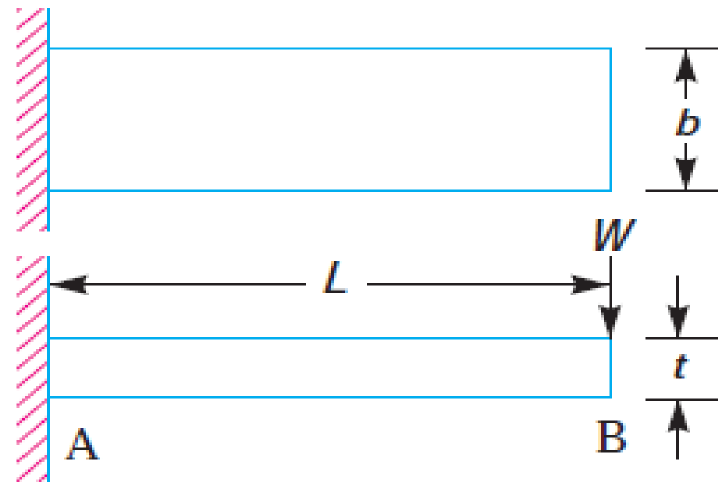
Leaf Springs

Leaf springs (also known as **flat springs) are made out of flat plates. The advantage of leaf** spring over helical spring is that the ends of the spring may be guided along a definite path as it deflects to act as a structural member in addition to energy absorbing device. Thus the leaf springs may carry lateral loads, brake torque, driving torque etc., in addition to shocks.

Consider a single plate fixed at one end and loaded at the other end as shown in Fig. This plate may be used as a flat spring.

Let



t = Thickness of plate,

b = Width of plate, and

L = Length of plate or distance of the load *W from the* cantilever end.

We know that the maximum bending moment at the cantilever end *A,*

$M = W.L$

and section modulus, $$Z = \frac{I}{y} = \frac{b\,t^3/12}{t/2} = \frac{1}{6} \times b.t^2$$

$\therefore$ Bending stress in such a spring,

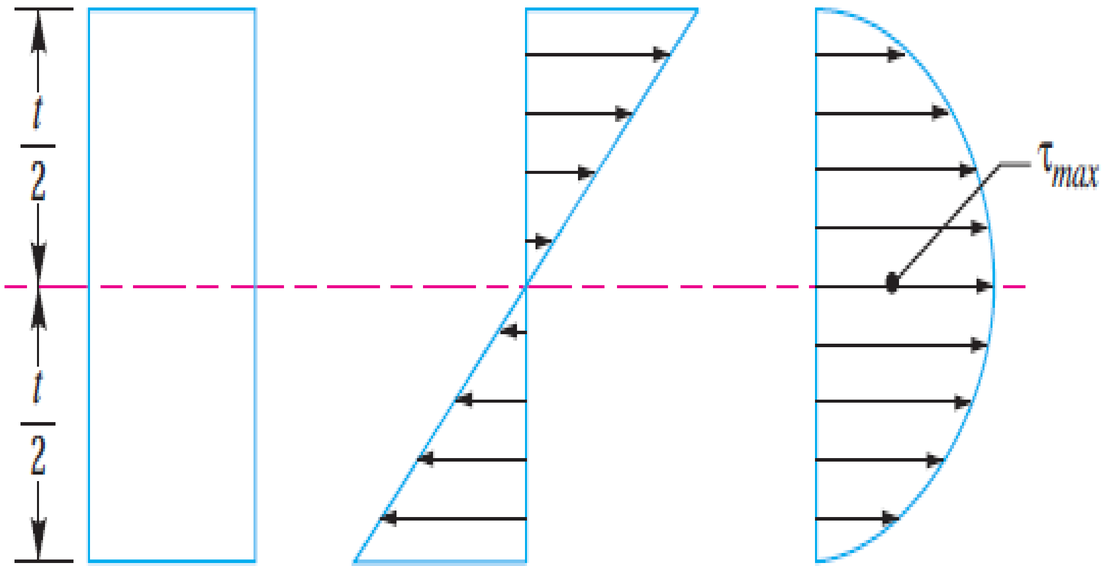
$$\sigma = \frac{M}{Z} = \frac{W.L}{\frac{1}{6} \times b.t^2} = \frac{6\,W.L}{b.t^2}$$

We know that the maximum deflection for a cantilever with concentrated load at the free end is given by

$$\delta = \frac{W.L^3}{3E.I} = \frac{W.L^3}{3E \times b.t^3/12} = \frac{4\,W.L^3}{E.b.t^3} \qquad \ldots(ii)$$

$$= \frac{2\,\sigma.L^2}{3\,E.t} \qquad \ldots\left(\because \sigma = \frac{6W.L}{b.t^2}\right)$$

It may be noted that due to bending moment, top fibres will be in tension and the bottom fibres are in compression, but the shear stress is zero at the extreme fibres and maximum at the centre, as shown in Fig. 23.26. Hence for analysis, both stresses need not to be taken into account simultaneously. We shall consider the bending stress only.

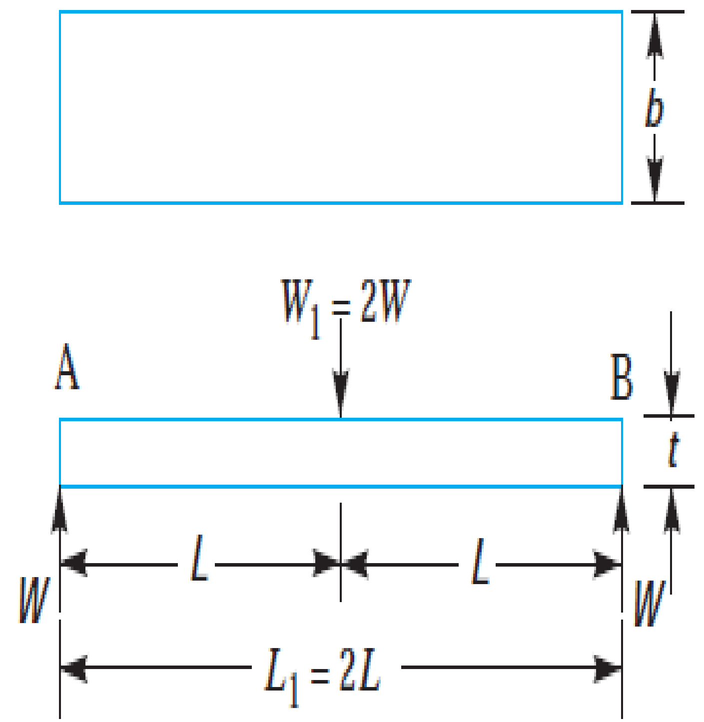


(*a*) Cross-section of plate.

(*b*) Bending stress diagram.

(*c*) Shear stress diagram.

If the spring is not of cantilever type but it is like a simply supported beam, with length $2L$ and load $2W$ in the centre, as shown in Fig. 23.27, then



Maximum bending moment in the centre,

$$M = W.L$$

Section modulus, $Z = b.t^2 / 6$

$$\therefore \text{ Bending stress, } \quad \sigma = \frac{M}{Z} = \frac{W.L}{b.t^2/6}$$

$$= \frac{6\,W.L}{b.t^2}$$

We know that maximum deflection of a simply supported beam loaded in the centre is given by

$$\delta = \frac{W_1 (L_1)^3}{48\, E.I} = \frac{(2W)\,(2L)^3}{48\, E.I} = \frac{W.L^3}{3\, E.I}$$

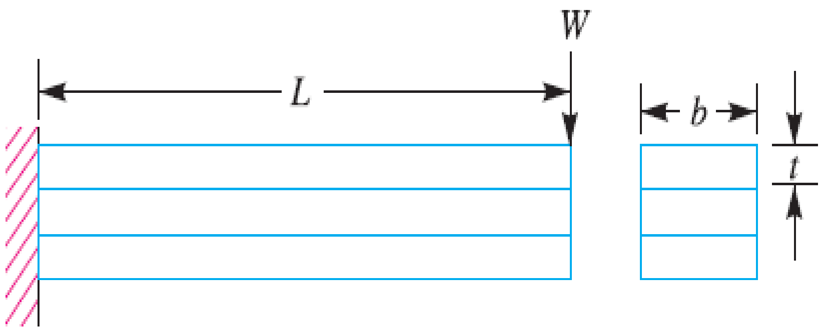
$$\ldots(\because \text{ In this case, } W_1 = 2W, \text{ and } L_1 = 2L)$$

From above we see that a spring such as automobile spring (semi-elliptical spring) with lengt 2*L and loaded in the centre by a load 2W, may be treated as a double cantilever.*
If the plate of cantilever is cut into a series of *n strips of width b and these are placed as shown* in Fig. then equations ***(i) and (ii) may be written as***

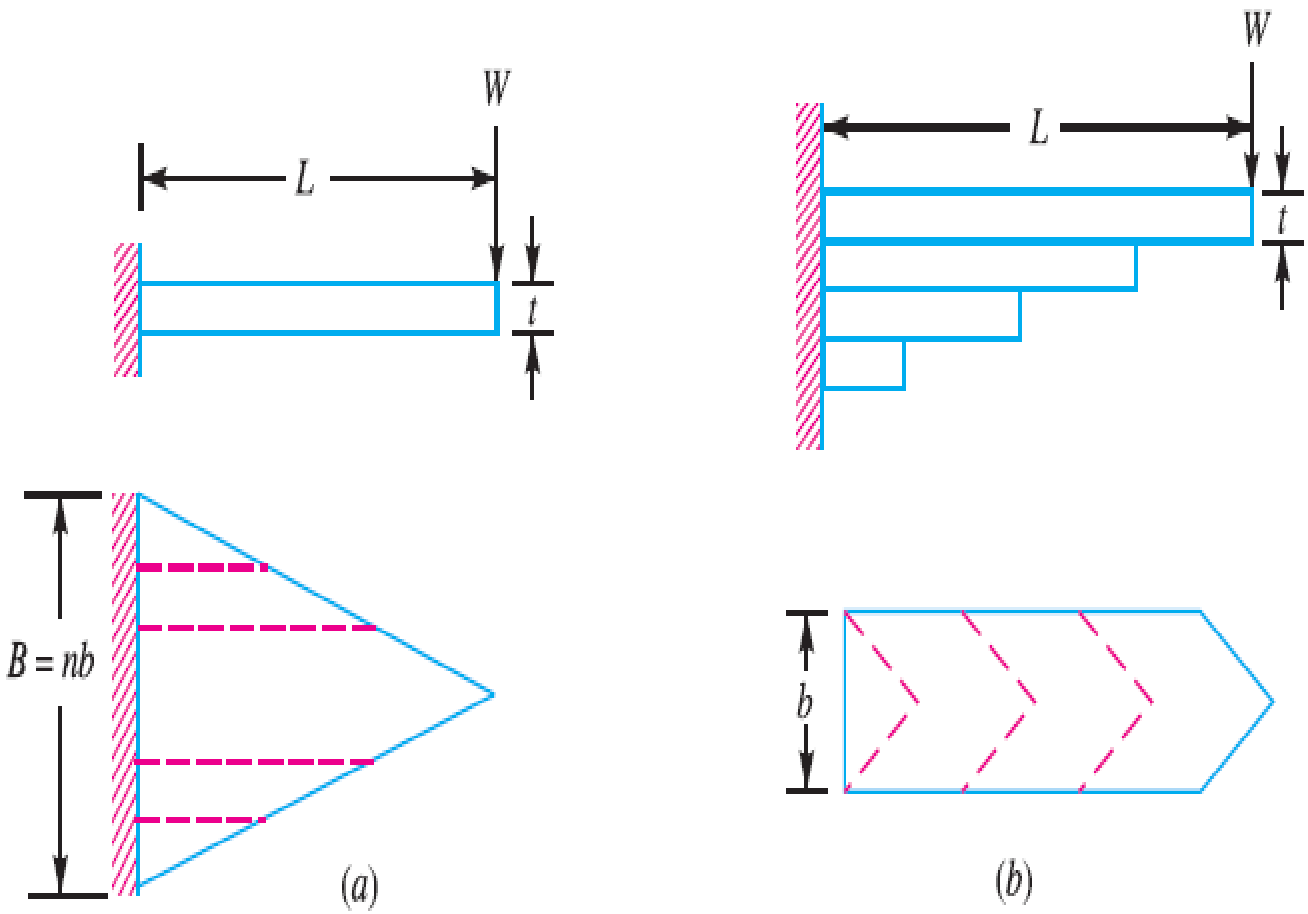
$$\sigma = \frac{6\,W.L}{n.b.t^2} \qquad \text{...}(iii)$$

and

$$\delta = \frac{4\,W.L^3}{n.E.b.t^3} = \frac{2\,\sigma.L^2}{3\,E.t} \qquad \text{...}(iv)$$



If a triangular plate is used as shown in Fig. 23.29 (*a*), the stress will be uniform throughout. If this triangular plate is cut into strips of uniform width and placed one below the other, as shown in Fig. 23.29 (*b*) to form a graduated or laminated leaf spring, then



(*a*) (*b*)

$$\sigma = \frac{6\,W.L}{n.b.t^2} \qquad \ldots(v)$$

and
$$\delta = \frac{6\,W.L^3}{n.E.b.t^3} = \frac{\sigma.L^2}{E.t} \qquad \ldots(vi)$$

where n = Number of graduated leaves.

A little consideration will show that by the above arrangement, the spring becomes compact so that the space occupied by the spring is considerably reduced.

When bending stress alone is considered, the graduated leaves may have zero width at the loaded end. But sufficient metal must be provided to support the shear. Therefore, it becomes necessary to have one or more leaves of uniform cross-section extending clear to the end. We see from equations (*iv*) and (*vi*) that for the same deflection, the stress in the uniform cross-section leaves (*i.e.* full length leaves) is 50% greater than in the graduated leaves, assuming that each spring element deflects according to its own elastic curve. If the suffixes $_F$ and $_G$ are used to indicate the full length (or uniform cross-section) and graduated leaves, then

$$\sigma_F = \frac{3}{2}\sigma_G$$

$$\frac{6 W_F . L}{n_F . b . t^2} = \frac{3}{2}\left[\frac{6 W_G . L}{n_G . b . t^2}\right] \quad \text{or} \quad \frac{W_F}{n_F} = \frac{3}{2} \times \frac{W_G}{n_G}$$

$$\therefore \quad \frac{W_F}{W_G} = \frac{3\, n_F}{2\, n_G} \qquad \text{...}(vii)$$

Adding 1 to both sides, we have

$$\frac{W_F}{W_G}+1=\frac{3\,n_F}{2\,n_G}+1 \quad \text{or} \quad \frac{W_F+W_G}{W_G}=\frac{3\,n_F+2\,n_G}{2\,n_G}$$

$$\therefore \qquad W_G=\left(\frac{2\,n_G}{3\,n_F+2\,n_G}\right)(W_F+W_G)=\left(\frac{2\,n_G}{3\,n_F+2\,n_G}\right)W \qquad \ldots(viii)$$

where W = Total load on the spring = $W_G + W_F$

W_G = Load taken up by graduated leaves, and

W_F = Load taken up by full length leaves.

From equation *(vii)*, we may write

$$\frac{W_G}{W_F}=\frac{2\,n_G}{3\,n_F}$$

or

$$\frac{W_G}{W_F}+1=\frac{2\,n_G}{3\,n_F}+1 \qquad \text{... (Adding 1 to both sides)}$$

$$\frac{W_G+W_F}{W_F}=\frac{2\,n_G+3\,n_F}{3\,n_F}$$

$$\therefore \qquad W_F=\left(\frac{3\,n_F}{2\,n_G+3\,n_F}\right)(W_G+W_F)=\left(\frac{3\,n_F}{2\,n_G+3\,n_F}\right)W \qquad \ldots(ix)$$

$$\therefore \qquad W_F = \left(\frac{3\,n_F}{2\,n_G + 3\,n_F}\right)(W_G + W_F) = \left(\frac{3\,n_F}{2\,n_G + 3\,n_F}\right) W \qquad \ldots(ix)$$

$\therefore$ Bending stress for full length leaves,

$$\sigma_F = \frac{6\,W_F.L}{n_F.b\,t^2} = \frac{6\,L}{n_F.b.t^2}\left(\frac{3\,n_F}{2\,n_G + 3\,n_F}\right) W = \frac{18\,W.L}{b.t^2\,(2\,n_G + 3\,n_F)}$$

Since $\sigma_F = \frac{3}{2}\,\sigma_G$, therefore

$$\sigma_G = \frac{2}{3}\,\sigma_F = \frac{2}{3} \times \frac{18\,W.L}{b.t^2\,(2\,n_G + 3\,n_F)} = \frac{12\,W.L}{b.t^2\,(2\,n_G + 3\,n_F)}$$

The deflection in full length and graduated leaves is given by equation (*iv*), *i.e.*

$$\delta = \frac{2\,\sigma_F \times L^2}{3\,E.t} = \frac{2\,L^2}{3\,E.t}\left[\frac{18\,W.L}{b.t^2\,(2\,n_G + 3\,n_F)}\right] = \frac{12\,W.L^3}{E.b.t^3\,(2\,n_G + 3\,n_F)}$$

Length of Leaf Spring Leaves

The length of the leaf spring leaves may be obtained as discussed below :

Let $2L_1$ = Length of span or overall length of the spring,

l = Width of band or distance between centres of U-bolts. It is the ineffective length of the spring,

n_F = Number of full length leaves,

n_G = Number of graduated leaves, and

n = Total number of leaves = $n_F + n_G$.

We have already discussed that the effective length of the spring,

$$2L = 2L_1 - l \quad \text{...(When band is used)}$$

$$= 2L_1 - \frac{2}{3}l \quad \text{... (When } U\text{-bolts are used)}$$

It may be noted that when there is only one full length leaf (*i.e. master leaf only), then the* number of leaves to be cut will be *n and when there are two full length leaves (including one master* leaf), then the number of leaves to be cut will be (*n – 1). If a leaf spring has two full length leaves, then* the length of leaves is obtained as follows :

$$\text{Length of smallest leaf} = \frac{\text{Effective length}}{n-1} + \text{Ineffective length}$$

$$\text{Length of next leaf} = \frac{\text{Effective length}}{n-1} \times 2 + \text{Ineffective length}$$

Similarly, length of $(n-1)$th leaf

$$= \frac{\text{Effective length}}{n-1} \times (n-1) + \text{Ineffective length}$$

The nth leaf will be the master leaf and it is of full length. Since the master leaf has eyes on both sides, therefore

$$\text{Length of master leaf} = 2\,L_1 + \pi\,(d+t) \times 2$$

where d = Inside diameter of eye, and

t = Thickness of master leaf.

The approximate relation between the radius of curvature (R) and the camber (y) of the spring is given by

$$R = \frac{(L_1)^2}{2y}$$

The exact relation is given by

$$y\,(2R + y) = (L_1)^2$$

where L_1 = Half span of the spring.

Note : The maximum deflection (δ) of the spring is equal to camber (y) of the spring.