UNSTEADY STATE HEAT TRANSFER

THERMAL DIFFUSIVITIES AND HEAT TRANSFER COEFFICIENTS

Objective

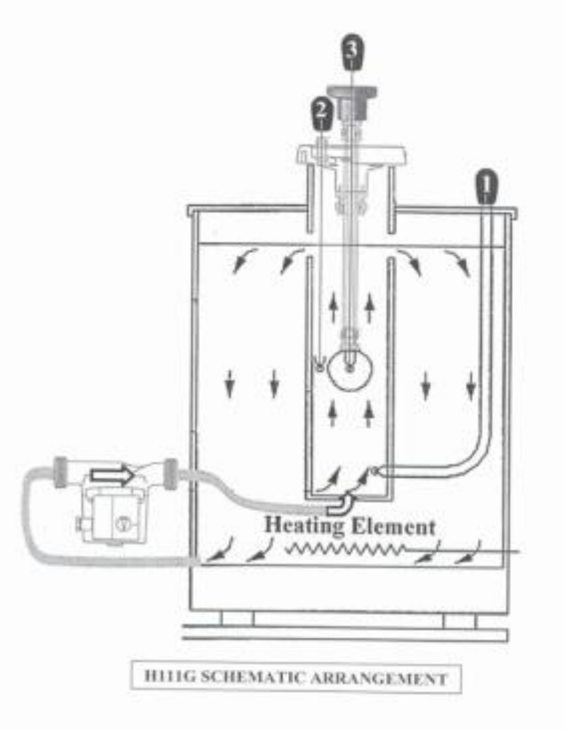
To observe unsteady state heat flow to or from the center of a solid shape (qualitative, using data acquisition) when a step change is applied to the temperature at the surface of the shape.

To determine the thermal conductivity of a solids and the heat transfer coefficient.
By using
Graphical method, analytical transient- temperature heat flow chart,
Method of Separation of variables, and
Polynomial approximation method,
Investigate the effect of shape, size, and material properties on unsteady heat flow.
Compare the results obtained using the above methods with literature values, and determine which method is more accurate.
Determine the temperature profiles as a function of position and time.
Investigate the Lumped Thermal Capacitance method of transient temperature analysis.

Apparatus

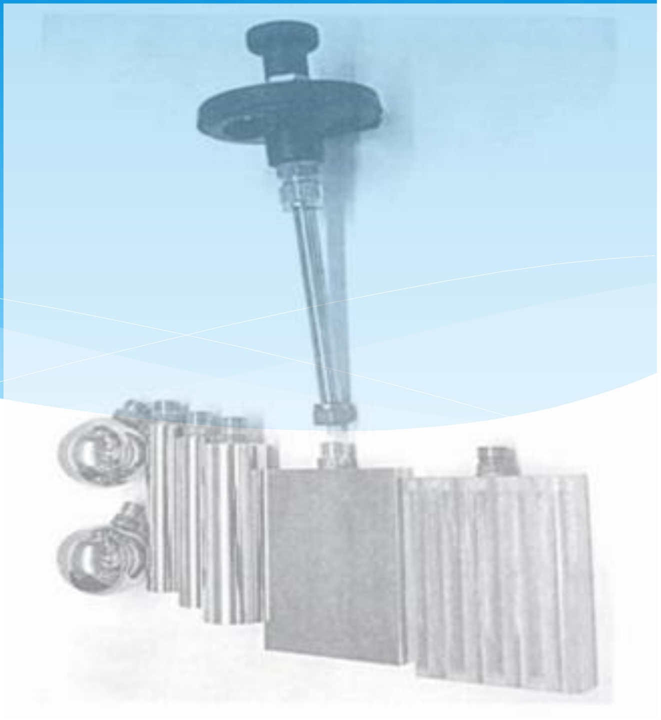
- The unit consists a stainless steel water bath and integral flow duct with external water circulating pump.

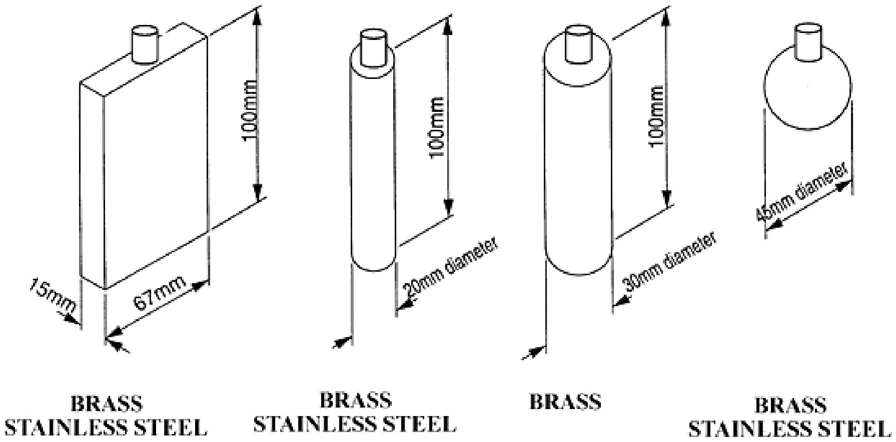
Adjustment of the thermostat allows the bath to be set to a constant temperature (T1) before beginning the experimental procedure.



Seven simple shapes of different materials (Stainless steel and brass) are provided as shown in Figure below.

Each is fitted with a thermocouples well at its geometric center and thermocouple into the carrier to sense water temperature adjacent to the shape to allow measurement of its core temperature (T_3) and the surface (T_2).





Introduction

* Heat transfer generally takes place by conduction, convection, and radiation.
* In absence of internal heat generation, when a cool, solid body is placed in a warm environment, energy in the form of heat flows into the body until it attains a thermal equilibrium with its surroundings.
* Starting at time, t = 0, the temperature at any point in the solid is uniform, and increases until the body reaches an isothermal steady state.
* The period of time required for the body to reach steady state is a function of the size, shape, and physical properties of the body as well as the conditions imposed on the surface of the body by the experimental conditions

* In another words, in unsteady transient heat conduction, temperature is a function of both time and spatial coordinates
* By using Fourier's Equation governs the temperature response of a body, and Thermal Energy equation (first law of thermodynamics for closed system), one can fined the thermal conductivity of any body by recording temperature change of its center with the time.

$$E = mC_p(T - T_b) = \rho V C_p(T - T_b)$$

$$\rho C_p(\Delta x * \Delta y * \Delta z) * \frac{dT}{dt} = (q)_{x,y,z} - (q)_{x+\Delta x,y+\Delta y,z+\Delta z}$$

$$V = (\Delta x * \Delta y * \Delta z)$$

* Rate of heat transfer by conduction from the center of the body out side or in opposite direction can be describe by Fourier's Law

$$q_{x,y,z} = -k\nabla T_{x,y,z} \ldots..[4]$$

* When the body is a metal semi infinite slab or cylinder or sphere, for one-dimensional case the governing energy equation is:

$$\rho C_p \frac{\partial T}{\partial t} = \frac{1}{r^m}\frac{\partial}{\partial r} k\left(r^m \frac{\partial T}{\partial r}\right)$$

Where for slab m=0

for cylinder m=1

for sphere m=2

From thermodynamics definition of the thermal diffusivity

$$\alpha = \frac{k}{\rho C_p}$$

The general governing convective equation may be modified to:

$$\frac{1}{\alpha}\frac{\partial T}{\partial t} = \frac{1}{r^m}\frac{\partial}{\partial r}\left(r^m \frac{\partial T}{\partial r}\right)$$

How can we characterize the object and the Environment

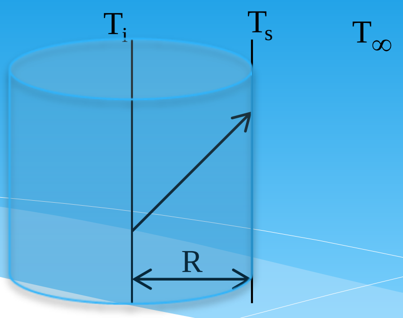
We know the heat transfer in the body mass by conduction to the surface should equal to the heat transfer from the body surface to the surrounding.

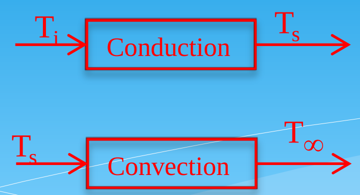
* If the body thermal conductivity is high or the body volume is small, then the temperature response is very fast.
* The temperature's response is a function the *internal resistance of the body material*.
* If the convection coefficient is very high, then the surface temperature of the body becomes very quickly identical to the surrounding temperature.
* Alternatively, for a low convection coefficient a large temperature difference exists between the surface and the surrounding.
* The value of the coefficient controls what is known as the *surface resistance* to heat transfer.
* In conclusion the instantaneous temperature variation within the system is dependent on the internal and surface resistances.
* The larger internal resistance or the smaller surface resistance, the larger temperature variation within the system, and vice versa.

Biot Number

$$q = k.(2\pi rl)\frac{T_i - T_s}{r}$$

$$q = h(2\pi rl)(T_s - T_\infty)$$





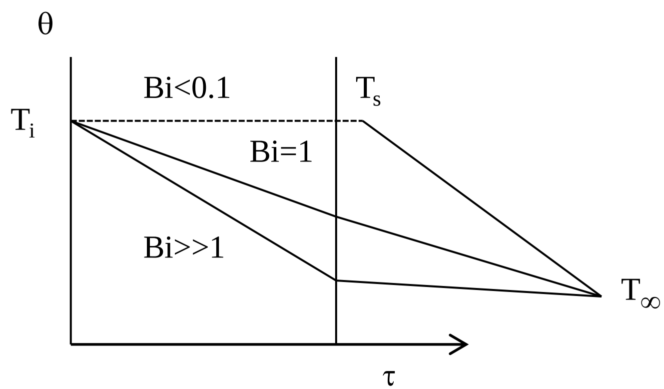
$$kA_s(T_i - T)_s = hV(T_s - T_\infty)$$

$$\boldsymbol{Bi = \frac{resistance\ to\ internal\ heat\ flow\ (conduction)}{resistance\ to\ external\ heat\ flow\ (convection)}}$$

$$-\frac{T_i - T_s}{T_s - T_\infty} = \frac{hR}{k} = Bit\ number$$

Where $R = \frac{V}{A_s}$ *Characteristic length*

This Figure shows **Meaning of Biot number value (**the relation between Biot number value and the temperature gradient of any solid object).



Low Biot Number less than 0.1

- This indicates that the thermal resistance is negligible compared to the convection resistance and $T_i \cong T_s$ and the solid may have a uniform temperature.
- This phenomenon is called (**Lumped Thermal Capacity**).
- Where in this case the heat transfer by convection from the surface will balance the change of the internal energy of the solid.

$$\rho V C_p \frac{dT}{dt} = -hA_s(T - T_\infty)$$

$$\frac{dT}{(T - T_\infty)} = -\frac{hA_s}{\rho V C_p} dt$$

At initial conditions

$t = 0 \quad T = T_i$

the integration of of the above eq.

$$ln\left(\frac{T_t - T_\infty}{T_i - T_\infty}\right) = -\frac{hA_s}{\rho V C_p} t$$

$$\frac{T - T_\infty}{T_i - T_\infty} = e^{-\frac{h}{\rho C_p R} t}$$

$\frac{1}{G}$

$G=\frac{rC_pR}{h}$ Time constant

This equation shows that the solid body temperature approaches the surrounding temperature T_∞ exponentially, which means that the solid temperature change rapidly at the beginning and slowdown after.
For a big value of (1/Γ) the solid reach T_∞ in very short time (very high conducting material).

$$\frac{T-T_\infty}{T_i-T_\infty}=e^{-\frac{t}{\Gamma}}$$

Fourier modules (Fo)

- If we rearrange the exponent term in the above equation

$$\frac{t}{\Gamma}=\frac{ht}{\rho C_pR}=\left(\frac{hR}{k}\right)\left(\frac{k}{\rho C_pR^2}t\right)$$

Fourier number (Fo)

$\frac{hR}{k}=Biot\ number$

$Fo=\frac{a}{R^2}t$

$\frac{k}{\rho C_p}=\alpha$

$\frac{t}{\Gamma}=Bi*Fo$

$$\theta=\frac{T-T_\infty}{T_i-T_\infty}=e^{-Bi*Fo}$$

Where the above Eq. is solution for transient conduction when we have the lumped-parameter (low Biot number) condition, and it can be portrayed graphically by plotting

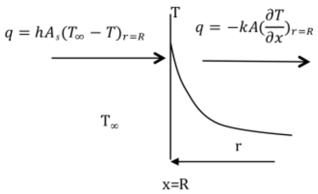
$$\theta = \frac{T - T_\infty}{T_i - T_\infty} \; vs \; Bi * \tau$$

where τ = Fourier number Fo

High Biot number (Bi > 0.1)

In this case the conduction resistance could not be neglected, and the temperature distribution in the solid object is a function of time and position within the body.

$$\begin{bmatrix} Heat\ convected \\ through\ the\ surface \end{bmatrix} = \begin{bmatrix} Heat\ conducted \\ into\ the\ solid \end{bmatrix}$$



$$hA_s(T - T_\infty)_{r=R} = -kA\left(\frac{\partial T}{\partial r}\right)_{r=R}$$

Take the boundary conditions

At r=0 T is finite and $\frac{\partial T}{\partial r} = 0$

At t = 0, $T=T_i$ (boundary condition)

If we rearrange the above eq and divide by (T_i-T_∞)

$$\frac{\partial\theta}{\partial r} = -\frac{hR}{k}\theta$$

But when **r do not equal R,** then

$$\theta = \frac{T_{r,t} - T_\infty}{T_i - T_\infty}$$

If we go back to governing equation and write it in dimensionless form by defining the following variables:

$$\frac{1}{\alpha}\frac{\partial T}{\partial t} = \frac{1}{r^{m}}\frac{\partial}{\partial r}\left(\quad ^{m}\frac{\partial T}{\partial r}\right)$$

$$= \frac{- \quad \infty}{- \quad \infty}$$

$$= \frac{\quad}{\quad p \quad ^{2}}$$

$$= -$$

$$\frac{\quad}{\quad} = \frac{1}{\quad ^{m}}\frac{\quad}{\quad}\left(\quad ^{m}\frac{\quad}{\quad}\right) \quad (1)$$

Where at

The initial conditions,

at t = 0 q = 1

at η = 1

$$\frac{\quad}{\quad} = \quad * \quad (2)$$

The resulting equation (2) contains only one dimensionless parameter, Bi, the Biot number for heat transfer. Therefore, the solution, in its dimensionless form, must be a function only of this parameter.

The transient heat transfer equations (1, and 2) maybe solved by:
Graphical method using Heisler Charts for different geometries.
Analytical method of separation variables or the Laplace Transform method.
Polynomial approximation method for different geometries.

Analysis

1- Graphical method may be used:

This method is based on the solution of Eq. (1). Graphs of this equation are presented in Appendix F of ref. 1.

1. Use the results obtained for the brass cylinder since its k value is given. Using Heisler chart together with the experimental results in terms of q and at/R^2

Find 1/Bi for each time interval.

2. Determine Bi for brass by interpolating a curve through the data points and reading the corresponding Bi from the graph. Knowing k and Bi determine h.

In order to determine k for the other cylinder having same radius:

3. Prepare a master plot of the theoretical solution of Eq. 1 in terms of q versus ht/(r Cp R) = Bi* t for several Bi value.
4. Re-plot the experimental data for the steel cylinder in this new graph using the value of h obtained for brass cylinder.
5. Obtain the Bi value from the curve that coincides with the experimental data and determine k from Bi and h.

Repeat the calculation to find h (heat transfer coefficient for other brass cylinders of different radius.

Same method may be repeated for slab and sphere objects.

By known h and k, one can find temperature as a function of position by using the provided graphs on Figures 7, 8, and 9.on your lab manual

Analytical method of separation variables or the Laplace Transform method:
Using the method of separation of variables[3,4] or the Laplace Transform method,[4,3] the solution of Eq. (1) is

$$\theta = 2 \sum_{m=1}^{\infty} \frac{Bi}{Bi^2+q_m^2} \frac{J_o(q_m \eta)}{J_o(q_m)} \exp\left[-(q_m^2)\tau\right] \quad (3)$$

where q_m are the roots of the transcendental characteristic equation,

$$q_m\, J_1(q_m) = Bi\, J_o(q_m) \quad (4)$$

The approximately by the single term, Eq. 3 become

$$\theta = \frac{2Bi}{(Bi^2+q_1^2)} \frac{J_0(q_1 \eta)}{J_0(q_1)} \exp\left[-(q_1^2 \tau\right)$$

Graph ln θ versus time (real time t), yield straight line of slope,

$$Slope = \frac{q_1^2 k}{\rho C_p R^2}$$

and Intercept,

$$I = \ln \left\{ \frac{q_1^2 Bi^2}{2Bi} \frac{J_0(q_1)}{J_0(q_1 \frac{r}{R})} \right\}$$

By solving equation (4) together with the slope and the intercept),
you may evaluate the three unknown parameters, or can be done graphically using the following table.