Shafts Subjected to Fluctuating Loads

In the previous articles we have assumed that the shaft is subjected to constant torque and bending moment. But in actual practice, the shafts are subjected to fluctuating torque and bending moments. In order to design such shafts like line shafts and counter shafts, the combined shock and fatigue factors must be taken into account for the computed twisting moment (*T) and bending moment (M). Thus for a shaft* subjected to combined bending and torsion, the equivalent twisting moment,

$$T_e = \sqrt{(K_m \times M)^2 + (K_t + T)^2}$$

and equivalent bending moment,

$$M_e = \frac{1}{2}\left[K_m \times M + \sqrt{(K_m \times M)^2 + (K_t \times T)^2}\right]$$

where K_m = Combined shock and fatigue factor for bending, and

K_t = Combined shock and fatigue factor for torsion.

Shafts Subjected to Axial Load in addition to Combined Torsion and Bending Loads

When the shaft is subjected to an axial load (F) in addition to torsion and bending loads as in propeller shafts of ships and shafts for driving worm gears, then the stress due to axial load must be added to the bending stress (σ_b). We know that bending equation is

$$\frac{M}{I} = \frac{\sigma_b}{y} \quad \text{or} \quad \sigma_b = \frac{M.y}{I} = \frac{M \times d/2}{\frac{\pi}{64} \times d^4} = \frac{32M}{\pi d^3}$$

and stress due to axial load

$$= \frac{F}{\frac{\pi}{4} \times d^2} = \frac{4F}{\pi d^2} \qquad \text{...(For round solid shaft)}$$

$$= \frac{F}{\frac{\pi}{4}\left[(d_o)^2 - (d_i)^2\right]} = \frac{4F}{\pi\left[(d_o)^2 - (d_i)^2\right]} \qquad \text{...(For hollow shaft)}$$

$$= \frac{F}{\pi (d_o)^2 (1 - k^2)} \qquad \ldots (\because k = d_i/d_o)$$

∴ Resultant stress (tensile or compressive) for solid shaft,

$$\sigma_1 = \frac{32M}{\pi d^3} + \frac{4F}{\pi d^2} = \frac{32}{\pi d^3}\left(M + \frac{F \times d}{8}\right) \qquad \ldots (i)$$

$$= \frac{32 M_1}{\pi d^3} \qquad \ldots \left(\text{Substituting } M_1 = M + \frac{F \times d}{8}\right)$$

In case of a hollow shaft, the resultant stress,

$$\sigma_1 = \frac{32M}{\pi (d_o)^3 (1 - k^4)} + \frac{4F}{\pi (d_o)^2 (1 - k^2)}$$

$$= \frac{32}{\pi (d_o)^3 (1 - k^4)}\left[M + \frac{F d_o (1 + k^2)}{8}\right] = \frac{32 M_1}{\pi (d_o)^3 (1 - k^4)}$$

$$\ldots \left[\text{Substituting for hollow shaft, } M_1 = M + \frac{F d_o (1 + k^2)}{8}\right]$$

In case of long shafts (slender shafts) subjected to compressive loads, a factor known as ***column factor*** (α) must be introduced to take the column effect into account.

∴ Stress due to the compressive load,

$$\sigma_c = \frac{\alpha \times 4F}{\pi d^2} \qquad \text{...(For round solid shaft)}$$

$$= \frac{\alpha \times 4F}{\pi (d_o)^2 (1 - k^2)} \qquad \text{...(For hollow shaft)}$$

The value of column factor (α) for compressive loads* may be obtained from the following relation :

$$\text{Column factor,} \quad \alpha = \frac{1}{1 - 0.0044 \, (L / K)}$$

This expression is used when the slenderness ratio (L / K) is less than 115. When the slenderness ratio (L / K) is more than 115, then the value of column factor may be obtained from the following relation :

**Column factor, $\alpha = \dfrac{\sigma_y (L/K)^2}{C \pi^2 E}$

where L = Length of shaft between the bearings,

K = Least radius of gyration,

σ_y = Compressive yield point stress of shaft material, and

C = Coefficient in Euler's formula depending upon the end conditions.

The following are the different values of C depending upon the end conditions.

C = 1, for hinged ends,

= 2.25, for fixed ends,

= 1.6, for ends that are partly restrained as in bearings.

Note: In general, for a hollow shaft subjected to fluctuating torsional and bending load, along with an axial load, the equations for equivalent twisting moment (T_e) and equivalent bending moment (M_e) may be written as

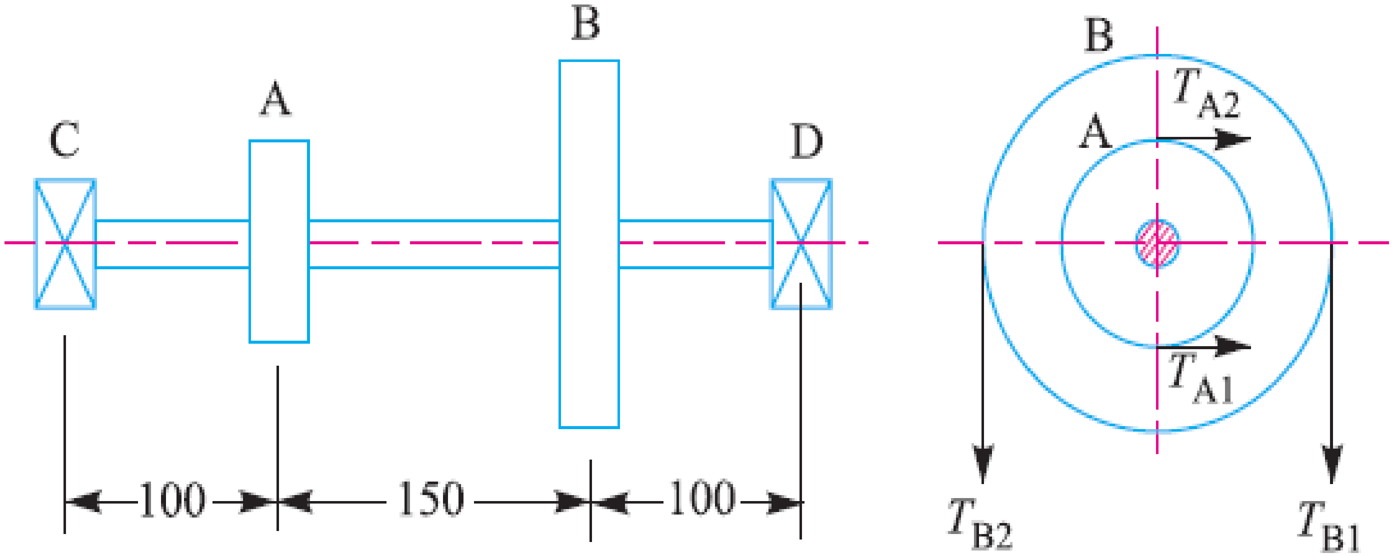
$$T_e = \sqrt{\left[K_m \times M + \frac{\alpha F d_o (1+k^2)}{8}\right]^2 + (K_t \times T)^2}$$

$$= \frac{\pi}{16} \times \tau (d_o)^3 (1-k^4)$$

and $$M_e = \frac{1}{2}\left[K_m \times M + \frac{\alpha F d_o (1+k^2)}{8} + \sqrt{\left\{K_m \times M + \frac{\alpha F d_o (1+k^2)}{8}\right\}^2 + (K_t \times T)^2}\right]$$

$$= \frac{\pi}{32} \times \sigma_b (d_o)^3 (1-k^4)$$

It may be noted that for a solid shaft, $k = 0$ and $d_0 = d$. When the shaft carries no axial load, then $F = 0$ and when the shaft carries axial tensile load, then $\alpha = 1$.



All dimensions in mm.