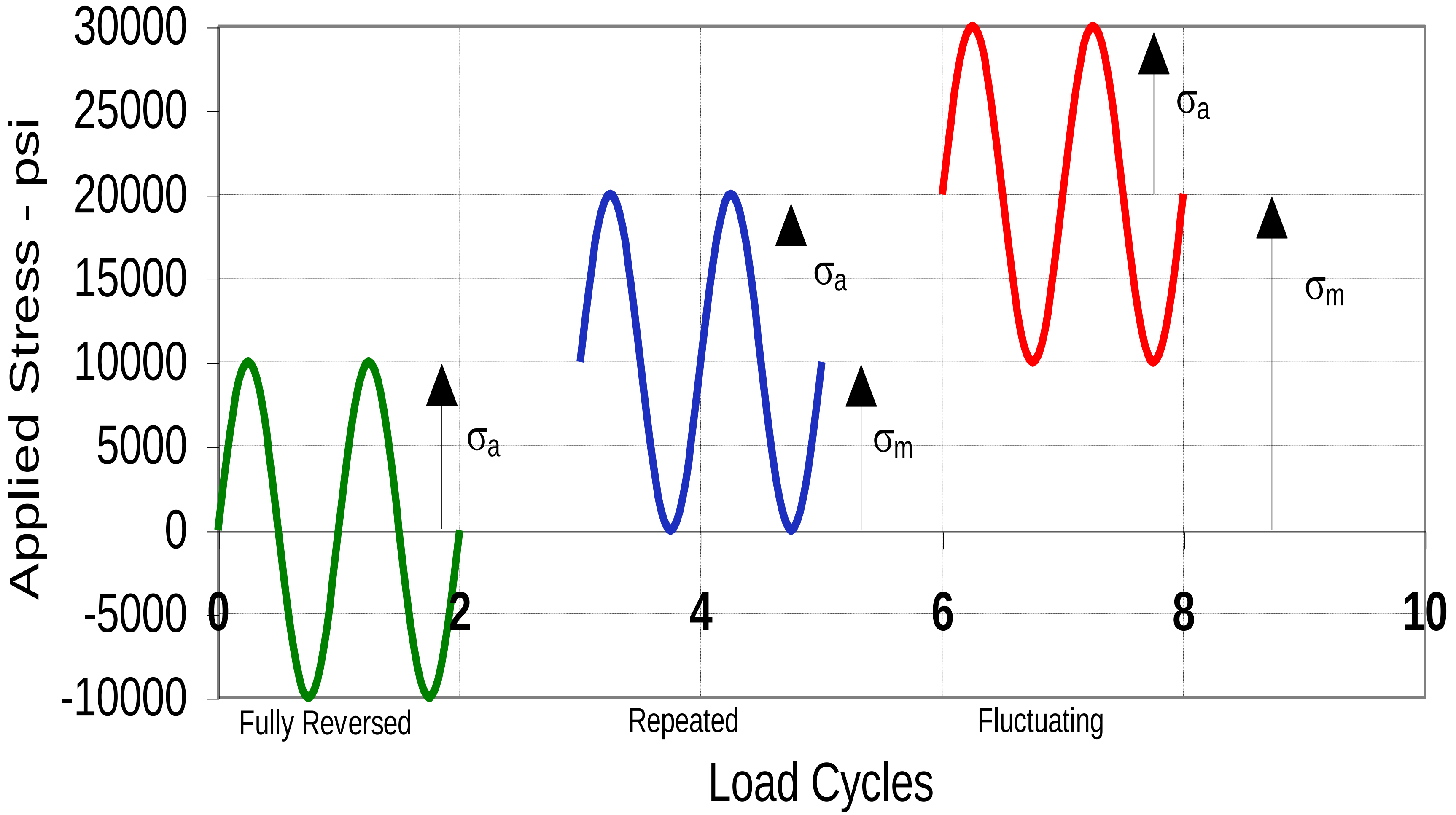
Fatigue Failure

- Occurs when stresses are changing throughout the life of a part.
- ____________________________ starts with a crack that propagates until a catastrophic failure occurs.
- This usually begins at a manufacturing defect or stress concentration that is subjected to tensile stresses for part of its load cycle.

Fatigue-Failure Models

- Stress-Life - cyclic stresses are kept below fatigue strength or endurance limit. Most widely used.
- Strain-Life - complicated not often used.
- ____________________ - sometimes used, gaining favor especially in the aircraft and aerospace industry.

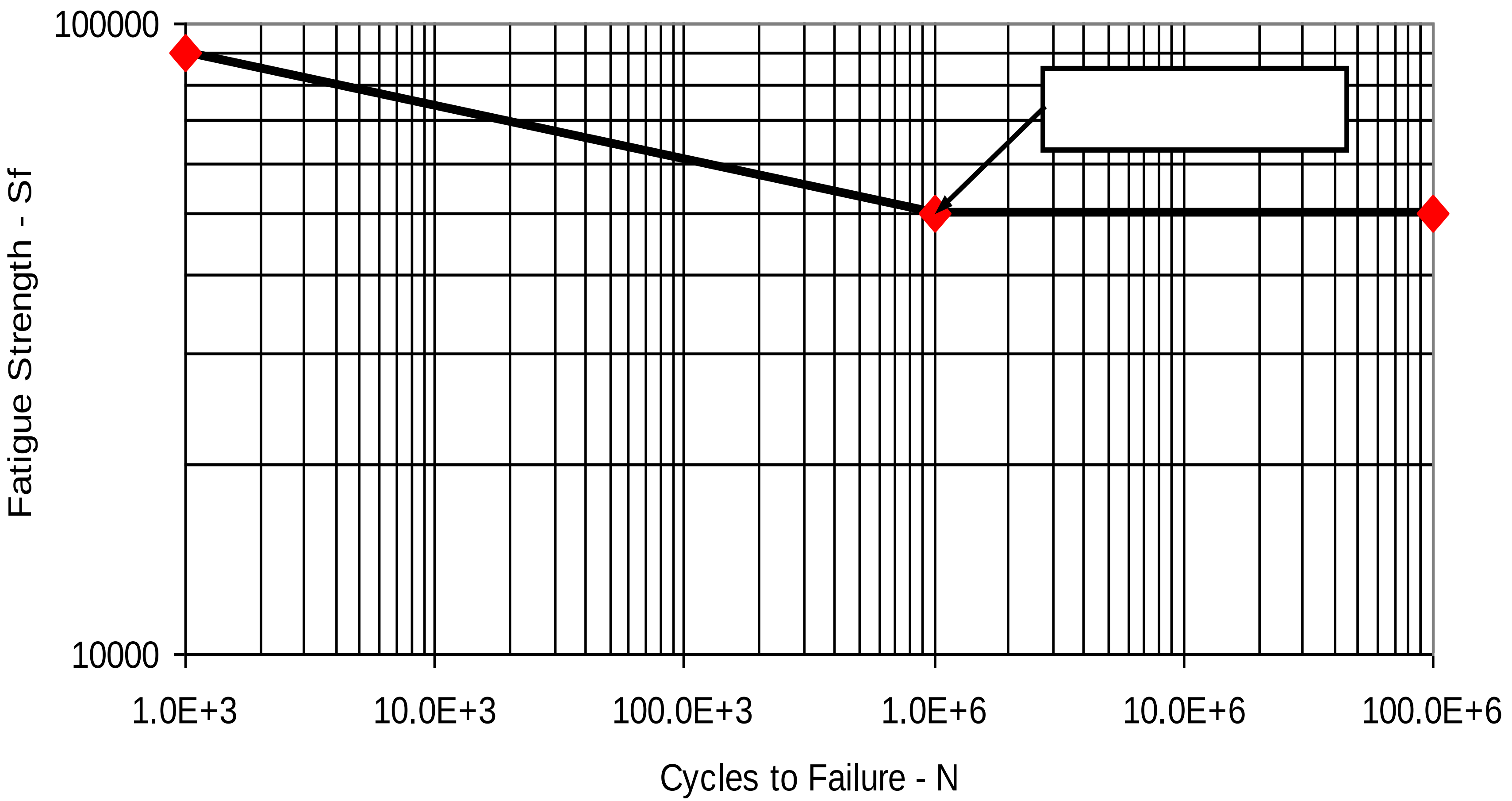
Fatigue Stresses



Fully Reversed Failure Criteria

- Most data comes from R. R. Moore rotating-beam test.
- Highly polished specimen of 0.3 inches in diameter is subjected to pure bending stresses that are alternated by rotating the specimen.
- Rotation is at 1725 rpm.
- Takes 1/2 day to reach 10^6 cycles.

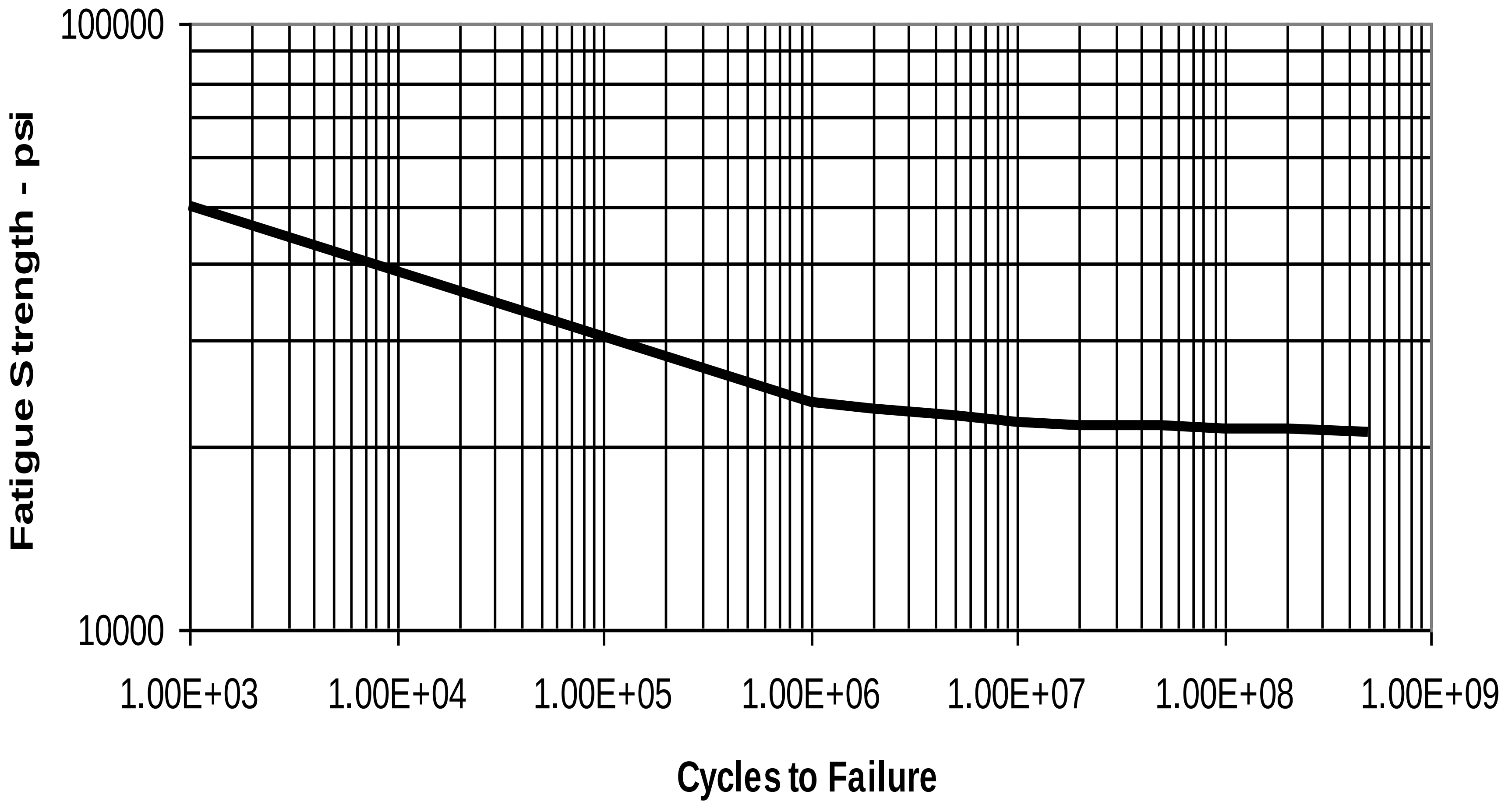
S-N Curve for Steel with Sut = 100 ksi



Endurance Limit and Fatigue Strength

- Endurance Limit of a Material ($S_{e'}$) - stress below which fatigue failure does not occur regardless of the number of stress cycles.
- ____________________________Strength of a Material ($S_{f'}$) -stress below which fatigue failure does not occur for a specified number of stress cycles.

Typical S-N Curve for Non-Ferrous Metals



Estimating $S_{e'}$ or $S_{f'}$

- For _______________:
  $S_{e'} = 0.5S_{ut}$ for $S_{ut}<200$ ksi (1400 MPa)
  $S_{e'} = 100$ ksi (700 MPa)
  for $S_{ut}>200$ ksi (1400 MPa)
- For _________________:
  $S_{e'} = 0.4S_{ut}$ for $S_{ut}<60$ ksi (400 MPa)
  $S_{e'} = 24$ ksi (160 MPa)
  for $S_{ut}>60$ ksi (400 MPa)

Estimating $S_{e'}$ or $S_{f'}$ cont'd

- For ______________________________:
  $S_{f'@5E8} = 0.4S_{ut}$ for $S_{ut}<48$ ksi (330 MPa)
  $S_{f'@5E8} = 19$ ksi (130 MPa)
  for $S_{ut}>330$ ksi (330 MPa)
- For ___________________________________:
  $S_{e'} = 0.4S_{ut}$ for $S_{ut}<40$ ksi (280 MPa)
  $S_{e'} = 14$ ksi (100 MPa)
  for $S_{ut}>40$ ksi (280 MPa)

Correction Factors to Endurance Limit and Fatigue Strength

- $S_e = C_{load} C_{size}\, C_{surf} C_{temp} C_{reliab}\, S_e'$
- $S_f = C_{load} C_{size}\, C_{surf} C_{temp} C_{reliab}\, S_f'$
- S_e - corrected endurance limit for a part
- S_f - corrected fatigue strength for a part

Correction Factors to Endurance Limit and Fatigue Strength

- C_{load} - load factor see eq. 6.7a pg 348
- C_{size} - size factor see eq's. 6.7a,b,c pg 348-349
- C_{surf} - surface factor see Fig's. 6-26, 6.27, and eq. 6.7e pg 349-352

Correction Factors to Endurance Limit and Fatigue Strength

- C_{temp} - temperature factor see eq. 6.7f pg 352-353
- C_{reliab} - reliability factor see Table 6.4 pg 353

Stress Concentration

- $K_f = 1+q(K_t + 1)$
- K_t - geometric stress concentration factor
- K_f - ______________________________
- q - notch sensitivity factor

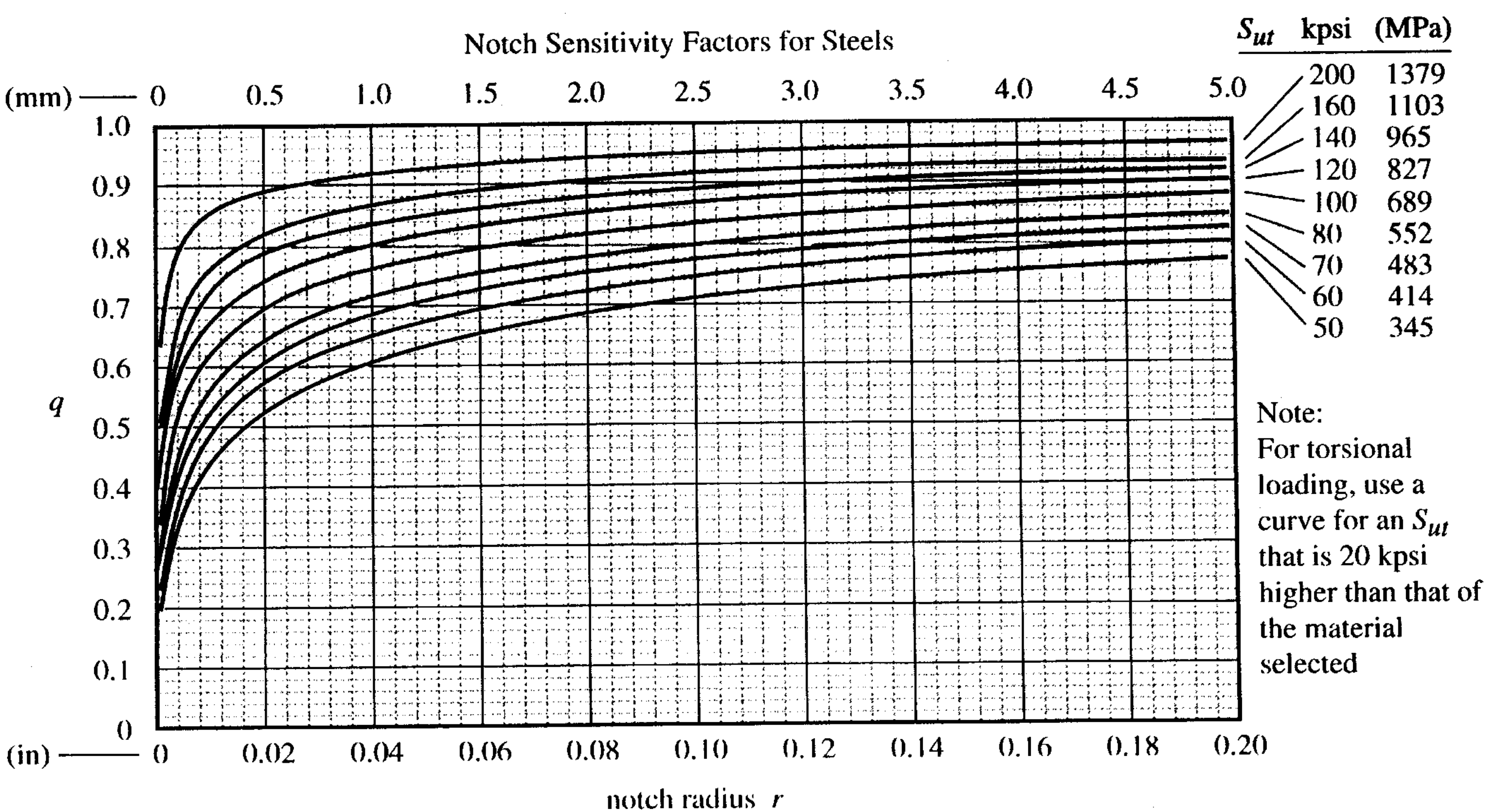


FIGURE 6-36 Part 1

Notch-Sensitivity Curves for Steels Calculated from Equation 6.13 using Data from Figure 6-35 as Originally Proposed by R. E. Peterson in "Notch Sensitivity," Chapter 13, in *Metal Fatigue* by G. Sines and J. Waisman, McGraw-Hill, New York, 1959.

Fluctuating Stresses

Modified-Goodman Diagram

Failure Lines for Fluctuating Stress

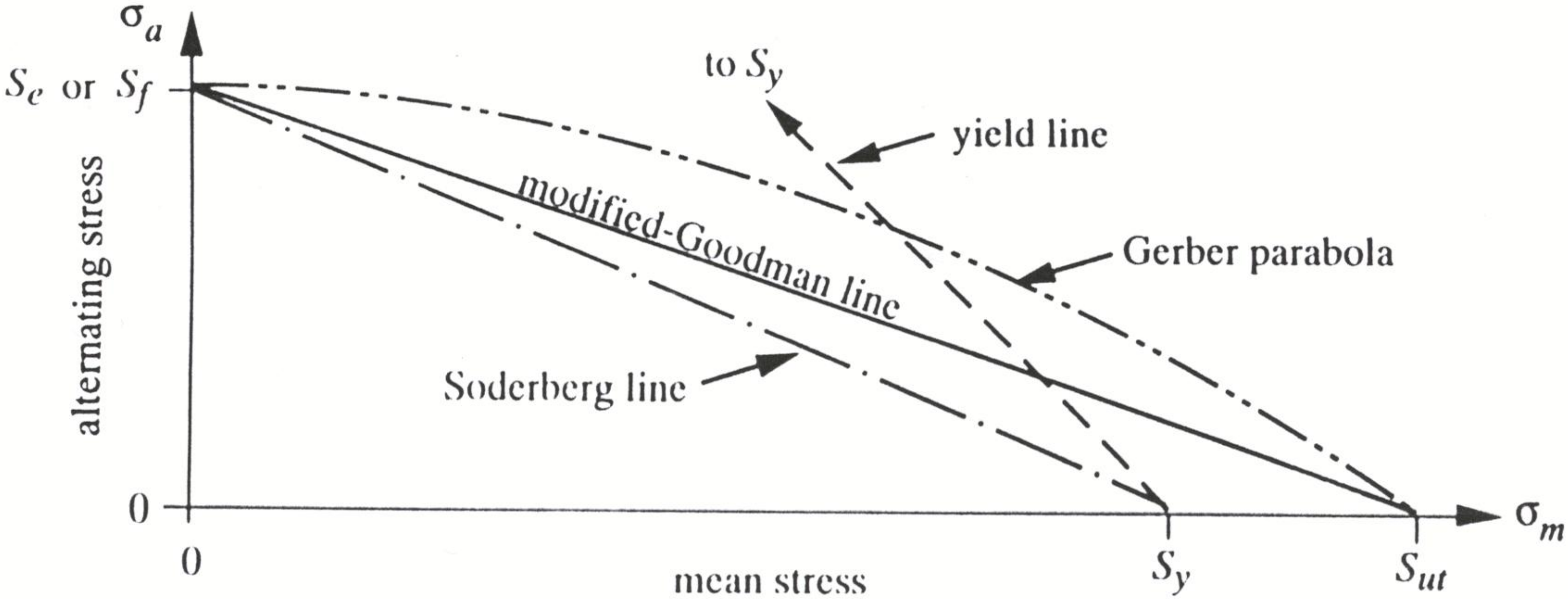


FIGURE 6-42

Various Failure Lines for Fluctuating Stresses

"Augmented" Modified-Goodman Diagram

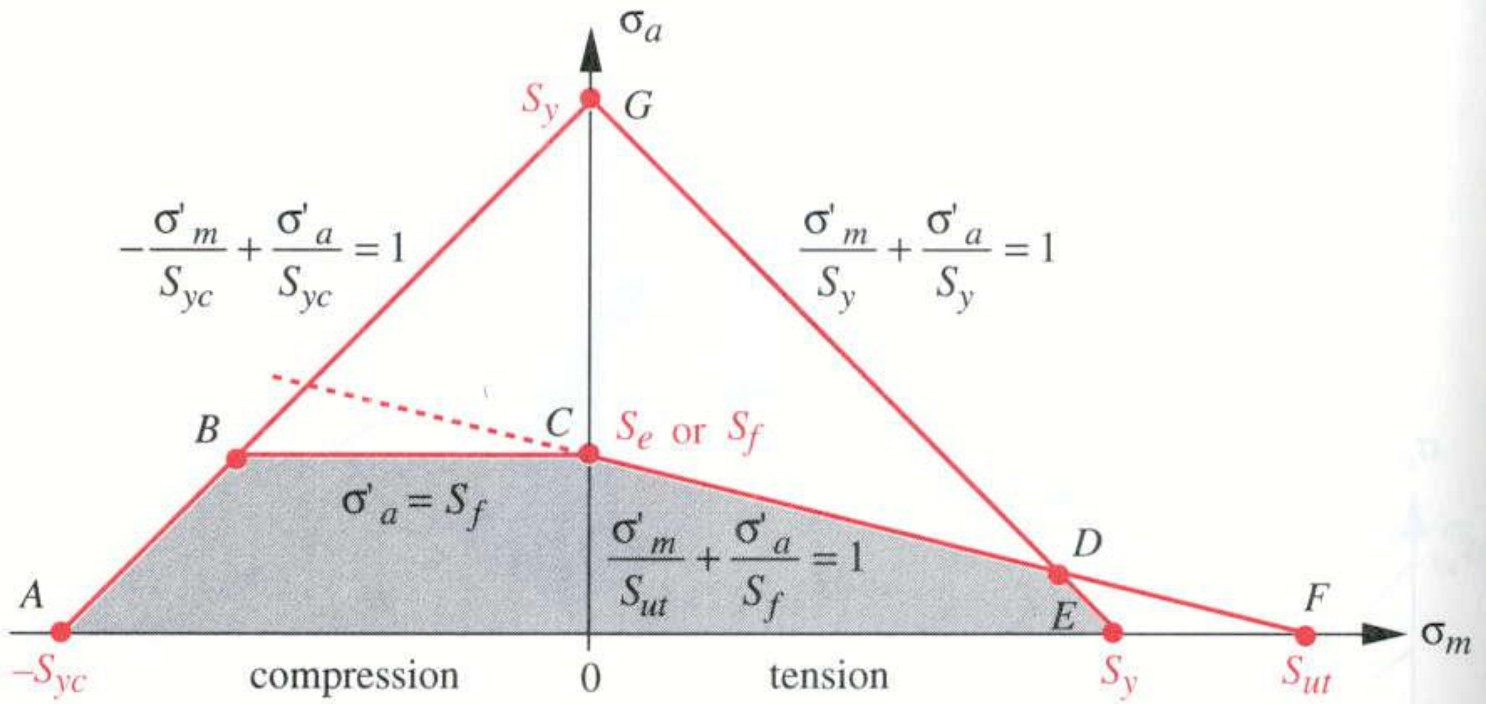


FIGURE 6-44

An "Augmented" Modified-Goodman Diagram

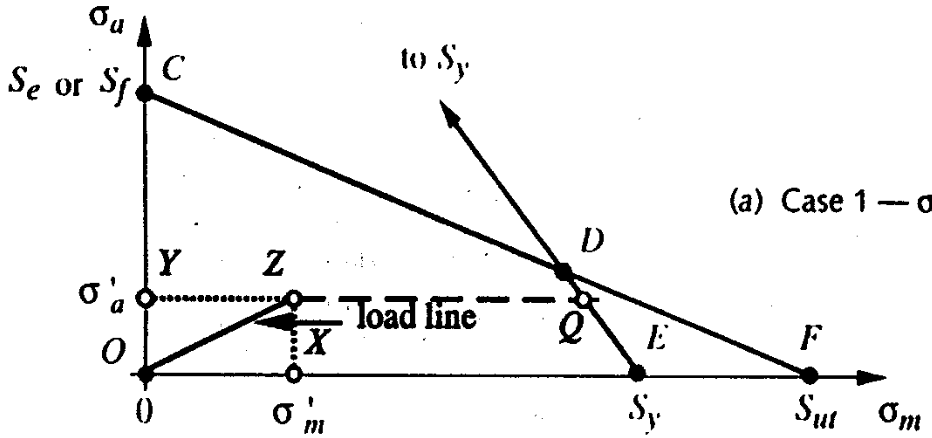
Stress Concentration

- Apply K_f to the ________________ components of stress.
- For ____________ materials apply K_t to the mean components of stress.

Stress Concentration - cont'd

- If $K_f|\sigma_{max_{nom}}| < S_y$ then: $K_{fm} = K_f$
- If $K_f|\sigma_{max_{nom}}| > S_y$ then:
  $K_{fm} = (S_y - K_f\sigma_{a_{nom}})/|\sigma_{m_{nom}}|$
- If $K_f|\sigma_{max_{nom}} - \sigma_{min_{nom}}| < 2S_y$ then: $K_{fm} = 0$

Factor of Safety for Fluctuating Stress - Case 1



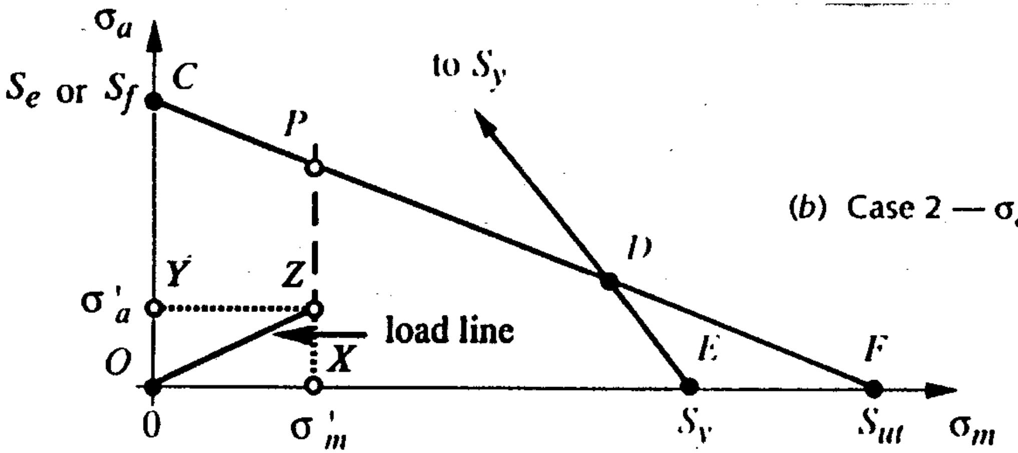
(a) Case 1 — σ_a constant and σ_m varies

For Case 1 failure occurs at point Q and the safety factor is the ratio of the lines YQ/YZ. To express this mathematically, we can solve equation 6.16*d* for the value of $\sigma'_{m@Q}$ and divide that by $\sigma'_{m@Z}$.

$$\sigma'_{m@Q} = \left(1 - \frac{\sigma'_a}{S_y}\right) S_y \qquad N_f = \frac{\sigma'_{m@Q}}{\sigma'_{m@Z}} = \frac{S_y}{\sigma'_m}\left(1 - \frac{\sigma'_a}{S_y}\right) \tag{6.18a}$$

If σ'_a were so large and σ'_m so small that point Q was on line CD instead of DE, then equation 6.16*c* should be used instead to determine the value of $\sigma'_{m@Q}$.

Factor of Safety for Fluctuating Stress - Case 2



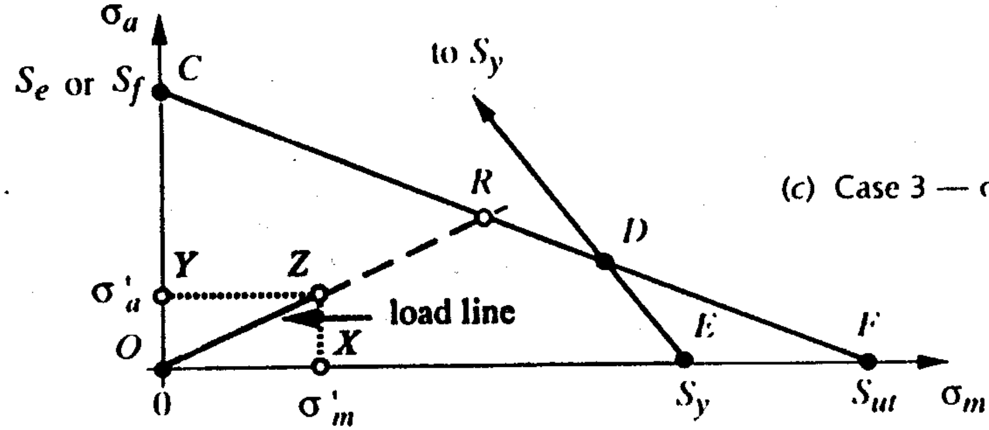
(*b*) Case 2 — σ_a varies and σ_m constant

FOR CASE 2 failure occurs at point P and the safety factor is the ratio of the lines XP/XZ. To express this mathematically, we can solve equation 6.16*c* for the value of $\sigma'_{a@P}$ and divide that by $\sigma'_{a@Z}$.

$$\sigma'_{a@P} = \left(1 - \frac{\sigma'_m}{S_{ut}}\right) S_f \qquad N_f = \frac{\sigma'_{a@P}}{\sigma'_{a@Z}} = \frac{S_f}{\sigma'_a}\left(1 - \frac{\sigma'_m}{S_{ut}}\right) \tag{6.18b}$$

If σ'_m were so large and σ'_a so small that point P was on line DE instead of CD, then equation 6.16*d* should be used instead to determine the value of $\sigma'_{a@P}$.

Factor of Safety for Fluctuating Stress - Case 3



FOR CASE 3 failure occurs at point R and the safety factor is the ratio of the lines OR/OZ or by similar triangles, either of the ratios $\sigma'_{m@R} / \sigma'_{m@Z}$ or $\sigma'_{a@R} / \sigma'_{a@Z}$. To express this mathematically, we can solve equations 6.16c and the equation of line OR simultaneously for the value of $\sigma'_{m@R}$ and divide that by $\sigma'_{m@Z}$.

from eq. 6.16c :

$$\sigma'_{a@R} = \left(1 - \frac{\sigma'_{m@R}}{S_{ut}}\right) S_f \tag{6.18c}$$

Factor of Safety for Fluctuating Stress - Case 3

from line *OR* : $$\sigma'_{a@R} = \left(\frac{\sigma'_{a@Z}}{\sigma'_{m@Z}}\right)\sigma'_{m@R} = \left(\frac{\sigma'_a}{\sigma'_m}\right)\sigma'_{m@R}$$

The simultaneous solution of these gives

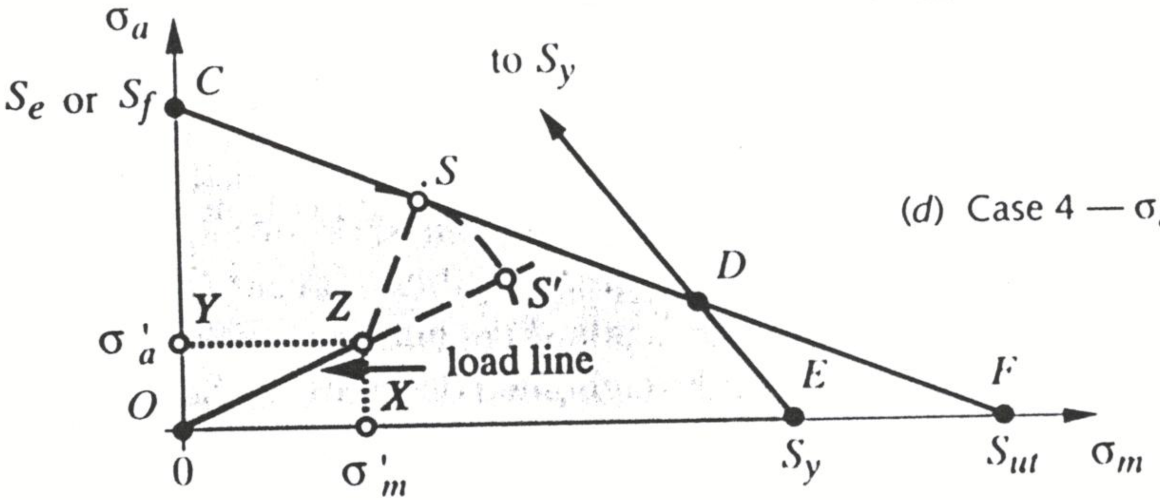
$$\sigma'_{m@R} = \frac{S_f}{\dfrac{\sigma'_a}{\sigma'_m} + \dfrac{S_f}{S_{ut}}} \qquad (6.18d)$$

which after substitution and some manipulation yields

$$N_f = \frac{\sigma'_{m@R}}{\sigma'_{m@Z}} = \frac{S_f S_{ut}}{\sigma'_a S_{ut} + \sigma'_m S_f} \qquad (6.18e)$$

There is also the possibility that point *R* may lie on line *DE* instead of *CD* in which case equation 6.16*d* should be substituted for 6.16*c* in the above solution.

Factor of Safety for Fluctuating Stress - Case 4



(*d*) Case 4 — σ_a and σ_m vary independently

FOR CASE 4 in which the future relationship between the mean and alternating stress components is either random or unknown, the point *S* on the failure line closest to the stress state at *Z* can be taken as a conservative estimate of the failure point. Line *ZS* is normal to *CD*, so its equation can be written and solved simultaneously with that of the line *CD* to find the coordinates of point *S* and the length *ZS*, which are

Factor of Safety for Fluctuating Stress - Case 4

$$\sigma'_{m@S} = \frac{S_{ut}\left(S_f^{\,2} - S_f\sigma'_a + S_{ut}\sigma'_m\right)}{S_f^{\,2} + S_{ut}^{\,2}}$$

$$\sigma'_{a@S} = -\frac{S_f}{S_{ut}}\left(\sigma'_{m@S}\right) + S_f$$

$$ZS = \sqrt{\left(\sigma'_m - \sigma'_{m@S}\right)^2 + \left(\sigma'_a - \sigma'_{a@S}\right)^2} \qquad (6.18f)$$

To establish a ratio for the safety factor, swing point S about point Z to be coincident with line OZR at point S'. The safety factor is the ratio OS'/OZ.

$$OZ = \sqrt{\left(\sigma'_a\right)^2 + \left(\sigma'_m\right)^2}$$

$$N_f = \frac{OZ + ZS}{OZ} \qquad (6.18g)$$

There is also the possibility that point S may lie on line DE instead of CD in which case equation 6.16*d* should be substituted for 6.16*c* in the above solution.

Fluctuating Stresses

Multiaxial Stresses in Fatigue

Fully Reversed Simple Multiaxial Stresses

$$\sigma'_a = \sqrt{\sigma_{1_a}^2 + \sigma_{2_a}^2 + \sigma_{3_a}^2 - \sigma_{1_a}\sigma_{2_a} - \sigma_{2_a}\sigma_{3_a} - \sigma_{1_a}\sigma_{3_a}} \tag{6.19a}$$

and for the two-dimensional case:

$$\sigma'_a = \sqrt{\sigma_{1_a}^2 - \sigma_{1_a}\sigma_{2_a} + \sigma_{2_a}^2} \tag{6.19b}$$

$$N_f = \frac{S_n}{\sigma'_a}$$

Fluctuating Simple Multiaxial Stresses

- Von Mises Method

$$\sigma'_a = \sqrt{\frac{\left(\sigma_{x_a} - \sigma_{y_a}\right)^2 + \left(\sigma_{y_a} - \sigma_{z_a}\right)^2 + \left(\sigma_{z_a} - \sigma_{x_a}\right)^2 + 6\left(\tau_{xy_a}^2 + \tau_{yz_a}^2 + \tau_{zx_a}^2\right)}{2}} \tag{6.22a}$$

$$\sigma'_m = \sqrt{\frac{\left(\sigma_{x_m} - \sigma_{y_m}\right)^2 + \left(\sigma_{y_m} - \sigma_{z_m}\right)^2 + \left(\sigma_{z_m} - \sigma_{x_m}\right)^2 + 6\left(\tau_{xy_m}^2 + \tau_{yz_m}^2 + \tau_{zx_m}^2\right)}{2}}$$

or for a biaxial stress state using:

$$\sigma'_a = \sqrt{\sigma_{x_a}^2 + \sigma_{y_a}^2 - \sigma_{x_a}\sigma_{y_a} + 3\tau_{xy_a}^2}$$

$$\sigma'_m = \sqrt{\sigma_{x_m}^2 + \sigma_{y_m}^2 - \sigma_{x_m}\sigma_{y_m} + 3\tau_{xy_m}^2} \tag{6.22b}$$

General Approach to High Cycle Fatigue Design

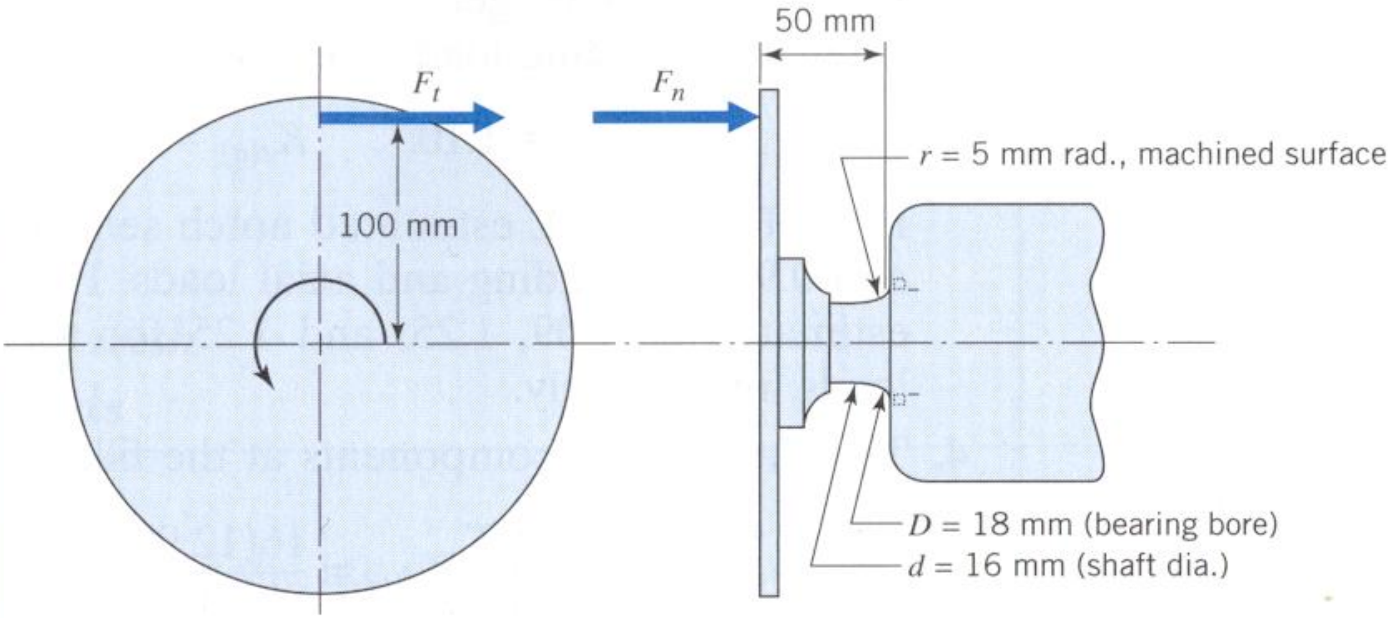
- Generate Modified Goodman Diagram
- Calculate alternating and mean components of stress at areas of concern on the part. Include appropriate stress concentration factors.
- Convert alternating and mean applied stresses to alternating and mean Von Mises Stresses.
- Plot these stresses on the Modified Goodman Diagram and find the factor of safety.

Example

- The figure pertains to the shaft of a disk sander that is made from steel having Su=900 MPa, and Sy=750 MPa. The most severe loading occurs when an object is held near the periphery of the disk with sufficient force to develop 12 N-m (which approaches the stall torque of the motor).
- Assume a coefficient of friction between the object and the disk is 0.6.
- What is the factor of safety with respect to eventual fatigue failure of the shaft?

Example

Schematic and Given Data:



f = 0.6 (between the object and the disk)
T = 12 N • m (friction torque)

S_u = 900 MPa

S_y = 750 MPa