Deflection of Helical Springs of Circular Wire

In the previous article, we have discussed the maximum shear stress developed in the wire. We know that

Total active length of the wire,

$$l = \text{Length of one coil} \times \text{No. of active coils} = \pi D \times n$$

Let θ = Angular deflection of the wire when acted upon by the torque T.

$\therefore$ Axial deflection of the spring,

$$\delta = \theta \times D/2 \qquad \ldots(i)$$

We also know that

$$\frac{T}{J} = \frac{\tau}{D/2} = \frac{G.\theta}{l}$$

$$\therefore \quad \theta = \frac{T.l}{J.G} \qquad \ldots\left(\text{considering } \frac{T}{J} = \frac{G.\theta}{l}\right)$$

where J = Polar moment of inertia of the spring wire

$= \frac{\pi}{32} \times d^4$, d being the diameter of spring wire.

and G = Modulus of rigidity for the material of the spring wire.

Now substituting the values of l and J in the above equation, we have

$$\theta = \frac{T.l}{J.G} = \frac{\left(W \times \frac{D}{2}\right)\pi D.n}{\frac{\pi}{32} \times d^4 G} = \frac{16 W.D^2.n}{G.d^4} \qquad \ldots(ii)$$

Substituting this value of θ in equation (*i*), we have

$$\delta = \frac{16 W.D^2.n}{G.d^4} \times \frac{D}{2} = \frac{8\,W.D^3.n}{G.d^4} = \frac{8\,W.C^3.n}{G.d} \qquad \ldots(\because C = D/d)$$

and the stiffness of the spring or spring rate,

$$\frac{W}{\delta} = \frac{G.d^4}{8\,D^3.n} = \frac{G.d}{8\,C^3.n} = \text{constant}$$

Eccentric Loading of Springs

Sometimes, the load on the springs does not coincide with the axis of the spring, *i.e. the spring* is subjected to an eccentric load. In such cases, not only the safe load for the spring reduces, the stiffness of the spring is also affected. The eccentric load on the spring increases the stress on one side of the spring and decreases on the other side. When the load is offset by a distance *e from the spring* axis, then the safe load on the spring may be obtained by multiplying the axial load by the factor

$$\frac{D}{2e + D}$$, where D is the mean diameter of the spring.

Surge in Springs

When one end of a helical spring is resting on a rigid support and the other end is loaded suddenly, then all the coils of the spring will not suddenly deflect equally, because some time is required for the propagation of stress along the spring wire. A little consideration will show that in the beginning, the end coils of the spring in contact with the applied load takes up whole of the deflection and then it transmits a large part of its deflection to the adjacent coils. In this way, a wave of compression propagates through the coils to the supported end from where it is reflected back to the deflected end. This wave of compression travels along the spring indefinitely. If the applied load is of fluctuating type as in the case of valve spring in internal combustion engines and if the time interval between the load applications is equal to the time required for the wave to travel from one end to the other end, then resonance will occur. This results in very large deflections of the coils and correspondingly very high stresses. Under these conditions, it is just possible that the spring may fail. This phenomenon is called surge.

It has been found that the natural frequency of spring should be atleast twenty times the frequency of application of a periodic load in order to avoid resonance with all harmonic frequencies upto twentieth order. The natural frequency for springs clamped between two plates is given by

$$f_n = \frac{d}{2\pi D^2 . n} \sqrt{\frac{6 G . g}{\rho}} \text{ cycles/s}$$

where *d = Diameter of the wire,*

D = Mean diameter of the spring,

n = Number of active turns,

G = Modulus of rigidity,

g = Acceleration due to gravity, and

ρ = Density of the material of the spring.

The surge in springs may be eliminated by using the following methods:

1. By using friction dampers on the centre coils so that the wave propagation dies out.
2. By using springs of high natural frequency.
3. By using springs having pitch of the coils near the ends different than at the centre to have different natural frequencies.

Energy Stored in Helical Springs of Circular Wire

We know that the springs are used for storing energy which is equal to the work done on it by
some external load.

Let *W = Load applied on the spring, and*

δ = Deflection produced in the spring due to the load *W.*

Assuming that the load is applied gradually, the energy stored in a spring is,

$$U = \frac{1}{2} W . \delta \qquad \ldots (i)$$

We have already discussed that the maximum shear stress induced in the spring wire,

$$\tau = K \times \frac{8 W . D}{\pi d^3} \text{ or } W = \frac{\pi d^3 . \tau}{8 K . D}$$

We know that deflection of the spring,

$$\delta = \frac{8 W . D^3 . n}{G . d^4} = \frac{8 \times \pi d^3 . \tau}{8 K . D} \times \frac{D^3 . n}{G . d^4} = \frac{\pi \tau . D^2 . n}{K . d . G}$$

Substituting the values of W and δ in equation (*i*), we have

$$U = \frac{1}{2} \times \frac{\pi\, d^3 . \tau}{8\, K . D} \times \frac{\pi\, \tau . D^2 . n}{K . d . G}$$

$$= \frac{\tau^2}{4\, K^2 . G} (\pi\, D . n) \left(\frac{\pi}{4} \times d^2 \right) = \frac{\tau^2}{4\, K^2 . G} \times V$$

where V = Volume of the spring wire

= Length of spring wire × Cross-sectional area of spring wire

$$= (\pi\, D . n) \left(\frac{\pi}{4} \times d^2 \right)$$

Note : When a load (say P) falls on a spring through a height h, then the energy absorbed in a spring is given by

$$U = P(h + \delta) = \frac{1}{2}\, W . \delta$$

where W = Equivalent static load *i.e.* the gradually applied load which shall produce the same effect as by the falling load P, and

δ = Deflection produced in the spring.