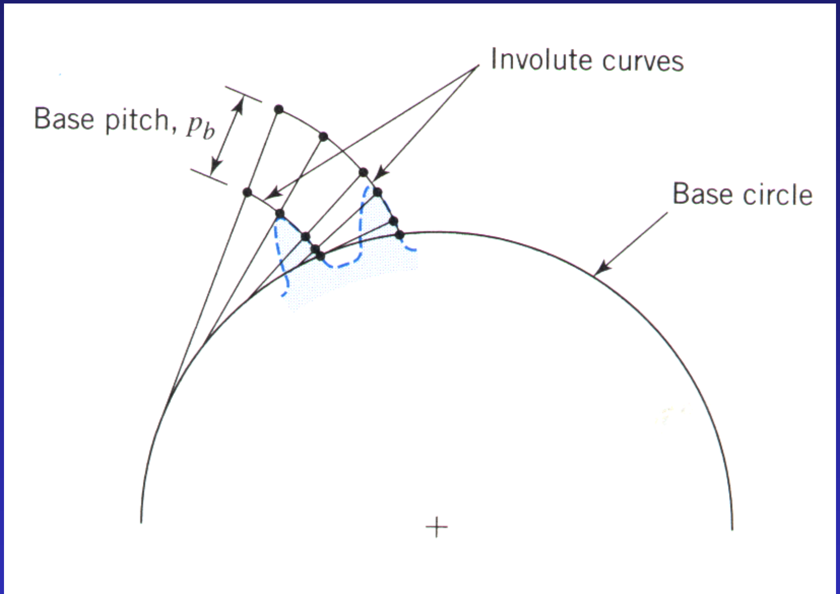
Nomenclature of Spur Gear Teeth



Backlash = $(\text{tooth spacing})_{\text{driven gear}} - (\text{tooth thickness})_{\text{driver}}$, measured on the pitch circle.

Fundamental Law and Involute Curve





Generation of the involute curve

r_G / r_P = constant (constant speed ratio) ⟶ All common normals have to intersect at the same point *P*

Useful Relations

$P = N / d$

P = diametral pitch, teeth per inch

N = number of teeth

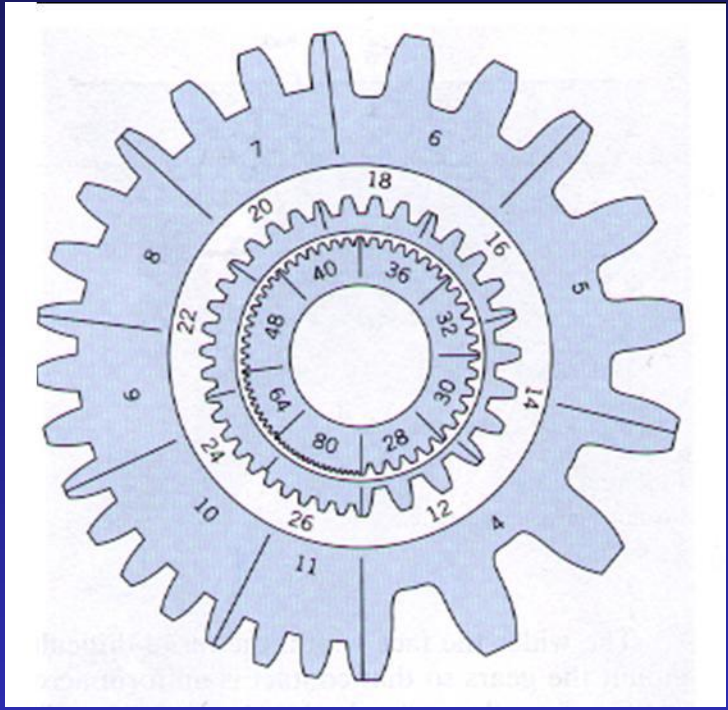
d = pitch diameter (gear diameter)

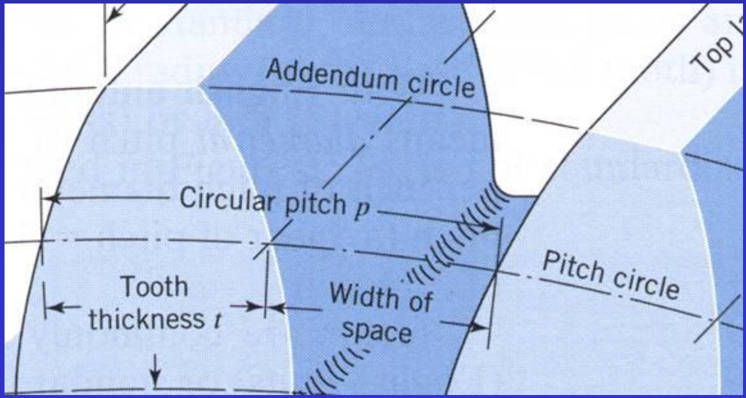
p (circular pitch) $= \pi d / N$

$Pp = \pi$

Metric system

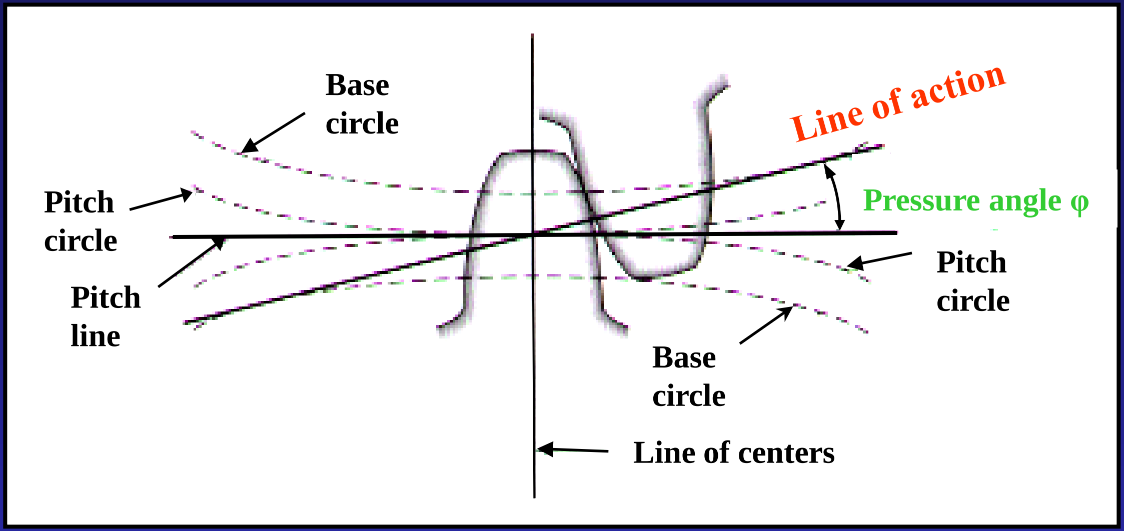
m (module, mm) = d / N



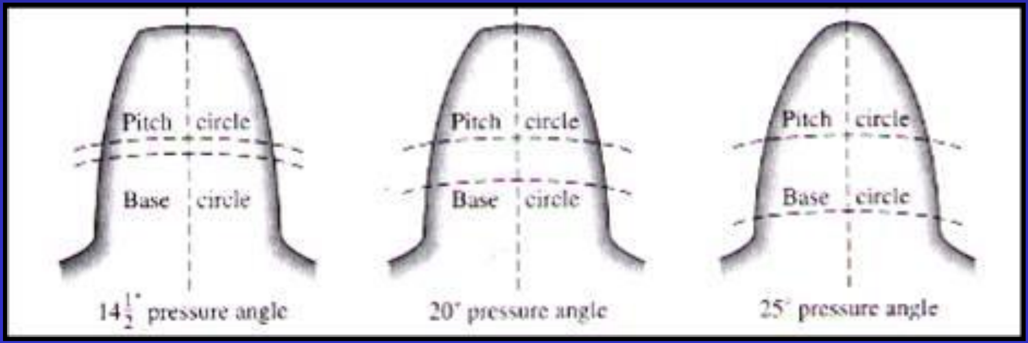


Standard Tooth Specifications

Pressure angle



Standard pressure angles, 14.5º (old), 20º, and 25º



Two mating gears must have the same diametral pitch, P, and pressure angle, φ.

Standard Tooth Specifications

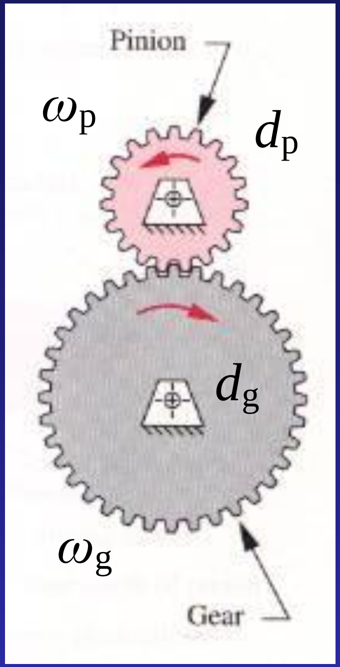
Standard Diametral Pitches

Coarse ($p_d < 20$)	Fine ($p_d > 20$)
1	20
1.25	24
1.5	32
1.75	48
2	64
2.5	72
3	80
4	96
5	120
6	
8	
10	
12	
14	
16	

Parameter	Coarse Pitch ($p_d < 20$)	Fine Pitch ($P_d > 20$)
Pressure angle φ	20^0 or 25^0	20^0
Addendum a	$1.000 / p_d$	$1.000 / p_d$
Dedendum b	$1.000 / p_d$	$1.000 / p_d$
Working depth	$2.000 / p_d$	$2.000 / p_d$
Whole depth	$2.250 / p_d$	$2.000 / p_d + 0.002$ *in.*
Tooth thickness	$1.571 / p_d$	$1.571 / p_d$
Fillet radius	$0.300 / p_d$	no standard

Power transmission, $2 \le P \le 16$

Kinematics



Spur, helical and bevel gears

$$P = (N_g / d_g) = (N_p / d_p)$$

$$(\omega_p / \omega_g) = (d_g / d_p) = (N_g / N_p) = VR \text{ (velocity ratio)}$$

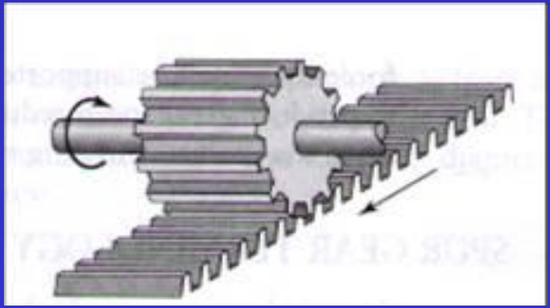
Rack and pinion

Displacement of the rack

$$\Delta s_{rack} = r(\Delta\theta) = \frac{(d_{pinion})(\Delta\theta_{pinion})}{2}$$, $\Delta\theta$ is in radians

Velocity of the rack

$$v_{rack} = \omega r = \frac{(d_{pinion})(\omega_{pinion})}{2}$$



Kinematics

Worm Gear Sets

N_g = number of teeth on the helical gear

N_w = number of threads on the worm, usually between 2-6

Speed ratio = N_g / N_w

Large reduction in one step, but lower efficiency due heat generation.

Kinematics of Gear Trains

Conventional gear trains

$$\frac{\omega_3}{\omega_2} = \frac{N_2}{N_3}\ , \quad \omega_3 = \omega_4\ , \quad \frac{\omega_5}{\omega_4} = \frac{N_4}{N_5}$$

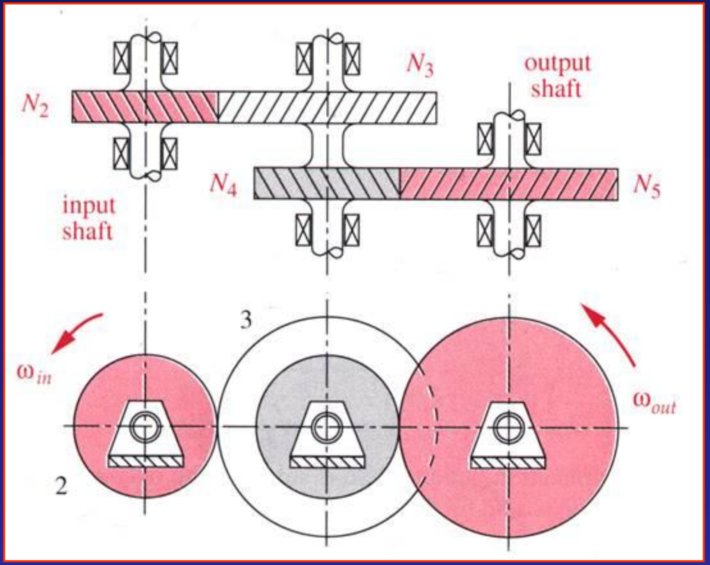
Speed ratio

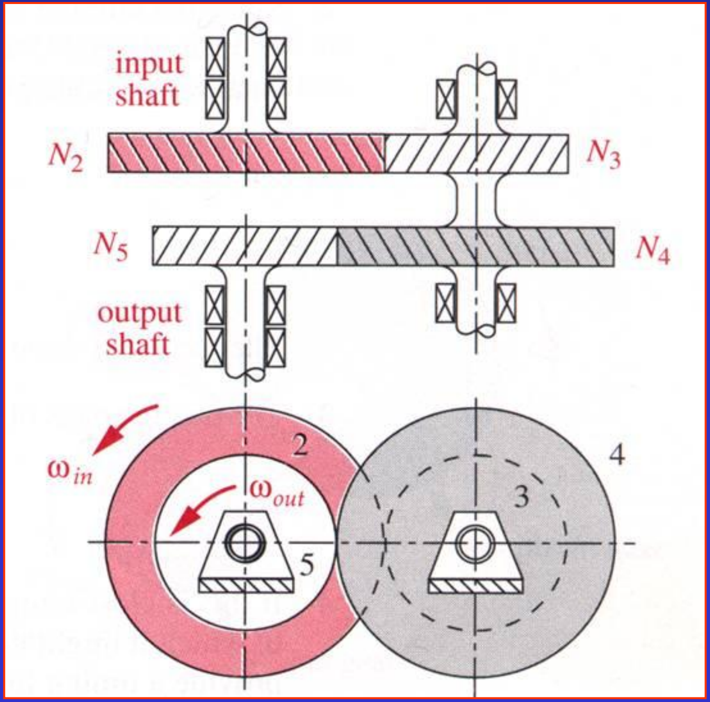
$$m_V = \left(-\frac{N_2}{N_3}\right)\left(-\frac{N_4}{N_5}\right) = \frac{\omega_5}{\omega_2} = \frac{\text{output}}{\text{input}}$$

$$m_V = \pm \frac{\text{product of number of teeth on driver gears}}{\text{product of number of teeth on driven gears}}$$

m_V = e = train value

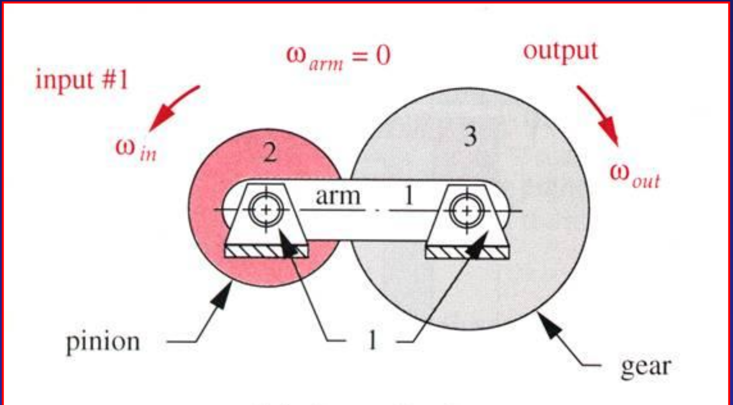
Reverted gear train – output shaft is concentric with the input shaft. Center distances of the stages must be equal.



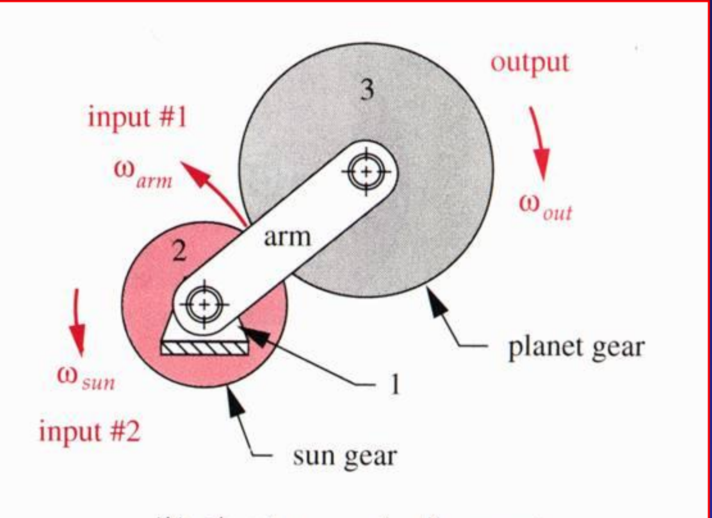


Kinematics of Gear Trains

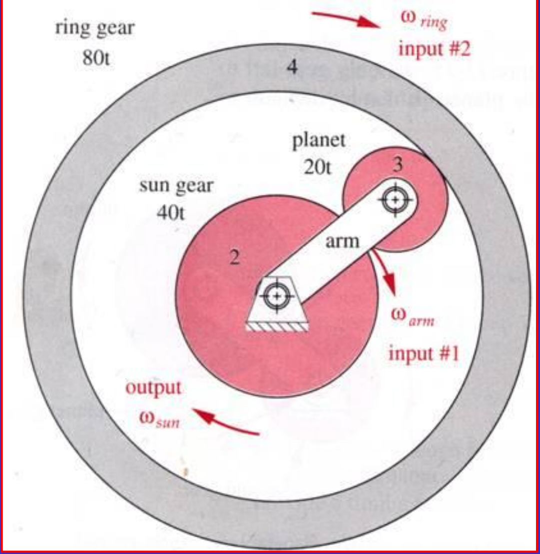
Planetary gear trains



(a) Conventional gearset



(b) Planetary or epicyclic gearset



$$\omega_{gear} = \omega_{arm} + \omega_{gear/arm}$$

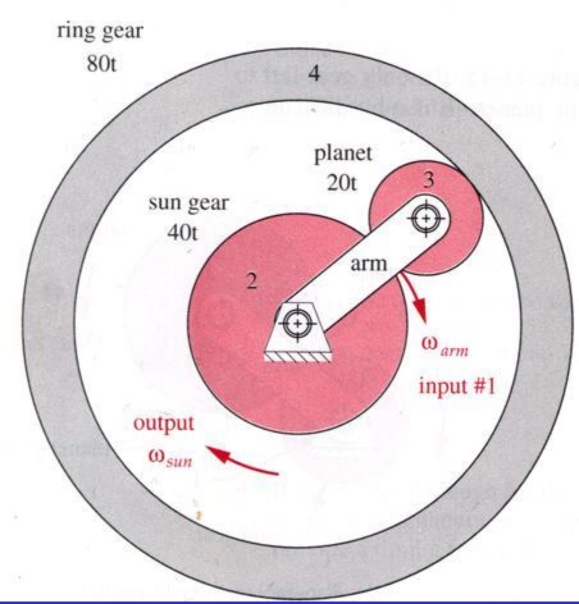
$$\omega_{F/arm} = \omega_F - \omega_{arm} \quad , \quad \omega_{L/arm} = \omega_L - \omega_{arm}$$

$$\frac{\omega_{L/arm}}{\omega_{F/arm}} = \frac{\omega_L - \omega_{arm}}{\omega_F - \omega_{arm}} = m_V = e \text{ (train value)}$$

$$\pm \frac{\text{product of number of teeth on driver gears}}{\text{product of number of teeth on driven gears}} = \frac{\omega_L - \omega_{arm}}{\omega_F - \omega_{arm}}$$

Kinematics of Gear Trains

Determine the speed of the sun gear if the arm rotates at 1 rpm. Ring gear is stationary.



$$\pm \frac{\text{product of number of teeth on driver gears}}{\text{product of number of teeth on driven gears}} = \frac{\omega_L - \omega_{arm}}{\omega_F - \omega_{arm}}$$

$$\left(-\frac{N_2}{N_3}\right)\left(+\frac{N_3}{N_4}\right) = \frac{\omega_L - \omega_{arm}}{\omega_F - \omega_{arm}}$$

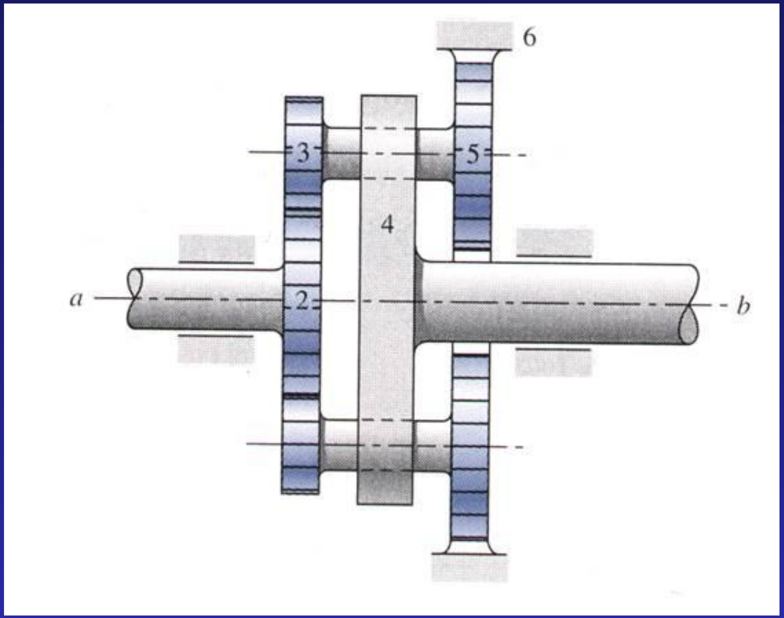
$$\left(-\frac{40}{20}\right)\left(+\frac{20}{80}\right) = \frac{0-1}{\omega_F - 1}$$

$$\omega_F = 3$$

2 degrees of freedom, two inputs are needed to control the system

Planetary Gear Trains - Example

For the speed reducer shown, the input shaft *a* is in line with output shaft *b*. The tooth numbers are N_2=24, N_3=18, N_5=22, and N_6=64. Find the ratio of the output speed to the input speed. Will both shafts rotate in the same direction? Gear 6 is a fixed internal gear.



Train value = $(-N_2 / N_3)(N_5 / N_6)$ = (-24/18)(22/64) = -.4583

$-.4583 = (\omega_L - \omega_{arm}) / (\omega_F - \omega_{arm}) = (0 - \omega_{arm}) / (1 - \omega_{arm})$

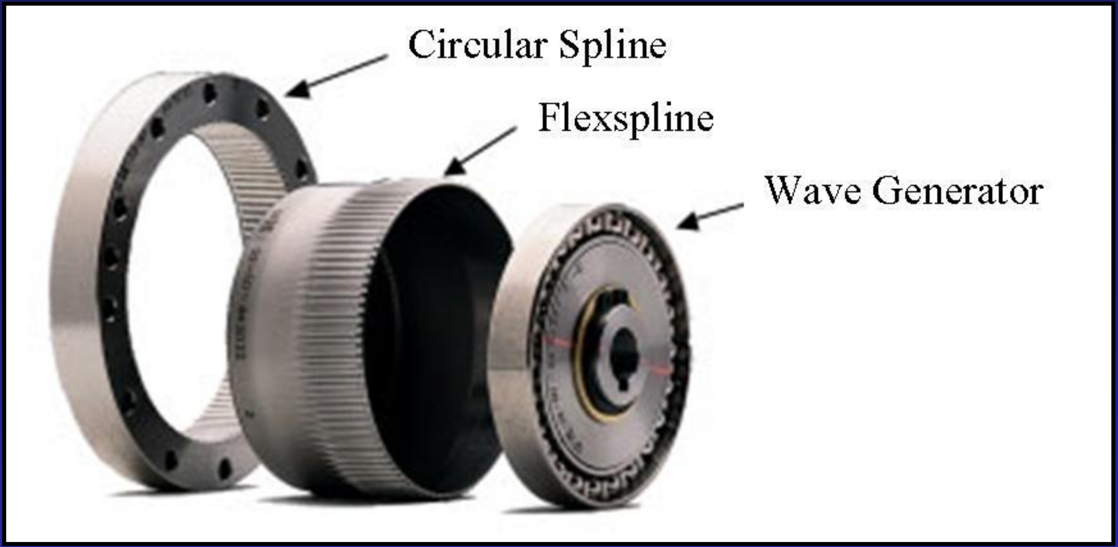
ω_{arm} = .125, reduction is 8 to 1

Input and output shafts rotate in the same direction

$$d_2 + d_3 = d_6 - d_5$$

Harmonic Drive

The mechanism is comprised of three components: Wave Generator, Flexspline, and Circular Spline.



Wave Generator

Consists of a steel disk and a specially design bearing. The outer surface has an elliptical shape. The ball bearing conforms to the same elliptical shape of the wave generator. The wave generator is usually the input.

Flexspline

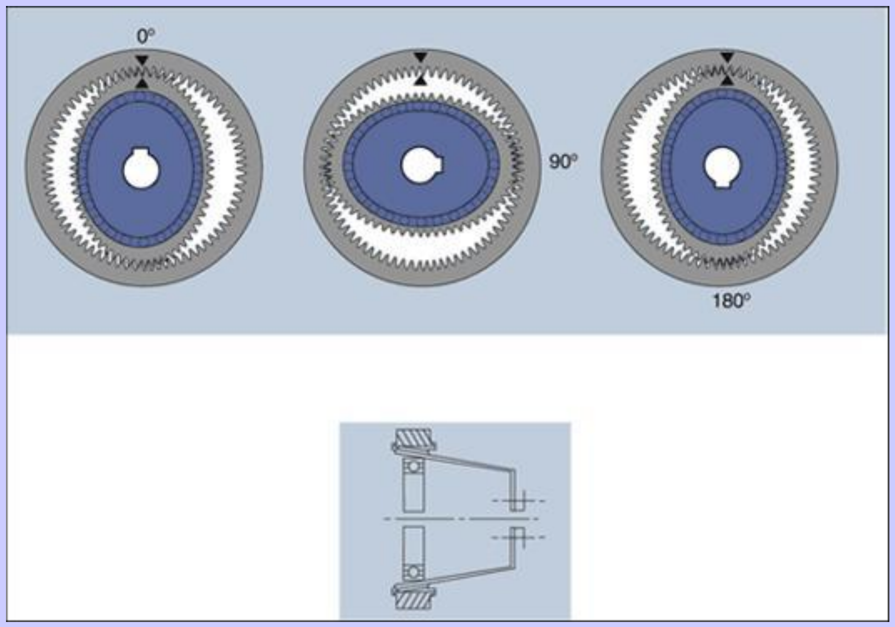
The Flexspline is a thin-walled steel cup with gear teeth on the outer surface near the open end of the cup. Flexspline is usually the output.

Circular Spline

Rigid internal circular gear, meshes with the external teeth on the Flexspline.

Harmonic Drive

Teeth on the Flexspline and circular spline simultaneously mesh at two locations which are 180° apart.



As the wave generator travels 180°, the flexspline shifts one tooth with respect to circular spline in the opposite direction.

The flexspline has two less teeth than the circular spline.

Gear Ratio = - ($N_{flex\ spline}$)/ 2

$\omega_{Wave\ Generator}$ = input , $\omega_{Flexspline}$ = output , $\omega_{Circular\ Spline}$ = 0