Laws of Radiation Heat Transfer

Contrast in Radiative Mode of Heat Transfer

- Any body (> absolute zero) emits radiation at various wavelengths.

- Transparent bodies radiate energy in spherical space.

- Non-transparent bodies radiate energy in hemi-spherical space.

- The radiation energy emitted by a body is distributed in space at various wavelengths.

- This complex phenomenon requires simplified laws for engineering use of radiation.

Planck Radiation Law

- The primary law governing blackbody radiation is the *Planck Radiation Law*.

- This law governs the intensity of radiation emitted by unit surface area into a fixed direction (solid angle) from the blackbody as a function of wavelength for a fixed temperature.

- The Planck Law can be expressed through the following equation.

$$E(\lambda, T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda kT}} - 1}$$

$h = 6.625 \times 10^{-27}$ erg-sec (Planck Constant)

$K = 1.38 \times 10^{-16}$ erg/K (Boltzmann Constant)

C = Speed of light in vacuum

Monochromatic emissive power E_λ

- All surfaces emit radiation in many wavelengths and some, including black bodies, over all wavelengths.

- The monochromatic emissive power is defined by:

- *dE = emissive power in the wave band in the infinitesimal wave band between λ and λ+dλ.*

$$dE = E(\lambda, T)d\lambda$$

The monochromatic emissive power of a blackbody is given by:

$$E(\lambda, T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda kT}} - 1}$$

Wein's Displacement Law:

- At any given wavelength, the black body monochromatic emissive power increases with temperature.

- The wavelength λ_{max} at which is a maximum decreases as the temperature increases.

- The wavelength at which the monochromatic emissive power is a maximum is found by setting the derivative of previous Equation with respect to λ.

$$\left. \frac{dE(\lambda,T)}{d\lambda} \right|_{\lambda_{max}} = \left. \frac{d\left\{ \frac{2hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda kT}} - 1} \right\}}{d\lambda} \right|_{\lambda_{max}} = 0$$

$$\lambda_{max} T = 2897.8 \mu m K$$

Stefan-Boltzmann Law

- The maximum emissive power at a given temperature is the **black body emissive power (E_b)**.

- Integrating this over all wavelengths gives E_b.

$$\int\limits_0^\infty E(\lambda, T)\,d\lambda = \int\limits_0^\infty \left\{ \frac{2hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda kT}} - 1} \right\} d\lambda$$

$$E(T) = \frac{2hc^2\pi^4}{15\left[\dfrac{hc}{k}\right]^4} T^4 = \sigma T^4$$

Relevance of E_b

- **Driving forces:** Heat transfer by radiation is driven by differences in emissive power (proportional to T^4), not just temperature differences (convection & conduction).

The *total (hemispherical) energy* emitted by a real body, regardless of the wavelengths, is given by:

$$Q_{emitted} = \varepsilon \sigma A T^4$$

- where ε is the emissivity of the body,
- A is the surface area,
- T is the temperature, and
- σ is the Stefan-Boltzmann constant, equal to 5.67×10^{-8} W/m^{2}K^4.
- Emissivity is a material property, ranging from 0 to 1, which measures how much energy a surface can emit with respect to an ideal emitter ($\varepsilon = 1$) at the same temperature

Emissivity

- A black body is an ideal emitter.

- The energy emitted by any real surface is less than the energy emitted by a black body at the same temperature.

- At a defined temperature, a black body has the highest monochromatic emissive power at all wavelengths.

- The ratio of the monochromatic emissive power E_λ to the monochromatic blackbody emissive power $E_{\beta\lambda}$ at the same temperature is the spectral hemispherical emissivity of the surface.

$$\varepsilon(\lambda) = \frac{E_\lambda}{E_{b\lambda}}$$

The total (hemispherical) emissive power is, then, given by

$$E = \int_0^\infty E_\lambda \, d\lambda = \int_0^\infty \varepsilon(\lambda) E_{b\lambda} \, d\lambda$$

Define total (hemispherical) emissivity, at a defined temperature

$$\varepsilon = \frac{\displaystyle\int_0^\infty E_\lambda \, d\lambda}{\displaystyle\int_0^\infty E_{b\lambda} \, d\lambda} = \frac{\displaystyle\int_0^\infty \varepsilon(\lambda) E_{b\lambda} \, d\lambda}{\displaystyle\int_0^\infty E_{b\lambda} \, d\lambda}$$

Here, ε can be interpreted as either the emissivity of a body, which is wavelength independent, i.e., ε_λ is constant, or as the average emissivity of a surface at that temperature.

A surface whose properties are independent of the wavelength is known as a gray surface.

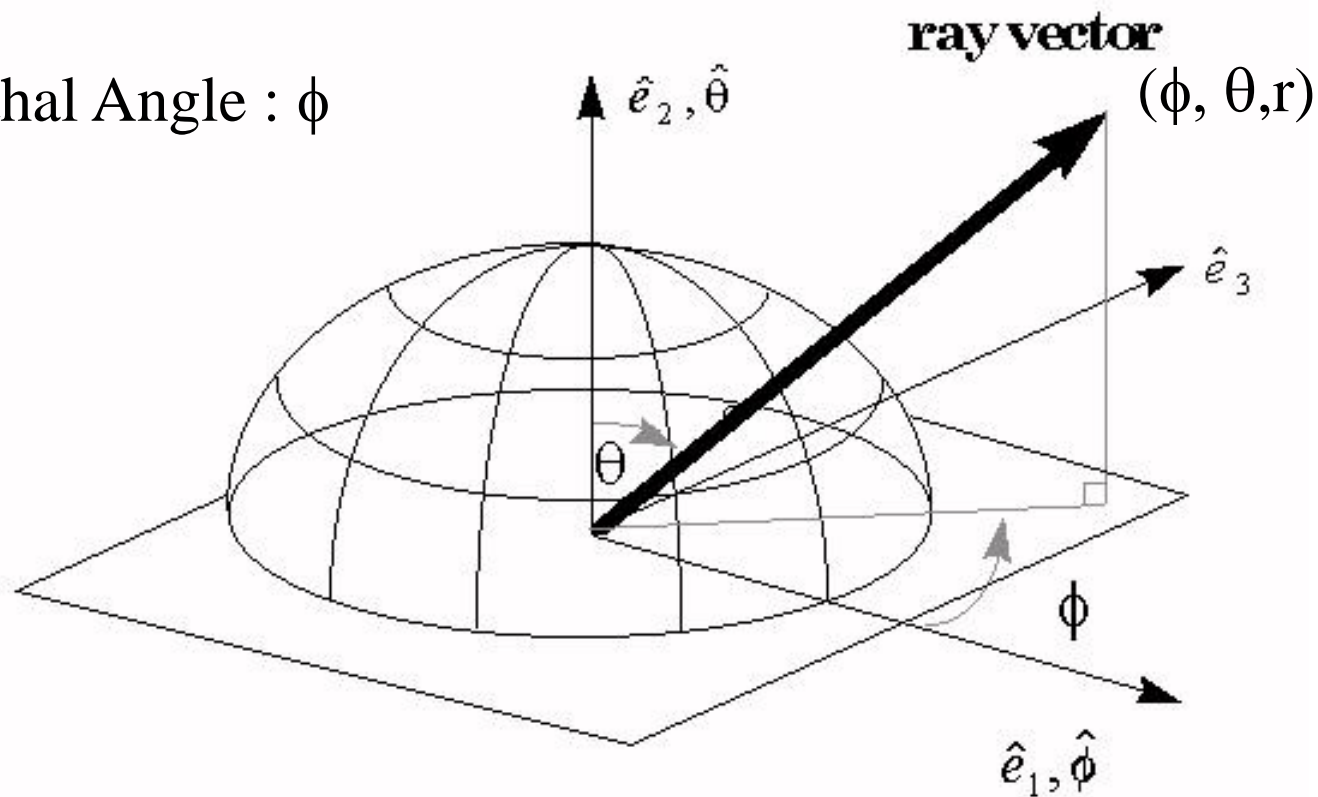The emissive power of a gray surface is given by $\quad E = \varepsilon E_b$
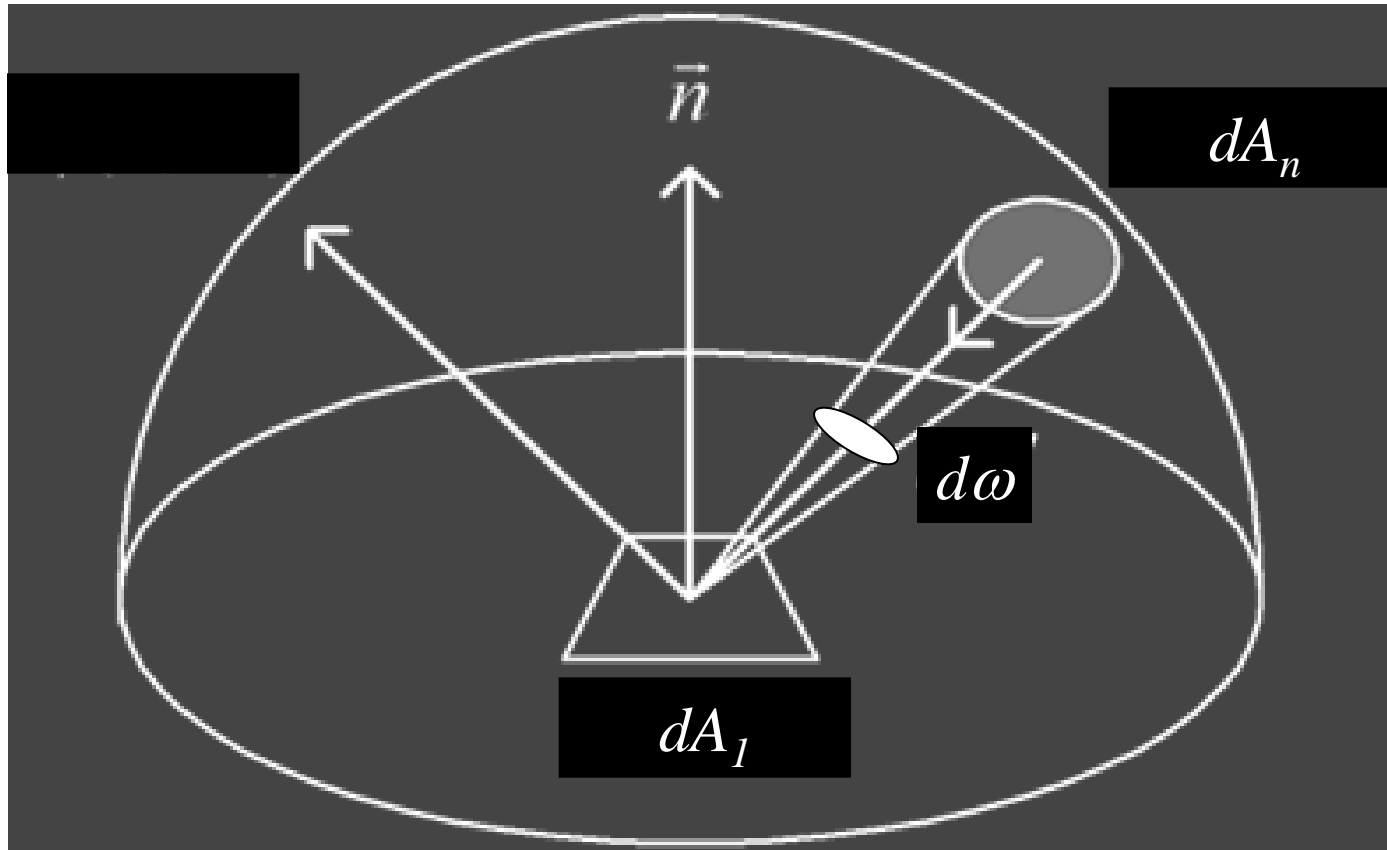
Spherical Geometry

dA
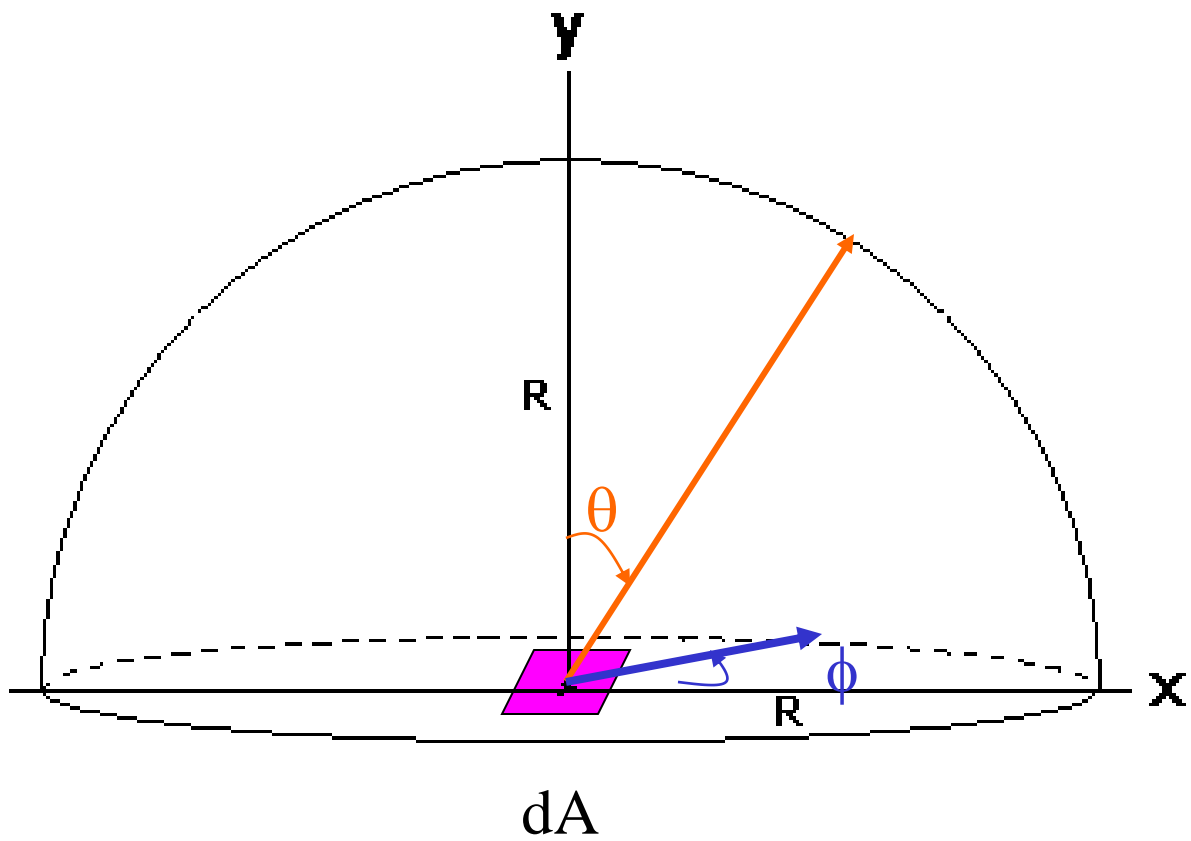
Vectors in Spherical Geometry

Zenith Angle : θ

Azimuthal Angle : φ

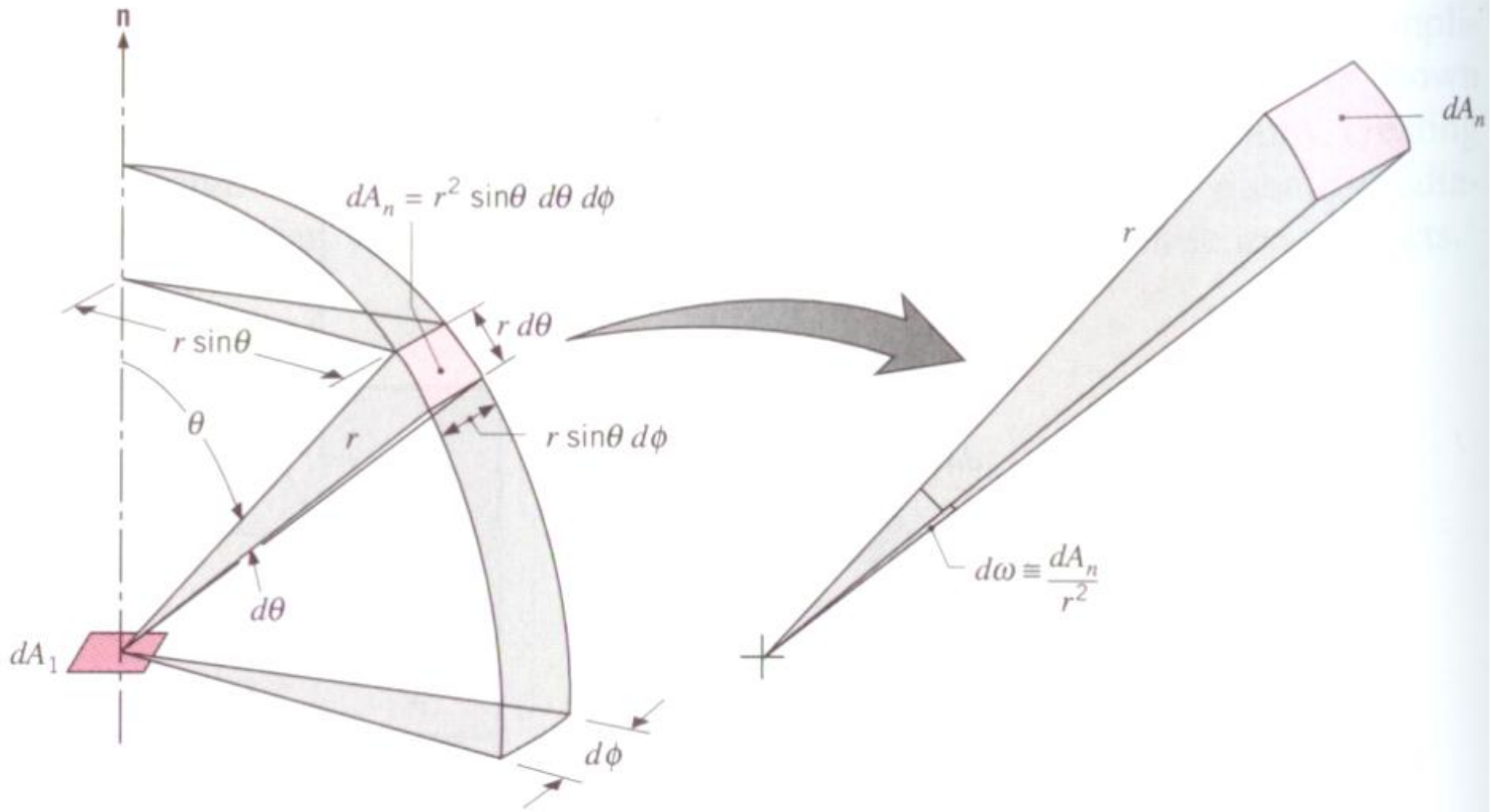# Definition of Solid Angle

Differential solid angle is defined by a region between the rays of a sphere.

It is measured as the ratio of the element of areas dA_n on the sphere to the square of the sphere's radius

$$d\omega = \frac{dA_n}{r^2}$$

Differential Solid Angle due to Spherical Differential Volume

$$dA_n = r^2 \sin\theta \, d\theta \, d\phi$$

$$d\omega \equiv \frac{dA_n}{r^2}$$

Emissive Intensity

- Solid Angle:

$$d\omega = \frac{dA_n}{r^2} \quad dA_n = r^2 \sin\theta d\theta d\phi$$

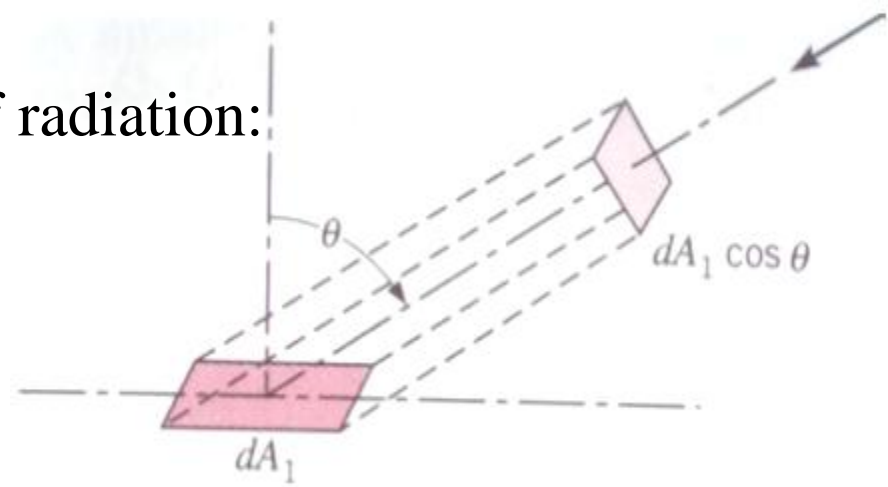$$d\omega = \sin\theta \, d\theta \, d\varphi$$

Emissive intensity

$$I_e(\theta, \varphi) = \frac{dq}{dA_n d\omega} \qquad W/m^2 Sr$$

Monochromatic Emissive intensity

$$I_{\lambda,e}(\lambda, \theta, \varphi) = \frac{dq}{dA_n d\omega d\lambda} \qquad W/m^2 Sr\mu m$$

Projection of dA_n normal to direction of radiation:

$$dA_n = dA_1 \cos \theta$$

Spectral Radiation flux

$$dq'' = \frac{I_{\lambda,e}(\lambda, \theta, \varphi) dA_n d\omega d\lambda}{dA_1}$$

$$dq'' = \frac{I_{\lambda,e}(\lambda, \theta, \varphi) dA_1 \cos \theta d\omega d\lambda}{dA_1}$$

$$dq'' = I_{\lambda,e}(\lambda, \theta, \varphi) \cos \theta d\omega d\lambda$$

The specrtal heat flux associated with emission into hypothetical hemisphere above dA_1

$$q''(\lambda) = \int_0^{2\pi} \int_0^{\pi/2} I_{\lambda,e}(\lambda,\theta,\varphi)\cos\theta \sin\theta d\theta d\phi$$

Total (hemispherical) heat flux associated with dA₁

$$q'' = \int_0^{\infty} q''(\lambda)d\lambda = \int_0^{\infty}\int_0^{2\pi}\int_0^{\pi/2} I_{\lambda,e}(\lambda,\theta,\varphi)\cos\theta \sin\theta d\theta d\phi d\lambda$$

Relation to Emission

Spectral emissive power is defined as the rate at which radiation of wavelength λ is emitted in all directions form a surface per unit wavelength dλ about λ.

$$E_\lambda(\lambda) = q^{''}(\lambda) = \int_0^{2\pi} \int_0^{\pi/2} I_{\lambda,e}(\lambda, \theta, \varphi) \cos\theta \sin\theta \, d\theta \, d\phi$$

The total hemispherical emissive power:

$$E = \int_0^\infty q^{''}(\lambda) d\lambda = \int_0^\infty \int_0^{2\pi} \int_0^{\pi/2} I_{\lambda,e}(\lambda, \theta, \varphi) \cos\theta \sin\theta \, d\theta \, d\phi \, d\lambda$$

# Radiative Heat Exchange between Two Differential Area Elements

- The elements dA_i and dA_j are isothermal at temperatures T_i and T_j respectively.

- The normals of these elements are at angles θ_i and θ_j respectively to their common normal.

- The total energy per unit time leaving dA_i and incident upon dA_2 is:

$$dQ_{i \to j} = I_{b,i} \cos \theta_i d\omega_i dA_i$$

$$d\omega_i = \frac{dA_j \cos \theta_j}{r^2}$$
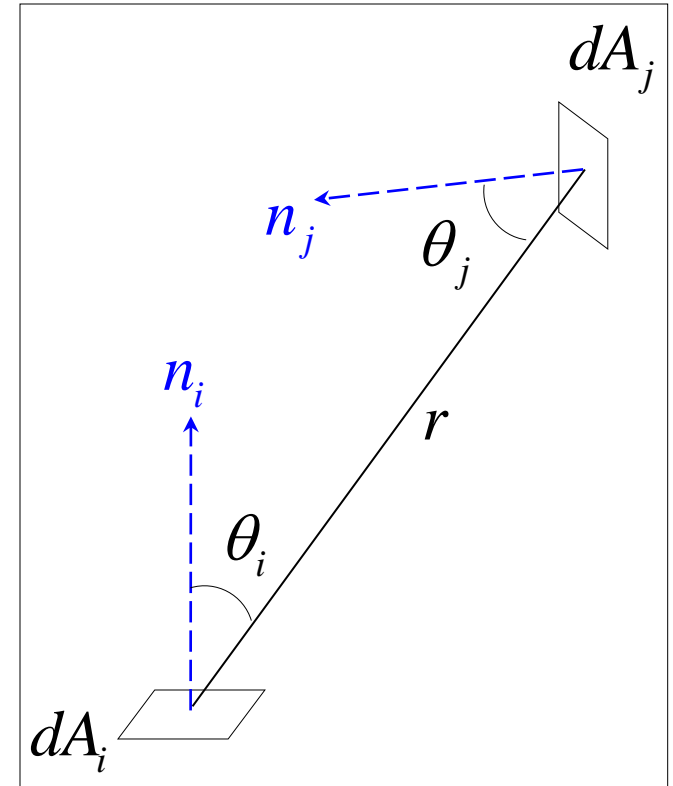
The monochromatic energy per unit time leaving dA_i and incident on dA_2 is
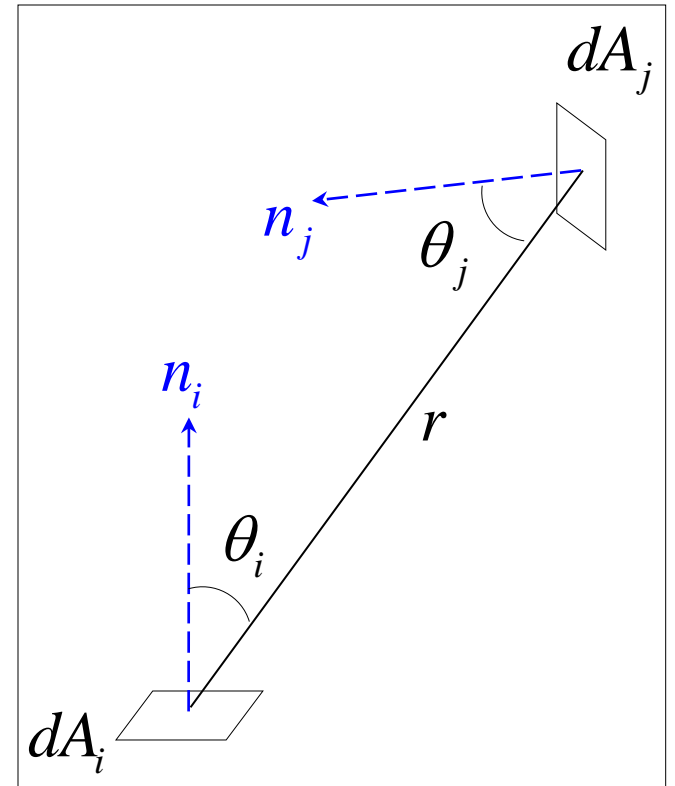
$$dQ_{\lambda,i \rightarrow j} = I_{\lambda b,i}(\lambda) \cos \theta_i d\omega_i dA_i d\lambda$$

• The total energy per unit time leaving dA_i and incident upon dA_2 is:

$$d^2 Q_{i \to j} = I_{b,i} \cos \theta_i \, d\omega_i \, dA_i$$

The monochromatic energy per unit time leaving dA_i and incident on dA_2 is:

$$d^3 Q_{\lambda, i \to j} = I_{\lambda b, i}(\lambda) \cos \theta_i \, d\omega_i \, dA_i \, d\lambda$$

$$d^2 Q_{\lambda, i \to j} = \int_0^\infty d^3 Q_{\lambda, i \to j} = \int_0^\infty I_{\lambda b, i}(\lambda) \cos \theta_i \, d\omega_i \, dA_i \, d\lambda$$

Absorptivity α, Reflectivity ρ, and Transmissivity τ

- Consider a semi-transparent sheet that receives incident radiant energy flux, also known as irradiation, G .

- Let dG represent the irradiation in the waveband λ to $\lambda + d\lambda$.

- Part of it may be absorbed, part of it reflected at the surface, and the rest transmitted through the sheet.

- Define monochromatic properties.
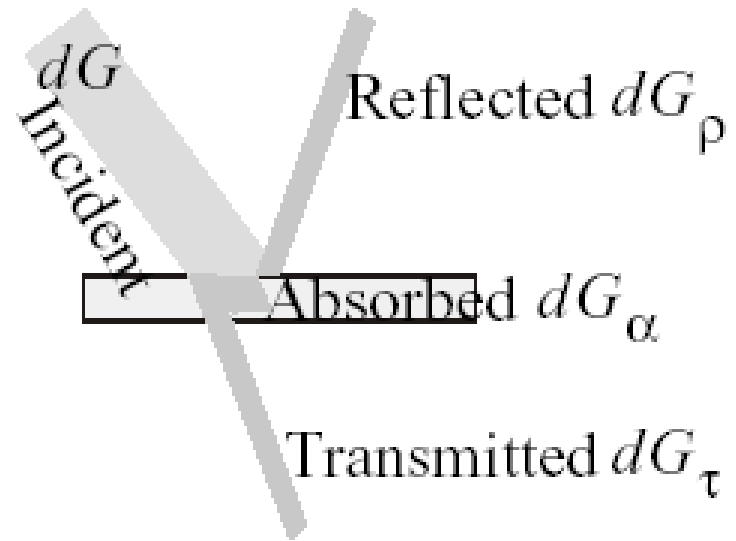
Monochromatic Absorptivity : $\quad \alpha_\lambda = \dfrac{dG_\alpha}{dG}$

Total (hemispherical) Absorptivity : $\quad \alpha = \displaystyle\int_0^\infty \alpha_\lambda d\lambda = \dfrac{G_\alpha}{G}$

Monochromatic reflectivity : $\quad \rho_\lambda = \dfrac{dG_\rho}{dG}$

Total reflectivity : $\quad \rho = \displaystyle\int_0^\infty \rho_\lambda d\lambda = \dfrac{G_\rho}{G}$

Monochromatic Transmissivity : $\quad \tau_\lambda = \dfrac{dG_\tau}{dG}$

Total Transmissivity : $\quad \tau = \displaystyle\int_0^\infty \tau_\lambda d\lambda = \dfrac{G_\tau}{G}$

Conservation of Irradiation

The total Irradiation = $\quad G = G_\alpha + G_\rho + G_\tau$

$$1 = \frac{G_\alpha}{G} + \frac{G_\rho}{G} + \frac{G_\tau}{G}$$

$$\alpha + \rho + \tau = 1$$

Blackbody Radiation

- The characteristics of a blackbody are :
- It is a perfect emitter.
- At any prescribed temperature it has the highest monochromatic emissive power at all wave lengths.
- A blackbody absorbs all the incident energy and there fore $\alpha = \alpha_\lambda = 1$.
- It is non reflective body (τ=0).
- It is opaque ($\tau = 0$).
- It is a diffuse emitter
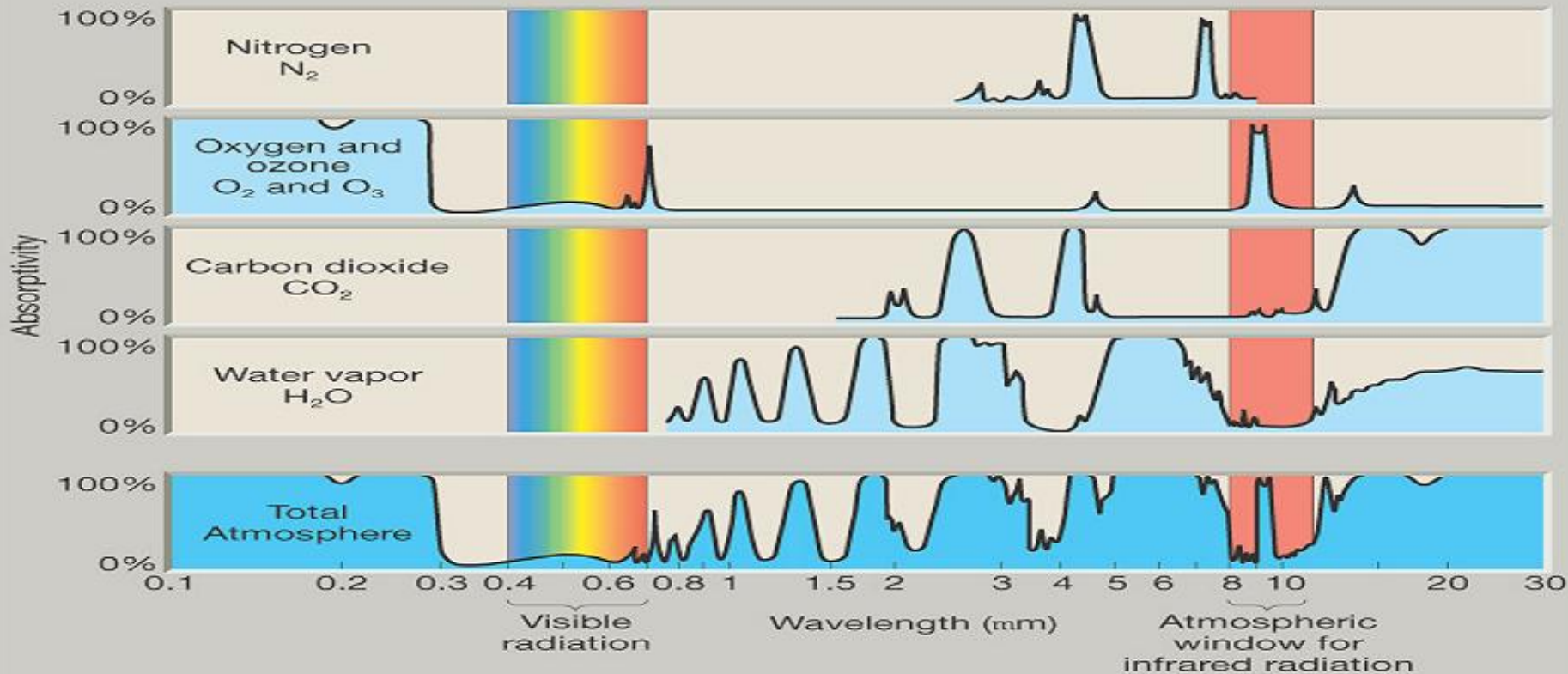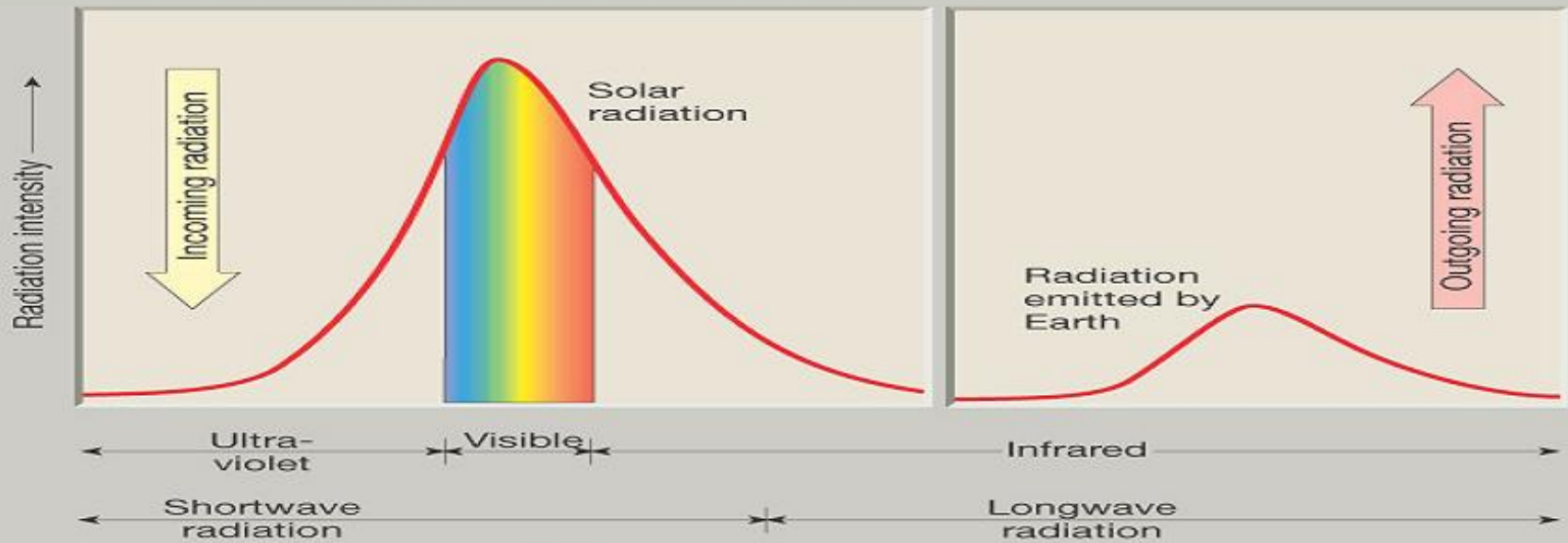
Radiative Heat Transfer

Consider the heat transfer between two surfaces, as shown in **Figure.**
What is the rate of heat transfer into Surface B?
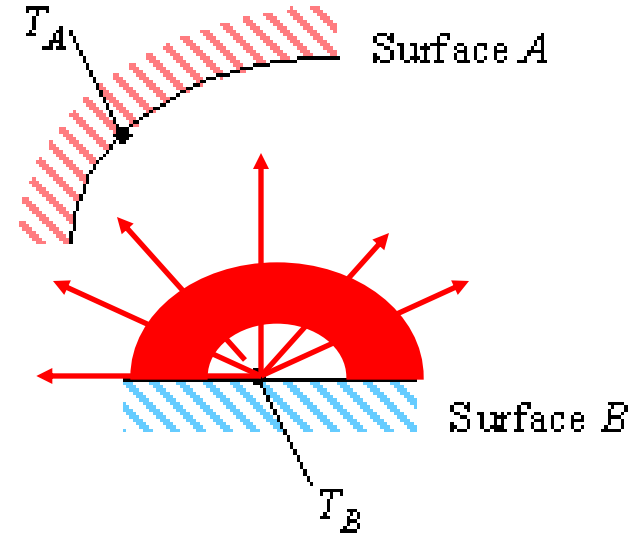To find this, we will first look at the emission from A to B.
Surface A emits radiation as described in

$$Q_{A,emitted} = \varepsilon_A \sigma A_A T_A^4$$

This radiation is emitted in all directions, and only a fraction of it will actually strike Surface B.
This fraction is called the shape factor, F.

The amount of radiation striking Surface B is therefore:

$$Q_{B,inceident} = F_{A \to B} \varepsilon_A \sigma A_A T_A^4$$

The only portion of the incident radiation contributing to heating Surface B is the absorbed portion, given by the absorptivity α_B:

$$Q_{B,absorbed} = \alpha_B F_{A \to B} \varepsilon_A \sigma A_A T_A^4$$

Above equation is the amount of radiation gained by Surface B from Surface A.

To find the net heat transfer rate at B, we must now subtract the amount of radiation emitted by B:

$$Q_{B,emitted} = \varepsilon_B \sigma A_B T_B^4$$

The net radiative heat transfer (gain) rate at Surface B is

$$Q_B = Q_{B,absorbed} - Q_{B,emitted}$$

$$Q_B = \alpha_B F_{A \to B} \varepsilon_A \sigma A_A T_A^4 - \varepsilon_B \sigma A_B T_B^4$$

Shape Factors

- Shape factor, F, is a geometrical factor which is determined by the shapes and relative locations of two surfaces.

- **Figure** illustrates this for a simple case of cylindrical source and planar surface.

- Both the cylinder and the plate are infinite in length.

- In this case, it is easy to see that the shape factor is reduced as the distance between the source and plane increases.

- The shape factor for this simple geometry is simply the cone angle (θ) divided by 2π
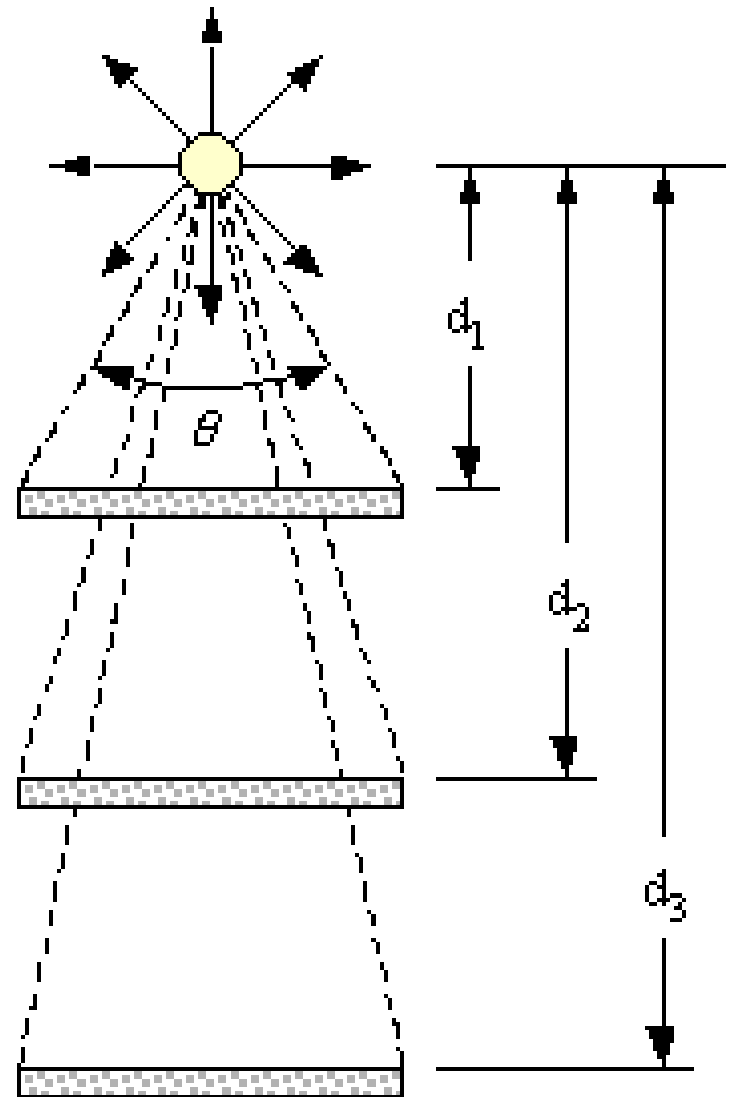
- Shape factors for other simple geometries can be calculated using basic theory of geometry.
- For more complicated geometries, the following two rules must be applied to find shape factors based on simple geometries.
- The first is the *summation rule*.
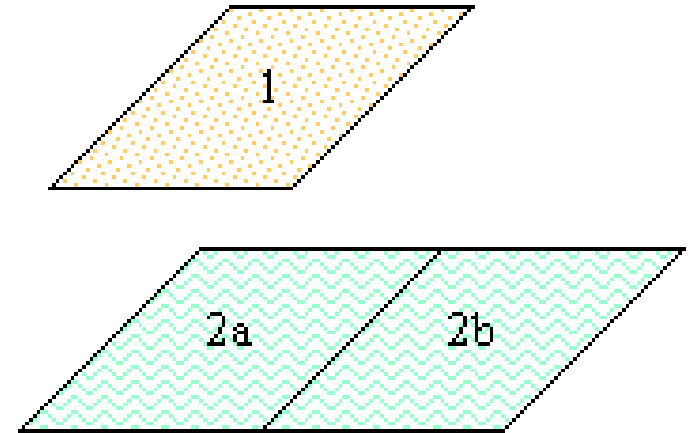
- This rule says that the shape factor from a surface (1) to another (2) can be expressed as a sum of the shape factors from (1) to (2a), and (1) to (2b).

$$F_{1\rightarrow 2} = F_{1\rightarrow 2a} + F_{1\rightarrow 2b}$$

- The second rule is the *reciprocity rule*, which relates the shape factors from (1) to (2) and that from (2) to (1) as follows:

$$A_1 F_{1\rightarrow 2} = A_2 F_{2\rightarrow 1}$$

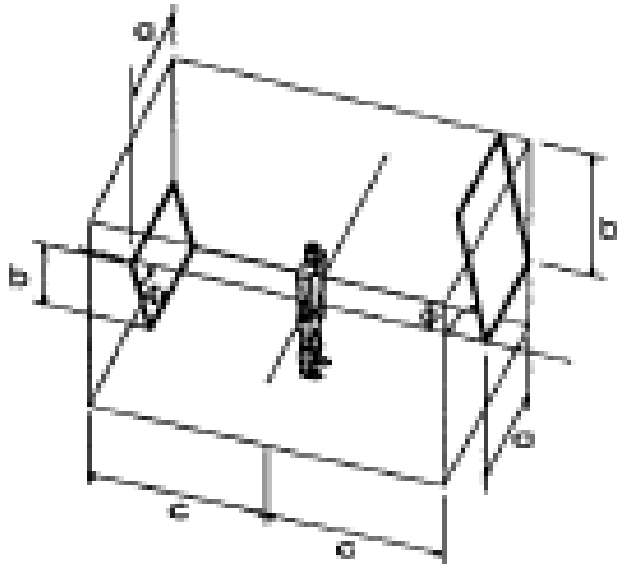Thus, if the shape factor from (1) to (2) is known, then the shape factor from (2) to (1) can be found by:

$$F_{1 \to 2} = \frac{A_2}{A_1} F_{2 \to 1}$$

If surface (2) totally encloses the surface 1:
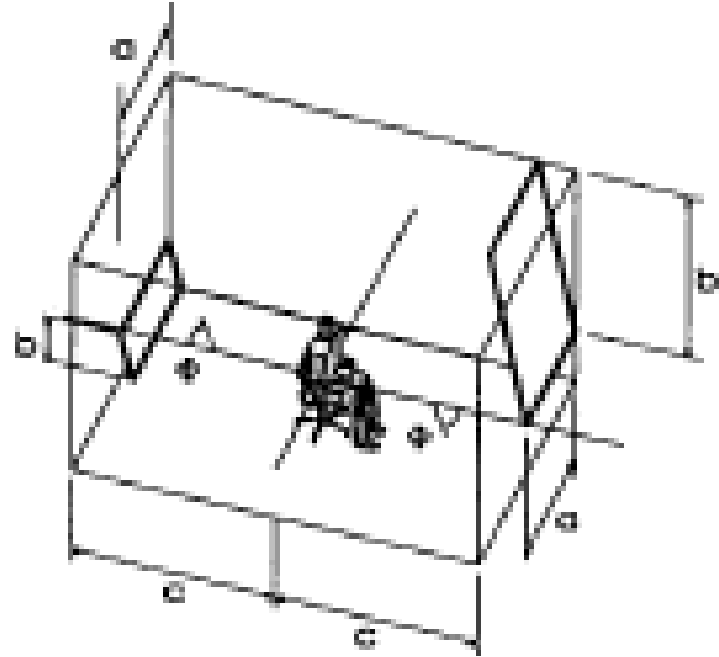
$$F_{1 \to 2} = 1$$

Geometrical Concepts in Radiation Heat Transfer

Human Shape Factors

Standing Person

Front/Top/Inclined ⊕
Back/Bottom/Inclined ⊕

Sitting Person

Wherever artificial climates are created for human occupation, the aim of the design is that individuals experience thermal comfort in the environment.

Among other factors thermal comfort depends on mean radiant temperature.

Flame to Furnace Wall Shape Factors