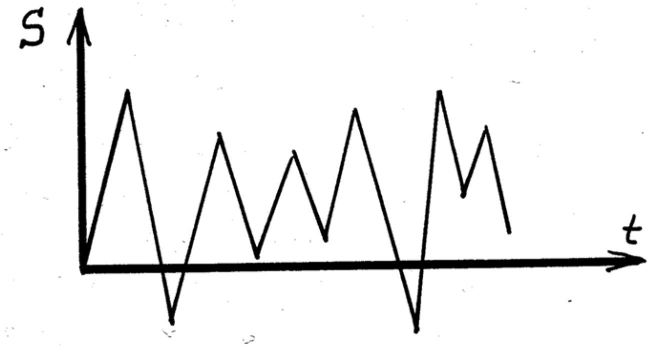
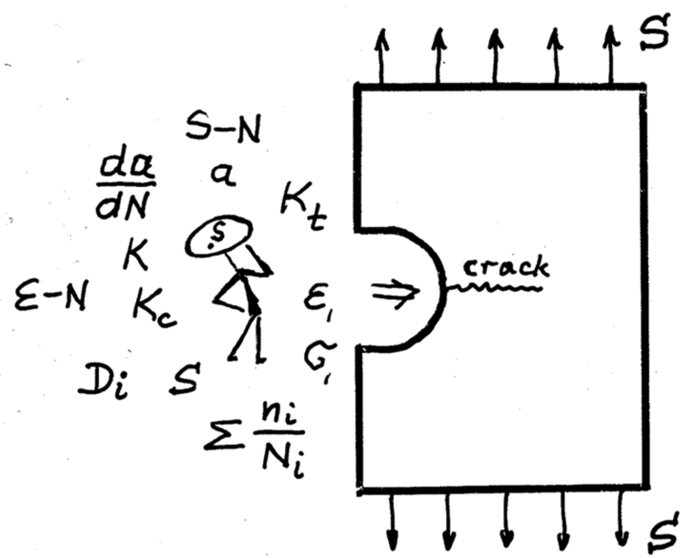
ME 322 - Mechanical Design 1

Partial notes – Part 4 (Fatigue)

FATIGUE - What is it?





N_i = ?

N_p = ?

N_T = ?

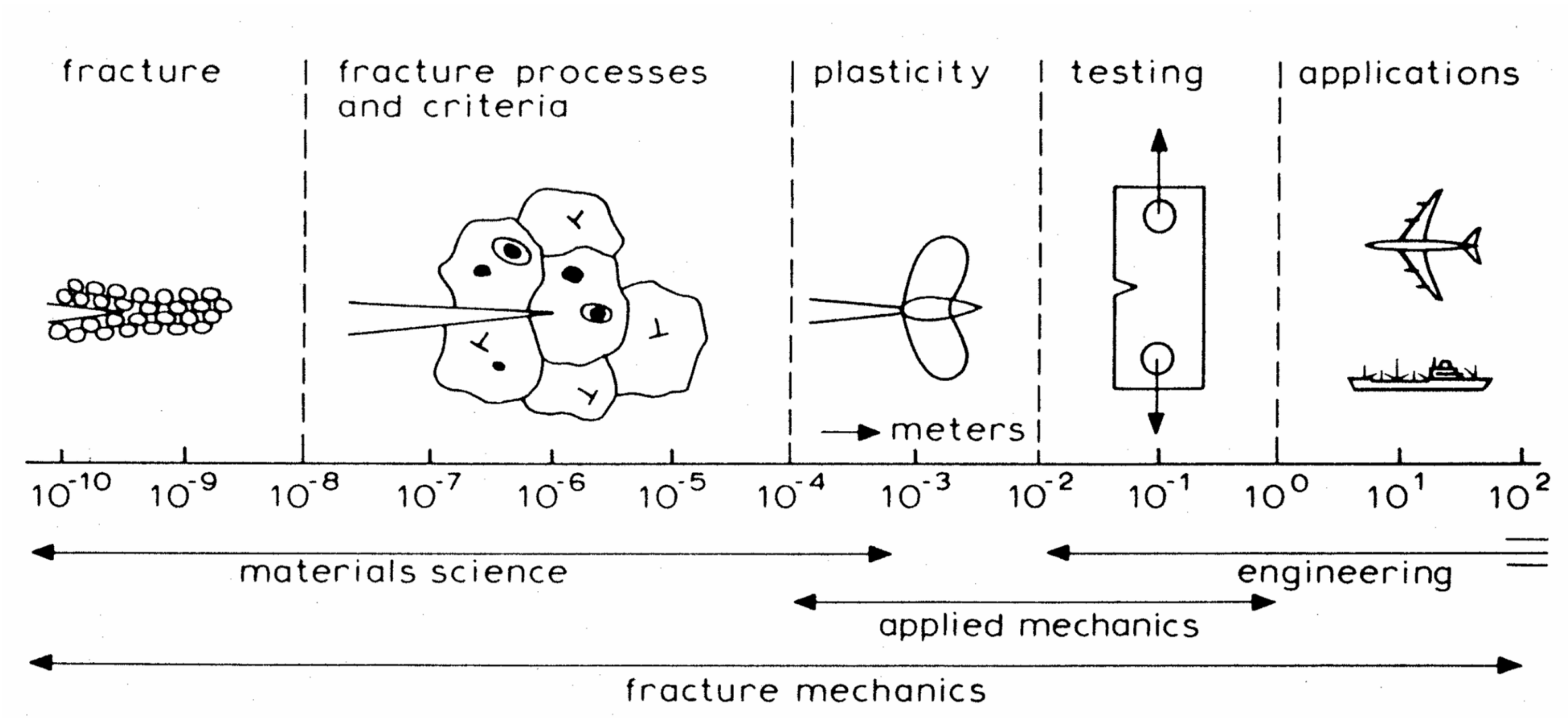
Metal Fatigue is a process which causes premature irreversible damage or failure of a component subjected to repeated loading.

Metallic Fatigue

- A sequence of several, very complex phenomena encompassing several disciplines:
  - motion of dislocations
  - surface phenomena
  - fracture mechanics
  - stress analysis
  - probability and statistics
- Begins as an consequence of reversed plastic deformation within a single crystallite but ultimately may cause the destruction of the entire component
- Influenced by a component's environment
- Takes many forms:
  - fatigue at notches
  - rolling contact fatigue
  - fretting fatigue
  - corrosion fatigue
  - creep-fatigue

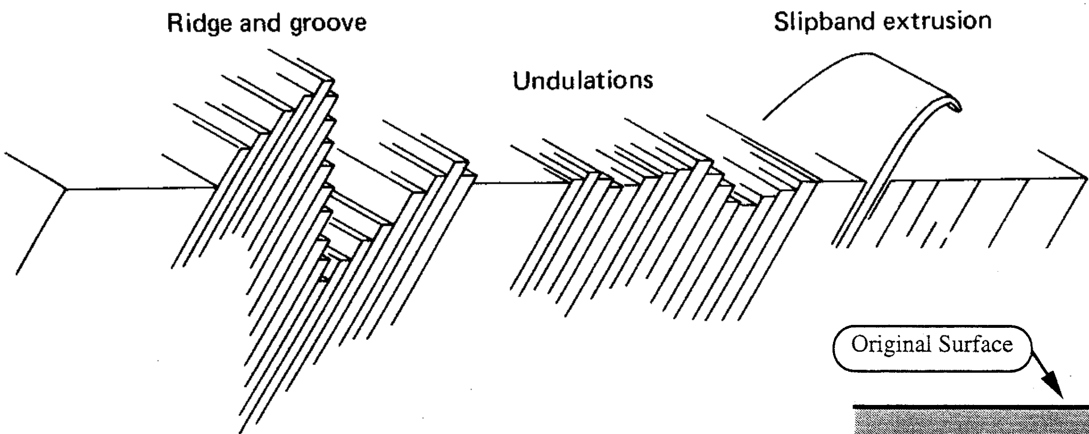
Fatigue is not cause of failure per se but leads to the final fracture event.

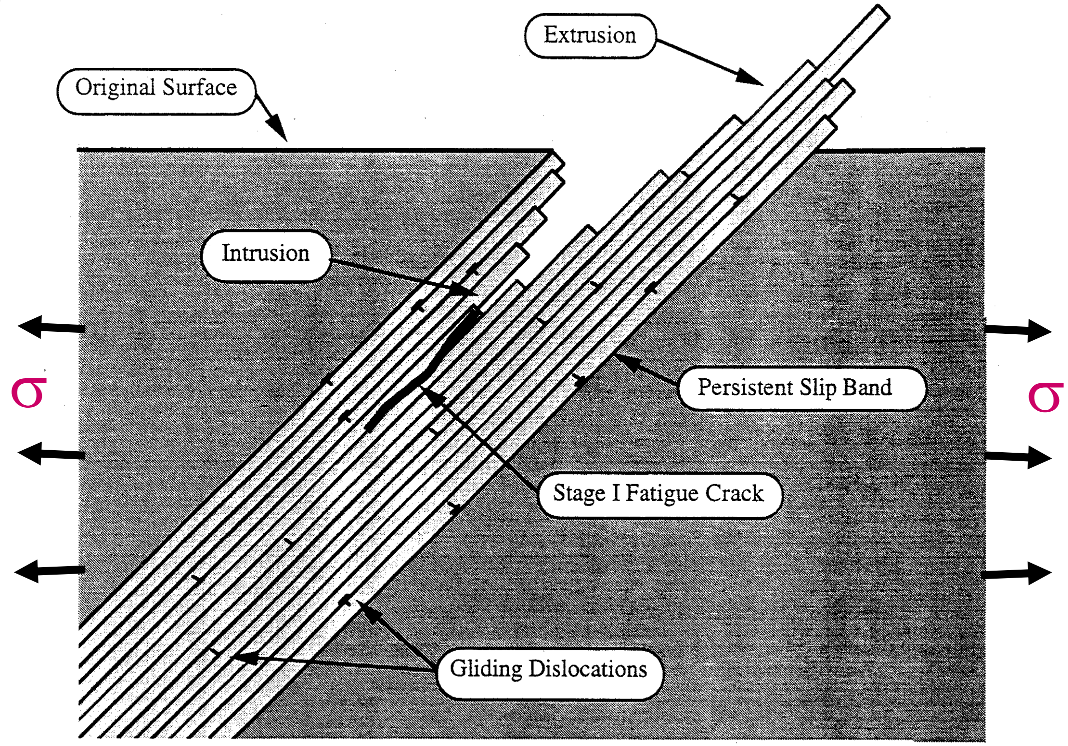
The Broad Field of Fracture Mechanics



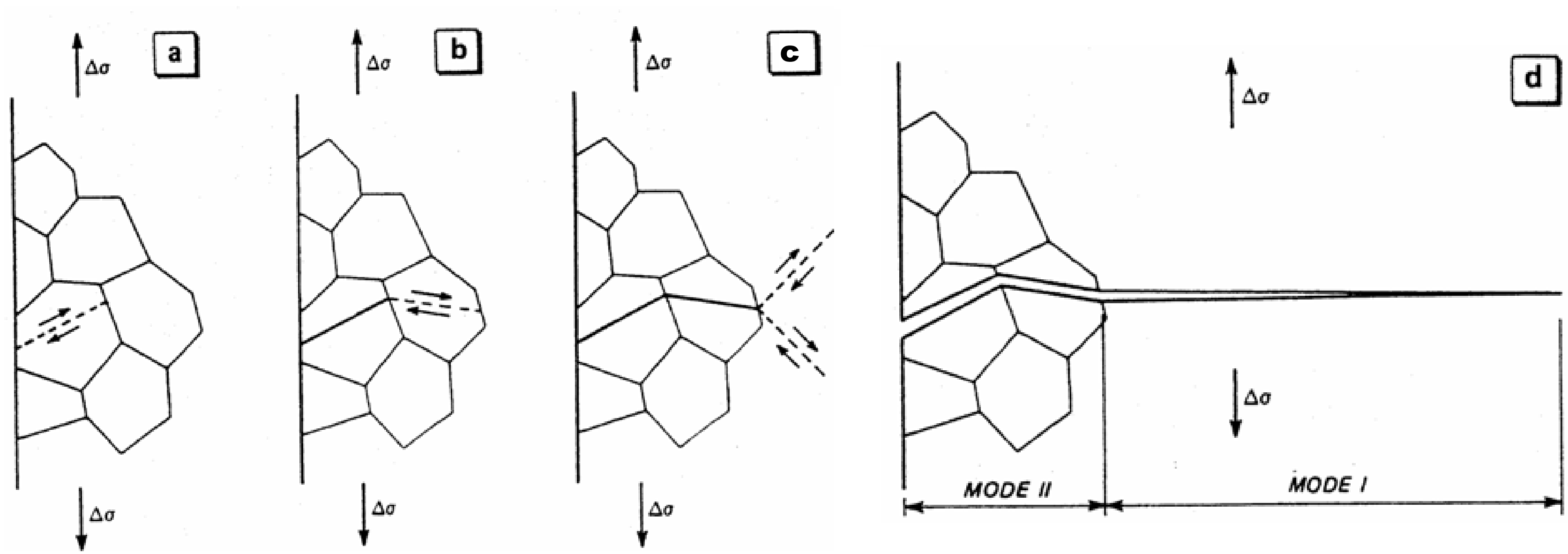
Intrusions and Extrusions:

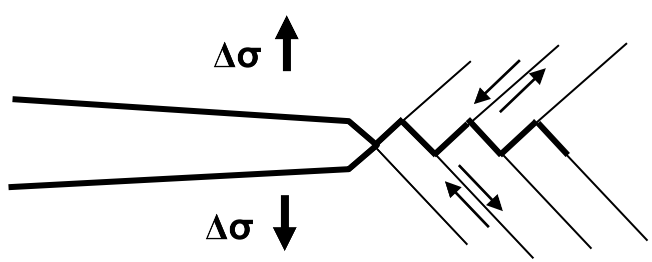
The Early Stages of Fatigue Crack Formation





Schematic of Fatigue Crack Initiation Subsequent Growth Corresponding and Transition From Mode II to Mode I





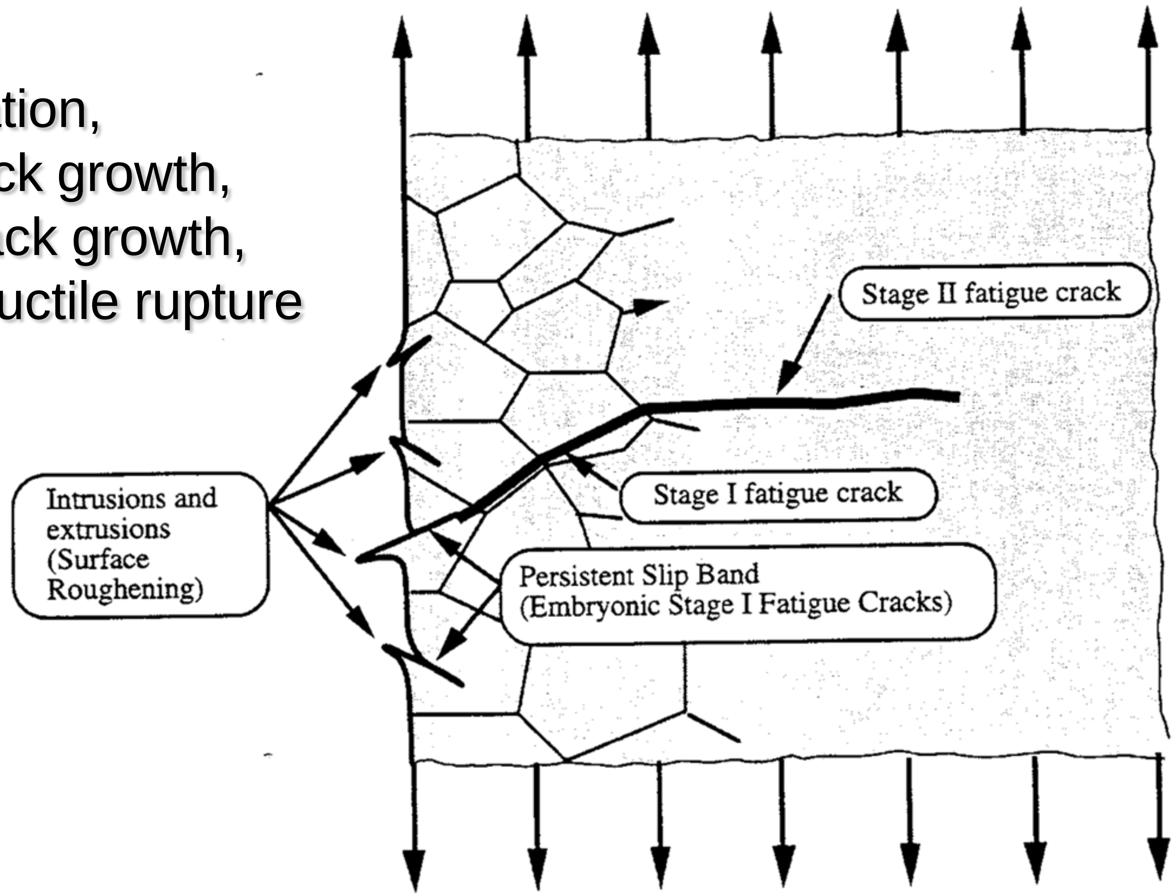
Locally, the crack grows in shear;

macroscopically it grows in tension.

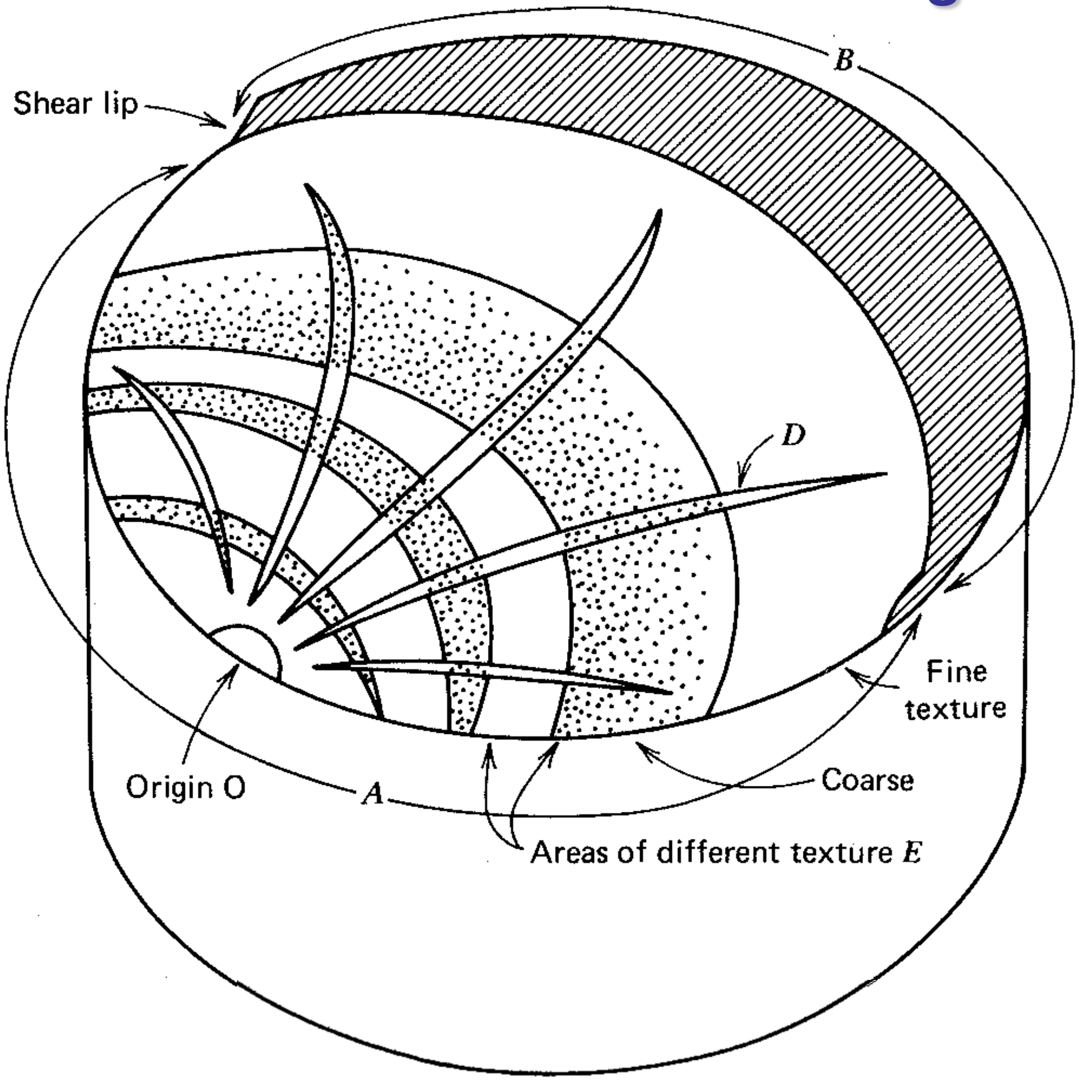
The Process of Fatigue

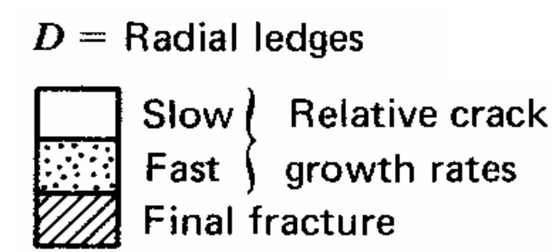
The Materials Science Perspective:

- Cyclic slip,
- Fatigue crack initiation,
- Stage I fatigue crack growth,
- Stage II fatigue crack growth,
- Brittle fracture or ductile rupture



Features of the Fatigue Fracture Surface of a Typical Ductile Metal Subjected to Variable Amplitude Cyclic Loading



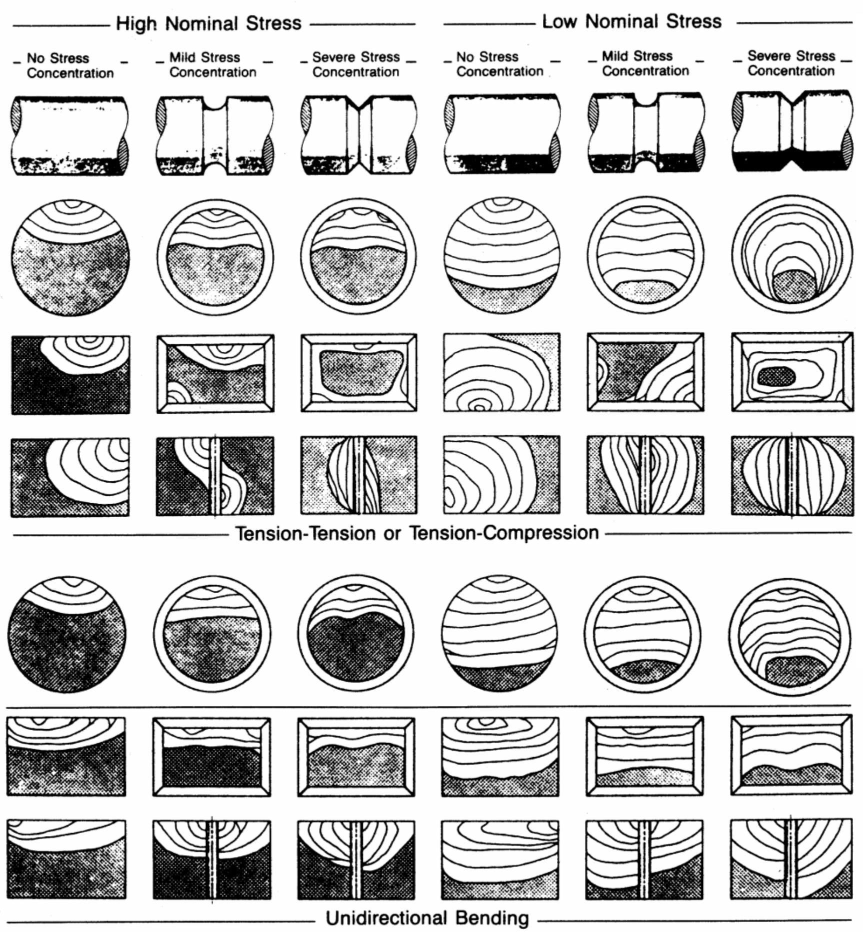


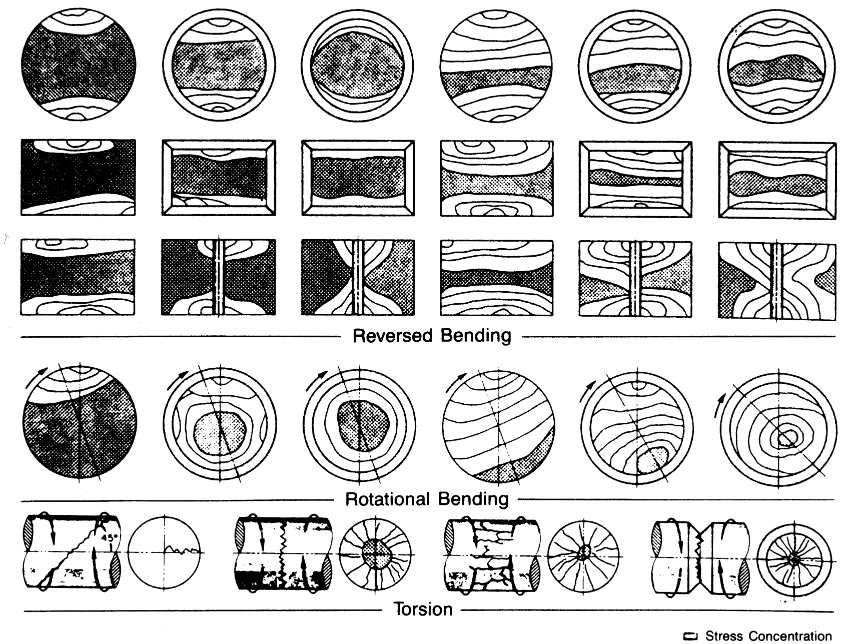
A – fatigue crack area

B – area of the final static failure

(Collins, ref. 22)

Appearance of Failure Surfaces Caused by Various Modes of Loading *(SAE Handbook)*





Factors Influencing Fatigue Life

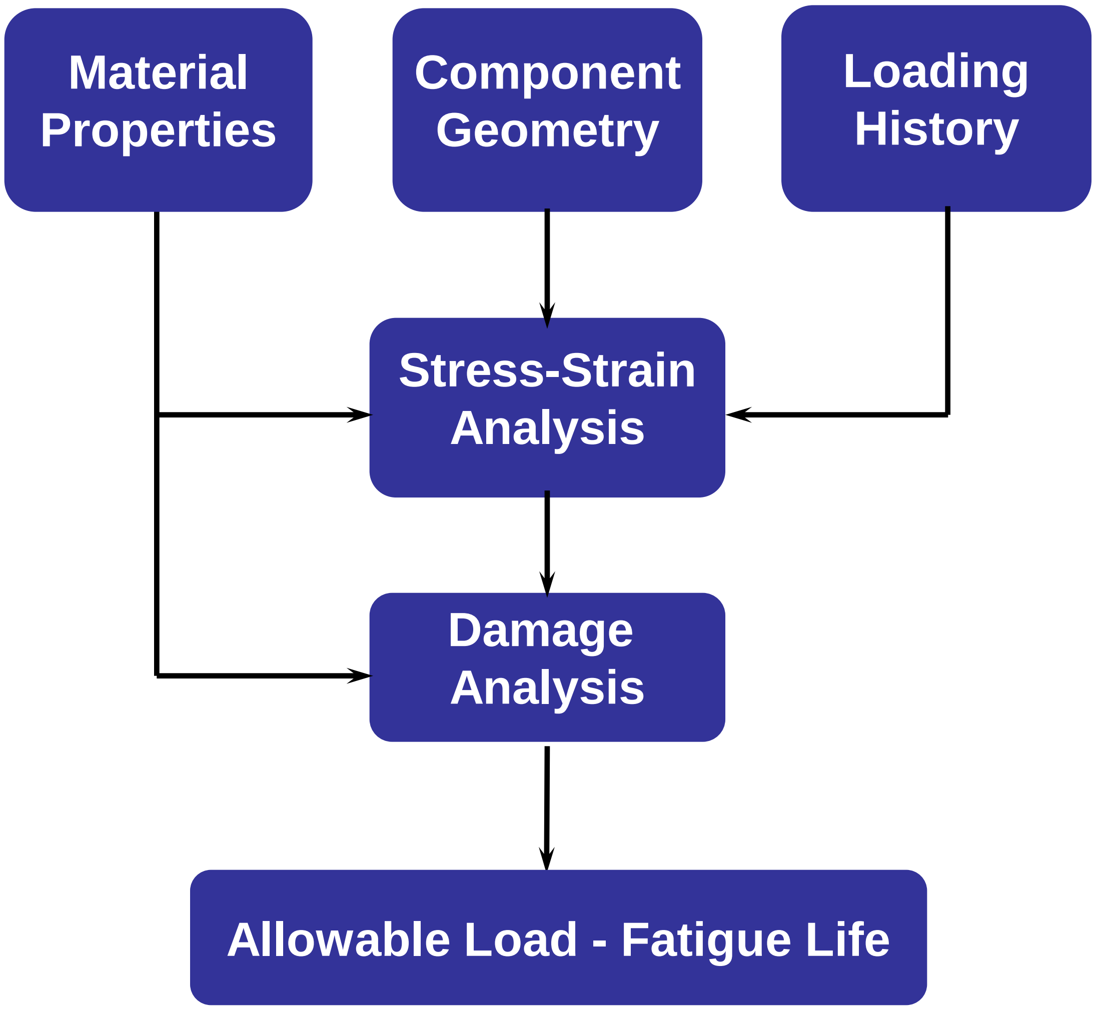
Applied Stresses

- **Stress range** – The basic cause of plastic deformation and consequently the accumulation of damage
- **Mean stress** – Tensile mean and residual stresses aid to the formation and growth of fatigue cracks
- **Stress gradients** – Bending is a more favorable loading mode than axial loading because in bending fatigue cracks propagate into the region of lower stresses

Materials

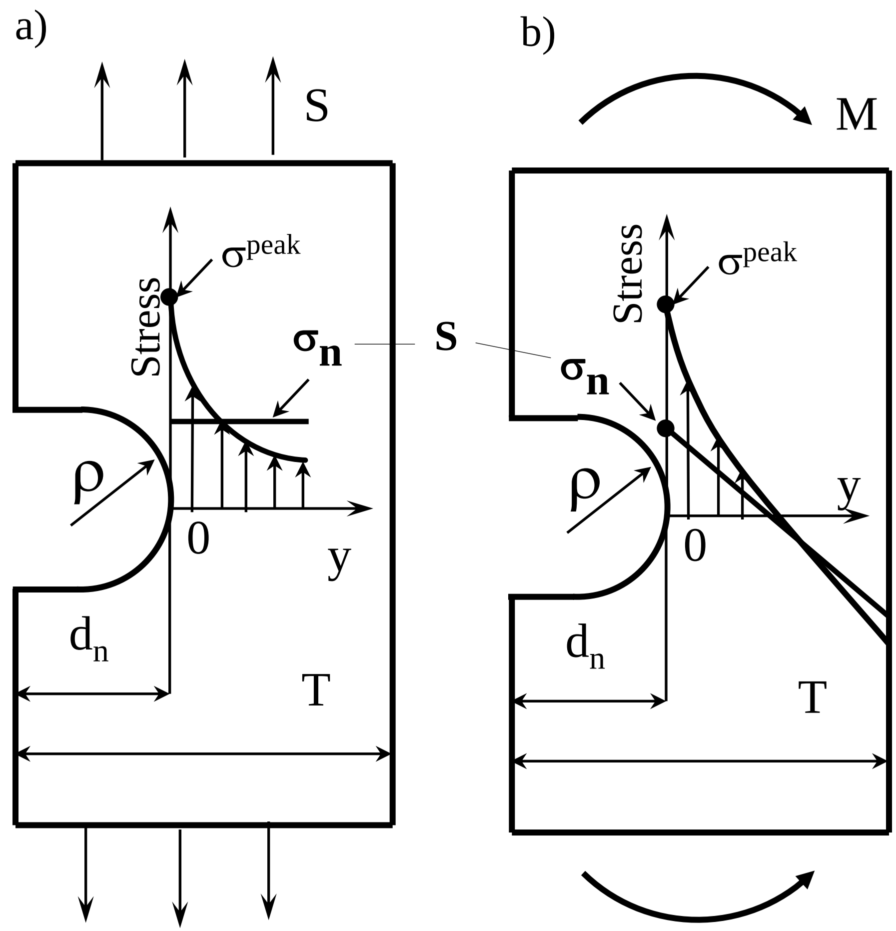
- **Tensile and yield strength** – Higher strength materials resist plastic deformation and hence have a higher fatigue strength at long lives. Most ductile materials perform better at short lives
- **Quality of material** – Metallurgical defects such as inclusions, seams, internal tears, and segregated elements can initiate fatigue cracks
- **Temperature** – Temperature usually changes the yield and tensile strength resulting in the change of fatigue resistance (high temperature decreases fatigue resistance)
- **Frequency** (rate of straining) – At high frequencies, the metal component may be self-heated.

Strength-Fatigue Analysis Procedure



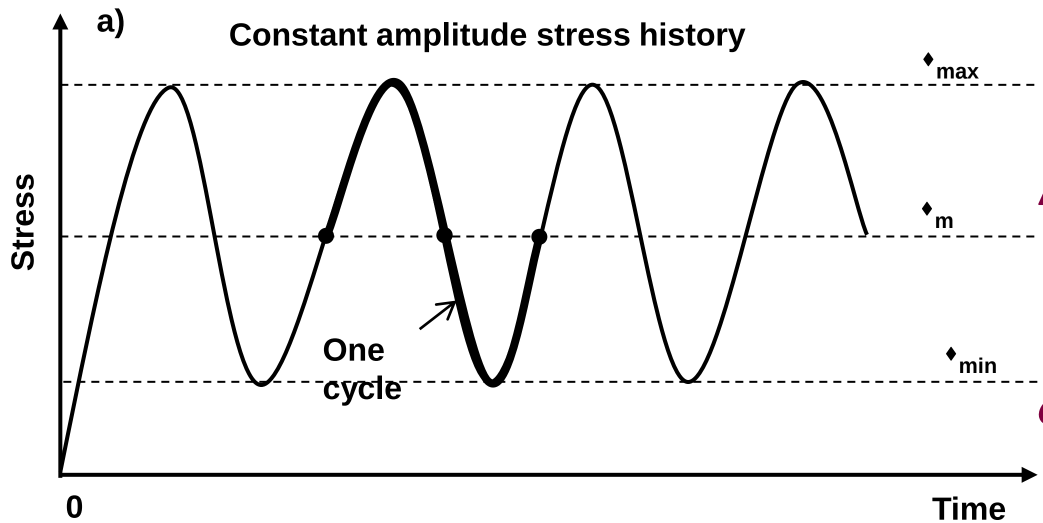
Information path in strength and fatigue life prediction procedures

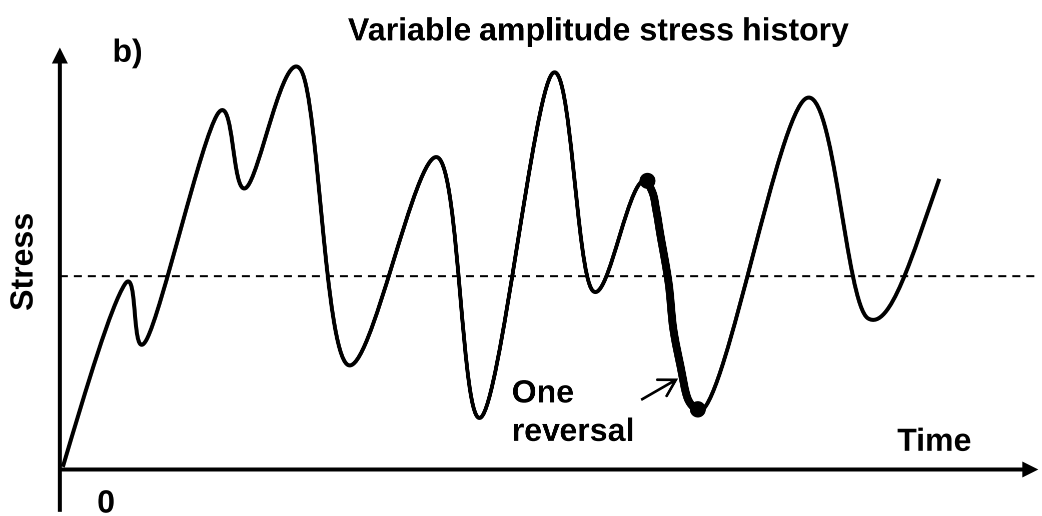
Stress Parameters Used in Static Strength and Fatigue Analyses



$$K_t = \frac{\sigma^{peak}}{\sigma_n} = \frac{\sigma^{peak}}{S}$$

Constant and Variable Amplitude Stress Histories;
Definition of a Stress Cycle & Stress Reversal





In the case of the peak stress history the important parameters are:

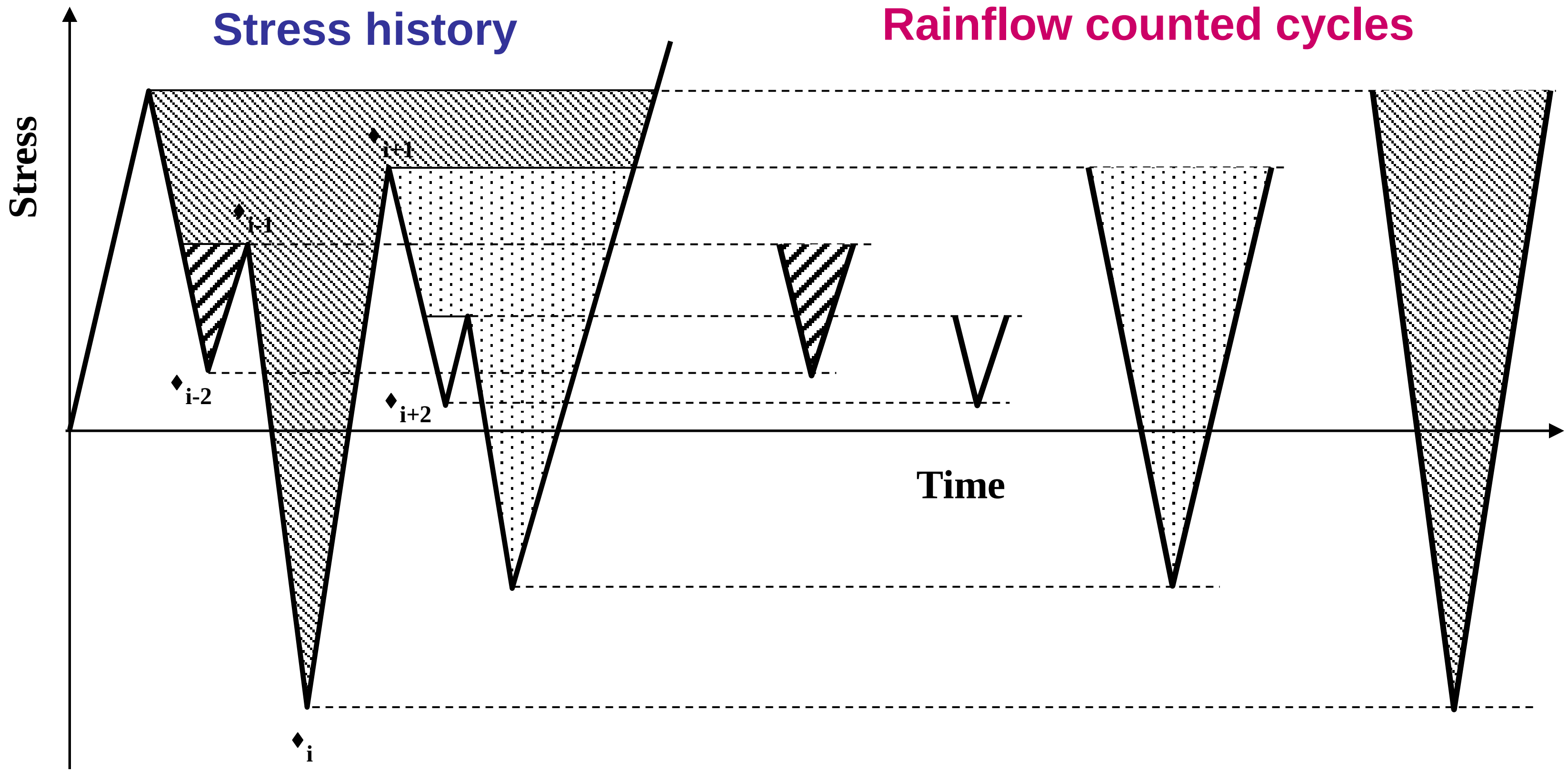
$$\Delta\sigma^{peak} = \sigma_{max}^{peak} - \sigma_{min}^{peak};$$

$$\sigma_a^{peak} = \frac{\Delta\sigma^{peak}}{2} = \frac{\sigma_{max}^{peak} - \sigma_{min}^{peak}}{2}$$

$$\sigma_m^{peak} = \frac{\sigma_{max}^{peak} + \sigma_{min}^{peak}}{2};$$

$$R = \frac{\sigma_{min}^{peak}}{\sigma_{max}^{peak}}$$

Stress History and the “Rainflow” Counted Cycles



A *rainflow counted cycle* is identified when any two adjacent reversals in thee stress history satisfy the following relation:

$$ABS\left|\sigma_{i-1} - \sigma_i\right| < ABS\left|\sigma_i - \sigma_{i+1}\right|$$

The Mathematics of the Cycle Rainflow Counting Method for Fatigue Analysis of Stress/Load Histories

A *rainflow counted cycle* is identified when any two adjacent reversals in thee stress history satisfy the following relation:

$$ABS\left|\sigma_{i-1} - \sigma_i\right| < ABS\left|\sigma_i - \sigma_{i+1}\right|$$

The stress amplitude of such a cycle is:

$$\sigma_a = \frac{ABS\left|\sigma_{i-1} - \sigma_i\right|}{2}$$

The stress range of such a cycle is:

$$\Delta\sigma = ABS\left|\sigma_{i-1} - \sigma_i\right|$$

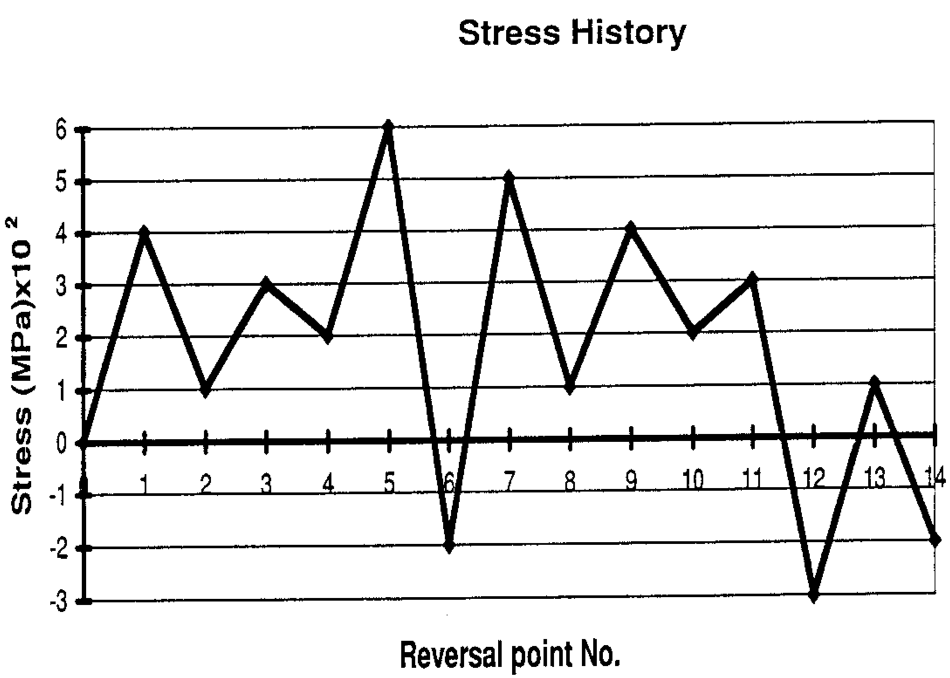
The mean stress of such a cycle is:

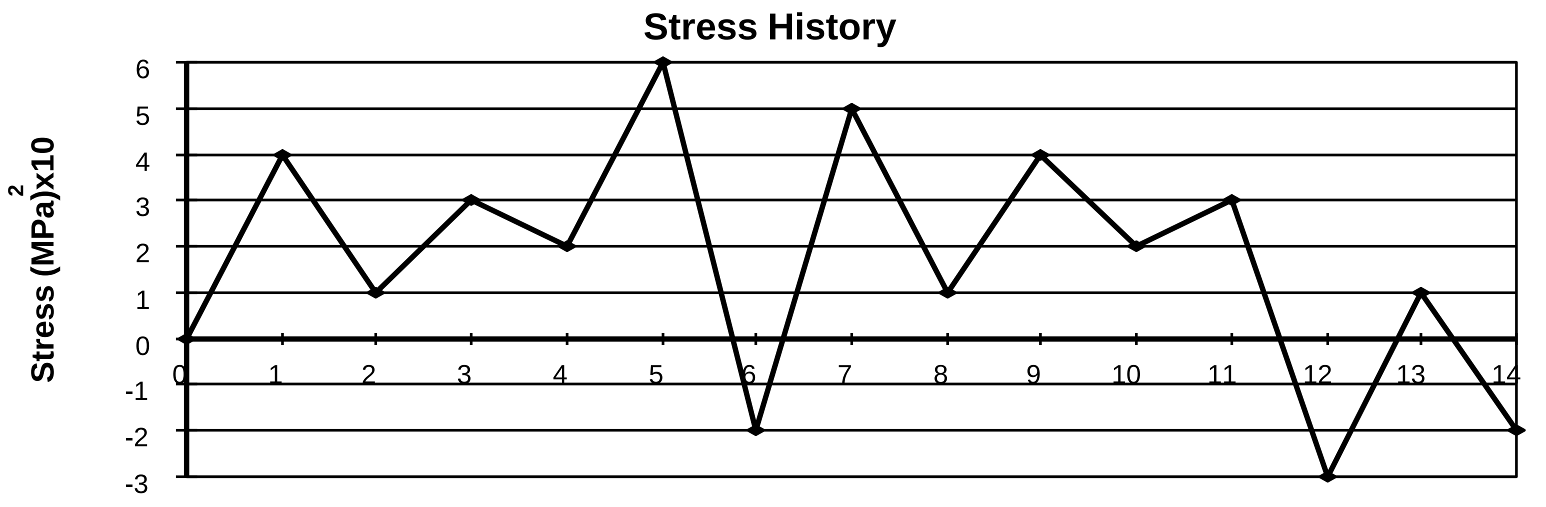
$$\sigma_m = \frac{\sigma_{i-1} + \sigma_i}{2}$$

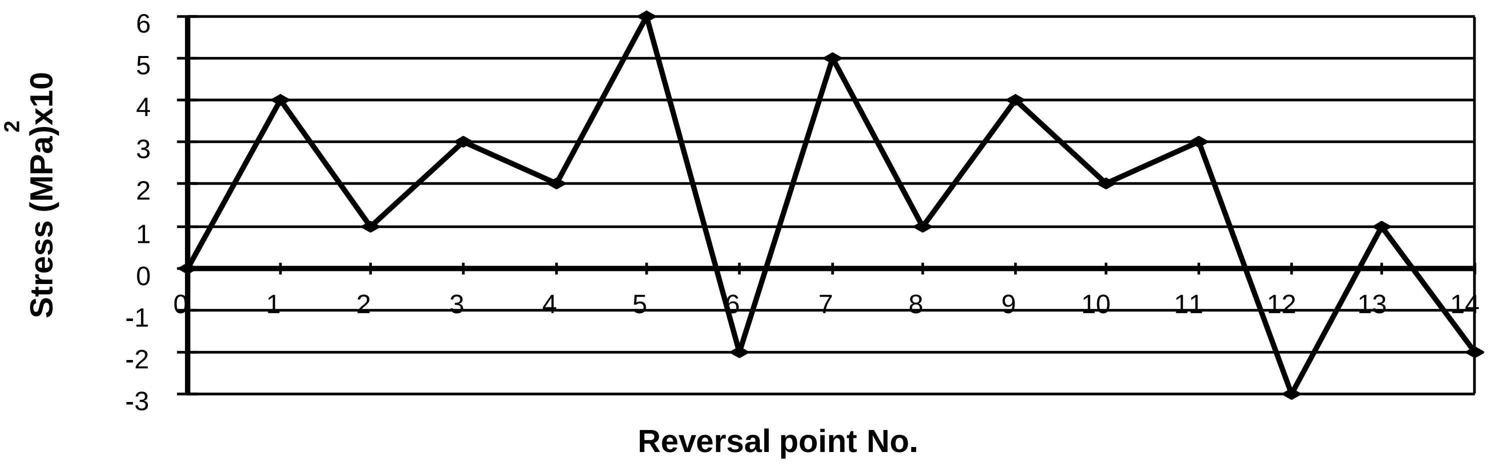
Example #1

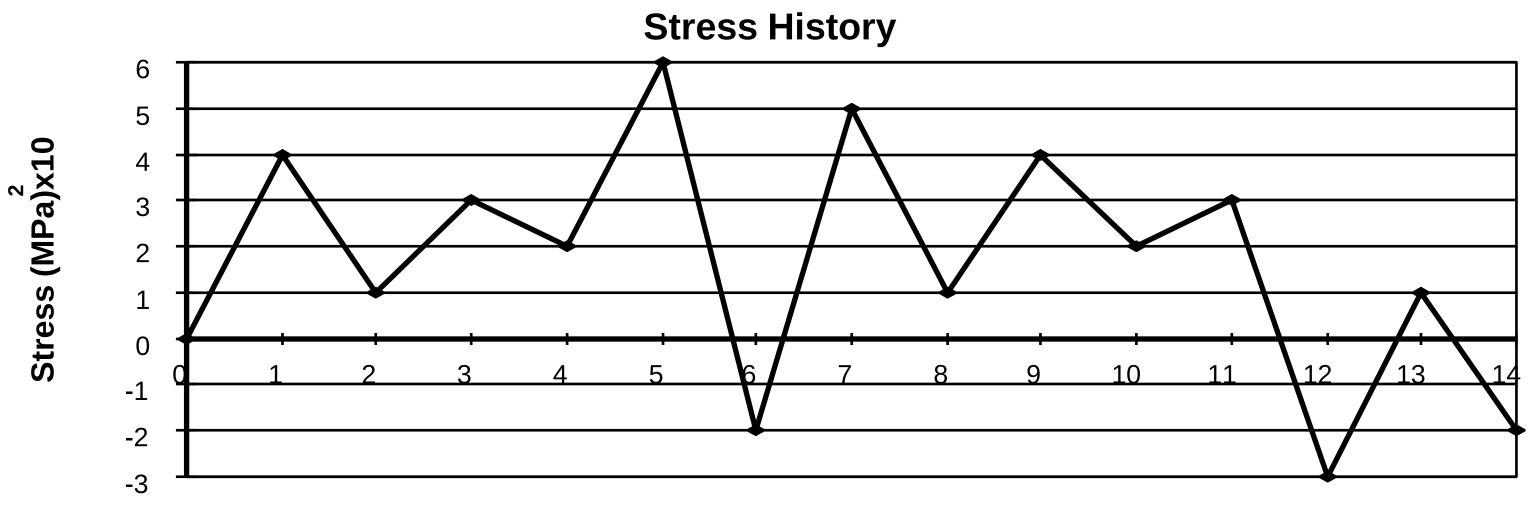
Determine stress ranges, ΔS_i, and corresponding mean stresses, $\Delta S_{m,i}$, for the stress history given below. Use the "rainflow" counting procedure.

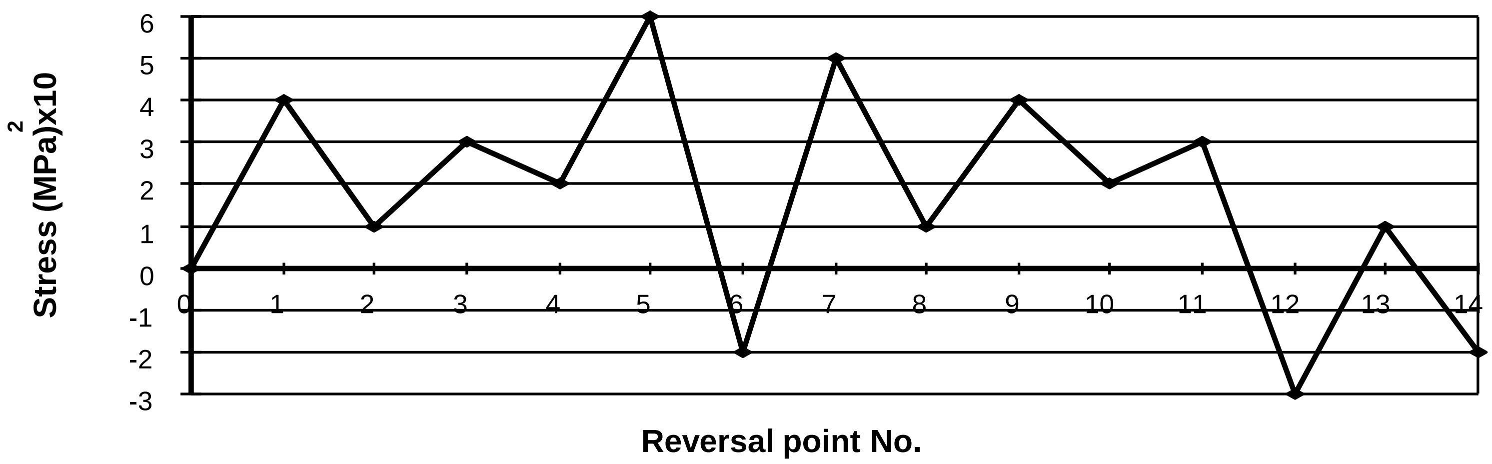
S_i = 0, 4, 1, 3, 2, 6, -2, 5, 1, 4, 2, 3, -3, 1, -2

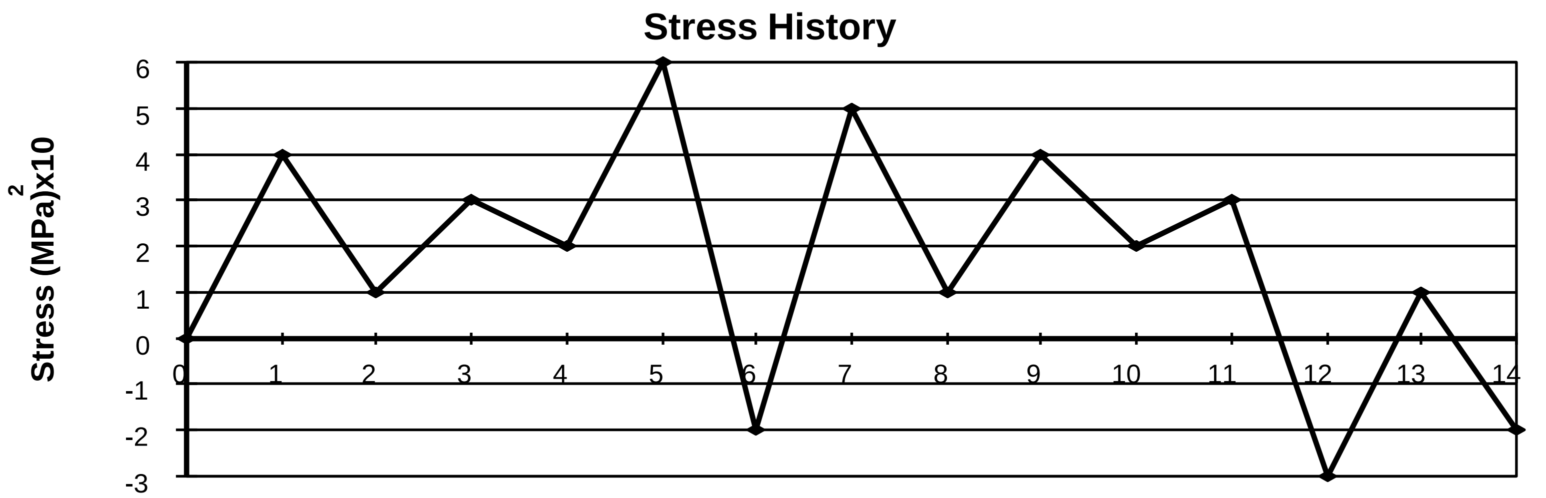


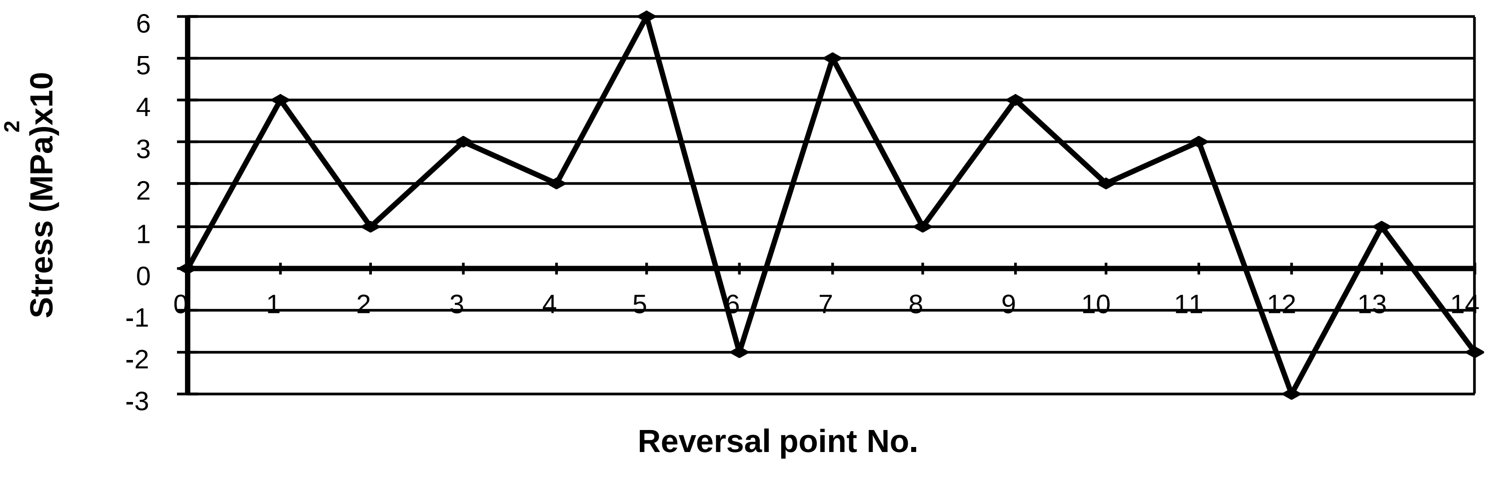

Stress History
Stress (MPa)x10^2
6
5
4
3
2
1
0
-1
-2
-3
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14


Stress (MPa)x10^2
6
5
4
3
2
1
0
-1
-2
-3
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
Reversal point No.

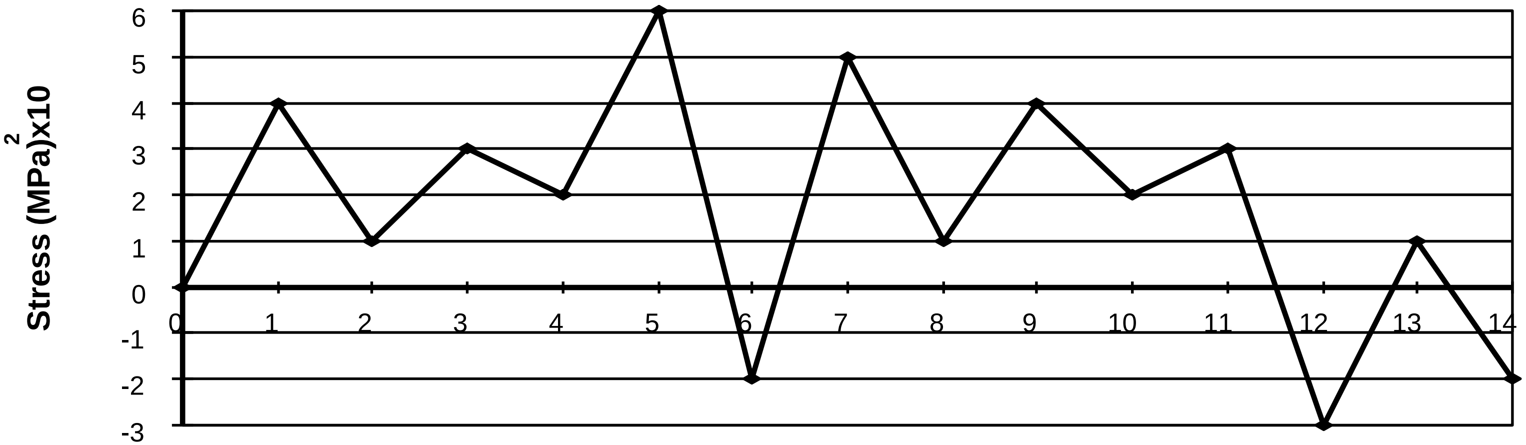

Stress History
Stress (MPa)x10 2
6
5
4
3
2
1
0
-1
-2
-3
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14

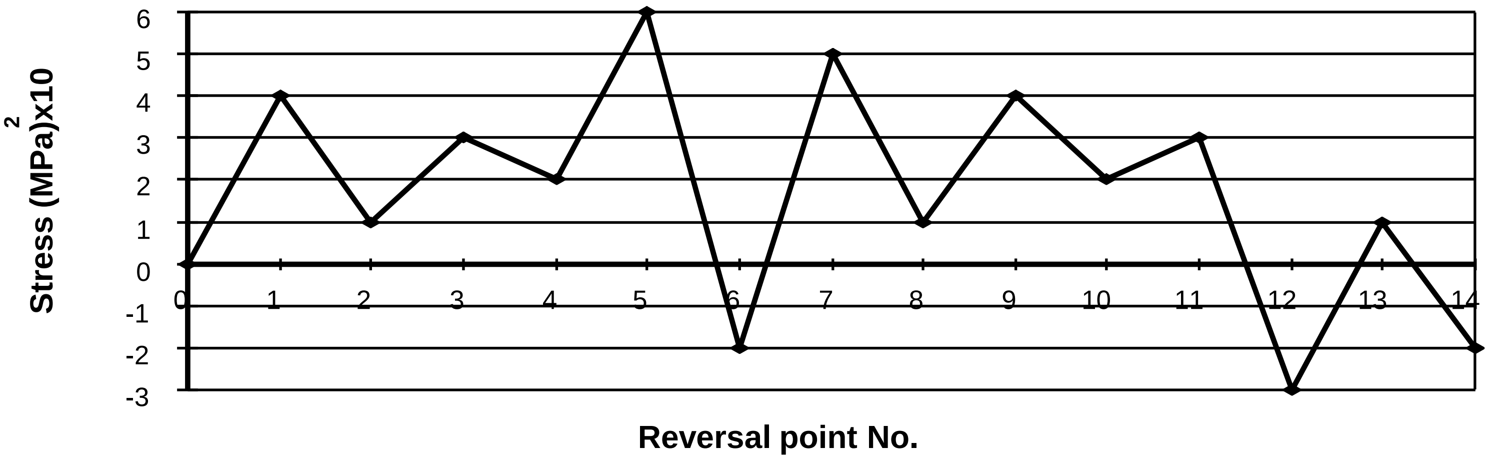

Stress (MPa)x10 2
6
5
4
3
2
1
0
-1
-2
-3
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
Reversal point No.

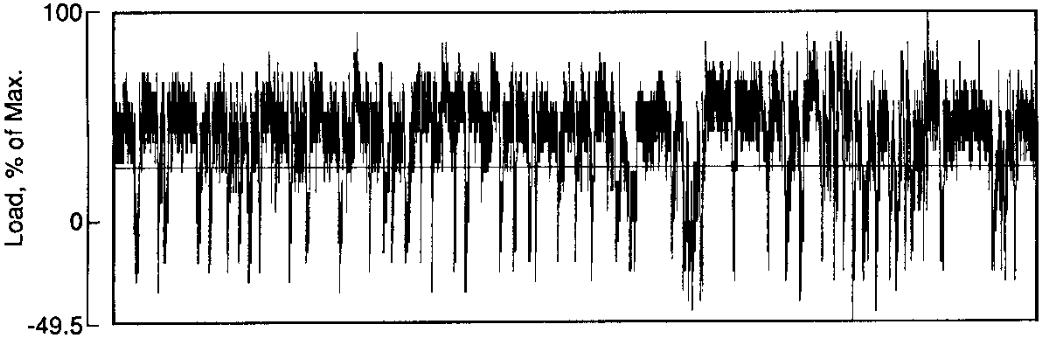

Stress History
Stress (MPa)x10 2
6
5
4
3
2
1
0
-1
-2
-3
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14


Stress (MPa)x10 2
6
5
4
3
2
1
0
-1
-2
-3
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
Reversal point No.

Stress History







Number of Cycles According to the Rainflow Counting Procedure *(N. Dowling, ref. 2)*

	Mean																			
Range	−15	−10	−5	0	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	All
20	4	1	5	2	2	5	—	—	3	6	15	27	29	32	22	12	6	2	—	173
25	2	4	3	9	8	10	4	6	2	7	17	37	36	43	33	13	7	1	2	244
30	1	1	5	3	1	1	4	3	—	4	13	20	20	23	20	8	6	1	—	134
35	1	1	4	2	3	2	—	1	3	2	8	17	16	11	11	7	2	—	—	91
40	—	1	1	1	2	1	1	—	—	4	7	15	16	9	8	2	—	—	—	68
45	—	1	—	4	3	—	—	—	—	2	1	9	7	2	3	1	—	—	—	33
50	—	—	2	2	2	1	—	—	—	2	2	3	3	1	1	1	1	—	—	21
55	—	—	1	1	—	—	—	—	—	2	2	4	4	2	—	1	—	1	—	18
60	—	1	1	—	—	—	—	—	—	1	1	3	2	1	—	—	—	—	—	10
65	—	—	—	—	—	—	—	—	—	—	2	1	—	—	—	—	—	—	—	3
70	—	—	—	—	—	—	—	—	—	—	2	—	1	—	—	—	—	—	—	3
75	—	—	—	—	—	—	1	—	—	—	1	2	—	—	—	—	—	—	—	4
80	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
85	—	—	—	—	1	—	1	3	3	—	—	—	—	—	—	—	—	—	—	8
90	—	—	—	—	—	—	—	—	4	—	—	—	—	—	—	—	—	—	—	4
95	—	—	—	—	—	1	—	1	4	1	—	—	—	—	—	—	—	—	—	7
100	—	—	—	—	—	—	—	5	3	1	—	—	—	—	—	—	—	—	—	9
105	—	—	—	—	—	—	—	3	3	3	—	—	—	—	—	—	—	—	—	9
110	—	—	—	—	—	—	—	—	2	3	—	—	—	—	—	—	—	—	—	5
115	—	—	—	—	—	—	—	—	3	—	—	—	—	—	—	—	—	—	—	3
120	—	—	—	—	—	—	1	—	1	1	—	—	—	—	—	—	—	—	—	3
125	—	—	—	—	—	—	—	—	2	—	—	—	—	—	—	—	—	—	—	2
130	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
135	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	1
140	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—

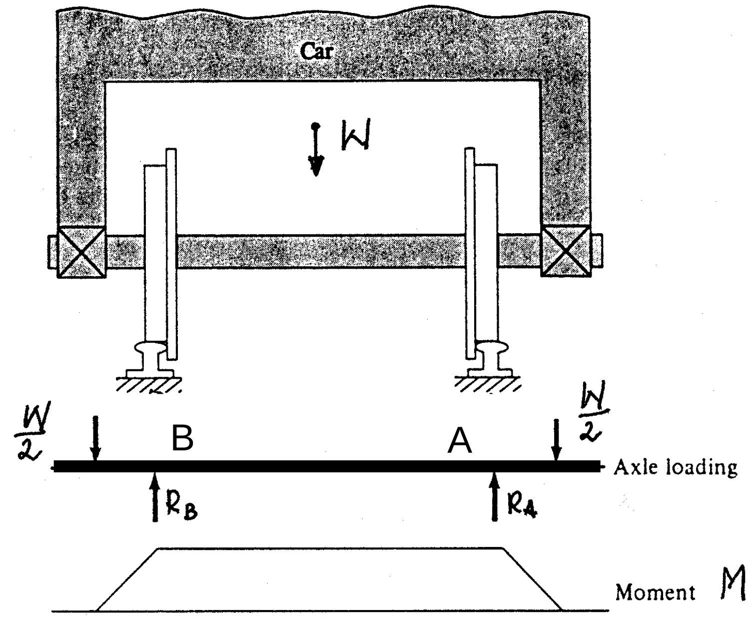
The Fatigue S-N method
(Nominal Stress Approach)

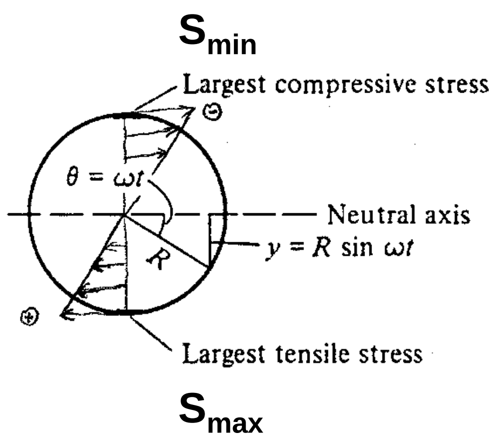
- The principles of the S-N approach (the nominal stress method)
- Fatigue damage accumulation
- Significance of geometry (notches) and stress analysis in fatigue evaluations of engineering structures
-
- Fatigue life prediction in the design process

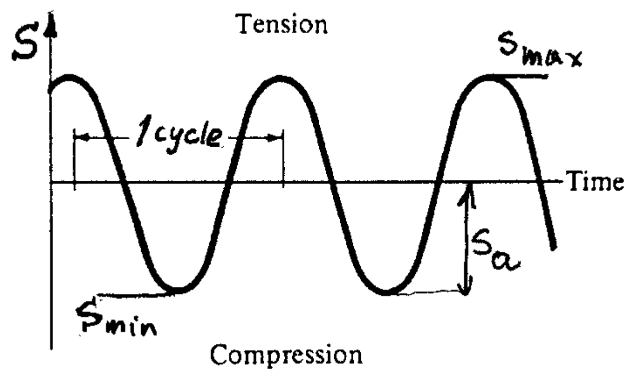
Wöhler's Fatigue Test

Note! In the case of smooth components such as the railway axle the nominal stress and the local peak stress are the same!

$$S = \sigma^{peak}$$

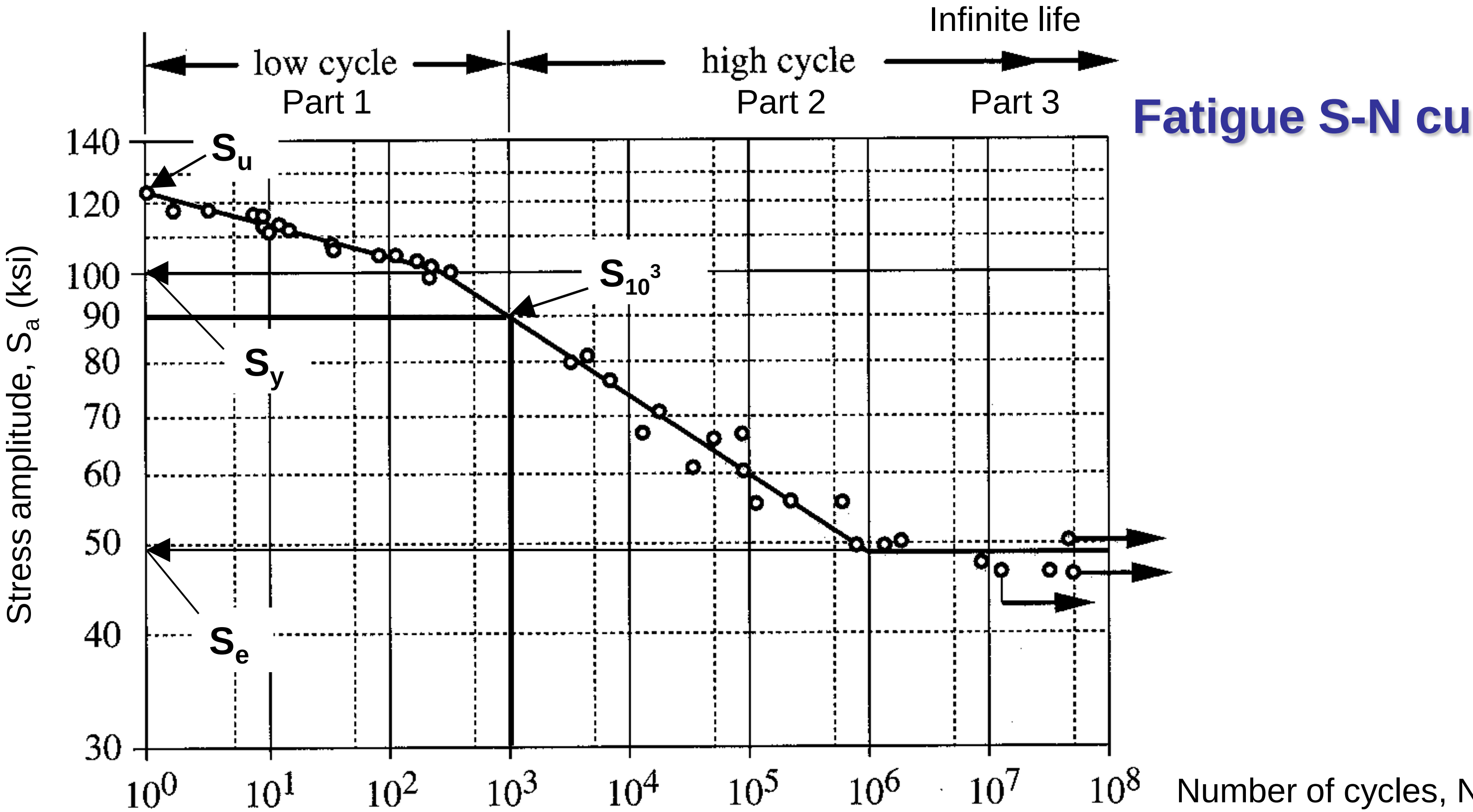






Railroad car axle. (*a*) Loading and moment diagrams. (*b*) Stress on axle. (*c*) Stress variation with time.

Fatigue S-N curve



Fully reversed axial S-N curve for AISI 4130 steel. Note the break at the LCF/HCF transition and the endurance limit

<u>Characteristic parameters of the S - N curve are:</u>

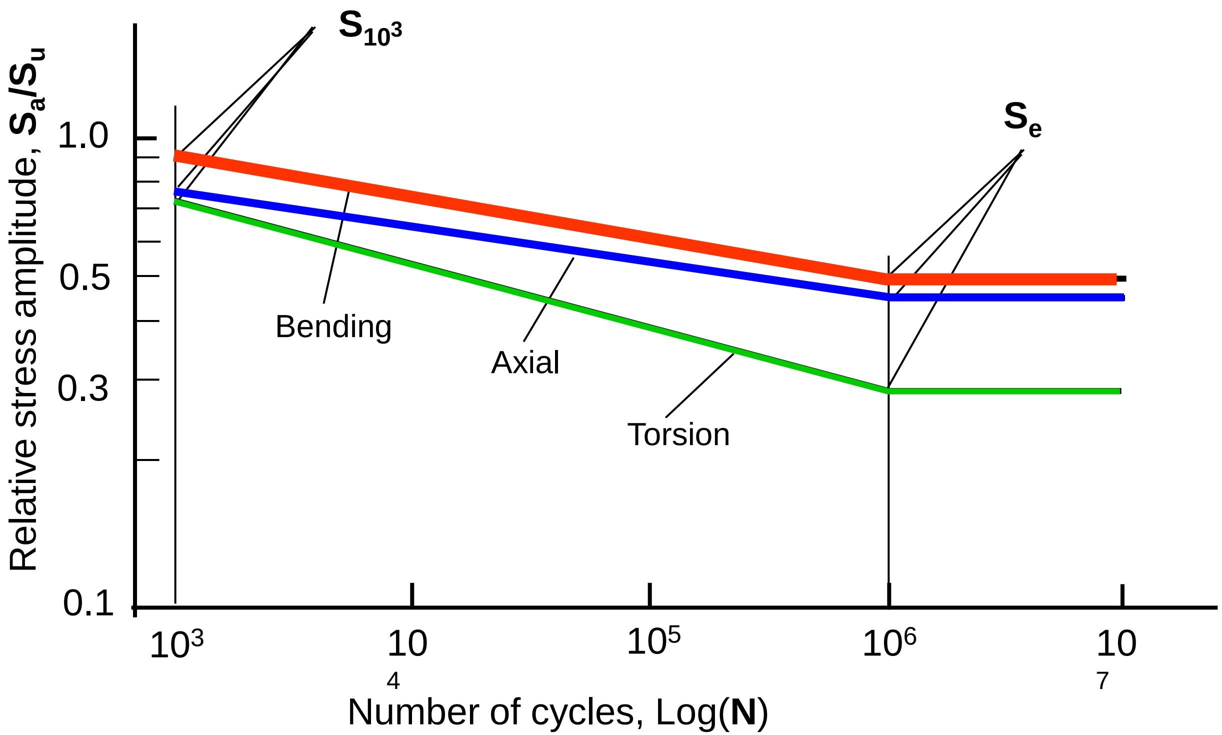
S_e - fatigue limit corresponding to N = 1 or 2×10^6 cycles for steels and N = 10^8 cycles for aluminum alloys,

S_{10^3} - fully reversed stress amplitude corresponding to N = 10^3 cycles

m - slope of the high cycle regime curve (Part 2)

$$S_a = C \cdot N^m = 10^A \cdot N^m$$

Most of available **S - N** fatigue data has been obtained from fully reversed rotational bending tests. However, material behavior and the resultant **S - N** curves are different for different types of loading. It concerns in particular the fatigue limit $\mathbf{S_e}$.



The stress endurance limit, $\mathbf{S_e}$, of **steels** (at 10^6 cycles) and the fatigue strength, $\mathbf{S_{10^3}}$ corresponding to 10^3 cycles for three types of loading can be approximated as (ref. 1, 23, 24):

$S_{10^3} = 0.90S_u$ and $S_e = S_{10^6} = 0.5\ S_u$ - bending

$S_{10^3} = 0.75S_u$ and $S_e = S_{10^6} = 0.35 - 0.45S_u$ - axial

$S_{10^3} = 0.72S_u$ and $S_e = S_{10^6} = 0.29\ S_u$ - torsion

Approximate endurance limit for various materials:

Magnesium alloys (at 10^8 cycles) $S_e = 0.35Su$

Copper alloys (at 10^8 cycles) $0.25S_u < S_e < 0.50S_u$

Nickel alloys (at 10^8 cycles) $0.35S_u < S_e < 0.50S_u$

Titanium alloys (at 10^7 cycles) $0.45S_u < S_e < 0.65S_u$

Al alloys (at $5x10^8$ cycles) $S_e = 0.45S_u$ (if $S_u \leq 48$ ksi) or $S_e = 19$ ksi (if $S_u > 48$ ksi)

Steels (at 10^6 cycles) $S_e = 0.5S_u$ (if $S_u \leq 200$ ksi) or $S_e = 100$ ksi (if $S_u > 200$ ksi)

Irons (at 106 cycles) $S_e = 0.4S_u$ (if $S_u \leq 60$ ksi) or $S_e = 24$ ksi (if $S_u > 60$ ksi)

S – N curve

$$S_a = C \cdot N^m = 10^A \cdot N^m \quad or \quad N = C^{-\frac{1}{m}} \left(S_a\right)^{\frac{1}{m}} = C^{-\frac{A}{m}} \left(S_a\right)^{\frac{1}{m}}$$

$$m = -\frac{1}{3} \log\left(\frac{S_{10^3}}{S_e}\right) \quad and \quad A = \log\left[\frac{\left(S_{10^3}\right)^2}{S_e}\right]$$

Fatigue Limit – Modifying Factors

For many years the emphasis of most fatigue testing was to gain an empirical understanding of the effects of various factors on the base-line S-N curves for ferrous alloys in the intermediate to long life ranges. The variables investigated include:

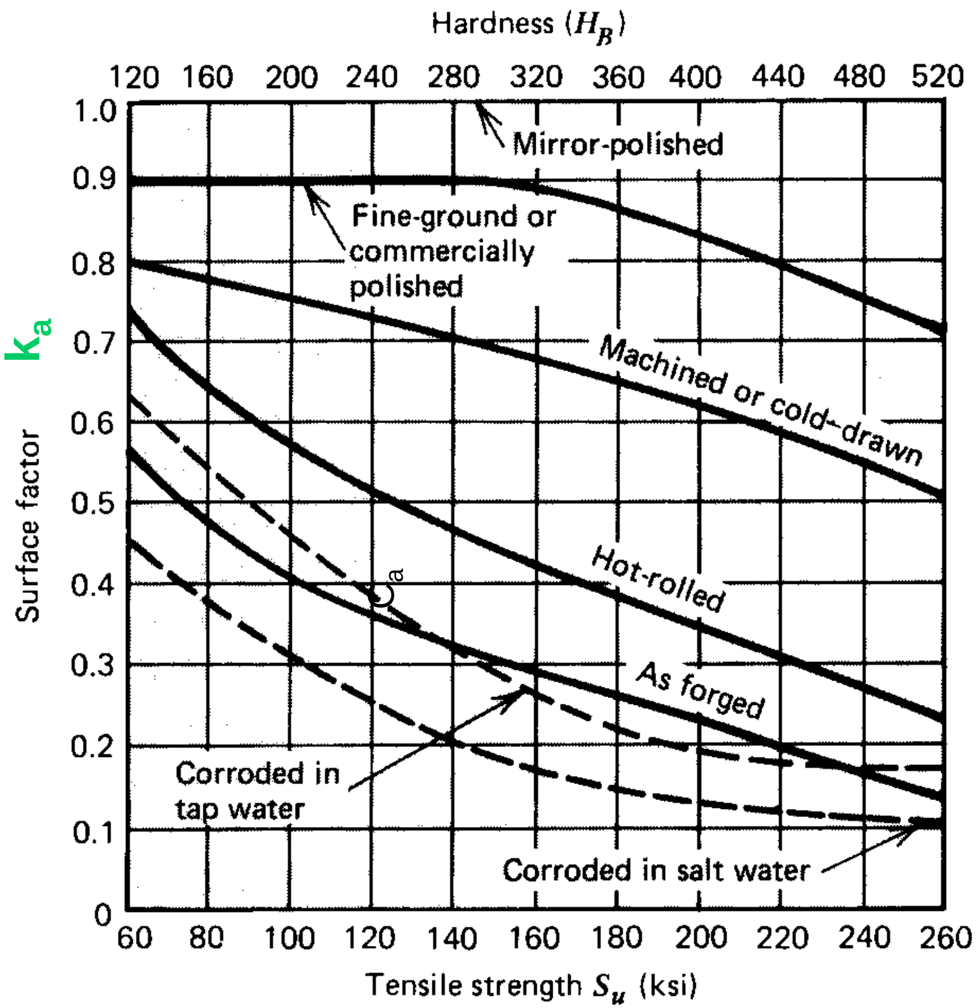
- **Rotational bending fatigue limit, S_e',**
- **Surface conditions, k_a,**
- **Size, k_b,**
- **Mode of loading, k_c,**
- **Temperature, k_d**
- **Reliability factor, k_e**
- **Miscellaneous effects (notch), k_f**

Fatigue limit of a machine part, S_e

$$S_e = k_a\, k_b\, k_c\, k_d\, k_e\, k_f \cdot S_e'$$

Surface Finish Effects on Fatigue Endurance Limit

The scratches, pits and machining marks on the surface of a material add stress concentrations to the ones already present due to component geometry. The correction factor for surface finish is sometimes presented on graphs that use a qualitative description of surface finish such as “polished” or “machined”.



Below a generalized empirical graph is shown which can be used to estimate the effect of surface finish in comparison with mirror-polished specimens [Shigley (23), Juvinal (24), Bannantine (1) and other textbooks].

Effect of various surface finishes on the fatigue limit of steel. Shown are values of the k_a, the ratio of the fatigue limit to that for polished specimens.

(from J. Bannantine, ref.1)

Size Effects on Endurance Limit

Fatigue is controlled by the weakest link of the material, with the probability of existence (or density) of a weak link increasing with material volume. The size effect has been correlated with the thin layer of surface material subjected to 95% or more of the maximum surface stress.

There are many empirical fits to the size effect data. A fairly conservative one is:

$$k_b = \frac{S_e}{S_e'} = \begin{cases} 1 & \text{if} \quad d \le 0.3\,in \\ 0.869 d^{-0.097} & \text{if} \quad 0.3\,in \le d \le 10.0\,in \end{cases}$$

or

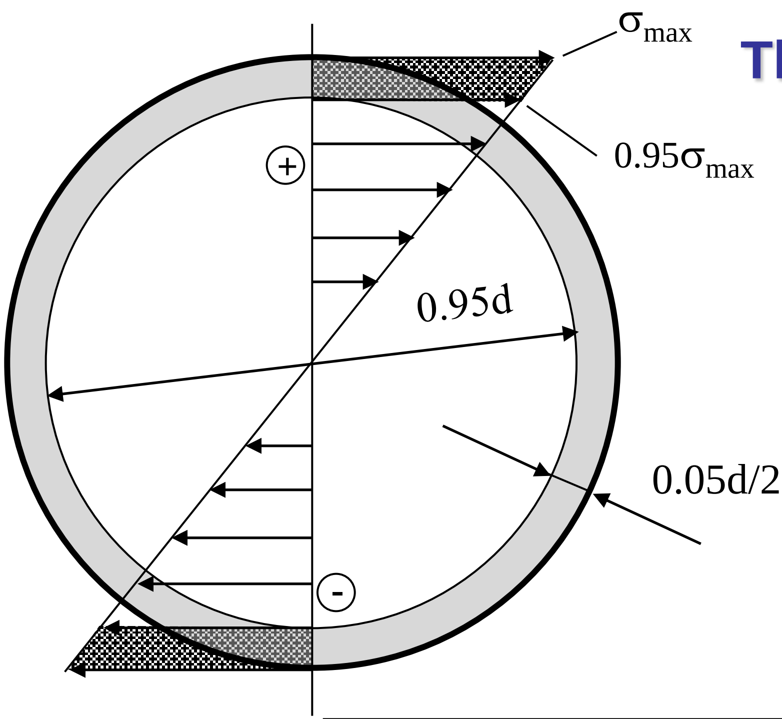
$$k_b = \frac{S_e}{S_e'} = \begin{cases} 1.0 & \text{if} \quad d \le 8\,mm \\ 1.189 d^{-0.097} & \text{if} \quad 8 \le d \le 250\,mm \end{cases}$$

- The size effect is seen mainly at very long lives.
- The effect is small in diameters up to 2.0 in (even in bending and torsion).

Stress effects in non-circular cross section members

In the case of non-circular members the approach is based on so called effective diameter, d_e.

The effective diameter, d_e, for non-circular cross sections is obtained by equating the volume of material stressed at and above 95% of the maximum stress to the same volume in the rotating-bending specimen.

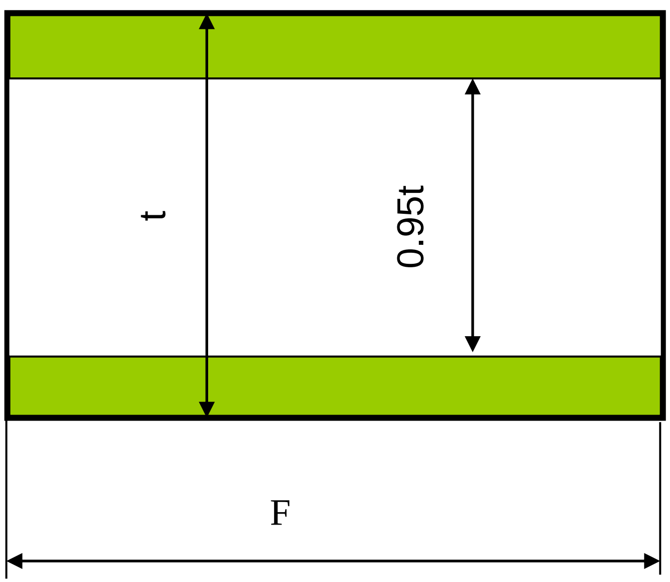


The effective diameter, d_e, for members with non-circular cross sections

The material volume subjected to stresses $\sigma \geq 0.95\sigma_{max}$ is concentrated in the ring of 0.05d/2 thick.

The surface area of such a ring is:

$$A_{0.95\sigma_{max}} = \frac{\pi}{4}\left[d^2 - (0.95d)^2\right] = 0.0766d^2$$



* rectangular cross section under bending

$$A = Ft$$

$$A_{0.95\sigma_{max}} = Ft - 0.95t = 0.05Ft$$

Equivalent diameter

$$0.0766d_e^2 = 0.05Ft$$

$$d_e = 0.808\sqrt{Ft}$$

Loading Effects on Endurance Limit

The ratio of endurance limits for a material found using axial and rotating bending tests ranges from 0.6 to 0.9.

$$S_{e(axial)} \approx (0.7 - 0.9) S_{e(bending)}$$

$$k_c = 0.7 - 0.9 \quad (suggested\ by\ Shigley\ k_c = 0.85)$$

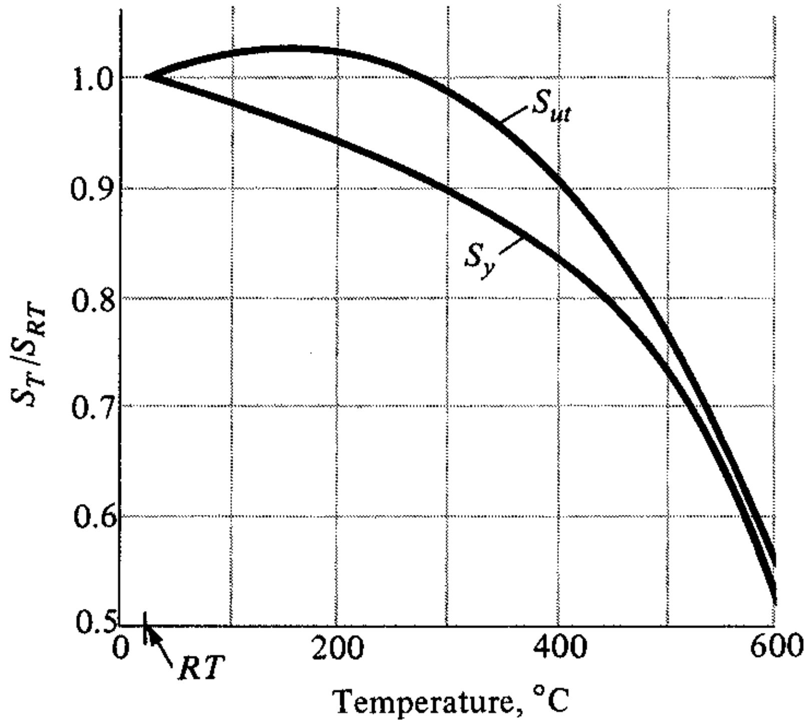
The ratio of endurance limits found using torsion and rotating bending tests ranges from 0.5 to 0.6. A theoretical value obtained from von Mises-Huber-Hencky failure criterion is been used as the most popular estimate.

$$\tau_{e(torsion)} \approx 0.577 S_{e(bending)}$$

$$k_c = 0.57 (suggested\ by\ Shigley\ k_c = 0.59)$$

Temperature Effect

A plot of the results of 145 tests of 21 carbon and alloy steels showing the effect of operating temperature on the yield strength S_y and the ultimate strength S_{ut}. The ordinate is the ratio of the strength at the operating temperature to the strength at room temperature. The standard deviations were $0.0442 \leq \hat{\sigma} \leq 0.152$ for S_y and $0.099 \leq \hat{\sigma} \leq 0.110$ for S_{ut}. *[Data source: E. A. Brandes (ed.), Smithells Metals Reference Book, 6th ed., Butterworth, London, 1983, pp. 22-128 to 22-131.]*



From: Shigley and Mischke, Mechanical Engineering Design, 2001

$$S_{e,T} = S_{e,RT}k_d = S_{e,RT}\frac{S_{u,T}}{S_{u,RT}}; \qquad k_d = \frac{S_{u,T}}{S_{u,RT}}$$

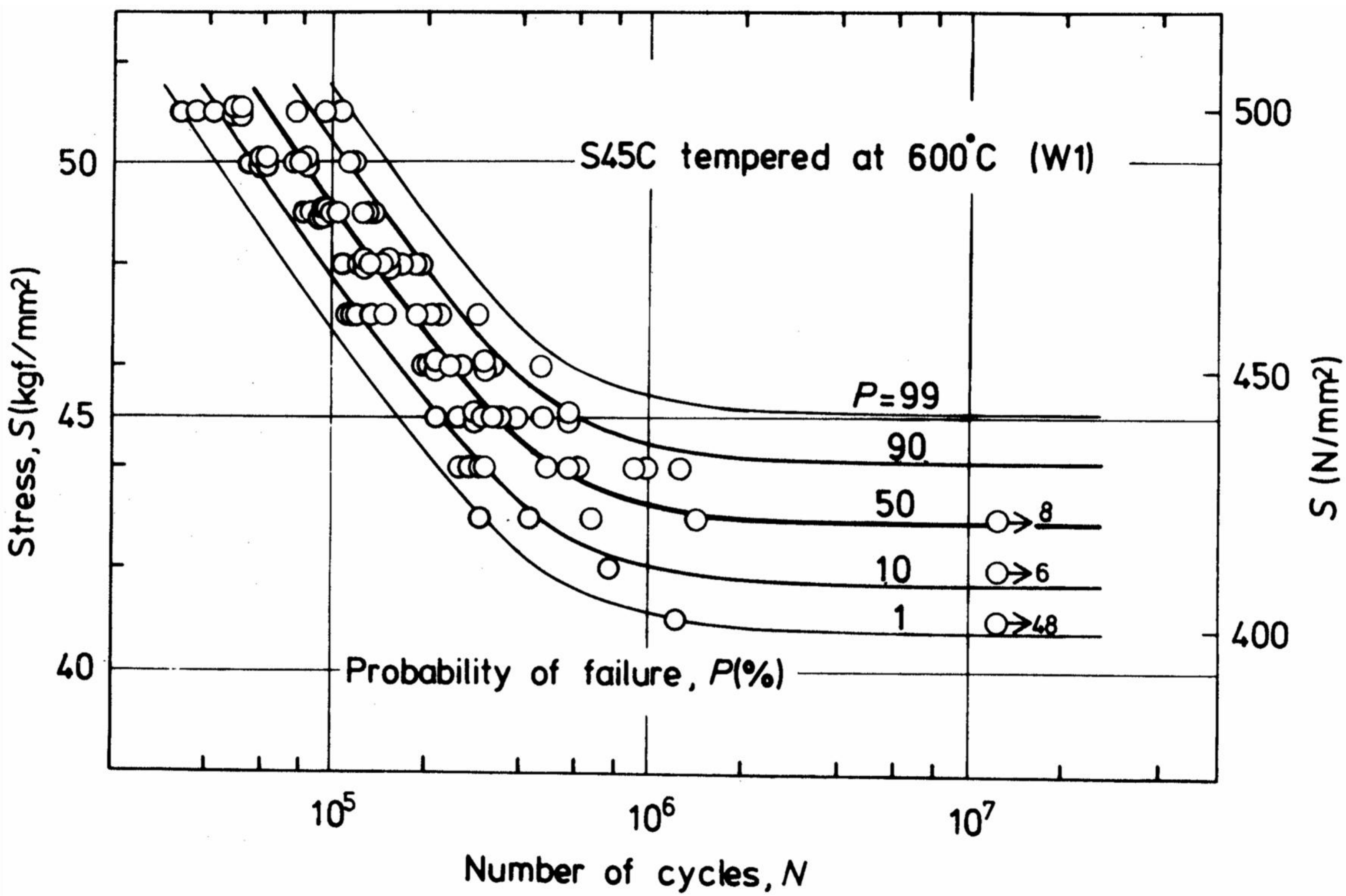
Reliability factor k_e

The reliability factor accounts for the scatter of reference data such as the rotational bending fatigue limit S_e'.

The estimation of the reliability factor is based on the assumption that the scatter can be approximated by the normal statistical probability density distribution.

$$k_e = 1 - 0.08 \times z_a$$

The values of parameter z_a associated with various levels of reliability can be found in Table 7-7 in the textbook by Shigley et.al.



S-N curves for assigned probability of failure; P - S - N curves

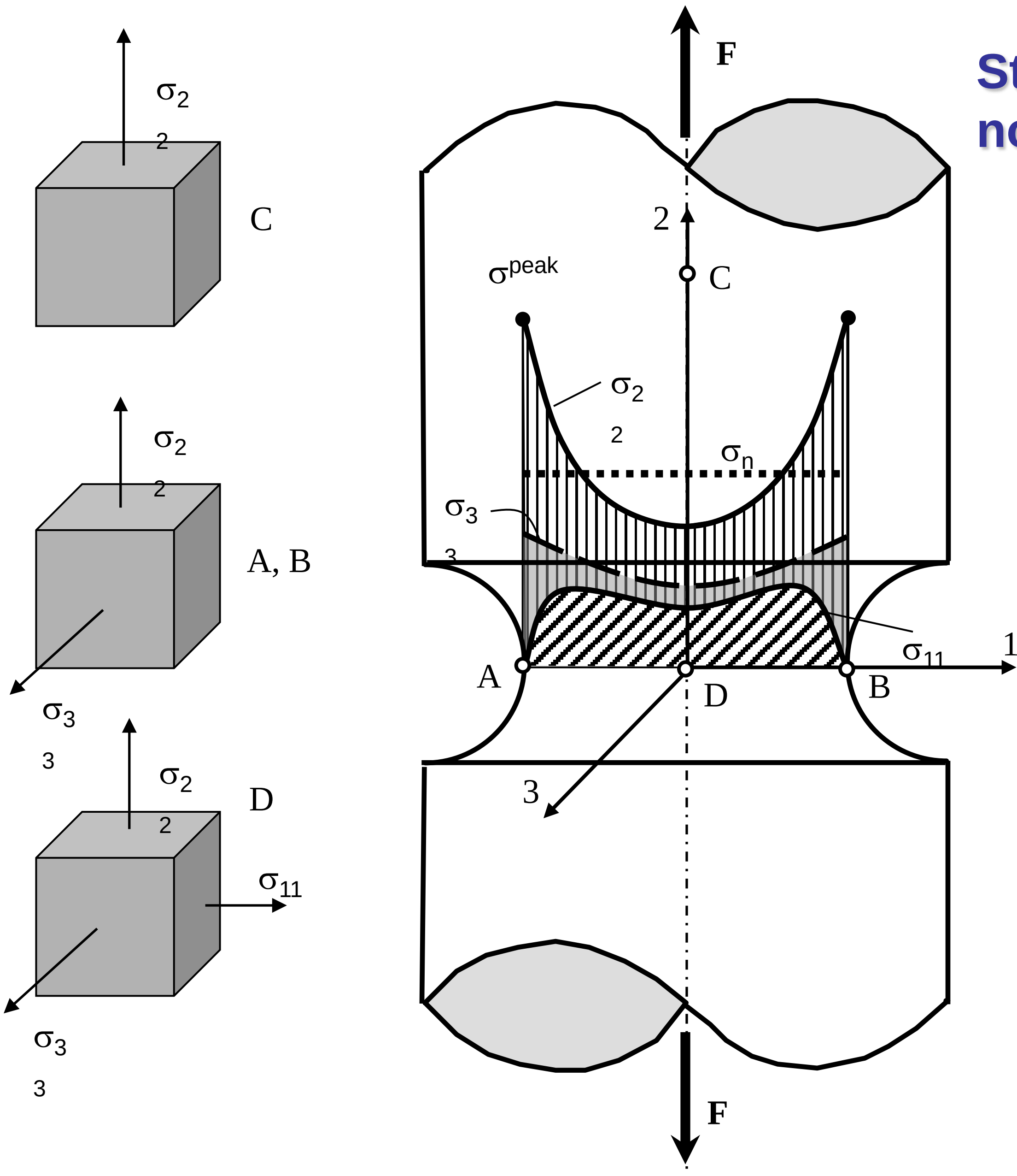
(source: S. Nishijima, ref. 39)

Stress concentration factor, K_t, and the notch factor effect, k_f

Fatigue notch factor effect k_f depends on the stress concentration factor K_t (geometry), scale and material properties and it is expressed in terms of the Fatigue Notch Factor K_f.

$$k_f = \frac{1}{K_f}$$

Stresses in axisymmetric notched body

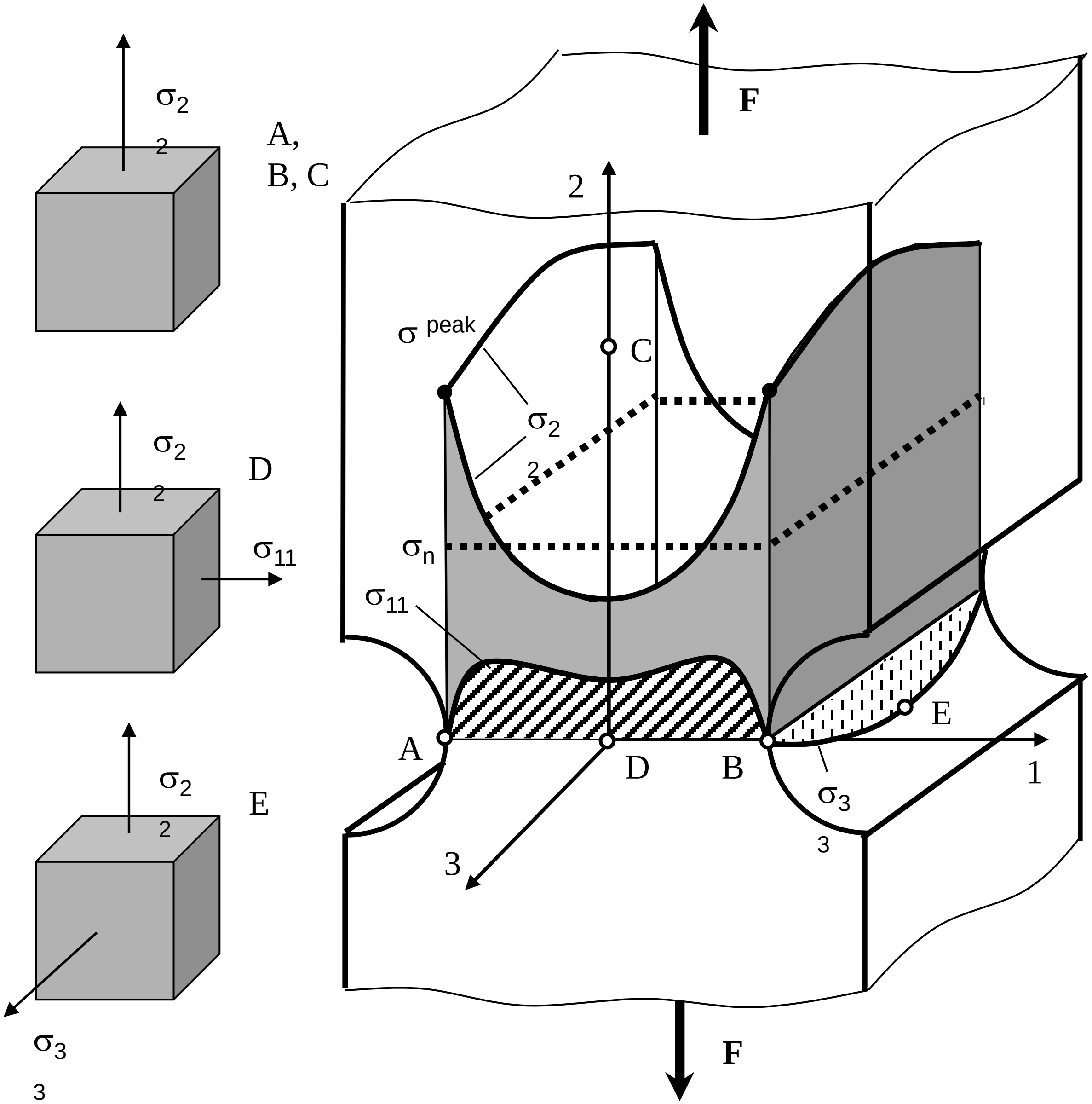


$$\sigma_n = S = \frac{F}{A}$$

and

$$\sigma^{peak} = K_t \sigma_n$$

Stresses in prismatic notched body



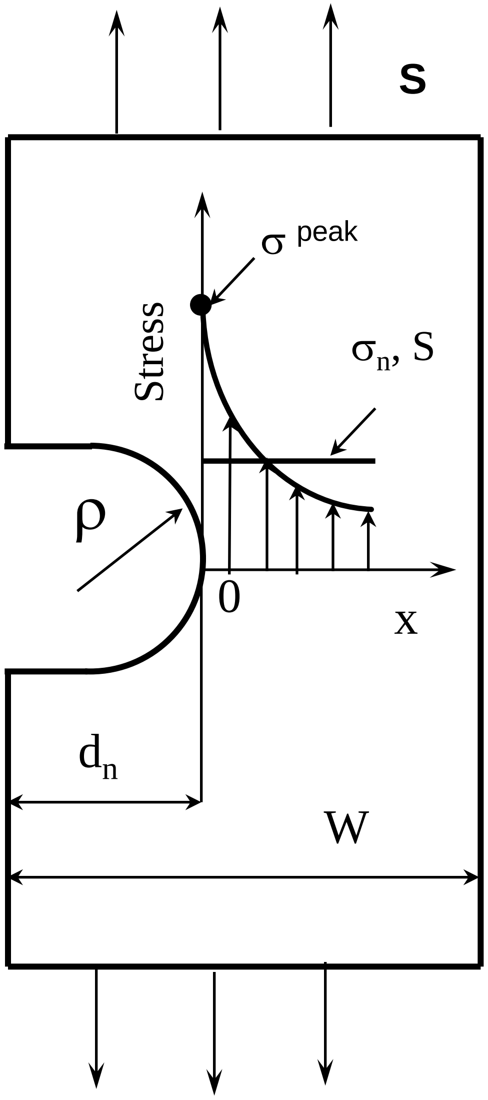
$$\sigma_n = S = \frac{F}{A}$$

and

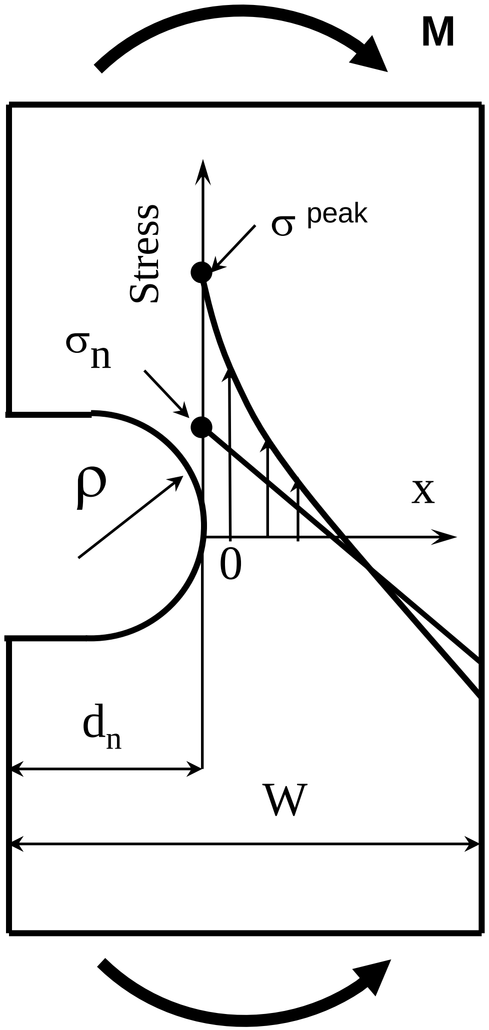
$$\sigma^{peak} = K_t \sigma_n$$

$$= K_t S$$

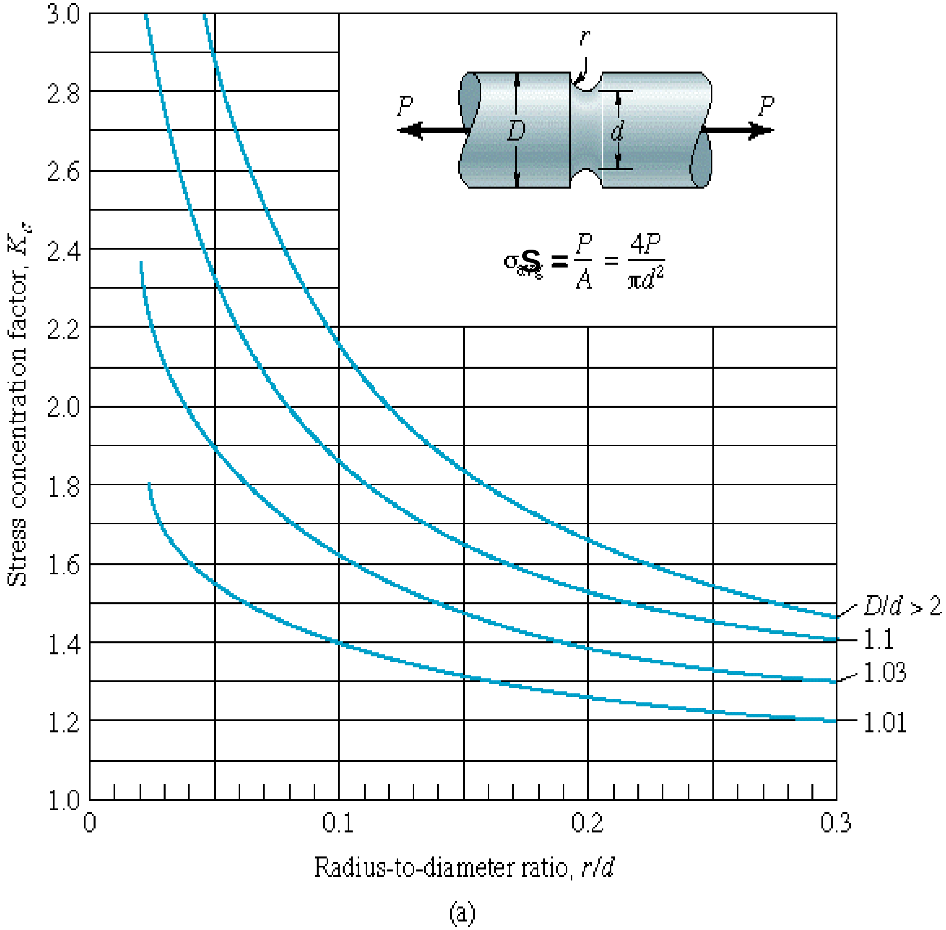
Stress concentration factors used in fatigue analysis



$$K_t = \frac{\sigma^{peak}}{\sigma_n} = \frac{\sigma^{peak}}{S}$$

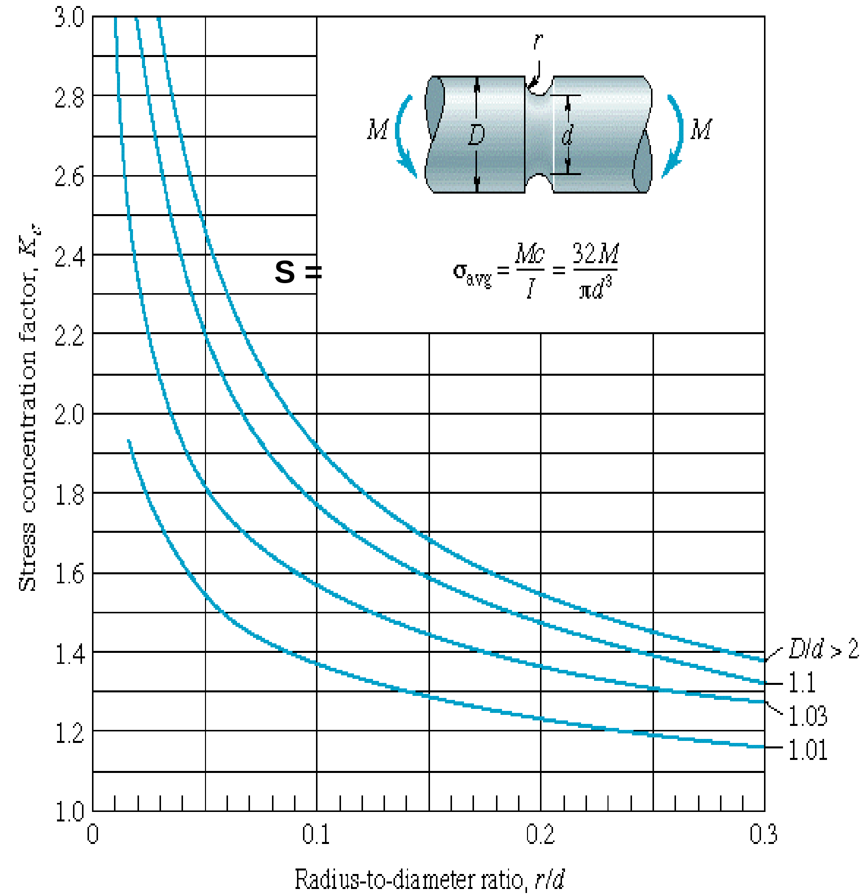


Stress concentration factors, K_t, in shafts



Axial load

Bending load



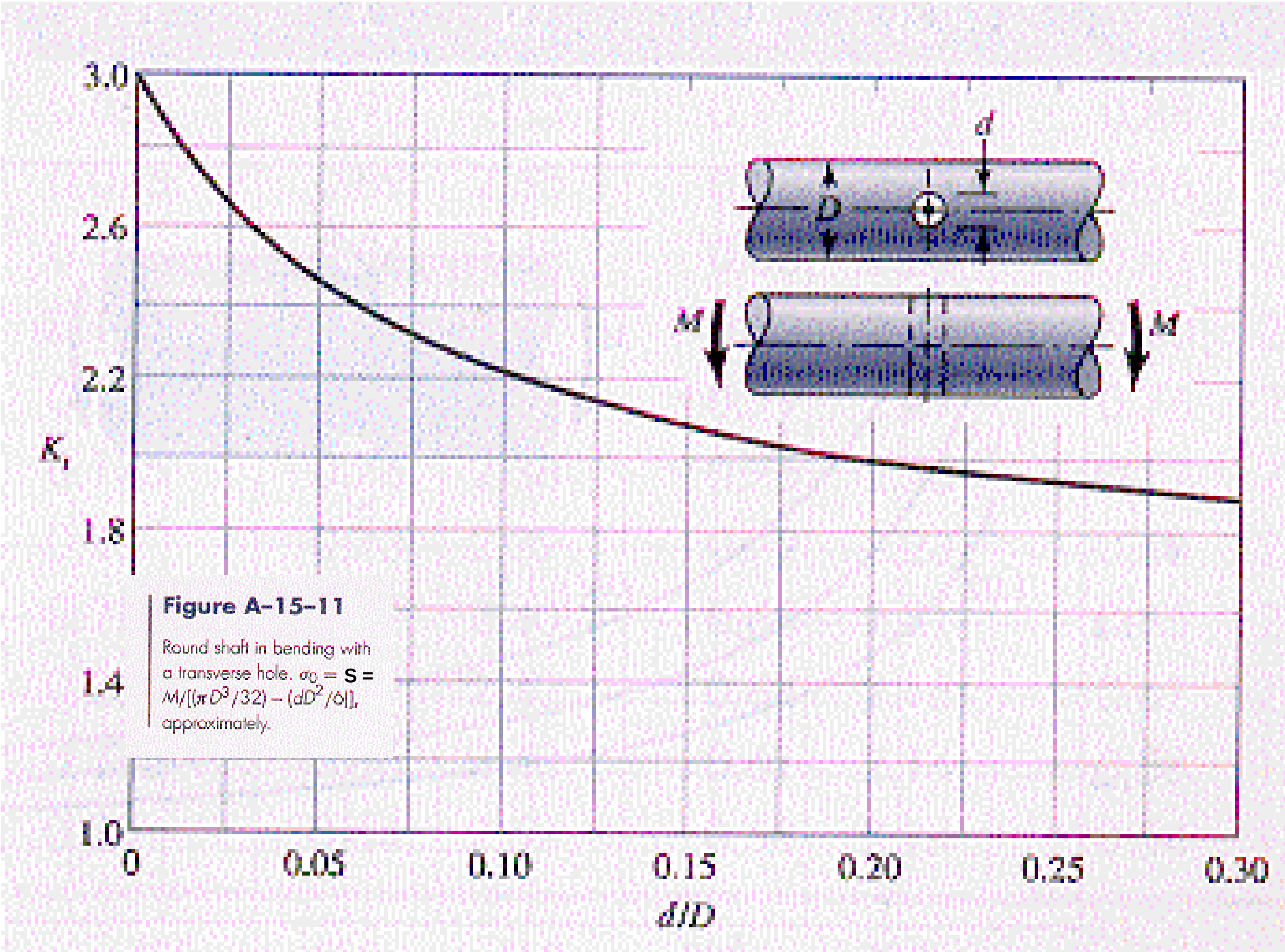
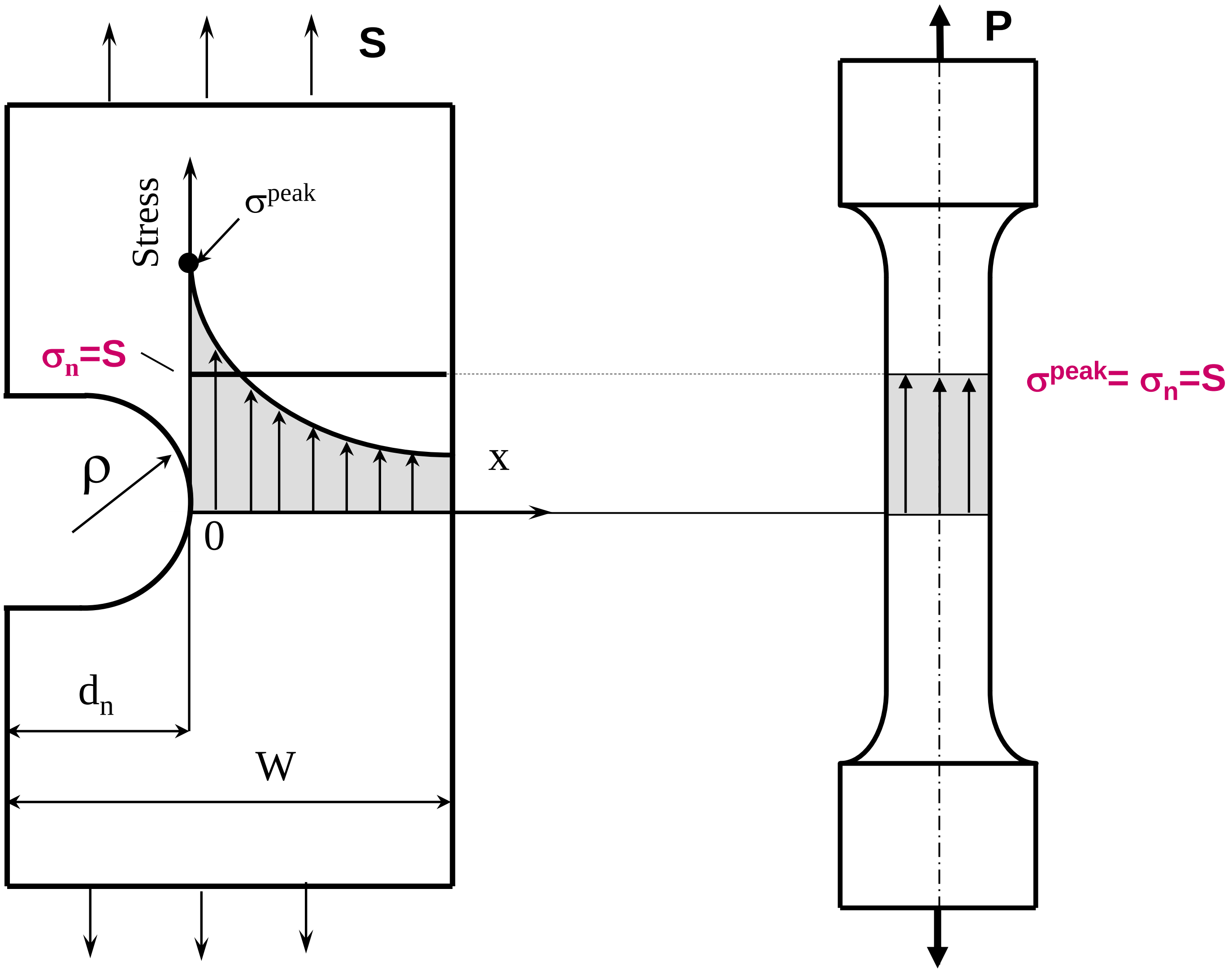


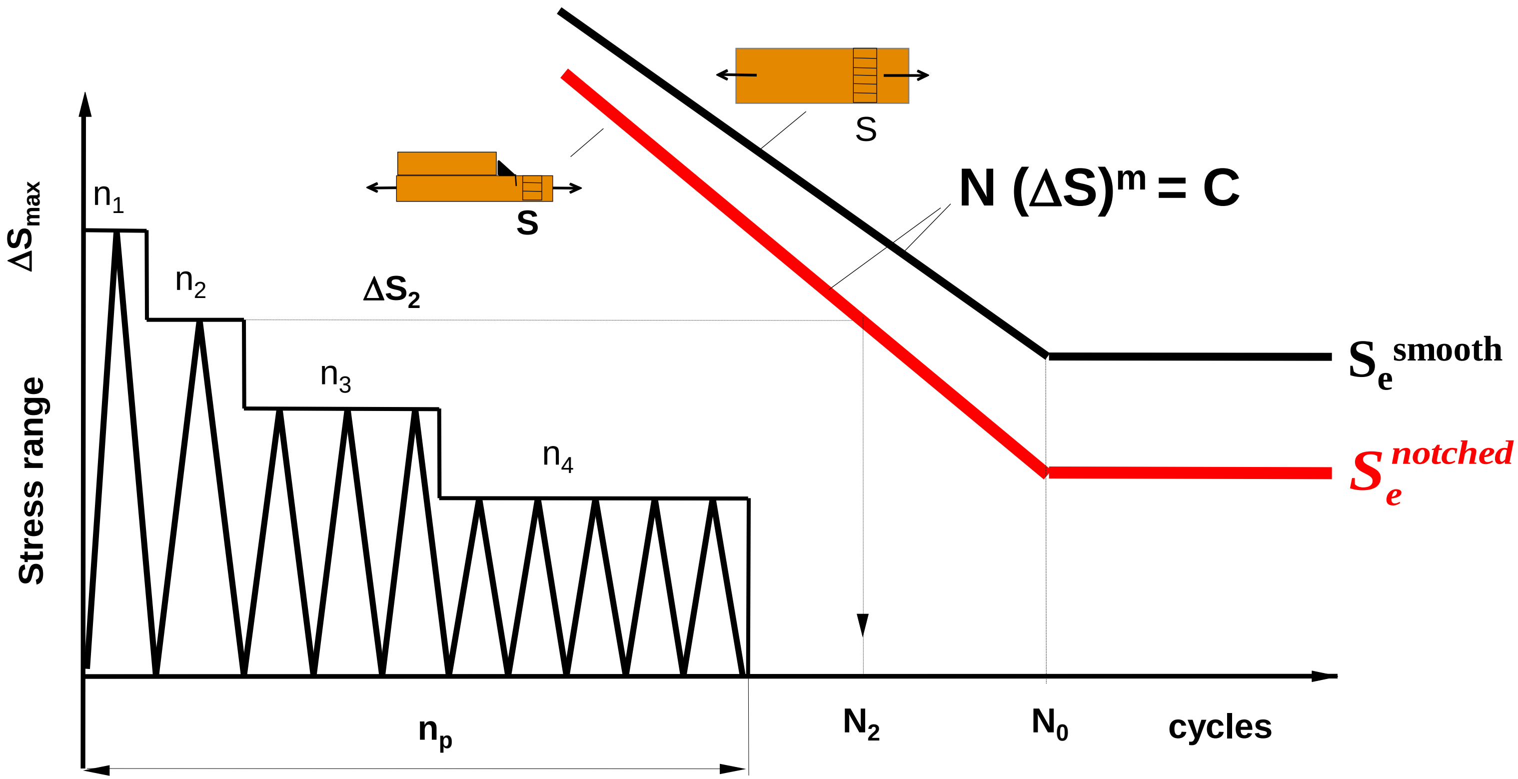
Figure A–15–11

Round shaft in bending with a transverse hole. $\sigma_0 = S = M/[(\pi D^3/32) - (dD^2/6)]$, approximately.

Similarities and differences between the stress field near the notch and in a smooth specimen



The Notch Effect in Terms of the Nominal Stress

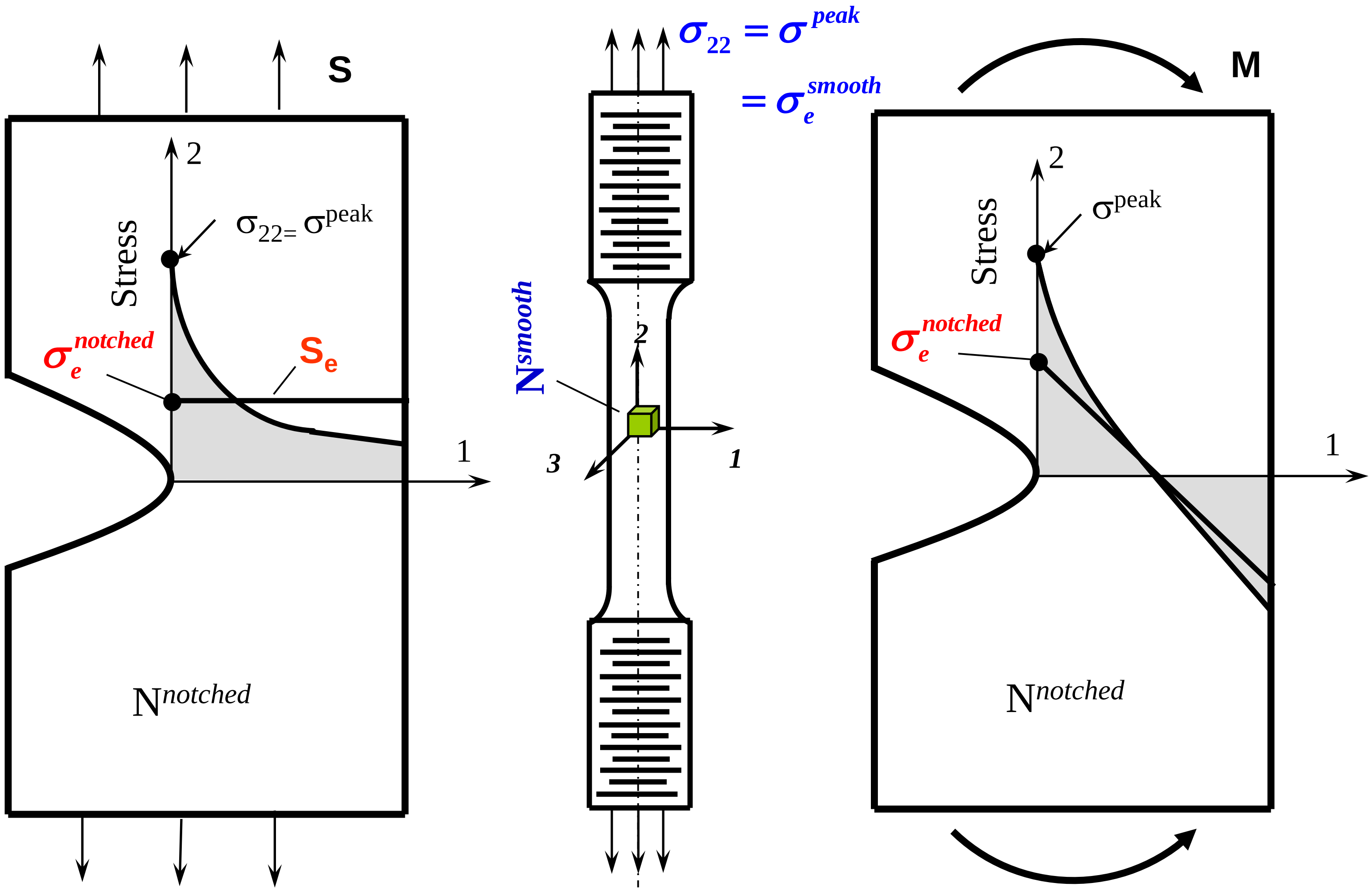


$$S_e^{notched} = \frac{S_e^{smooth}}{K_f}$$

K_f – **Fatigue notch factor!**

$K_f \neq K_t$ **!!!**

Definition of the fatigue notch factor K_f



$$K_f = \frac{\sigma_e^{smooth}}{\sigma_e^{notched}} \quad \textbf{for} \quad N^{smooth} = N^{notched}$$

PETERSON's approach

$$K_f = 1 + \frac{K_t - 1}{1 + a/r} = 1 + q(K_t - 1)$$

$$q = \frac{1}{1 + a/r};$$

a – constant,
r – notch tip radius;

$$a = \left[\frac{300}{S_u}\right]^{1.8} \times 10^{-3} \ [in.]$$

$$for \ S_u \ in \ [in.]$$

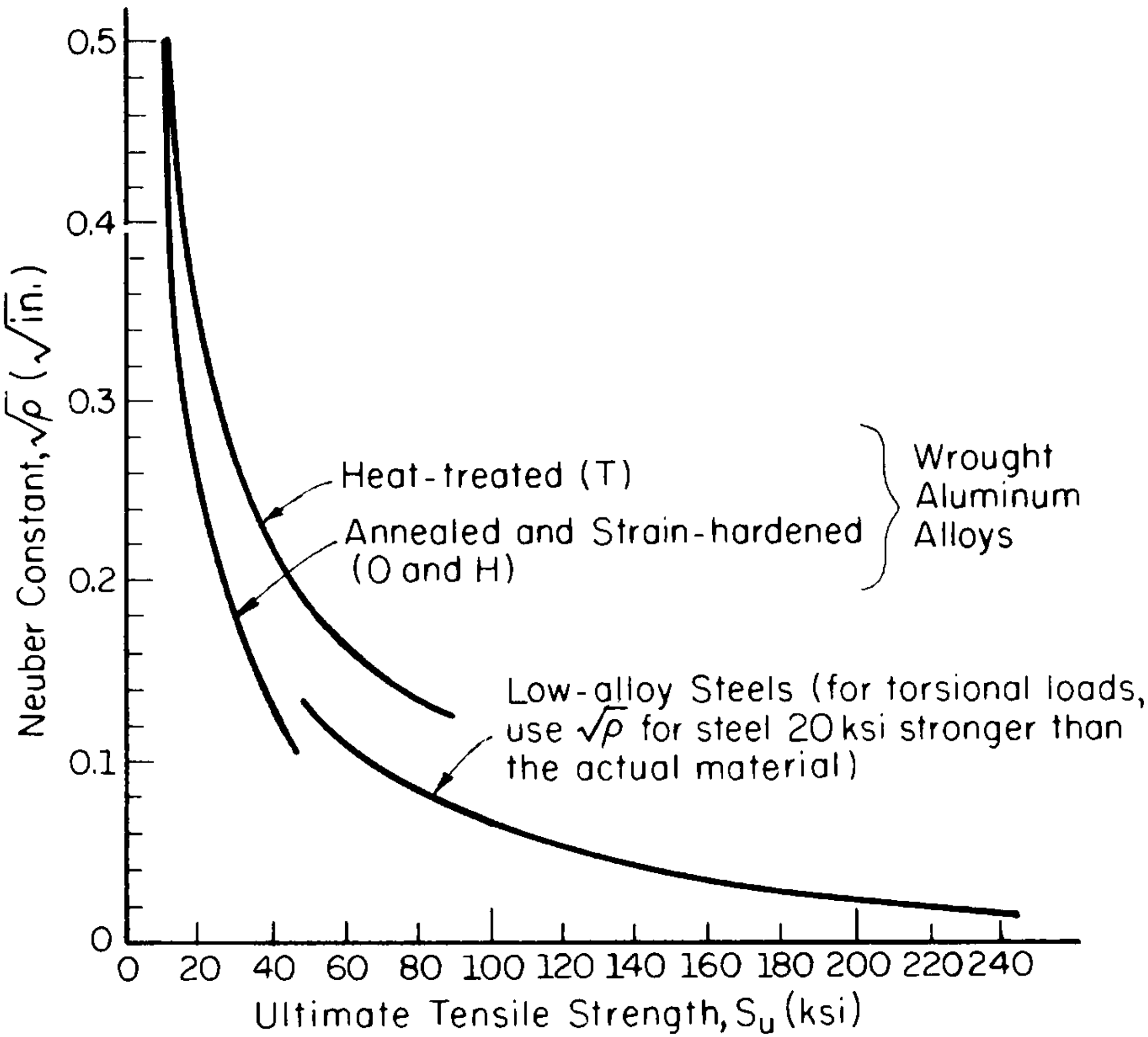
NEUBER's approach

$$K_f = 1 + \frac{K_t - 1}{1 + \sqrt{\rho/r}}$$

ρ – constant,

r – notch tip radius

The Neuber constant ‘ρ’ for steels and aluminium alloys



Curves of notch sensitivity index ‘q’ versus notch radius

(McGraw Hill Book Co, from ref. 1)

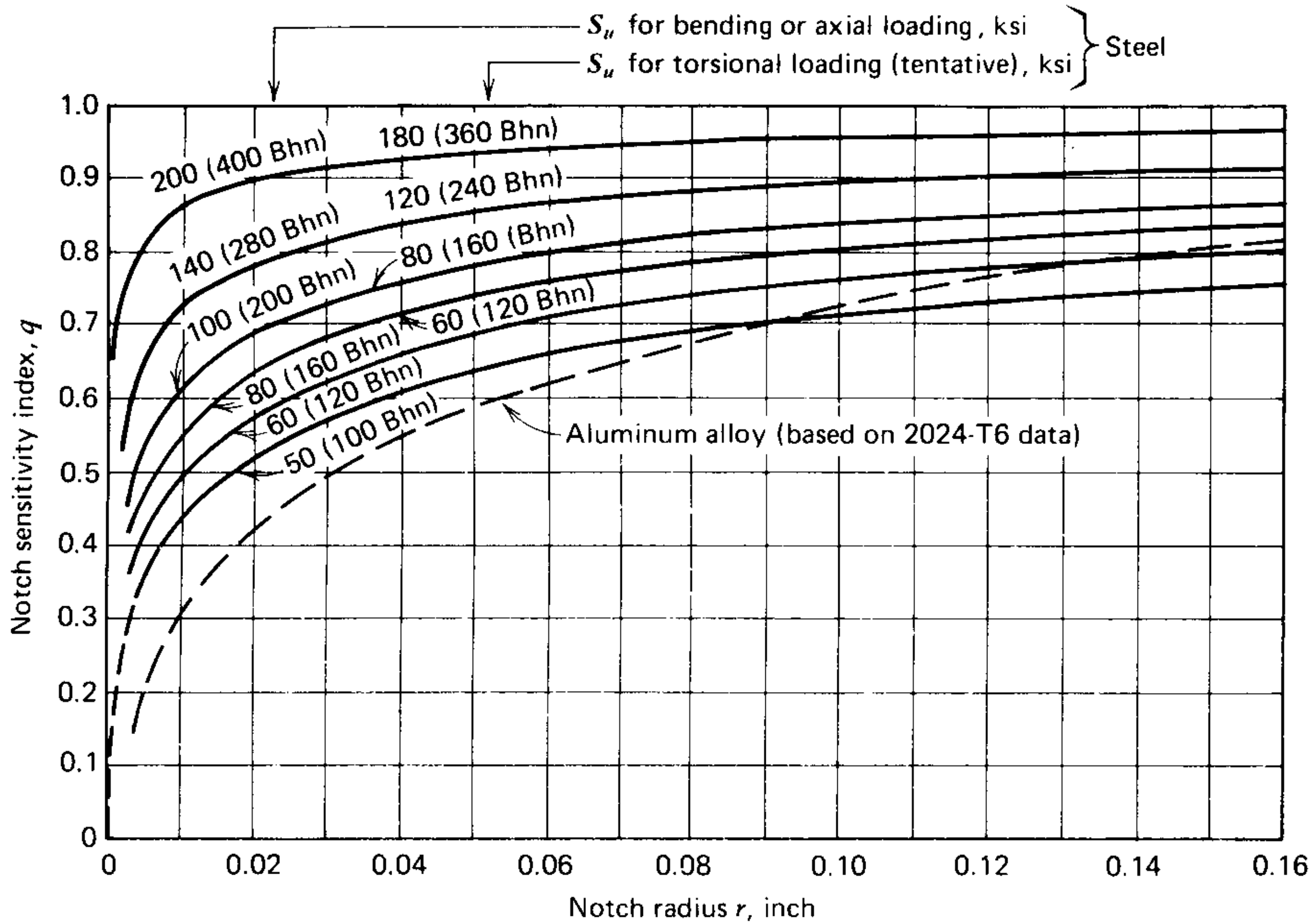


Illustration of the notch/scale effect

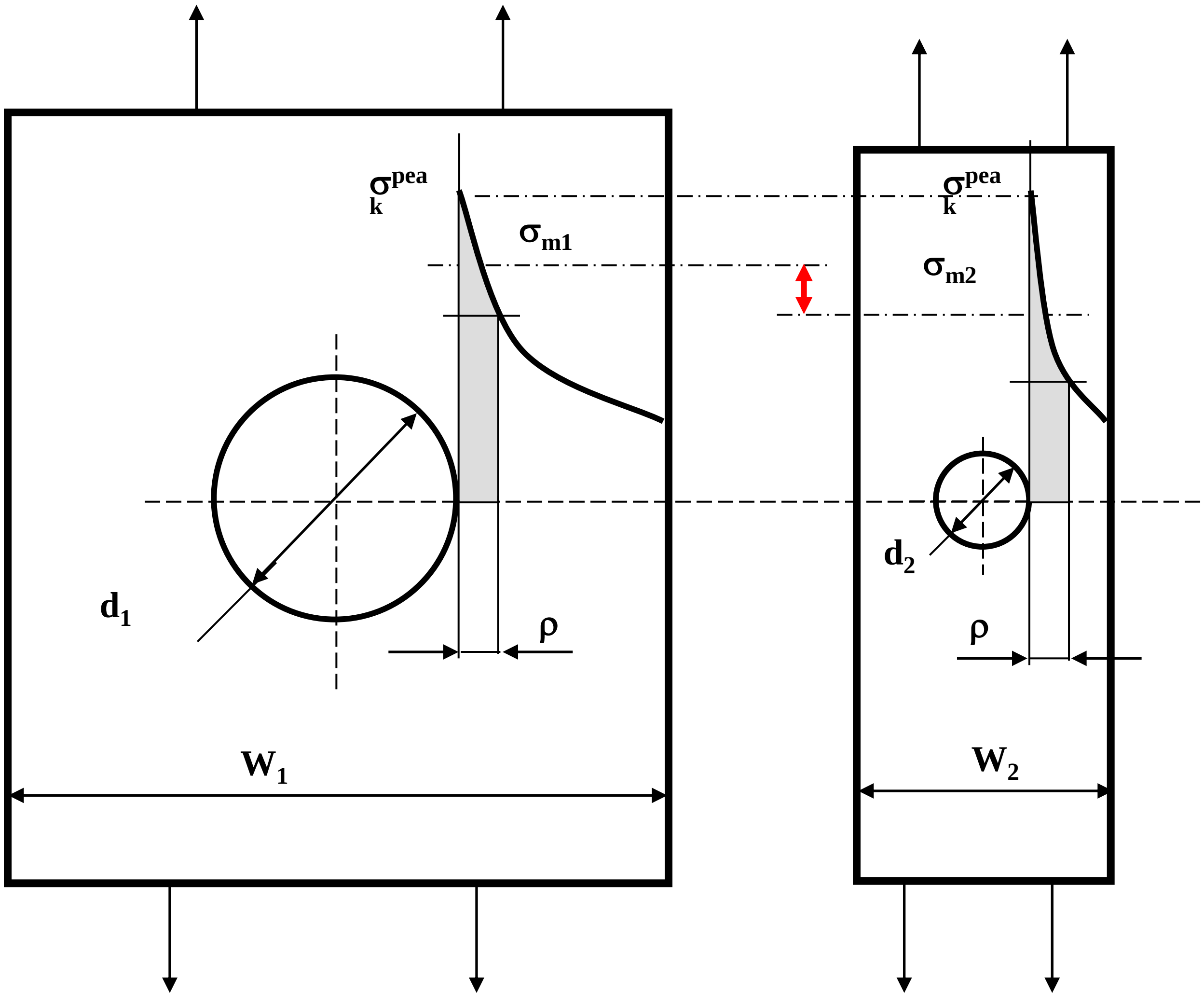


Plate 1

W_1 = 5.0 in

d_1 = 0.5 in.

S_u = 100 ksi

K_t = 2.7

q = 0.97

K_{f1} = 2.65

Plate 2

W_2= 0.5 in

d_2 = 0.5 i

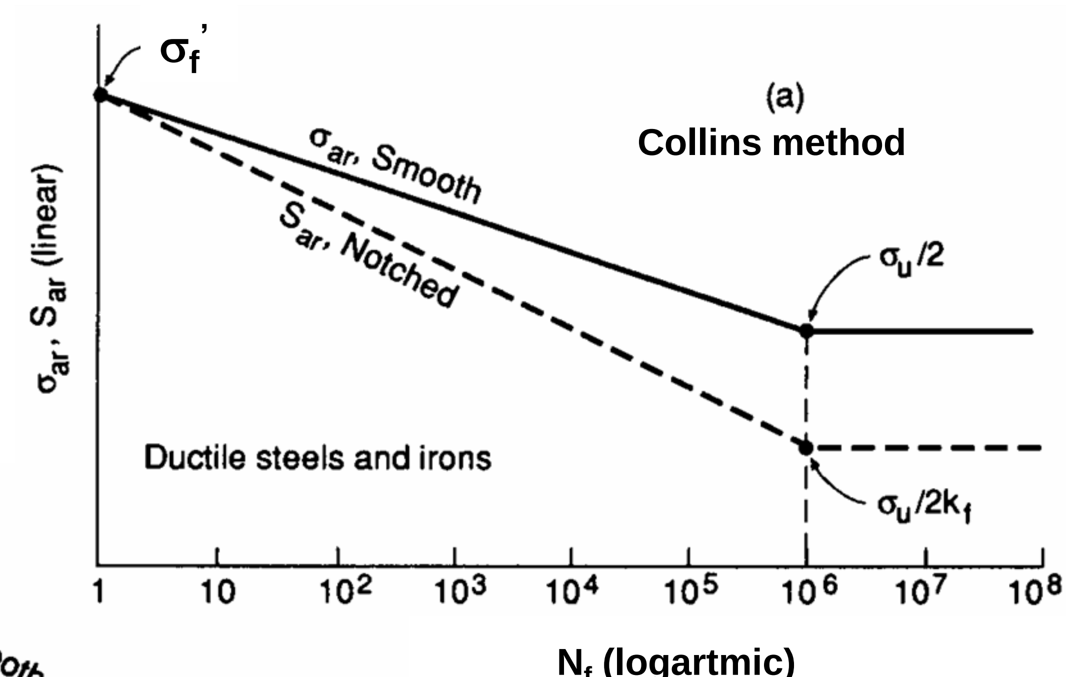
S_u = 100 ksi

K_t = 2.7

q = 0.78

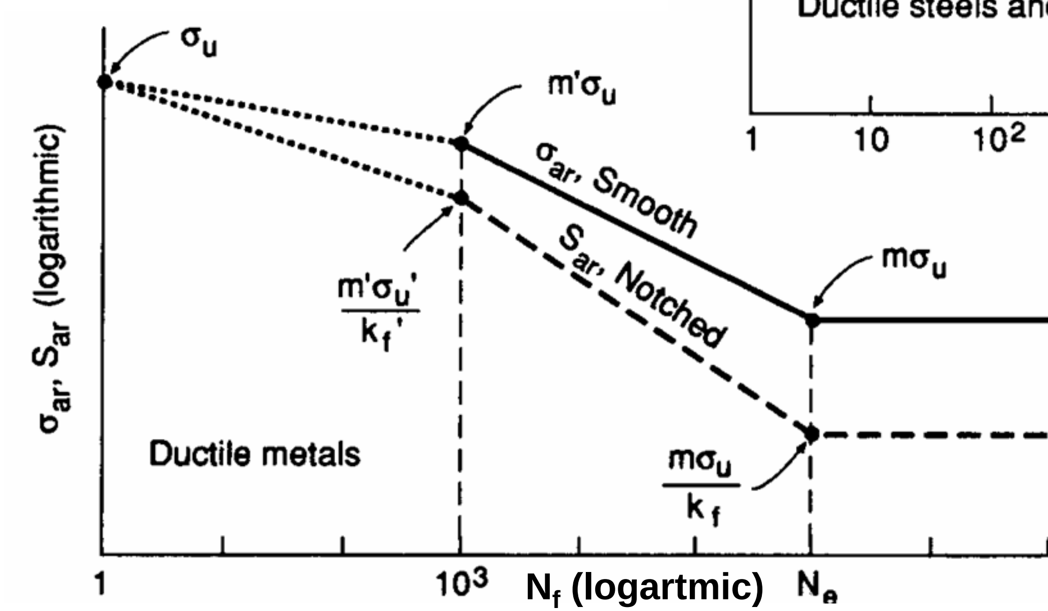
K_{f1} = 2.32

Procedures for construction of approximate fully reversed S-N curves for smooth and notched components



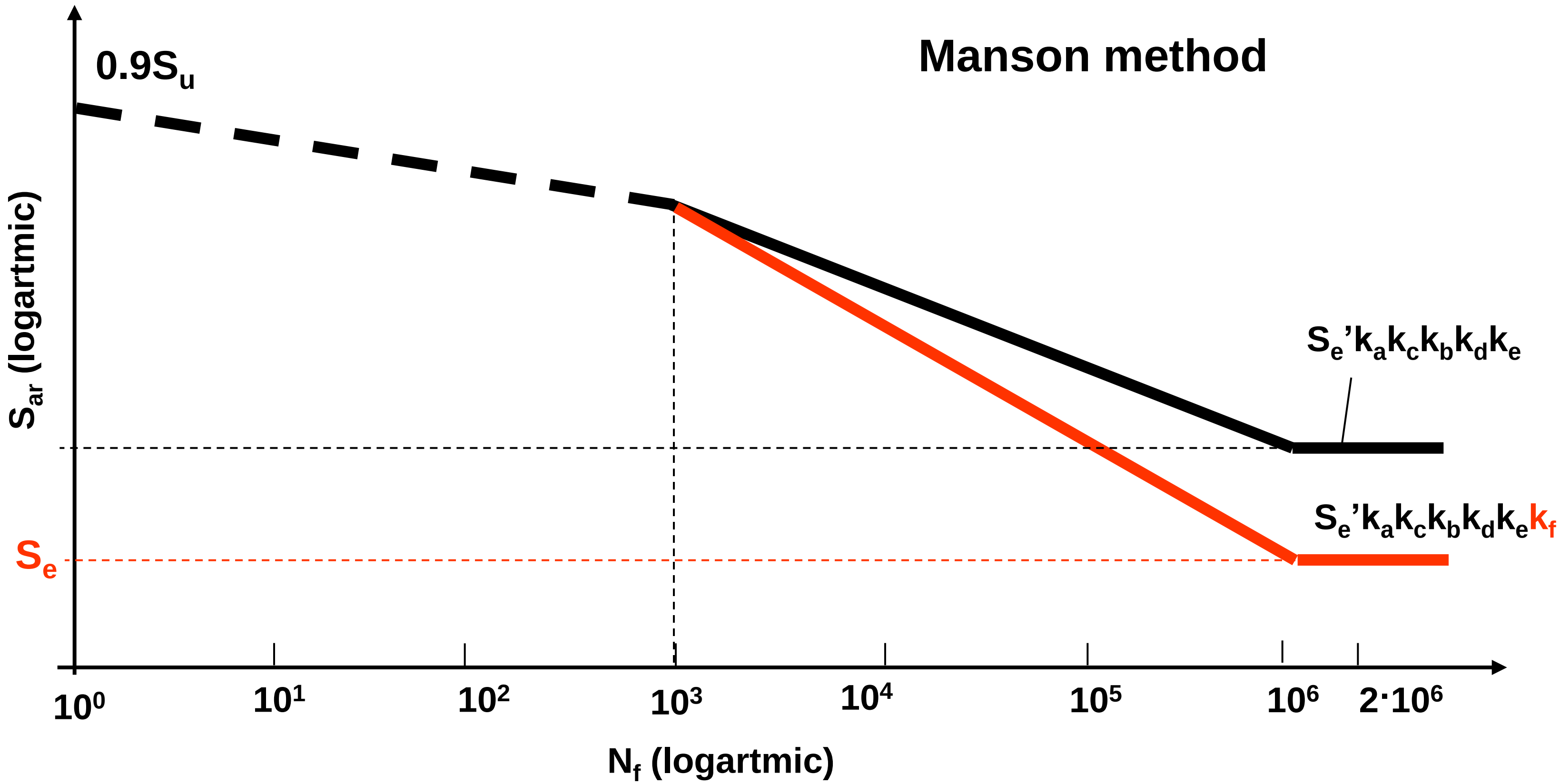
(b)

Juvinal/Shigley method



S_{ar}, σ_{ar} – nominal/local stress amplitude at zero mean stress σ_m=0 (fully reversed cycle)!

Procedures for construction of approximate fully reversed S-N curves for smooth and notched components



S_{ar}, σ_{ar} – nominal/local stress amplitude at zero mean stress σ_m=0 (fully reversed cycle)!

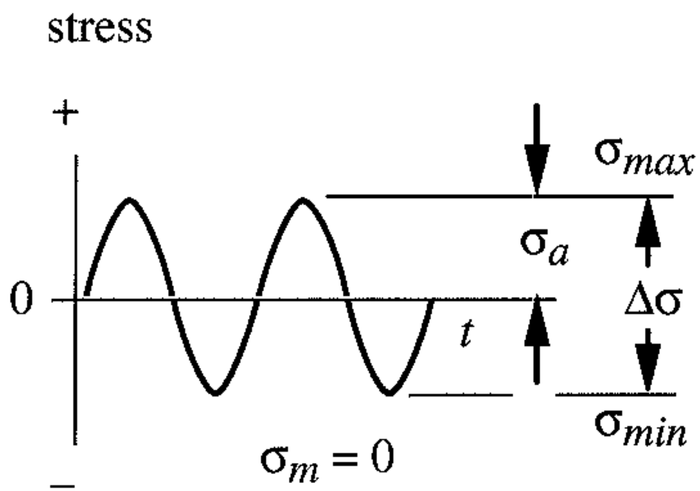
NOTE!

• The empirical relationships concerning the **S –N** curve data are only estimates! Depending on the acceptable level of uncertainty in the fatigue design, actual test data may be necessary.

• The most useful concept of the **S - N** method is the **endurance limit**, which is used in "**infinite-life",** or **"safe stress**" design philosophy.

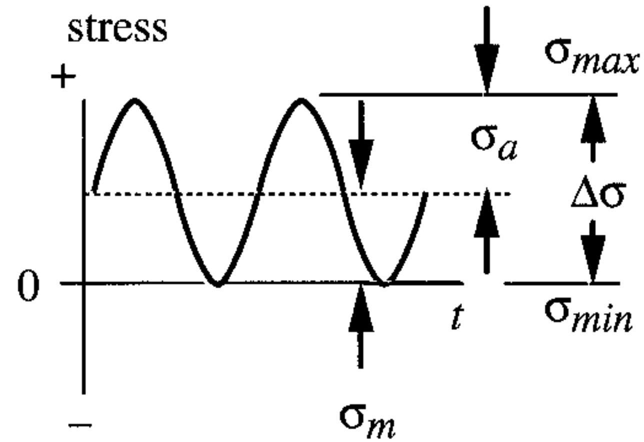
• In general, the **S – N** approach should not be used to estimate lives below 1000 cycles (**N < 1000**).

Constant amplitude cyclic stress histories



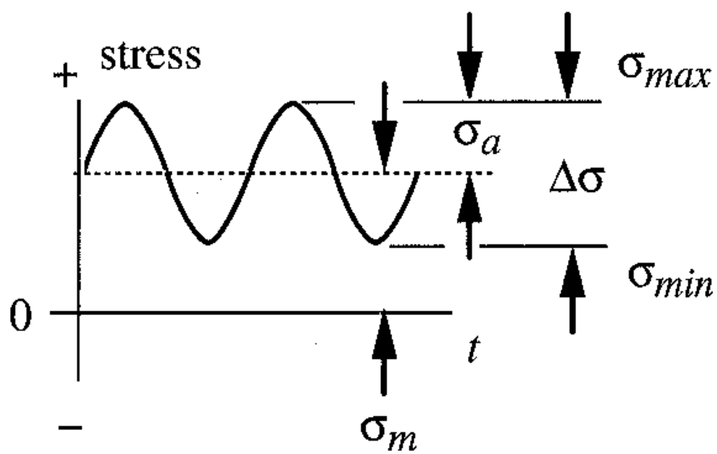
Fully reversed

$\sigma_m = 0$, R = -1



Pulsating

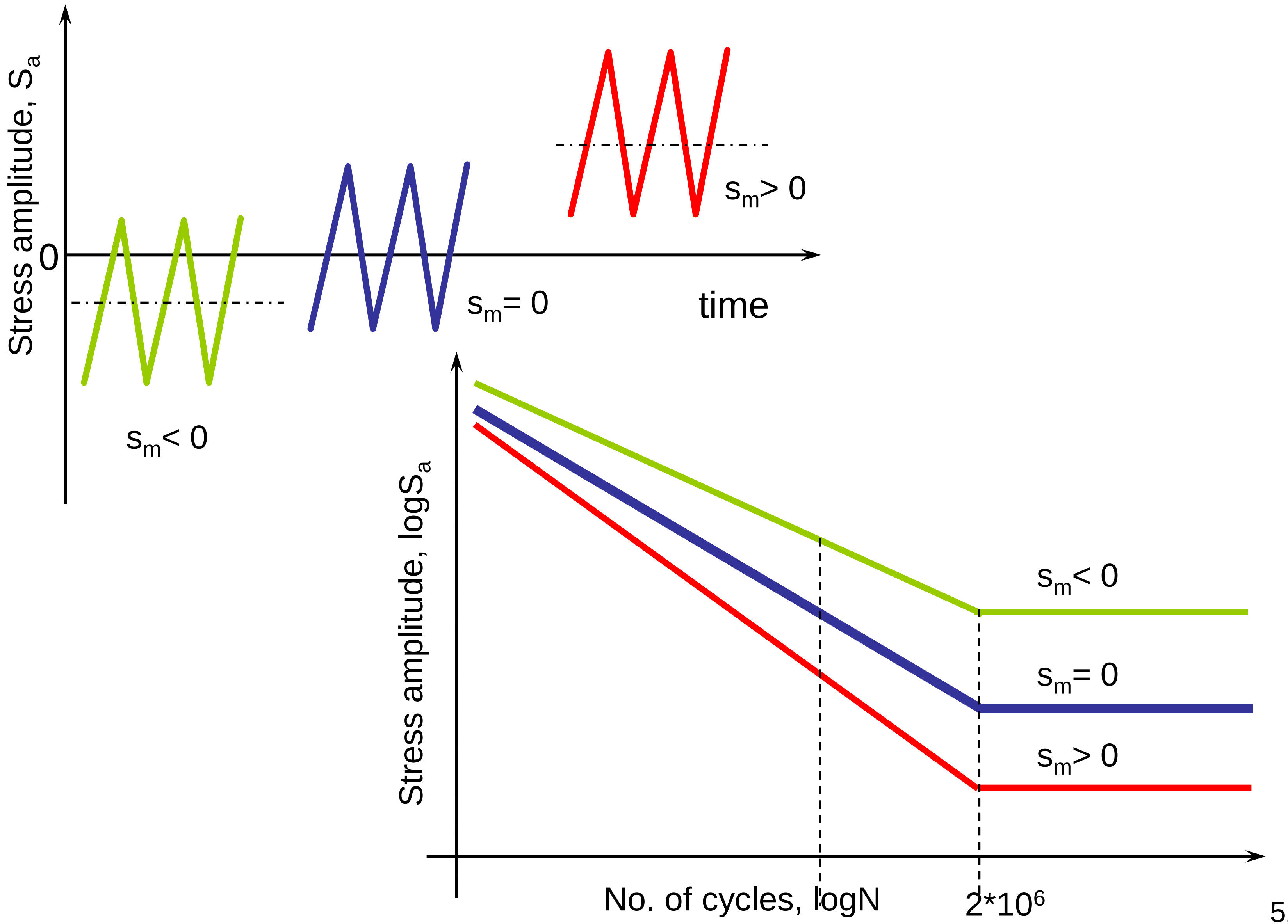
$\sigma_m = \sigma_a$ R = 0



Cyclic

$\sigma_m > 0$ R > 0

Mean Stress Effect

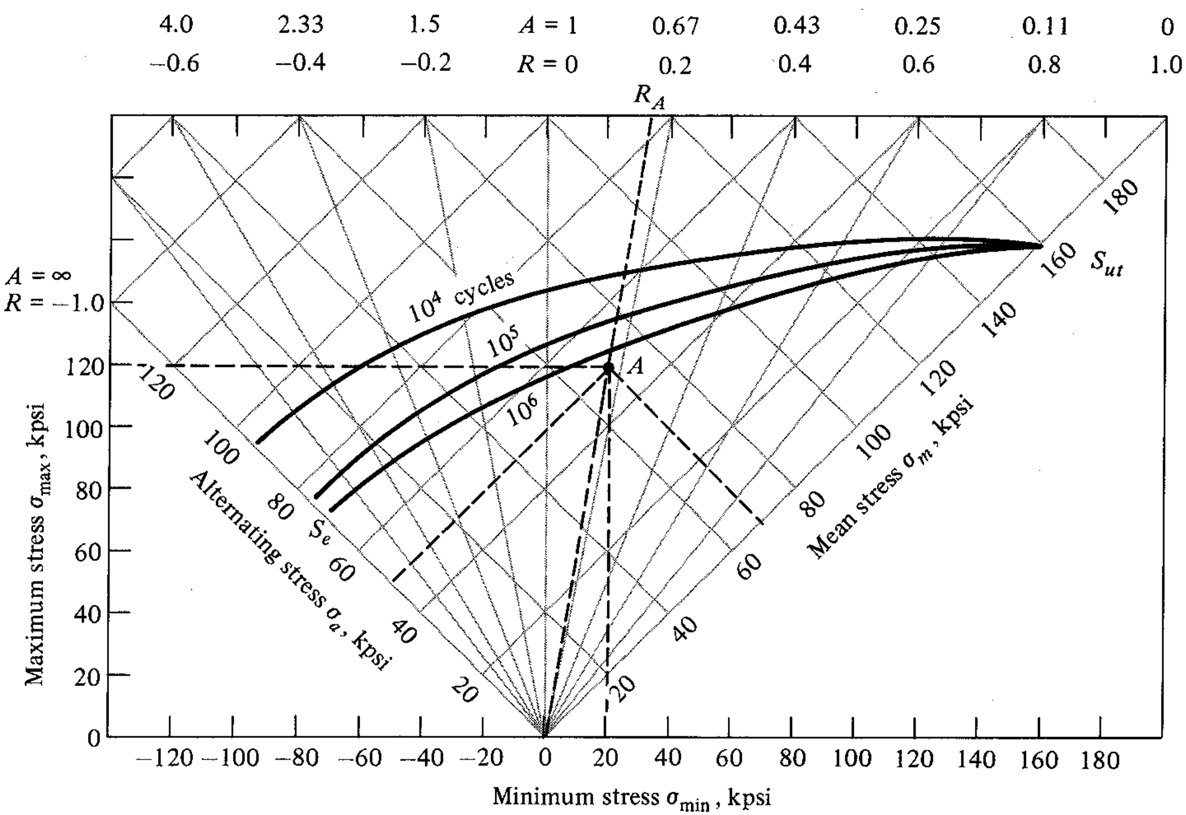


The tensile mean stress is in general detrimental while the compressive mean stress is beneficial or has negligible effect on the fatigue durability.

Because most of the S – N data used in analyses was produced under zero mean stress (R = -1) therefore it is necessary to translate cycles with non- zero mean stress into equivalent cycles with zero mean stress producing the same fatigue life.

There are several empirical methods used in practice:

The Hiagh diagram was one of the first concepts where the mean stress effect could be accounted for. The procedure is based on a family of S_a – S_m curves obtained for various fatigue lives.



Steel AISI 4340,

S_y = 147 ksi *(Collins)*

Mean Stress Correction for Endurance Limit

Gereber (1874) $$\frac{S_a}{S_e} + \left(\frac{S_m}{S_u}\right)^2 = 1$$

Goodman (1899) $$\frac{S_a}{S_e} + \frac{S_m}{S_u} = 1$$

Soderberg (1930) $$\frac{S_a}{S_e} + \frac{S_m}{S_y} = 1$$

Morrow (1960) $$\frac{S_a}{S_e} + \frac{S_m}{\sigma_f'} = 1$$

S_a – stress amplitude applied at the mean stress $S_m \neq 0$ and fatigue life N = 1-2x10^6 cycles.

S_m- mean stress

S_e- fatigue limit at S_m=0

S_u- ultimate strength

σ_f'- true stress at fracture

Mean stress correction for arbitrary stress amplitude applied at non-zero mean stress

Gereber (1874) $$\frac{S_a}{S_{ar}} + \left(\frac{S_m}{S_u}\right)^2 = 1$$

Goodman (1899) $$\frac{S_a}{S_{ar}} + \frac{S_m}{S_u} = 1$$

Soderberg (1930) $$\frac{S_a}{S_{ar}} + \frac{S_m}{S_y} = 1$$

Morrow (1960) $$\frac{S_a}{S_{ar}} + \frac{S_m}{\sigma_f^{'}} = 1$$

S_a – stress amplitude applied at the mean stress $S_m \neq 0$ and resulting in fatigue life of N cycles.

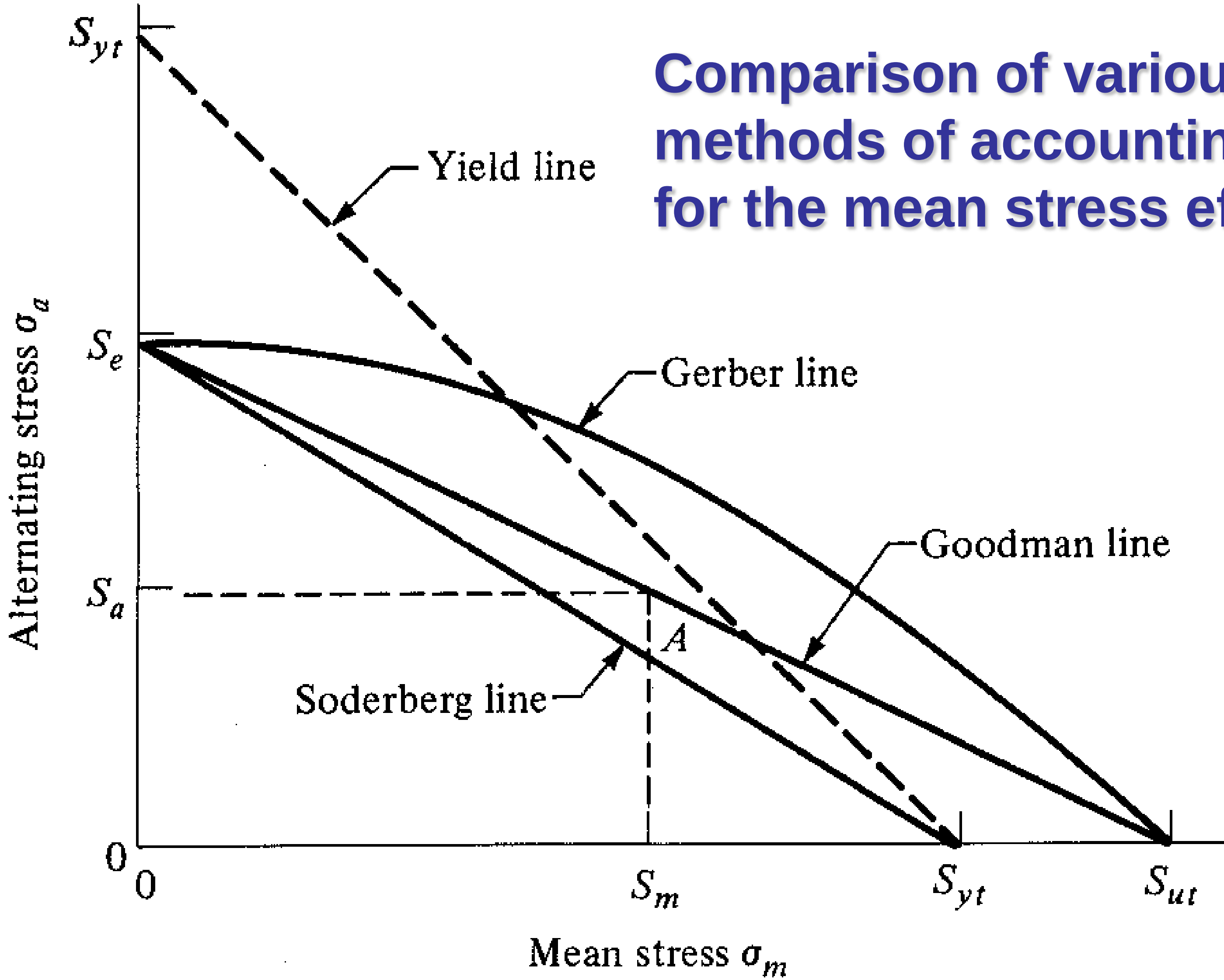
S_m- mean stress

S_{ar}- fully reversed stress amplitude applied at mean stress S_m=0 and resulting in the same fatigue life of N cycles

S_u- ultimate strength

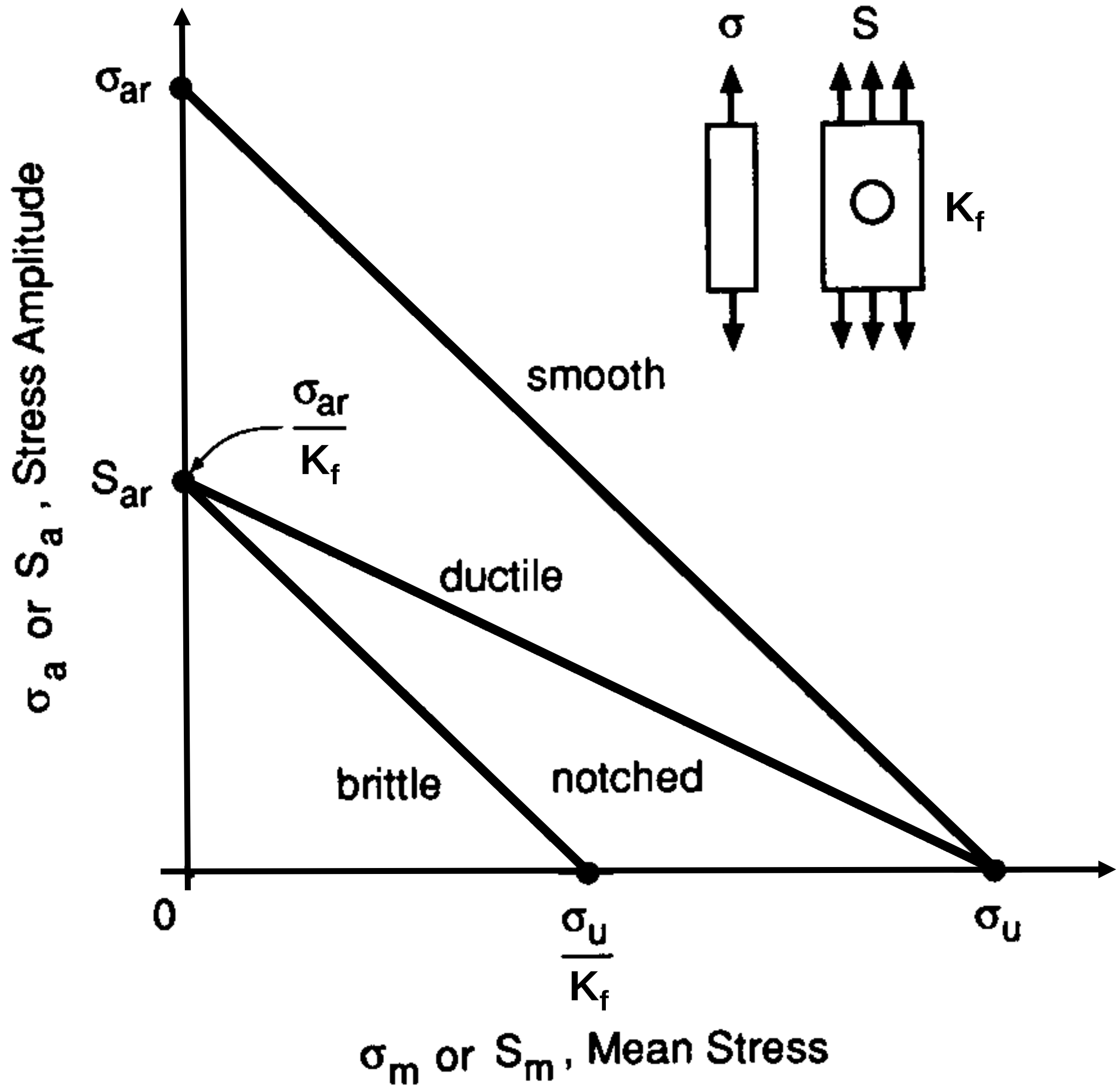
$\sigma_f^{'}$- true stress at fracture

Comparison of various methods of accounting for the mean stress effect



Most of the experimental data lies between the Goodman and the yield line!

Approximate Goodman's diagrams for ductile and brittle materials



The following generalisations can be made when discussing mean stress effects:

1. The Söderberg method is very conservative and seldom used.

3. Actual test data tend to fall between the Goodman and Gerber curves.

3. For hard steels (i.e., brittle), where the ultimate strength approaches the true fracture stress, the Morrow and Goodman lines are essentially the same. For ductile steels (of > S,,) the Morrow line predicts less sensitivity to mean stress.

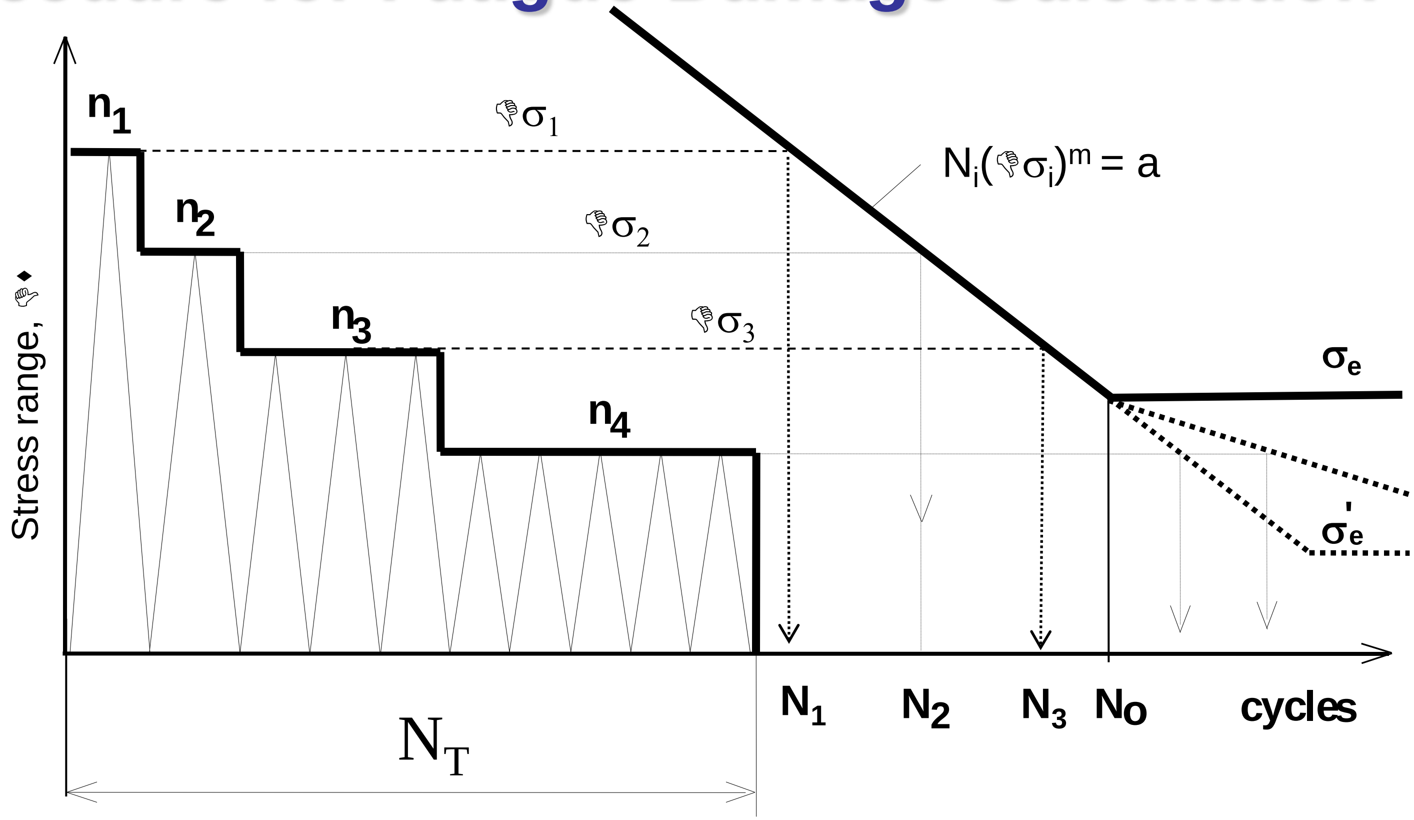
4. For most fatigue design situations, R < 1 (i.e., small mean stress in relation to alternating stress), there is little difference in the theories.

5. In the range where the theories show a large difference (i.e., R values approaching 1), there is little experimental data. In this region the yield criterion may set design limits.

6. The mean stress correction methods have been developed mainly for the cases of tensile mean stress.

For finite-life calculations the endurance limit in any of the equations can be replaced with a fully reversed alternating stress level corresponding to that finite-life value!

Procedure for Fatigue Damage Calculation



$$D = \frac{n_1 \, cycles \, applied \, at \, \Delta\sigma_1}{N_1 \, cycles \, to \, failure \, at \, \Delta\sigma_1} + \frac{n_2 \, cycles \, applied \, at \, \Delta\sigma_2}{N_2 \, cycles \, to \, failure \, at \, \Delta\sigma_2} .... + ... \frac{n_i \, cycles \, applied \, at \, \Delta\sigma_i}{N_i \, cycles \, to \, failure \, at \, \Delta\sigma_i}$$

$$D = D_{n1} + D_{n2} + ... + D_{ni} = \frac{n_1}{N_1} + \frac{n_2}{N_2} + ... + \frac{n_i}{N_i} = \sum_1^i \frac{n_i}{N_i}$$

$$L_R = \frac{1}{D} = \frac{1}{n_1/N_1 + n_2/N_2 + ...n_i/N_i}$$

n_1 - number of cycles of stress range $\Delta\sigma_1$

n_2 - number of cycles of stress range $\Delta\sigma_2$

n_i - number of cycles of stress range $\Delta\sigma_i$,

$$D_1 = \frac{1}{N_1}$$ - damage induced by one cycle of stress range $\Delta\sigma_1$,

$$D_{n1} = \frac{n_1}{N_1}$$ - damage induced by n_1 cycles of stress range $\Delta\sigma_1$,

$$D_2 = \frac{1}{N_2}$$ - damage induced by one cycle of stress range $\Delta\sigma_2$,

$$D_{n2} = \frac{n_2}{N_2}$$ - damage induced by n_2 cycles of stress range $\Delta\sigma_2$,

$$D_i = \frac{1}{N_i}$$ - damage induced by one cycle of stress range $\Delta\sigma_i$,

$$D_{ni} = \frac{n_i}{N_i}$$ - damage induced by n_i cycles of stress range $\Delta\sigma_i$,

Total Damage Induced by the Stress History

$$D = \frac{n_1 \, cycles\, applied\, at\, \Delta\sigma_1}{N_1 \, cycles\, to\, failure\, at\, \Delta\sigma_1} = \frac{n_2 \, cycles\, applied\, at\, \Delta\sigma_2}{N_2 \, cycles\, to\, failure\, at\, \Delta\sigma_2} .... = ... \frac{n_i \, cycles\, applied\, at\, \Delta\sigma_i}{N_i \, cycles\, to\, failure\, at\, \Delta\sigma_i}$$

$$D = D_{n1} + D_{n2} + ... + D_{ni} = \frac{n_1}{N_1} + \frac{n_2}{N_2} + ... + \frac{n_i}{N_i} = \sum_1^i \frac{n_i}{N_i}$$

It is usually assumed that fatigue failure occurs when the cumulative damage exceeds some critical value such as D =1,

i.e. if D > 1 - fatigue failure occurs!

For D < 1 we can determine the remaining fatigue life:

$$L_R = \frac{1}{D} = \frac{1}{n_1/N_1 + n_2/N_2 + ... n_i/N_i}$$

L_R - number of repetitions of the stress history to failure

$$N = L_R\left(n_1 + n_2 + n_3 + ... + n_i\right)$$

N - total number of cycles to failure

$$N_j = \bar{a}\left(\Delta\sigma_j\right)^{-m} \qquad \text{if} \quad \Delta\sigma_j > \sigma_e \ .$$

It is assumed that stress cycles lower than the fatigue limit, $\Delta\sigma_j < \sigma_e$, produce no damage ($N_j=\infty$) in the case of constant amplitude loading however in the case of variable amplitude loading the extension of the S-N curve with the slope 'm+2" is recommended. The total damage produced by the entire stress spectrum is equal to:

$$D = \sum_{j=1}^{j} D_j$$

It is assumed that the component fails if the damage is equal to or exceeds unity, i.e. when $D \geq 1$. This may happen after a certain number of repetitions, BL (blocks), of the stress spectrum, which can be calculated as:

$$BL = 1/D.$$

Hence, the fatigue life of a component in cycles can be calculated as:

$$N = BL \times N_T,$$

where, N_T is the spectrum volume or the number of cycles extracted from given stress history.

$$N_T = (NOP - 1)/2$$

If the record time of the stress history or the stress spectrum is equal to Tr, the fatigue life can be expressed in working hours as:

$$T = BL \times Tr.$$

Main Steps in the S-N Fatigue Life Estimation Procedure

- Analysis of external forces acting on the structure and the component in question,
- Analysis of internal loads in chosen cross section of a component,
- Selection of individual notched component in the structure,
- Selection (from ready made family of S-N curves) or construction of S-N curve adequate for given notched element (corrected for all effects),
- Identification of the stress parameter used for the determination of the S-N curve (nominal/reference stress),
- Determination of analogous stress parameter for the actual element in the structure, as described above,
- Identification of appropriate stress history,
- Extraction of stress cycles (rainflow counting) from the stress history,
- Calculation of fatigue damage,
- Fatigue damage summation (Miner- Palmgren hypothesis),
- Determination of fatigue life in terms of number of stress history repetitions, N_{blck}, (No. of blocks) or the number of cycles to failure, N.
- The procedure has to be repeated several times if multiple stress concentrations or critical locations are found in a component or structure.

Example #2

An unnotched machine component undergoes a variable amplitude stress history S_i given below. The component is made from a steel with the ultimate strength S_{uts}=150 ksi, the endurance limit S_e=60 ksi and the fully reversed stress amplitude at N_{1000}=1000 cycles given as S_{1000}=110 ksi.

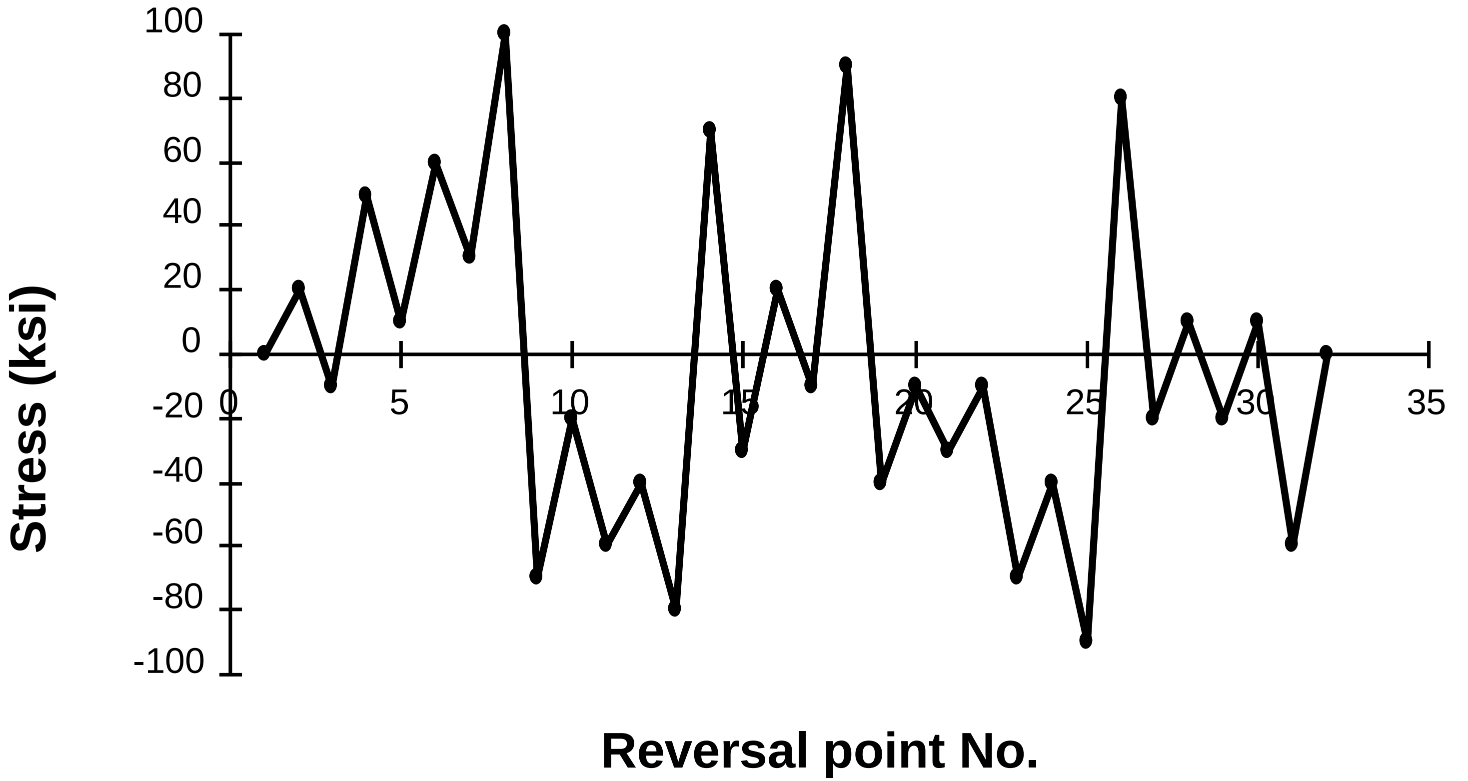
Determine the expected fatigue life of the component.

Data: K_t=1, S_Y=100 ksi S_{uts}=150 ksi, S_e=60 ksi, S_{1000}=110 ksi

The stress history:

S_i = 0, 20, -10, 50, 10, 60, 30, 100, -70, -20, -60, -40, -80, 70, -30, 20, -10, 90, -40, 10, -30, -10, -70, -40, -90, 80, -20, 10, -20, 10, 0

Stress History



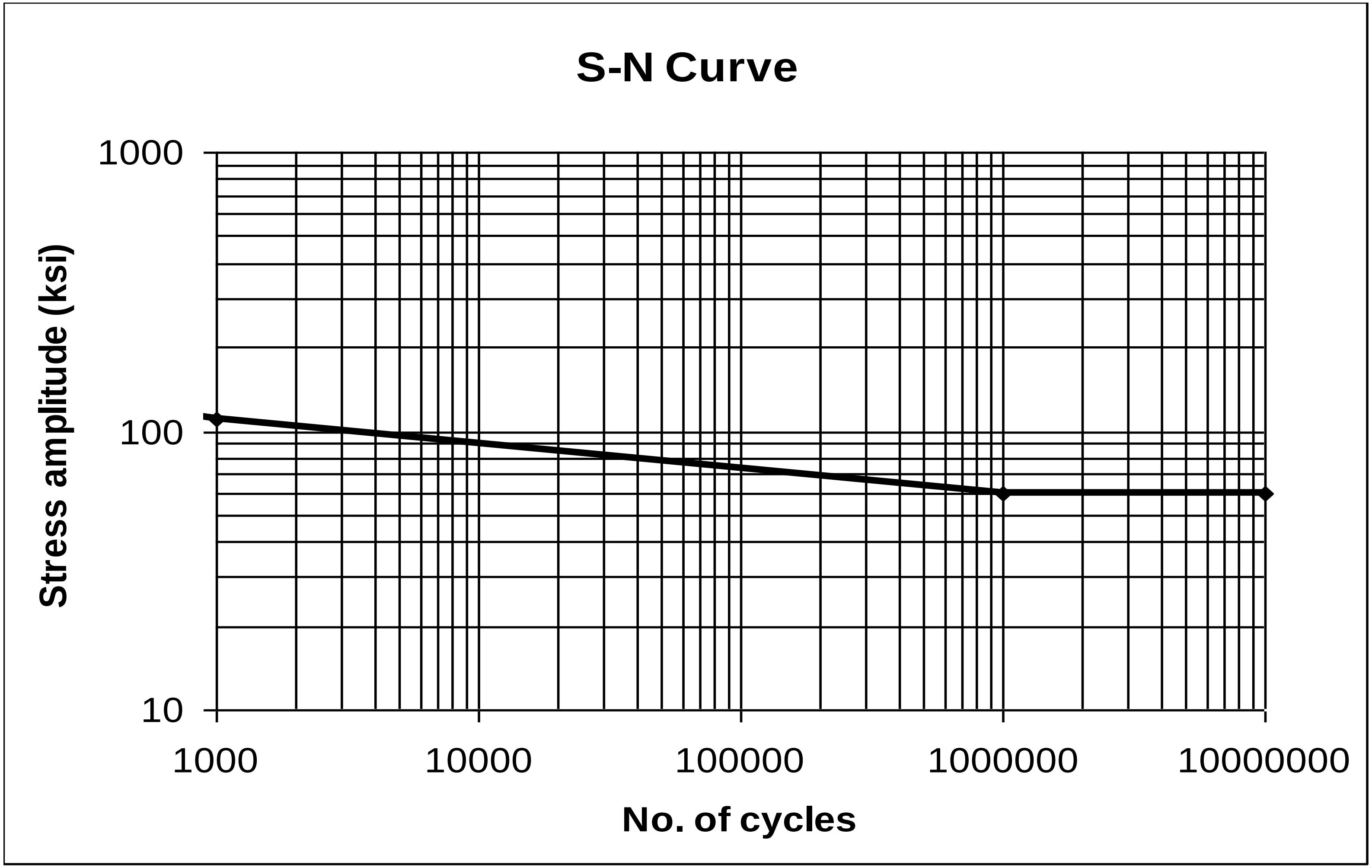
$$N\left(S_a\right)^m = C$$

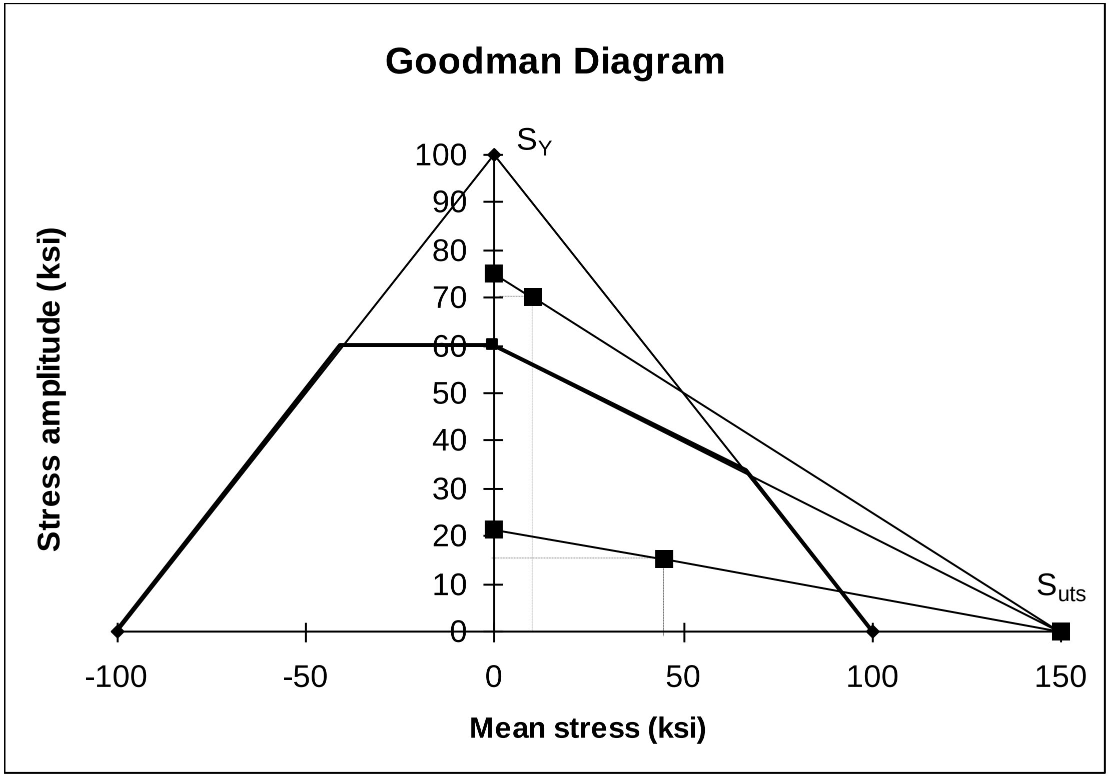
$$\log N + m\log S_a = \log C$$

$$\begin{cases} \log 1000 + m\log 110 = \log C \\ \log 10^6 + m\log 60 = \log C \end{cases}$$

$$\begin{cases} 3 + m\log 110 = \log C \\ 6 + m\log 60 = \log C \end{cases}$$

$$C = 1.886\times 10^{26} \quad m = 11.4$$


S-N Curve
Stress amplitude (ksi)
1000
100
10
1000
10000
100000
1000000
10000000
No. of cycles



$$\frac{S_a}{S_e} + \frac{S_m}{S_{uts}} = 1 \quad \textit{for fatigue endurance} \qquad S_{a,r(at\, S_m = 0)} = \left(1 - \frac{S_m}{S_{uts}}\right)^{-1} S_a$$

$$\frac{S_a}{S_{a,r}} + \frac{S_m}{S_{uts}} = 1 \quad \textit{for any stress amplitude} \qquad S_{a,r(at\, S_m = 0)} = \frac{S_a}{1 - \dfrac{S_m}{S_{uts}}}$$

Calculations of Fatigue Damage

a) Cycle No.11

$S_{a,r}$=87.93 ksi

$N_{11}= C\times(S_{a,N})^{-m}= 1.866\times10^{26}\times87.93^{-11.4}$=12805 cycles

D_{11}=**0.000078093**

b) Cycle No. 14

$S_{a,r}$=75.0 ksi

$N_{14}= C\times(S_{a,N})^{-m}= 1.866\times10^{26}\times75.0^{-11.4}$=78561 cycles

D_{14}=**0.000012729**

c) Cycle no. 15

$S_{a,r}$=98.28 ksi

$N_{14}= C\times(S_{a,N})^{-m}= 1.866\times10^{26}\times98.28^{-11.4}$=3606 cycles

D_{14}=**0.00027732**

	Results of "rainflow" counting				Damage	
No.	**ΔS**	**S_m**	**S_a**	**$S_{a,r\ (Sm=0)}$**	**$D_i=1/N_i=1/C*S_a^{-m}$**	
1	30	5	15	15.52	2.0155E-13	0
2	40	30	20	25.00	4.6303E-11	0
3	30	45	15	21.43	7.9875E-12	0
4	20	-50	10	10.00	1.3461E-15	0
5	50	-45	25	25.00	4.6303E-11	0
6	30	5	15	15.52	2.0155E-13	0
7	100	20	50	57.69	6.3949E-07	0
8	20	-20	10	10.00	1.3461E-15	0
9	30	-25	15	15.00	1.3694E-13	0
10	30	-55	15	15.00	1.3694E-13	0
11	**170**	**5**	**85**	**87.93**	**7.8039E-05**	**7.80E-05**
12	30	-5	15	15.00	1.3694E-13	0
13	30	-5	15	15.00	1.3694E-13	0
14	**140**	**10**	**70**	**75.00**	**1.2729E-05**	**1.27E-05**
15	**190**	**5**	**95**	**98.28**	**0.00027732**	**0.000277**
n_0=15				D =	0.00036873	3.677E-04
	D=0.000370			D=3.677E-04		
	L_R = 1/D =2712.03			**L_R = 1/D =2719.61**		
$N=n_0*L_R$=15*2712.03=40680				**$N=n_0*L_R$=15*2719.61=40794**		