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# Bode Plots

# Get to know your logs!

dB	ratio			dB	ratio
-20	0.100			20	10.000
-10	0.316			10	3.162
-5	0.562			5	1.778
-3	0.708			3	1.413
-2	0.794			2	1.259
-1	0.891			1	1.122

- Engineers are very conservative. A “margin” of 3dB is a factor of 2 (power)!
- Knowing a few logs by memory can help you calculate logs of different ratios by employing properties of log. For instance, knowing that the ratio of 2 is 3 dB, what’s the ratio of 4?

# Bode Plot Overview

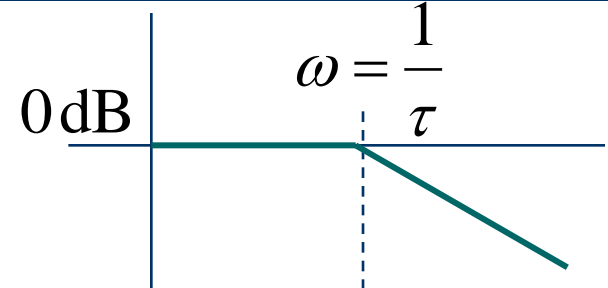
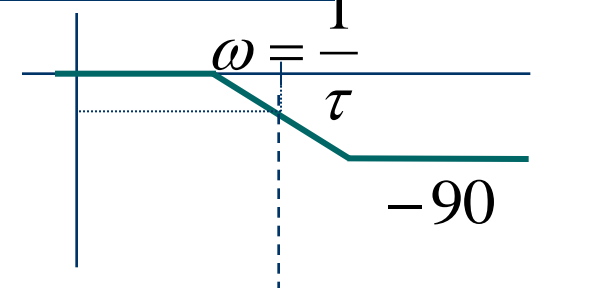
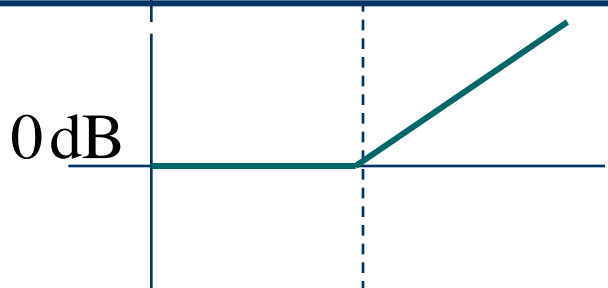
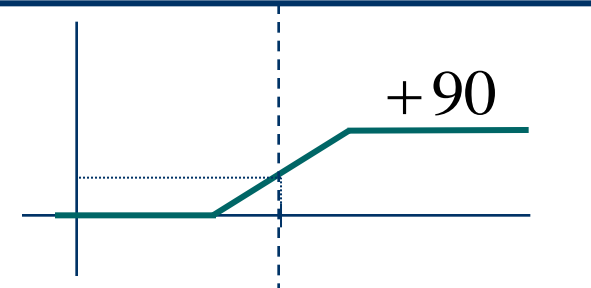
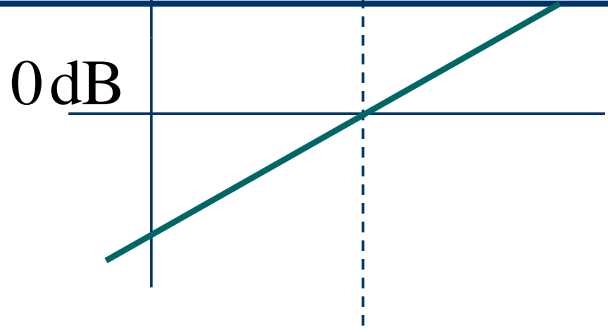
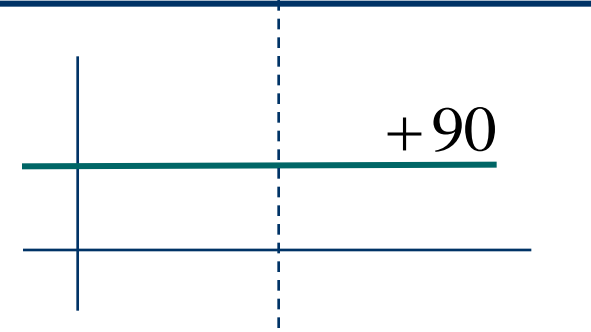
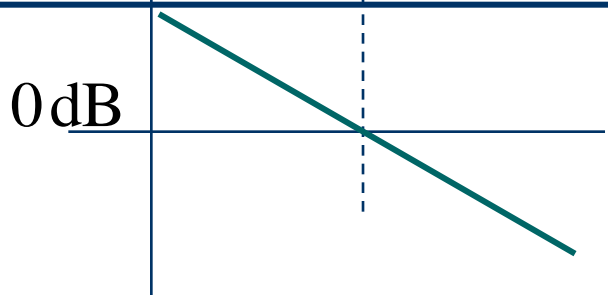
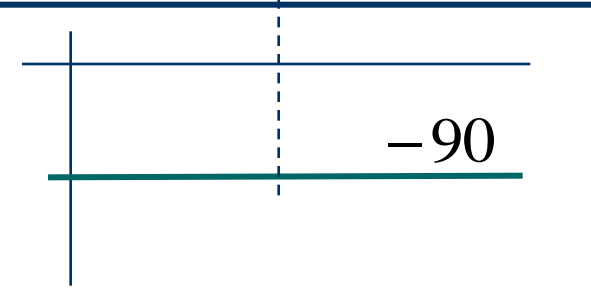
- Technique for estimating a complicated transfer function (several poles and zeros) quickly

$$H(\omega) = G_0 (j\omega)^K \frac{(1 + j\omega\tau_{z1})(1 + j\omega\tau_{z2}) \cdots (1 + j\omega\tau_{zn})}{(1 + j\omega\tau_{p1})(1 + j\omega\tau_{p2}) \cdots (1 + j\omega\tau_{pm})}$$

- Break frequencies :

$$\omega_i = \frac{1}{\tau_i}$$

# Summary of Individual Factors

<ul style="list-style-type: none"> <li>Simple Pole: <math>\frac{1}{1 + j\omega\tau}</math></li> </ul>		
<ul style="list-style-type: none"> <li>Simple Zero: <math>1 + j\omega\tau</math></li> </ul>		
<ul style="list-style-type: none"> <li>DC Zero: <math>j\omega\tau</math></li> </ul>		
<ul style="list-style-type: none"> <li>DC Pole: <math>\frac{1}{j\omega\tau}</math></li> </ul>		

# Example

- Consider the following transfer function

$$H(j\omega) = \frac{10^{-5} j\omega(1 + j\omega\tau_2)}{(1 + j\omega\tau_1)(1 + j\omega\tau_3)}$$

$$\tau_1 = 100 \text{ ns}$$

$$\tau_2 = 10 \text{ ns}$$

$$\tau_3 = 100 \text{ ps}$$

- Break frequencies: invert time constants

$$\omega_1 = 10 \text{ Mrad/s}$$

$$\omega_2 = 100 \text{ Mrad/s}$$

$$\omega_3 = 10 \text{ Grad/s}$$

$$H(j\omega) = \frac{\frac{j\omega}{10^5} \left(1 + j \frac{\omega}{\omega_2}\right)}{\left(1 + j \frac{\omega}{\omega_1}\right) \left(1 + j \frac{\omega}{\omega_3}\right)}$$

# Breaking Down the Magnitude

- Recall log of products is sum of logs

$$\begin{aligned}
 |H(j\omega)|_{\text{dB}} &= 20 \log \left| \frac{\frac{j\omega}{10^5} (1 + j \frac{\omega}{\omega_2})}{(1 + j \frac{\omega}{\omega_1})(1 + j \frac{\omega}{\omega_3})} \right| \\
 &= 20 \log \left| \frac{j\omega}{10^5} \right| + 20 \log \left| 1 + j \frac{\omega}{\omega_2} \right| \\
 &\quad - 20 \log \left| 1 + j \frac{\omega}{\omega_1} \right| - 20 \log \left| 1 + j \frac{\omega}{\omega_3} \right|
 \end{aligned}$$

- Let's plot each factor separately and add them graphically

# Breaking Down the Phase

- Since  $\angle a \cdot b = \angle a + \angle b$

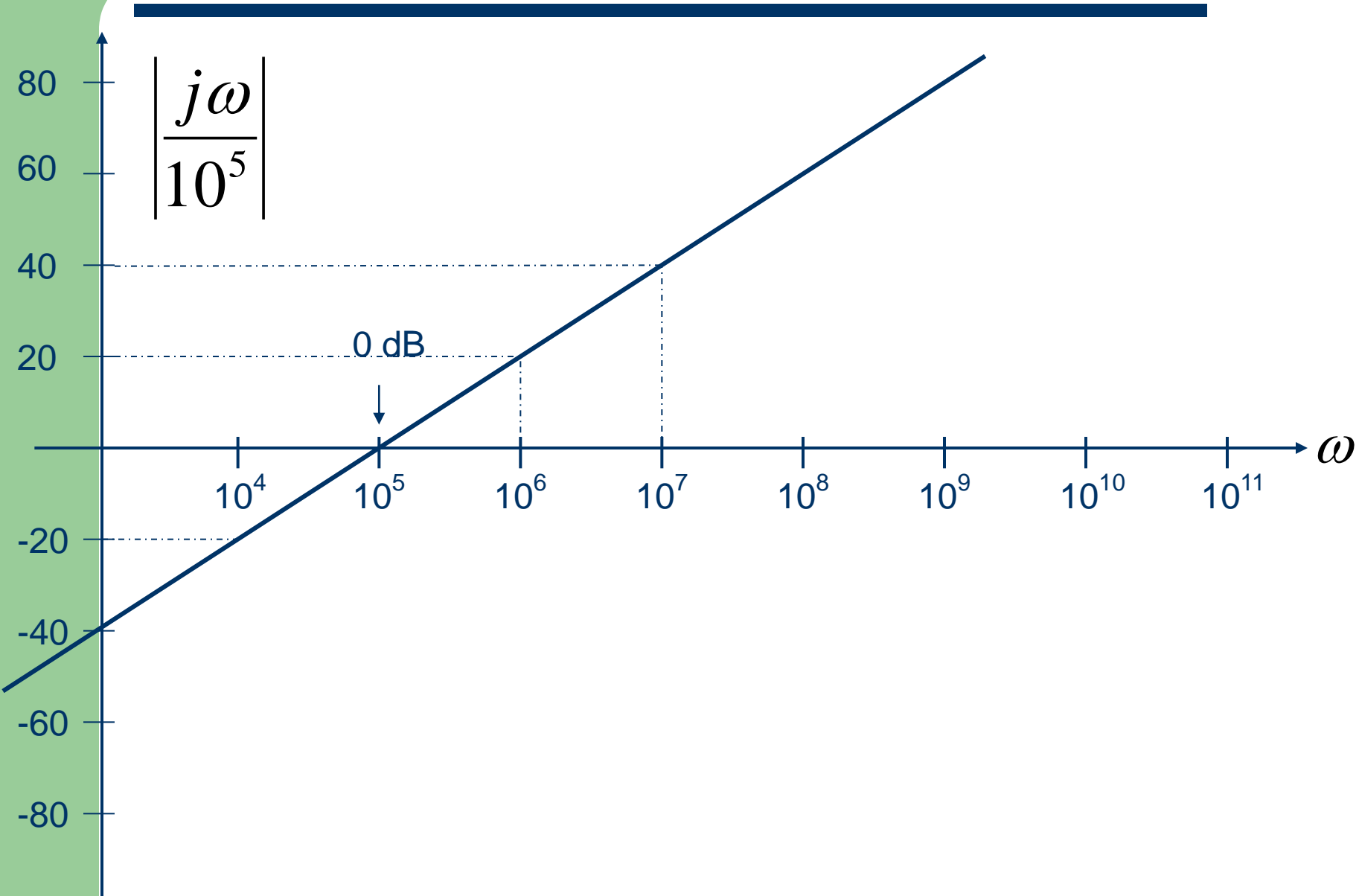
$$\angle H(j\omega) = \angle \frac{10^{-5} j\omega(1 + j\omega\tau_2)}{(1 + j\omega\tau_1)(1 + j\omega\tau_3)}$$

$$\angle H(j\omega) = \angle \frac{j\omega}{10^5} + \angle 1 + j \frac{\omega}{\omega_2}$$

$$- \angle 1 + j \frac{\omega}{\omega_1} - \angle 1 + j \frac{\omega}{\omega_3}$$

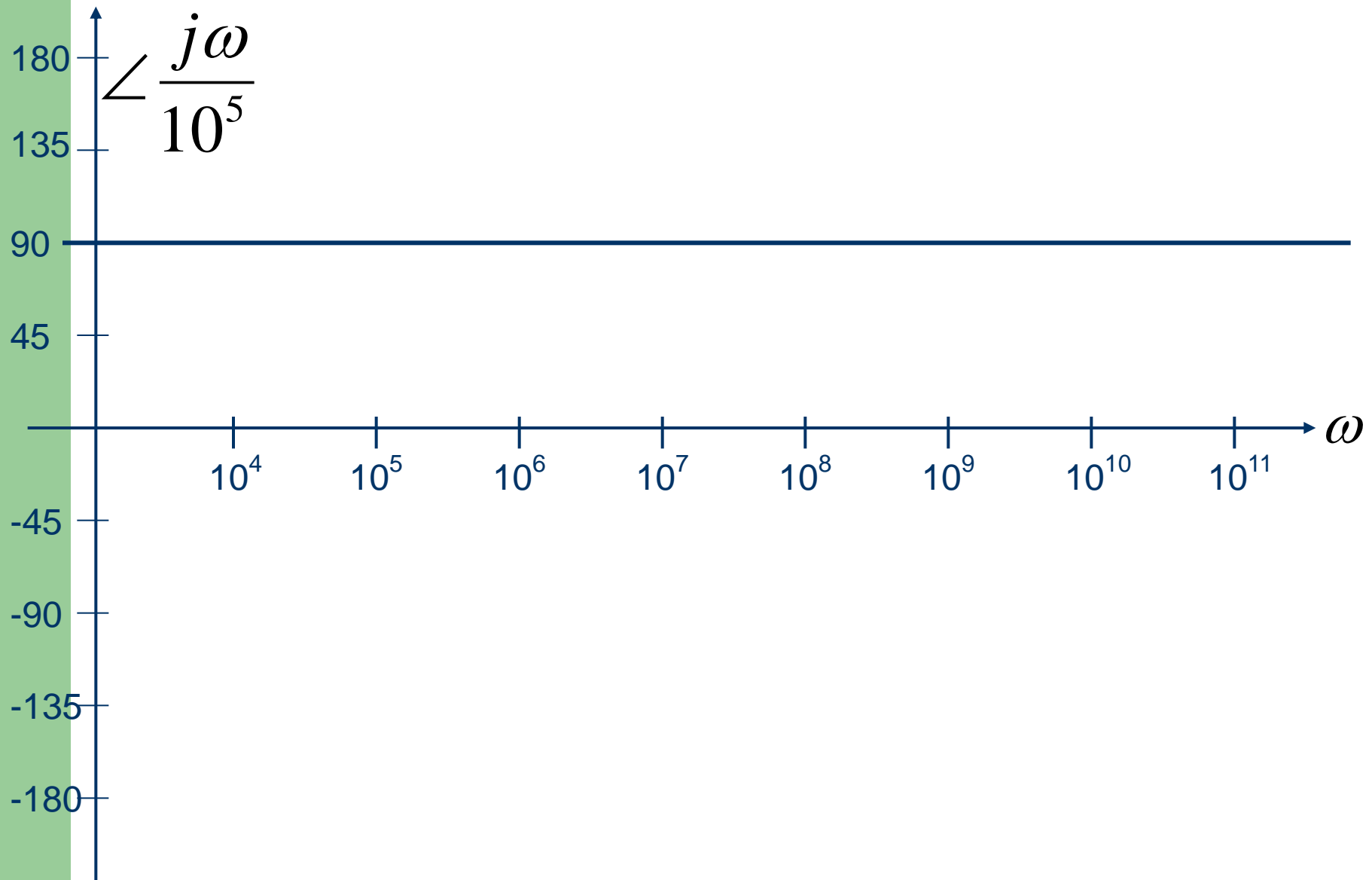
- Let's plot each factor separately and add them graphically

# Magnitude Bode Plot: DC Zero

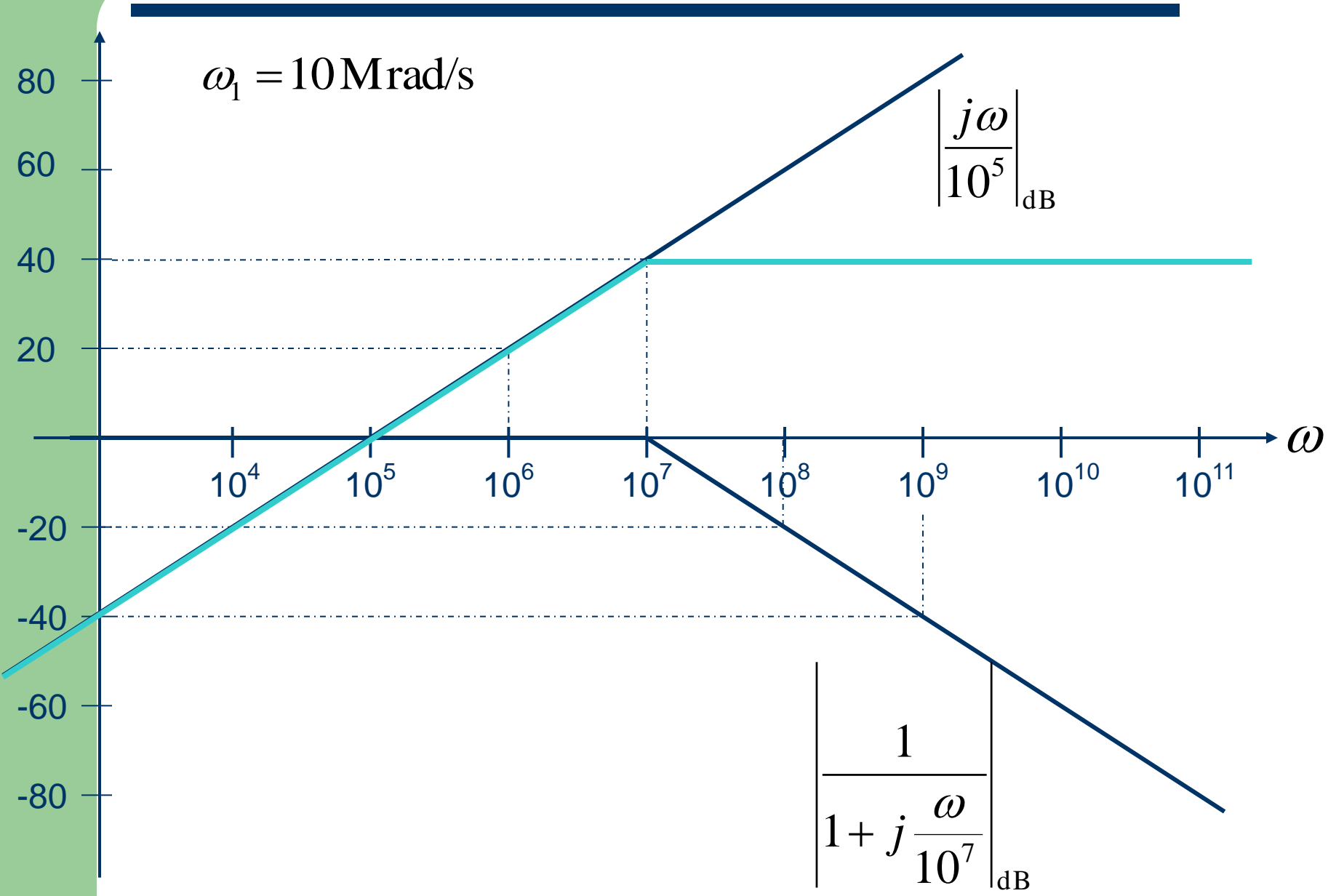




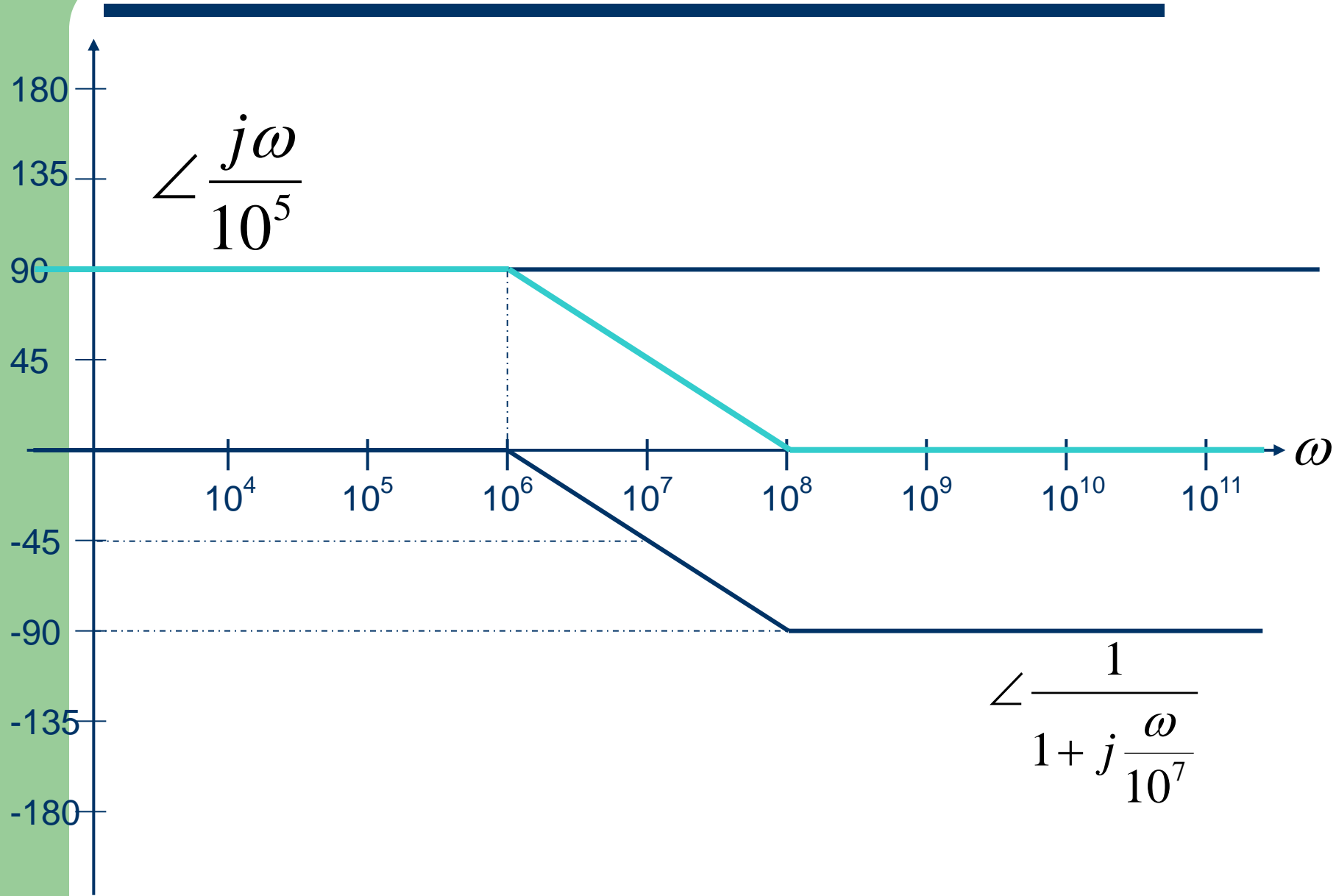
# Phase Bode Plot: DC Zero



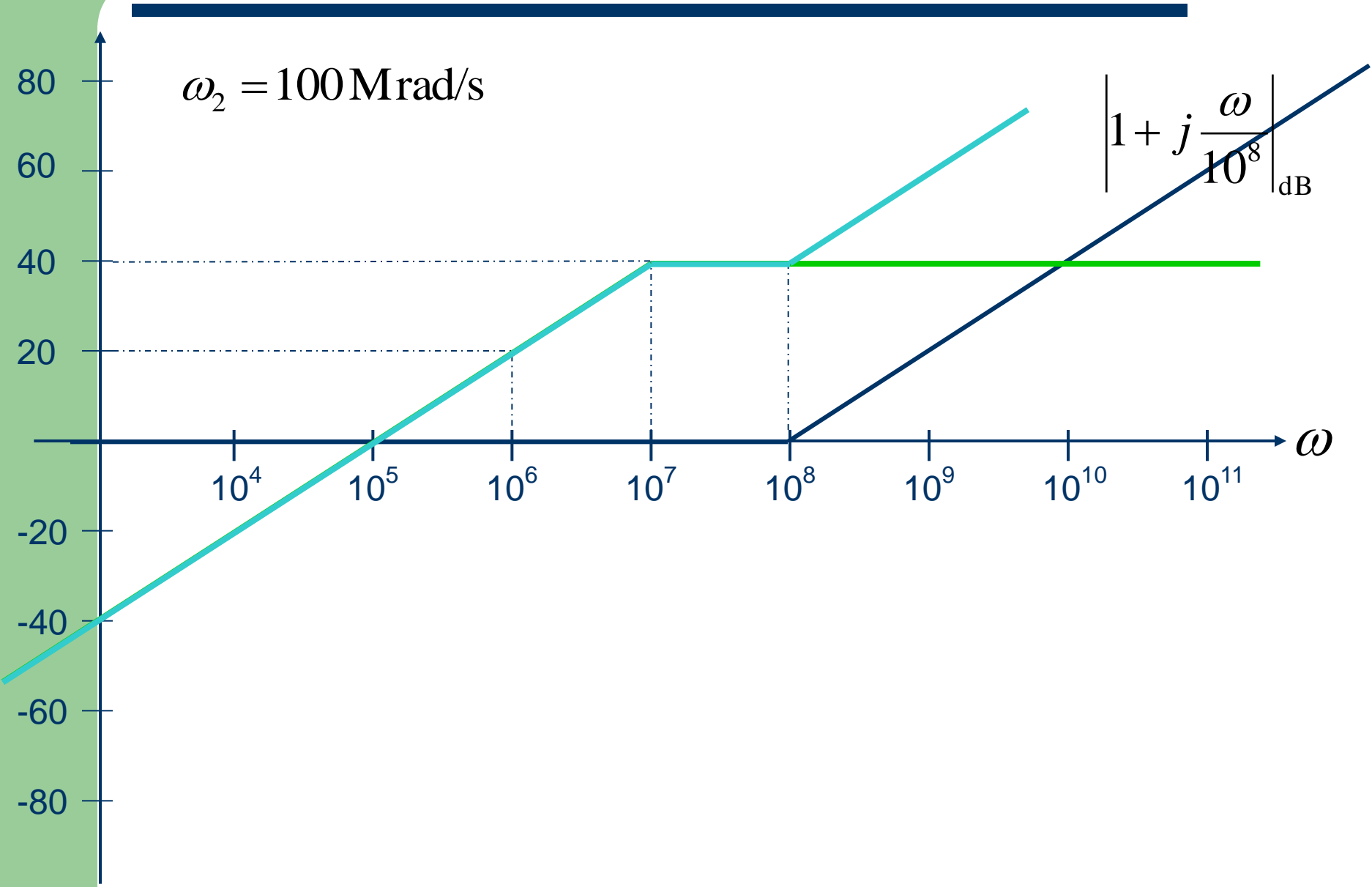
# Magnitude Bode Plot: Add First Pole



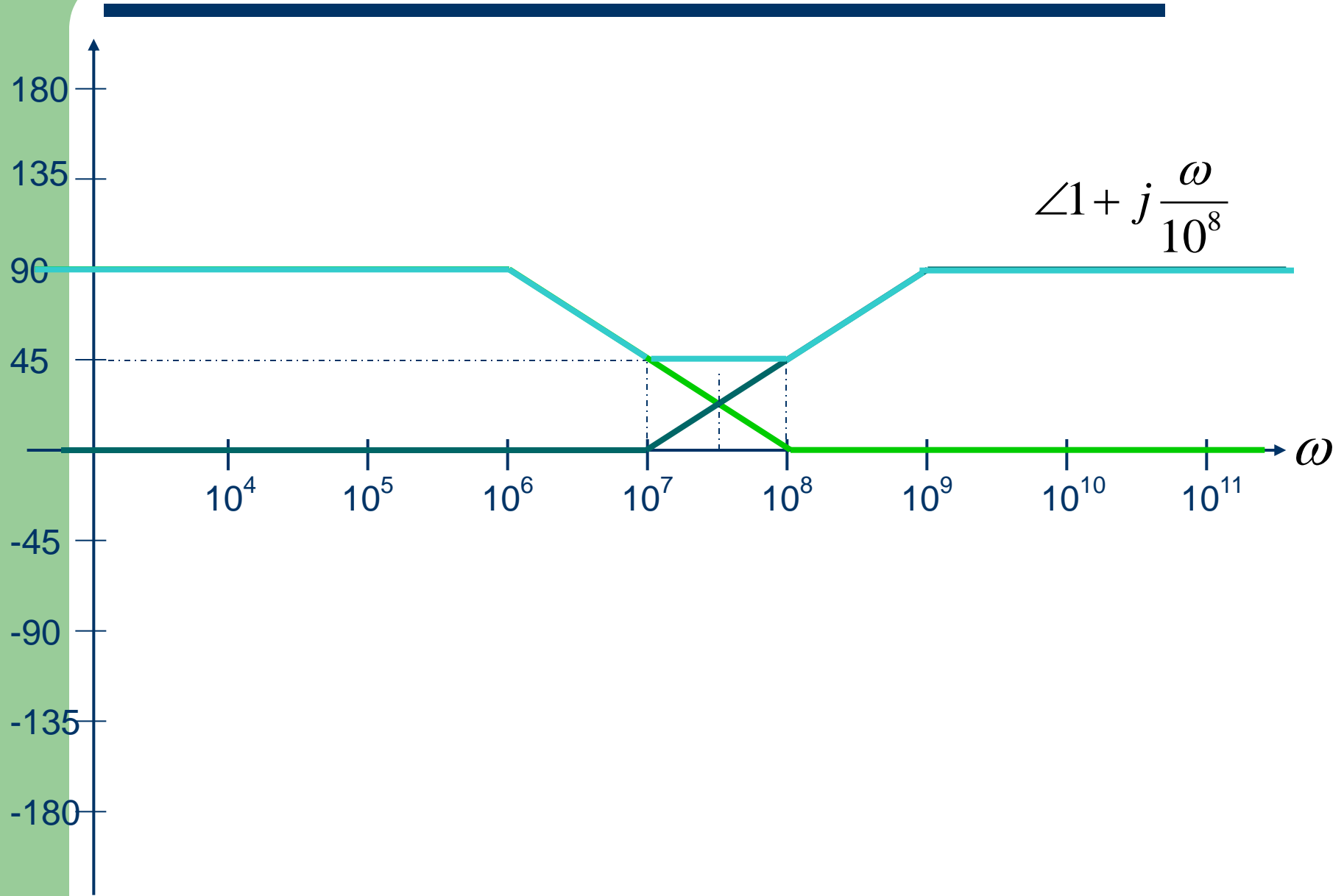
# Phase Bode Plot: Add First Pole



# Magnitude Bode Plot: Add 2<sup>nd</sup> Zero



# Phase Bode Plot: Add 2<sup>nd</sup> Zero



# Magnitude Bode Plot: Add 2<sup>nd</sup> Pole

