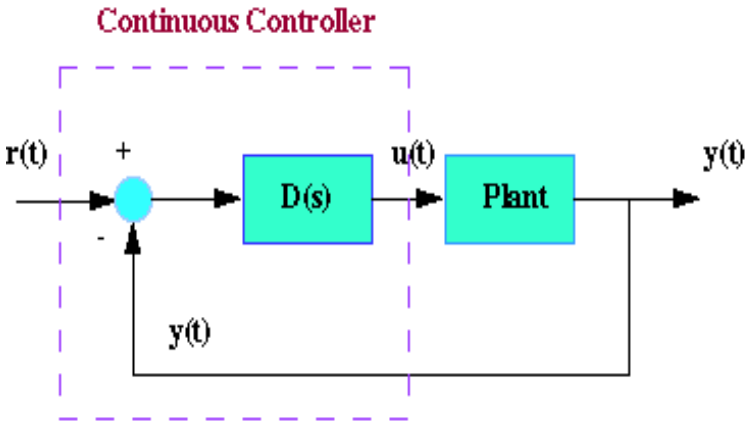
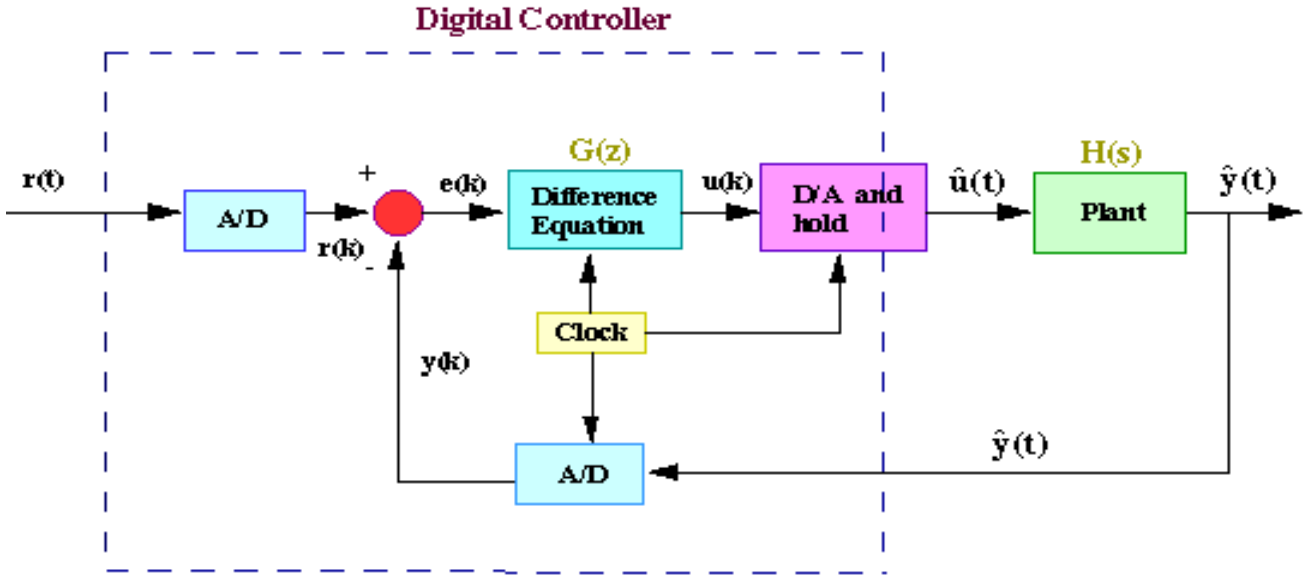
Digital Control Systems

Continuous vs. Discrete Time





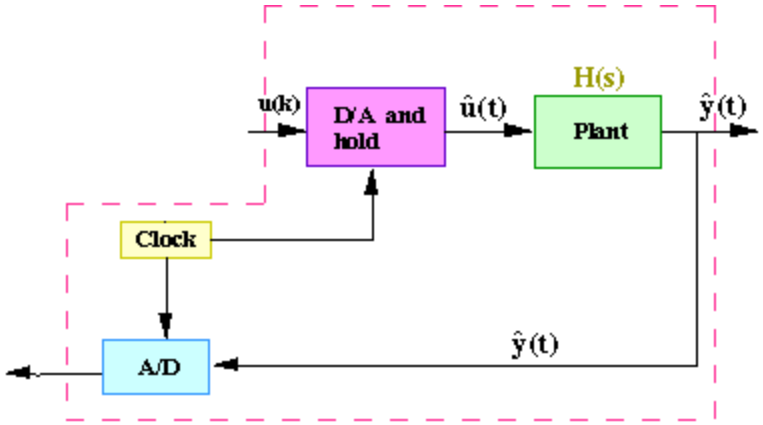
Advantages

- Improved sensitivity
- Use digital components
- Control algorithms easily modified
- Many systems inherently digital

Disadvantages

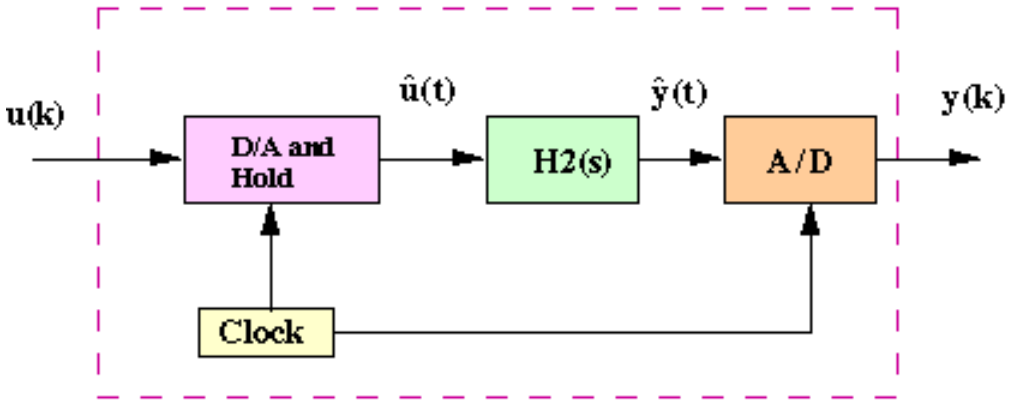
- Develop complex math algorithms
- Lose information during conversions

Digital Control Systems:
Zero-Order Hold

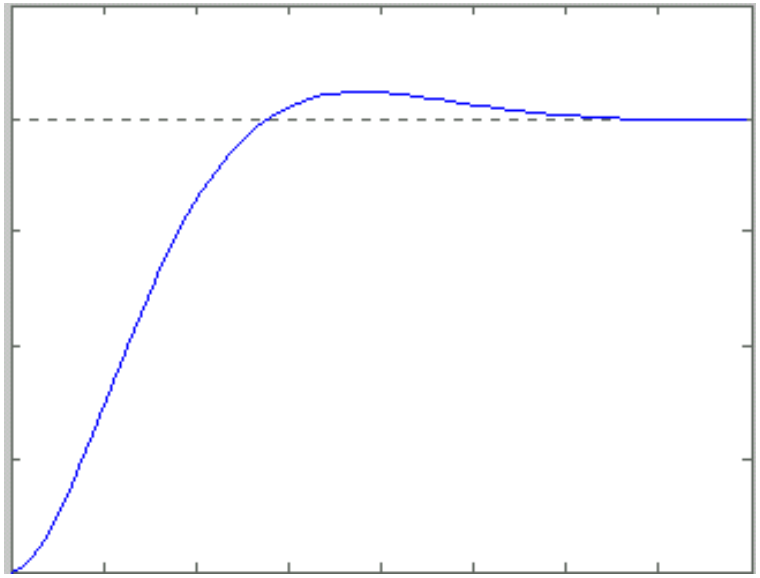


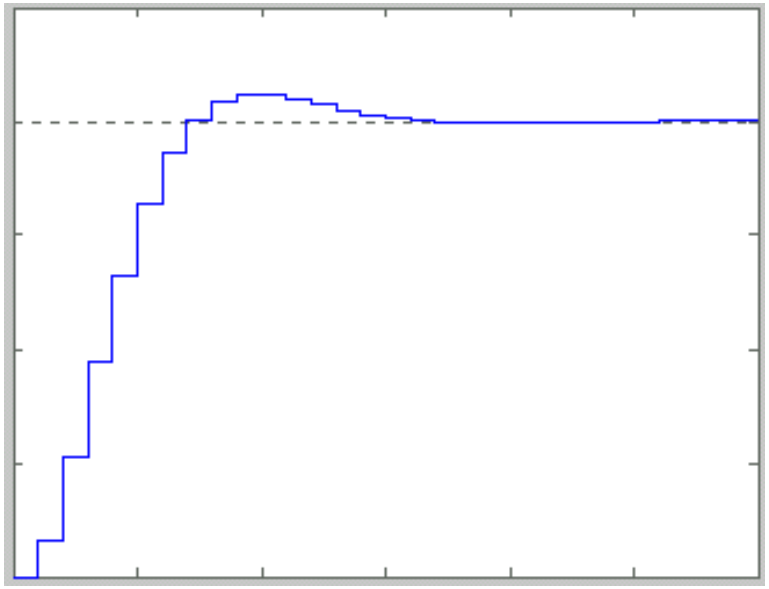
$$H2(s) = \frac{1 - e^{-sT}}{s}$$

Hzoh(z)



Digital Control Systems:
Zero-Order Hold (cont)





Digital Control Systems:
The z-transform

Def'n of z-Transform:

$$Z\{f(t)\} = F(z) = \sum_{k=0}^{\infty} \left(f(kT)\, z^{-k} \right)$$

Relationship b/w s-plane and z-plane:

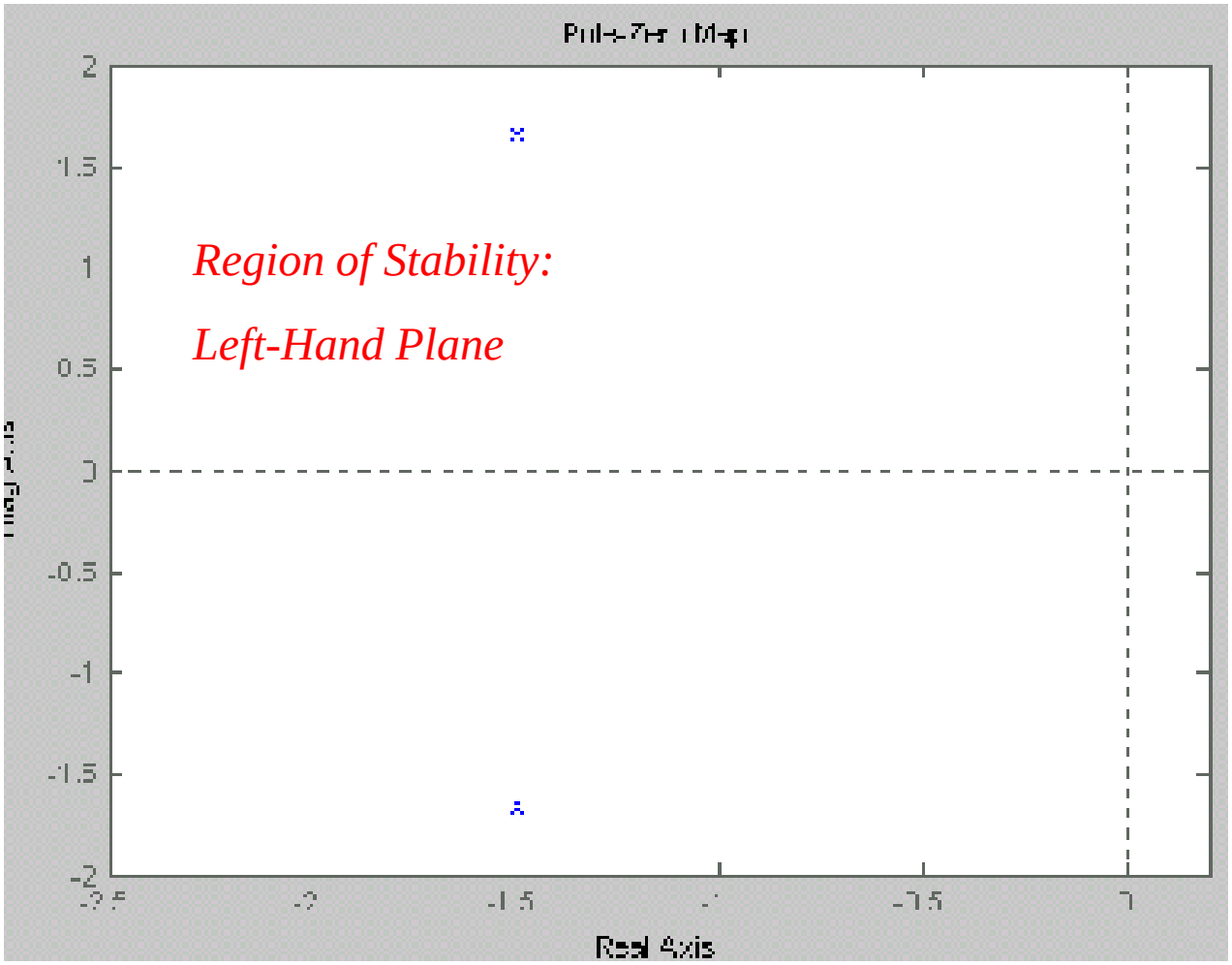
$$z = e^{sT}$$

Digital Control Systems:
The z-transform (cont)

Several common z-Transforms

x(t)	X(s)	X(z)
$u(t)$	$\frac{1}{s}$	$\frac{z}{z-1}$
t	$\frac{1}{s^2}$	$\frac{Tz}{(z-1)^2}$
e^{-at}	$\frac{1}{s+a}$	$\frac{z}{z-e^{-aT}}$
$\sin(\omega t)$	$\frac{\omega}{s^2+\omega^2}$	$\frac{z\cdot\sin(\omega T)}{z^2-2z\cdot\cos(\omega T)+1}$

Digital Control Systems:
Stability in s-plane

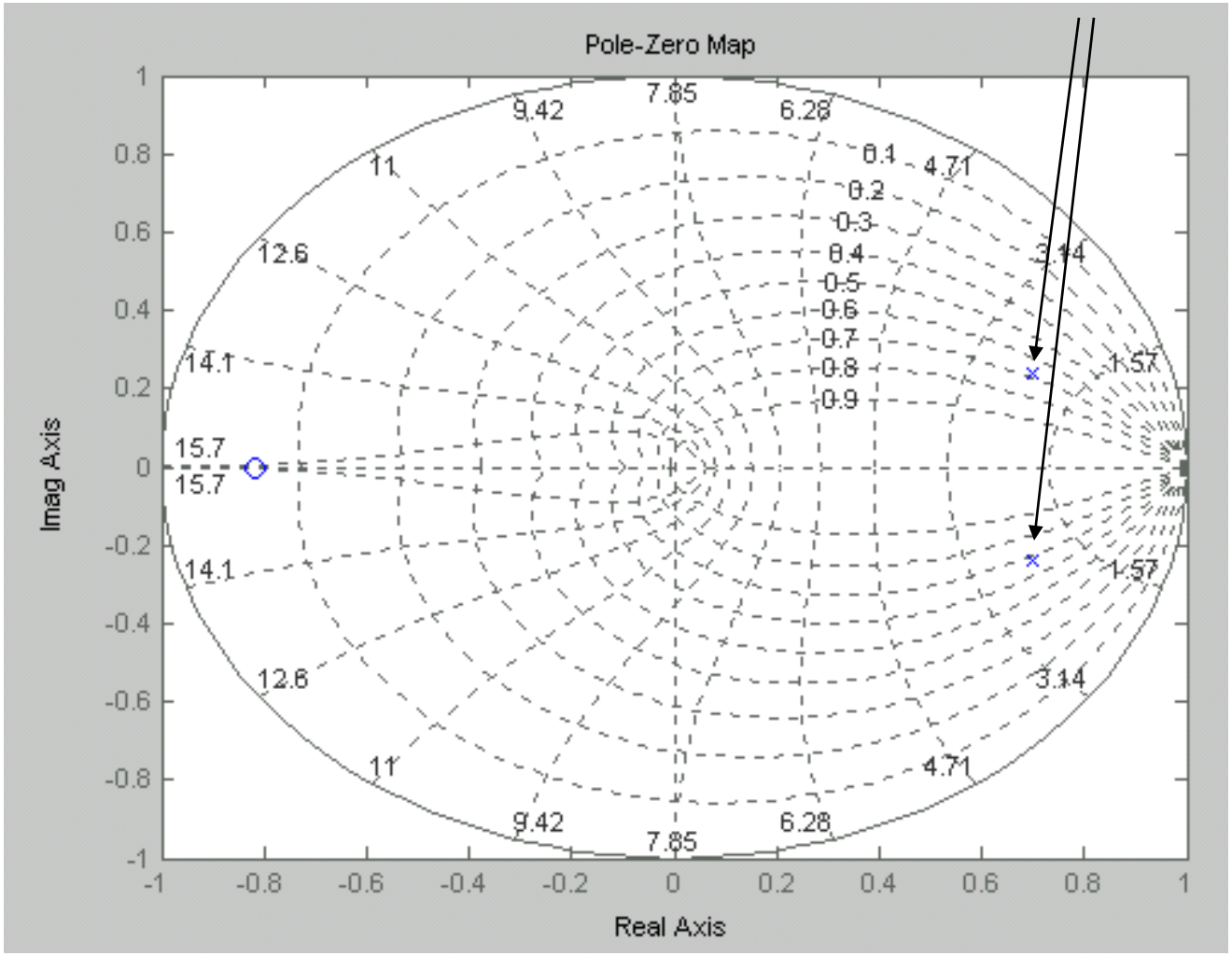


Digital Control Systems:

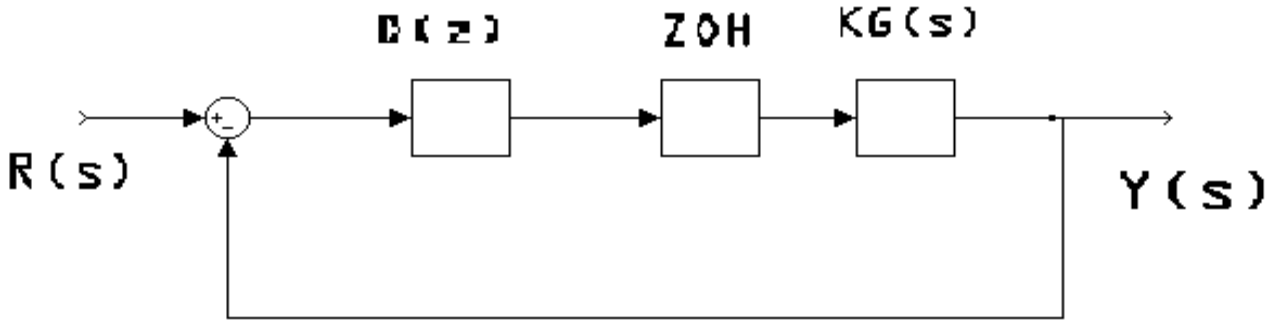
Stability in z-plane

Region of Stability:

Unit-Circle



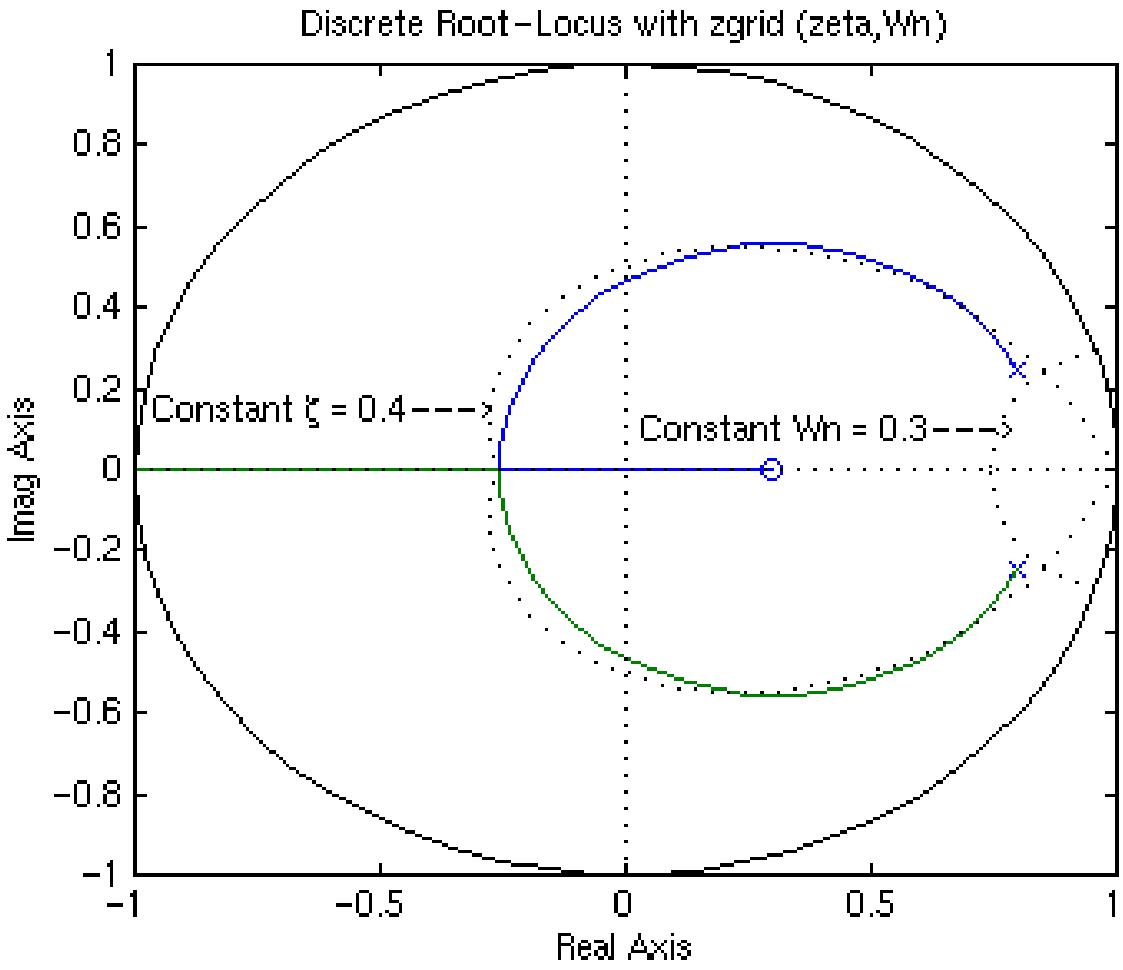
Digital Control Systems:
Root-Locus



$$\frac{Y(z)}{R(z)} = \frac{KG(z)\,D(z)}{1 + KG(z)\,D(z)}$$

$$1 + KG(z)\,D(z) = 0$$

Digital Control Systems:
Root-Locus (cont)



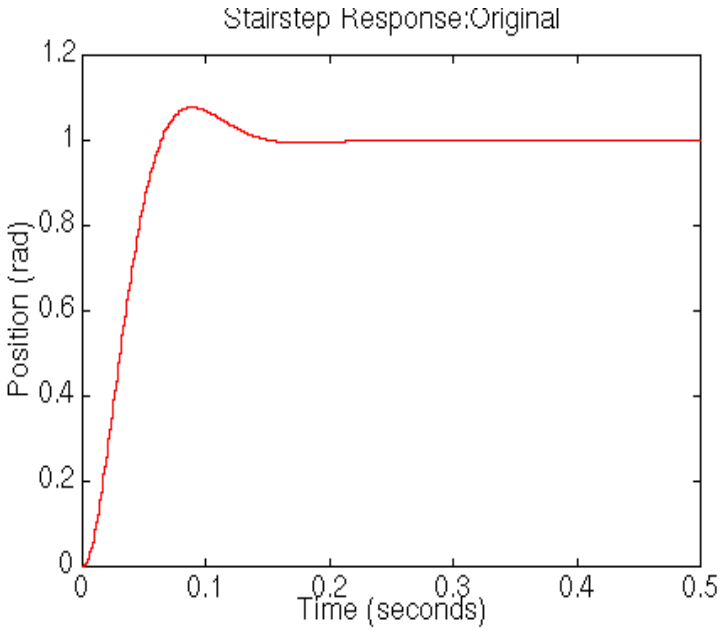
Digital Control Systems:
MATLAB to the rescue

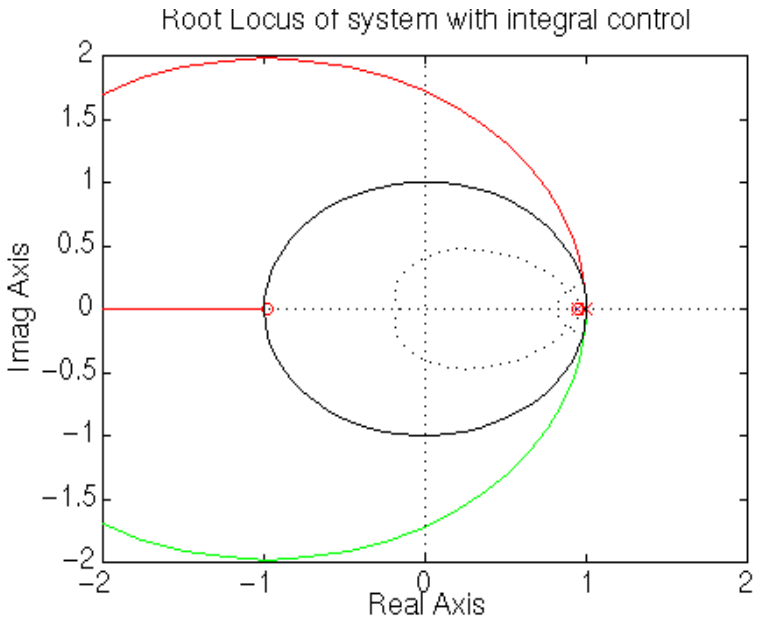
- **c2d**
  - conversion of continuous-time models to discrete time
- **zgrid**
  - generate z-plane grid lines for a root locus or pole-zero map over an existing map
- **dstep**
  - Step response of discrete time system
- **stairs**
  - Connects the elements from **dstep** to form stairstep graph

Digital Control Systems:

Design Example

Root Locus Design for Digital DC Motor Position Control





Digital Control Systems:
Design Example