# Helical Springs Subjected to Fatigue Loading

The helical springs subjected to fatigue loading are designed by using the *Soderberg line method. The spring materials are usually tested for torsional endurance strength under a repeated stress that varies from zero to a maximum. Since the springs are ordinarily loaded in one direction only (the load in springs is never reversed in nature), therefore a modified Soderberg diagram is used for springs, as shown in Fig.

The endurance limit for reversed loading is shown at point *A where the mean shear stress is* equal to *$\tau e$ / 2 and the variable shear stress is also equal to $\tau e$ / 2. A line drawn from A to B (the yield* point in shear, *$\tau y$) gives the Soderberg's failure stress line. If a suitable factor of safety (F.S.) is* applied to the yield strength ($\tau$*y), a safe stress line CD may be drawn parallel to the line AB, as shown* in Fig. Consider a design point *P on the line CD. Now the value of factor of safety may be* obtained as discussed below :

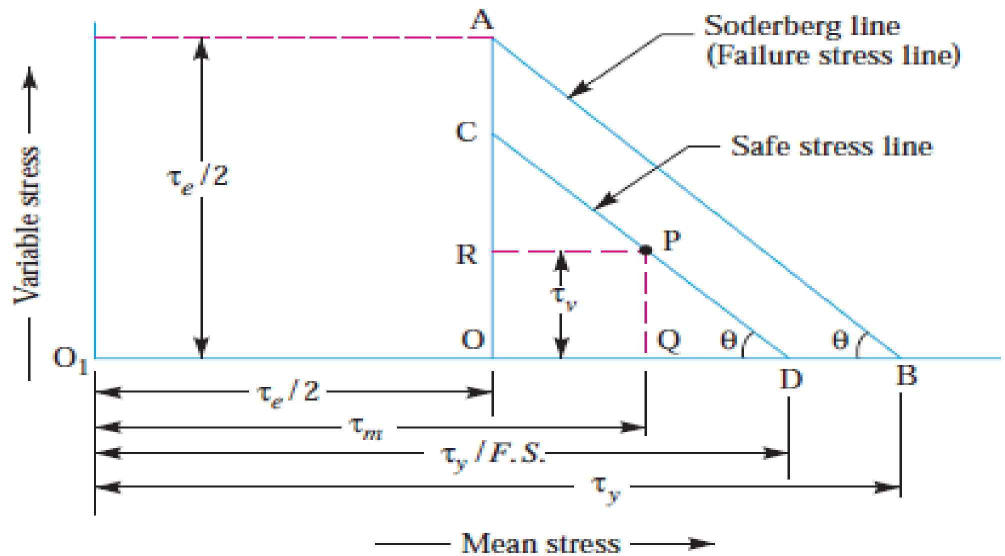


**Fig. 23.19.** Modified Soderberg method for helical springs.

From similar triangles *PQD* and *AOB*, we have

$$\frac{PQ}{QD} = \frac{OA}{OB} \quad \text{or} \quad \frac{PQ}{O_1D - O_1Q} = \frac{OA}{O_1B - O_1O}$$

$$\frac{\tau_v}{\dfrac{\tau_y}{F.S.} - \tau_m} = \frac{\tau_e / 2}{\tau_y - \dfrac{\tau_e}{2}} = \frac{\tau_e}{2\tau_y - \tau_e}$$

or
$$2\,\tau_v \,.\, \tau_y - \tau_v \,.\, \tau_e = \frac{\tau_e \,.\, \tau_y}{F.S.} - \tau_m . \tau_e$$

$\therefore$
$$\frac{\tau_e \,.\, \tau_y}{F.S.} = 2\,\tau_v . \tau_y - \tau_v . \tau_e + \tau_m . \tau_e$$

Dividing both sides by $\tau_e . \tau_y$ and rearranging, we have

$$\frac{1}{F.S.} = \frac{\tau_m - \tau_v}{\tau_y} + \frac{2\tau_v}{\tau_e} \qquad \ldots(i)$$

**Notes : 1.** From equation *(i)*, the expression for the factor of safety (*F.S.*) may be written as

$$F.S. = \frac{\tau_y}{\tau_m - \tau_v + \frac{2\,\tau_v . \tau_y}{\tau_e}}$$

**2.** The value of mean shear stress ($\tau_m$) is calculated by using the shear stress factor ($K_S$), while the variable shear stress is calculated by using the full value of the Wahl's factor ($K$). Thus

Mean shear stress,

$$\tau_m = K_s \times \frac{8\,W_m \times D}{\pi d^3}$$

where $$K_S = 1 + \frac{1}{2C}; \text{ and } W_m = \frac{W_{max} + W_{min}}{2}$$

and variable shear stress, $$\tau_v = K \times \frac{8\,W_v \times D}{\pi\, d^3}$$

where $$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C}; \text{ and } W_v = \frac{W_{max} - W_{min}}{2}$$