

# Performance of a second-order system

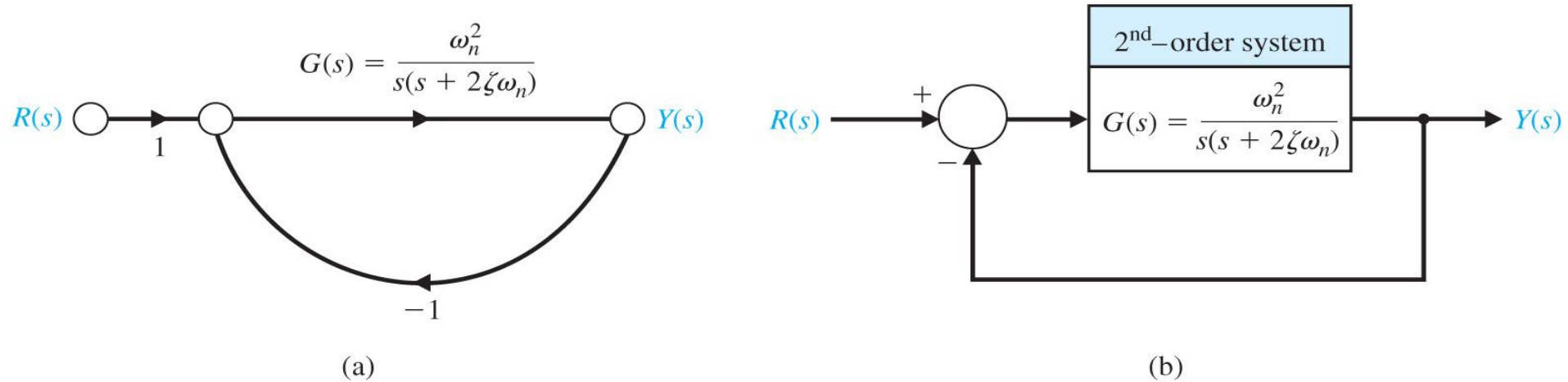


Figure: 05-04

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# Response to unit step input

$$Y(s) = \frac{G(s)}{1 + G(s)} R(s)$$

$$Y(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} R(s)$$

$$Y(s) = \frac{\omega_n^2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}$$

$$y(t) = 1 - \frac{1}{\beta} e^{-\zeta\omega_n t} \sin(\omega_n \beta t + \theta)$$

$$\beta = \sqrt{1 - \zeta^2}$$

$$\theta = \cos^{-1} \zeta$$

- **Natural frequency  $\omega_n$  - the frequency of natural oscillation that would occur for two complex poles if the damping were equal to zero**
- **Damping ratio  $\zeta$  - a measure of damping for second-order characteristic equation**

# Characteristic equation

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

$$s_1 = -\zeta\omega_n + \omega_n \sqrt{\zeta^2 - 1}$$

$$s_2 = -\zeta\omega_n - \omega_n \sqrt{\zeta^2 - 1}$$

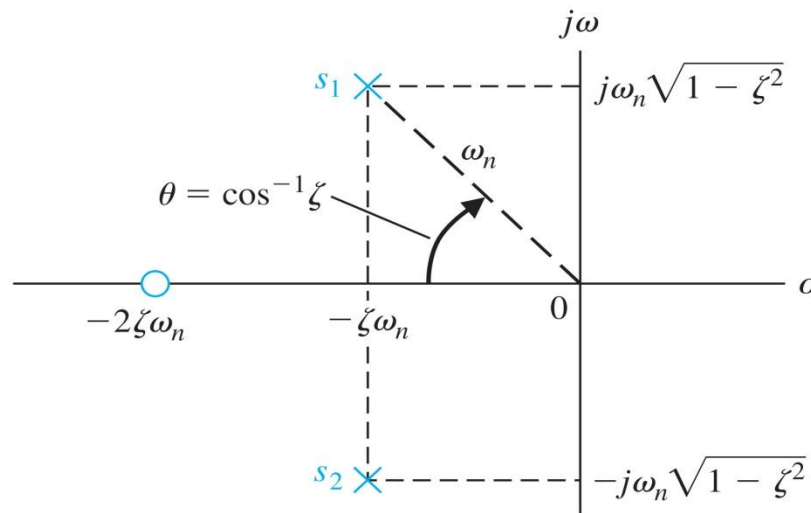


Figure: 02-09

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# Finding $\omega_n$ and $\zeta$ for a second-order system

$$G(s) = \frac{36}{s^2 + 4.2s + 36}$$

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

$$s_1 = -\zeta\omega_n + \omega_n \sqrt{\zeta^2 - 1}$$

$$s_2 = -\zeta\omega_n - \omega_n \sqrt{\zeta^2 - 1}$$

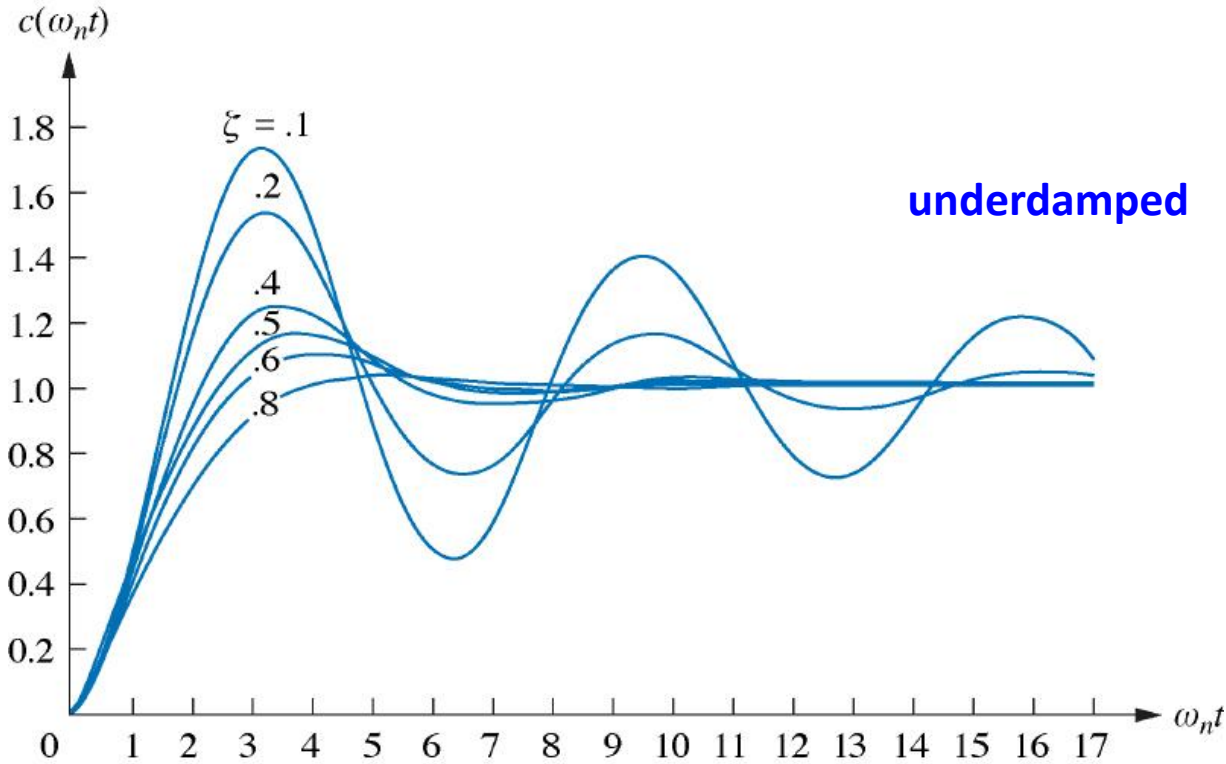
$$\omega_n^2 = 36$$

$$2\zeta\omega_n = 4.2$$

$$\omega_n = 6$$

$$\zeta = 0.35$$

# Second-order responses for $\zeta$



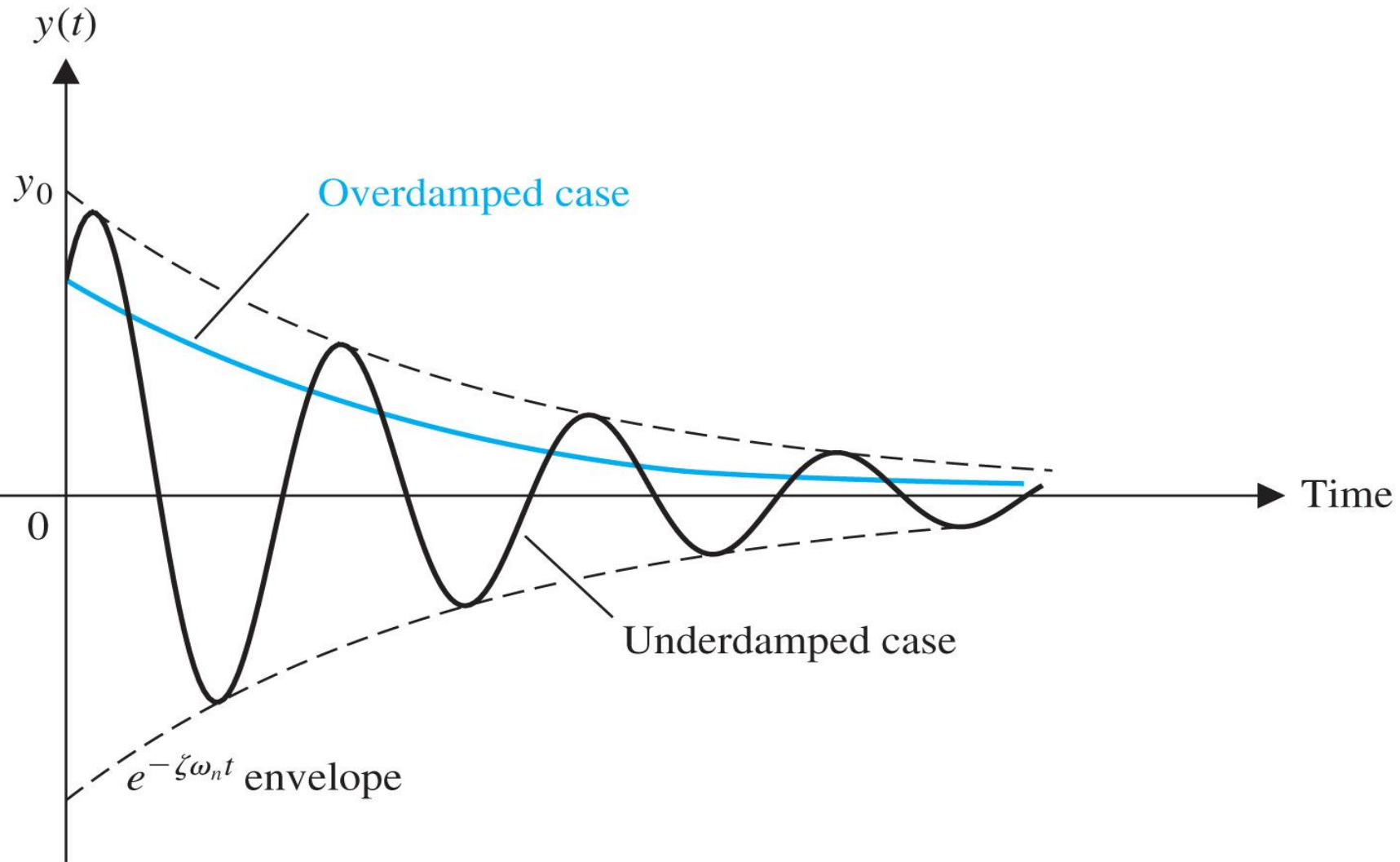
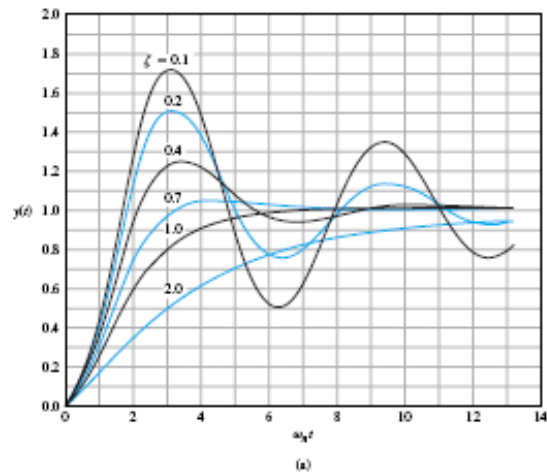


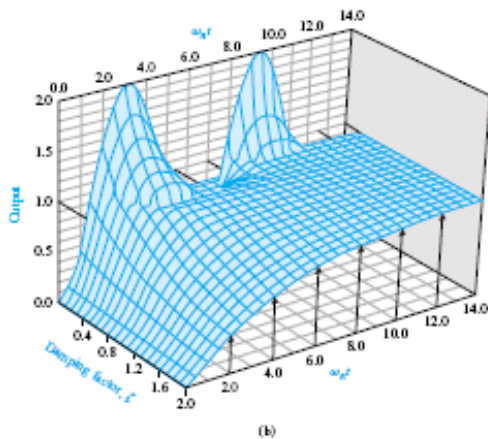
Figure: 02-12

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# Transit response



For step input as  
a function of  $\zeta$



For step input as  
a function of  $\zeta$  and  $\omega_n t$



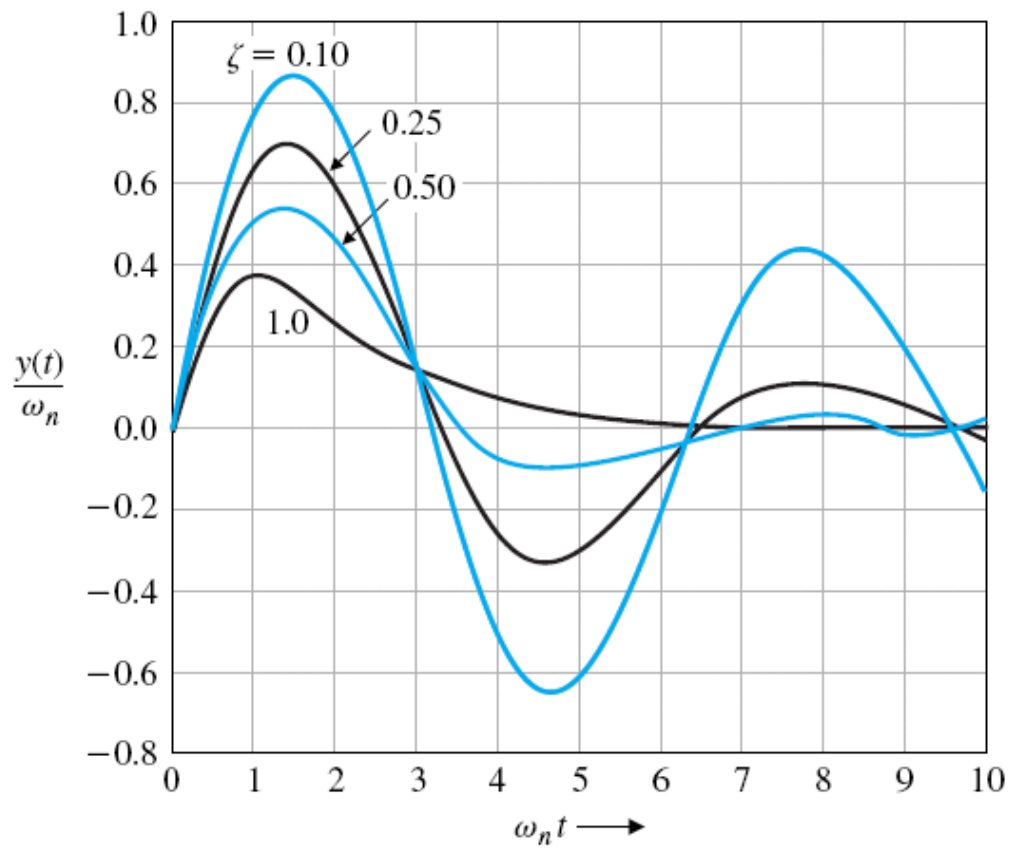
# Unit impulse response

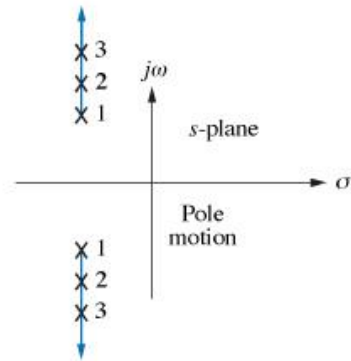
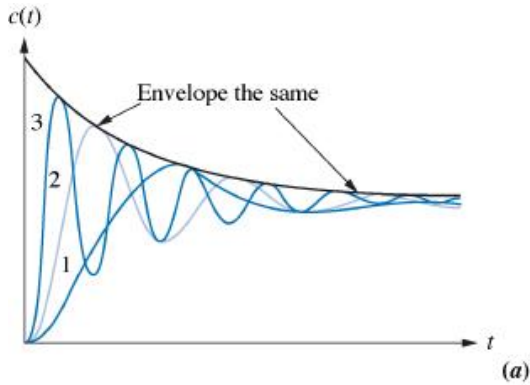
$$Y(s) = \frac{\omega_n^2}{(s^2 + 2\zeta\omega_n s + \omega_n^2)} R(s)$$

$$R(s)=1$$

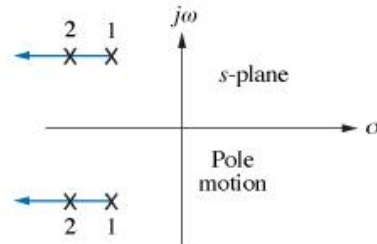
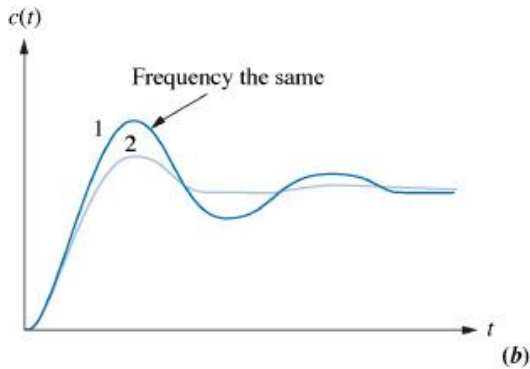
$$T(s)=Y(s)$$

$$y(t) = \frac{\omega_n}{\beta} e^{-\xi\omega_n t} \sin \omega_n \beta t$$

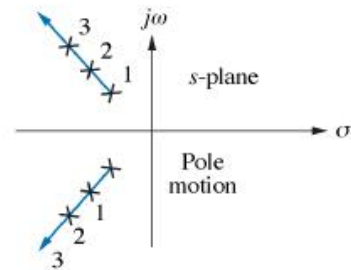
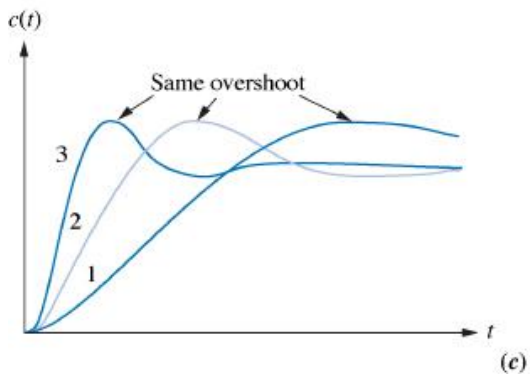




Constant real part



Constant imaginary part



Constant damping ratio

# Standard performance measures

$$T_s(s) = 4\tau = \frac{4}{\zeta\omega_n}$$

Settling time

$$T_p = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}}$$

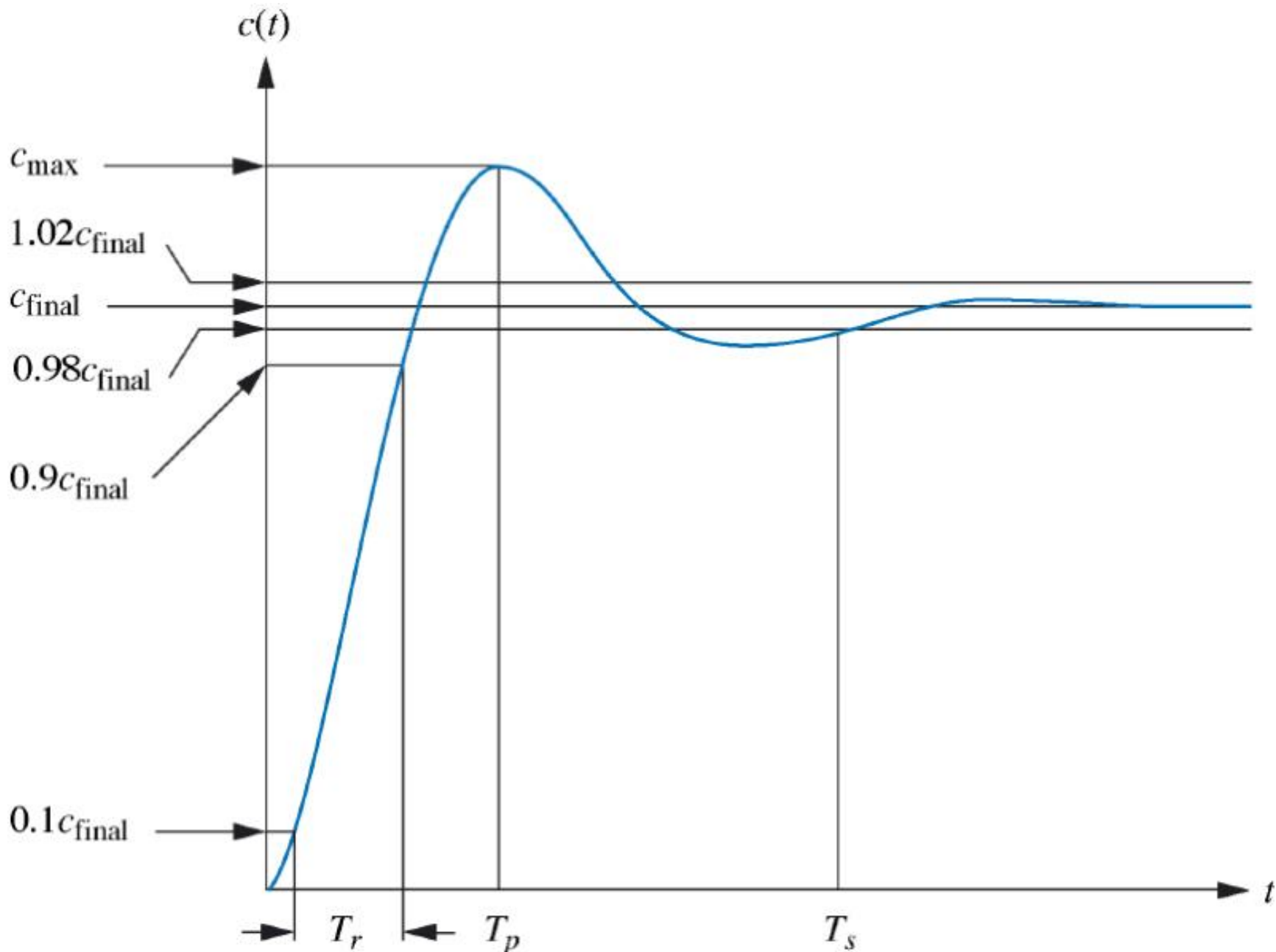
Peak time

$$M_{pt} = 1 + e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}}$$

Peak response

$$P.O. = 100e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}}$$

Percent overshoot

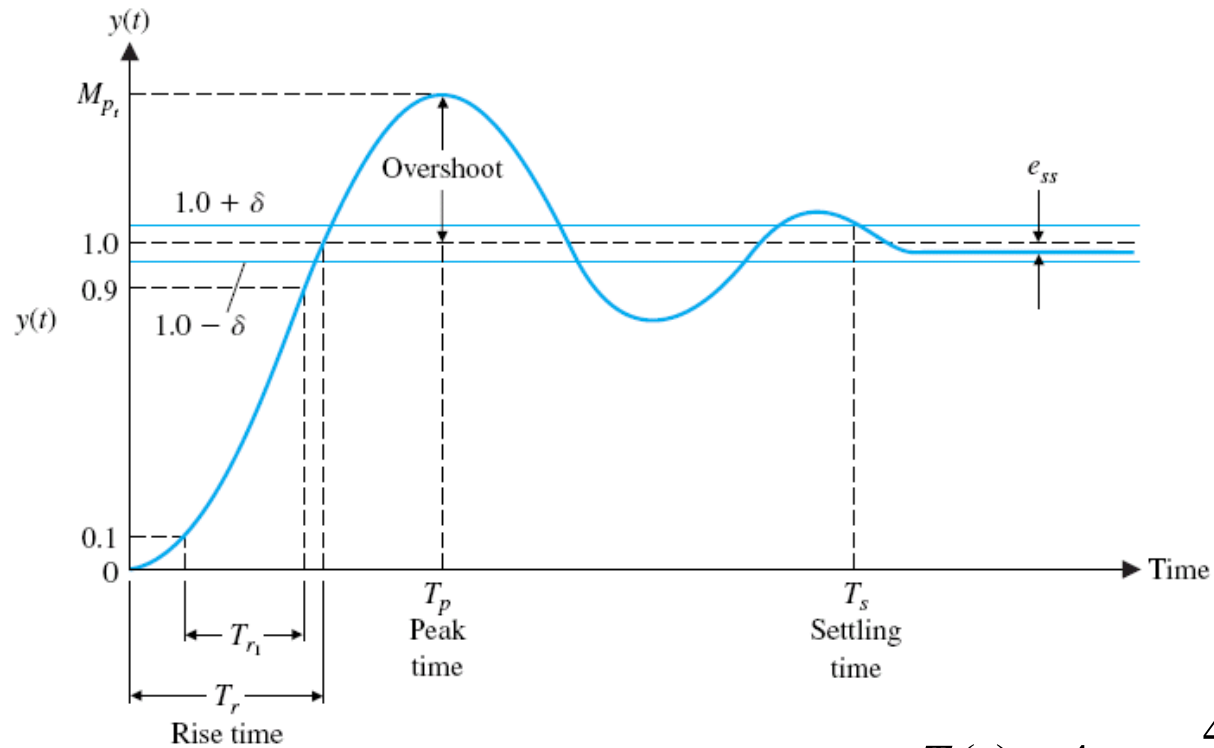


# Settling time

- The settling time is defined as the time required for a system to settle within a certain percentage of the input amplitude.

$$T_s(s) = 4 \tau = \frac{4}{\zeta \omega_n}$$

# Settling time



$$T_s(s) = 4\tau = \frac{4}{\zeta\omega_n}$$

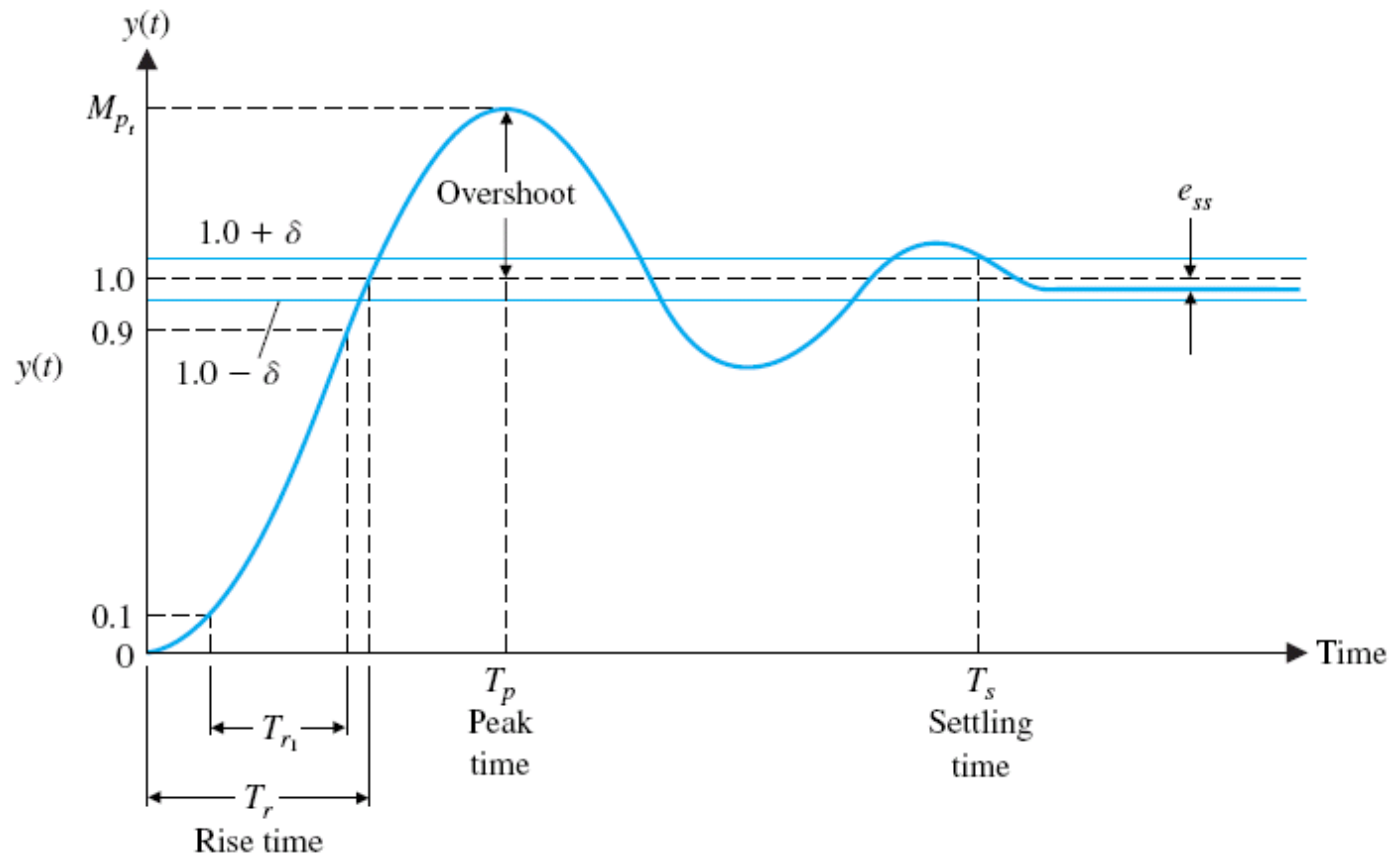
# Rise time

- **The time it takes for a signal to go from 10% of its value to 90% of its final value**

$$T_r(s) = \frac{2.16\zeta + 0.60}{\omega_n} \quad 0.3 \leq \zeta \leq 0.8$$



# Rise time

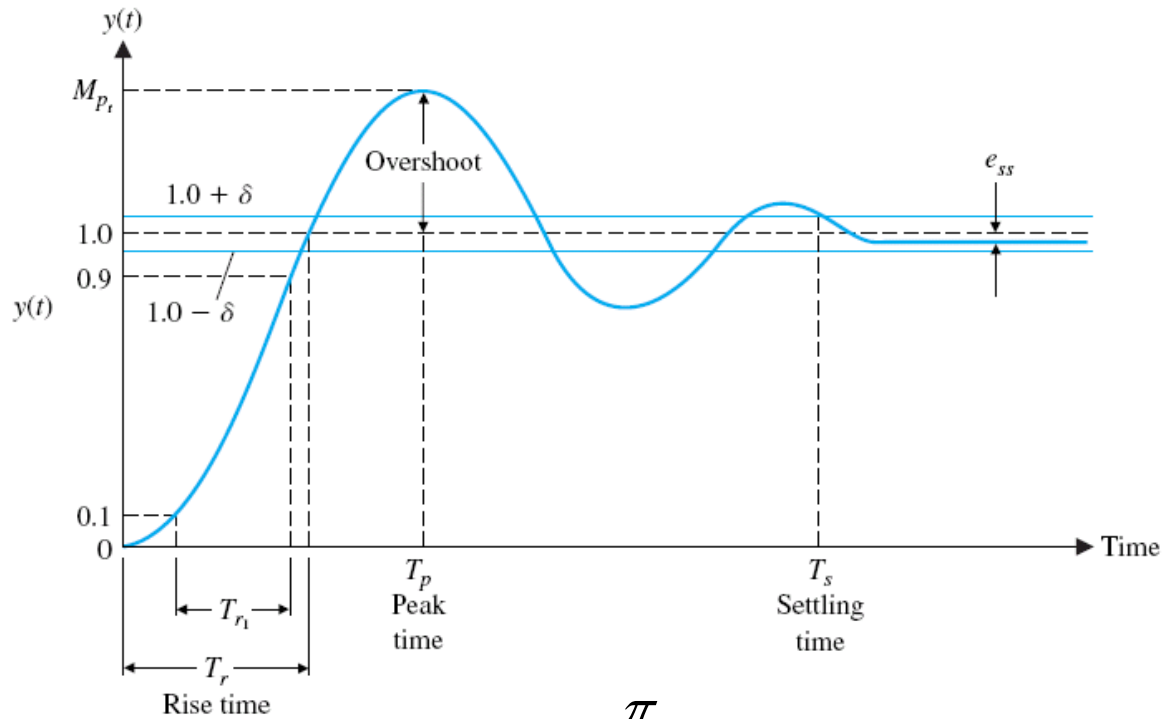


# Peak time

- **Peak time is the time required by a signal to reach its maximum value.**

$$T_p = \frac{\pi}{\omega_n \sqrt{1 - \zeta^2}}$$

# Peak time



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# Percent overshoot

- Percent Overshoot is defined as:

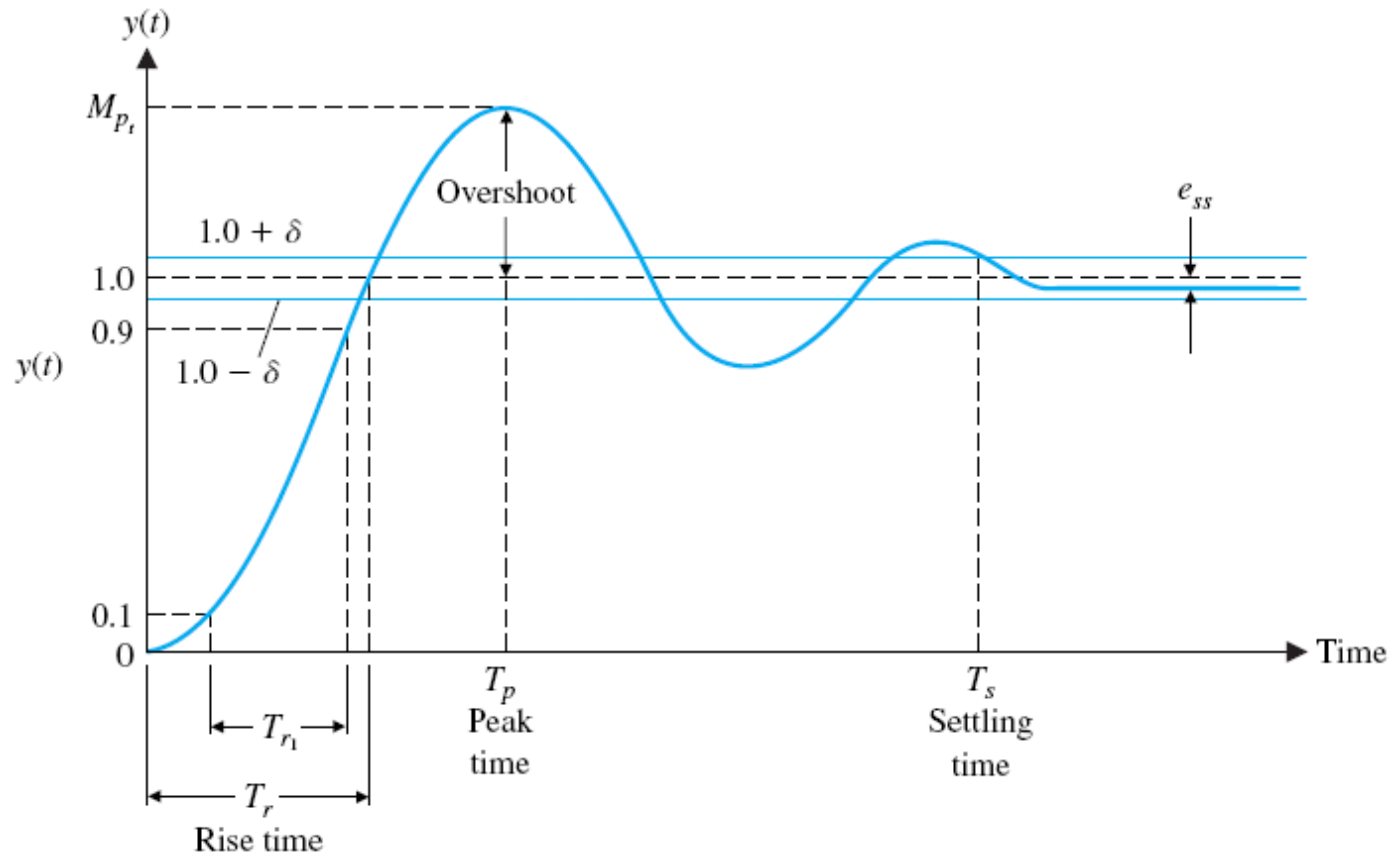
$$P.O. = [(M_{pt} - fv) / fv] * 100\%$$

$M_{pt}$  = The peak value of the time response

$fv$  = Final value of the response

$$P.O. = 100e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}}$$

# Percent overshoot



# Percent overshoot and normalized peak time versus $\zeta$

