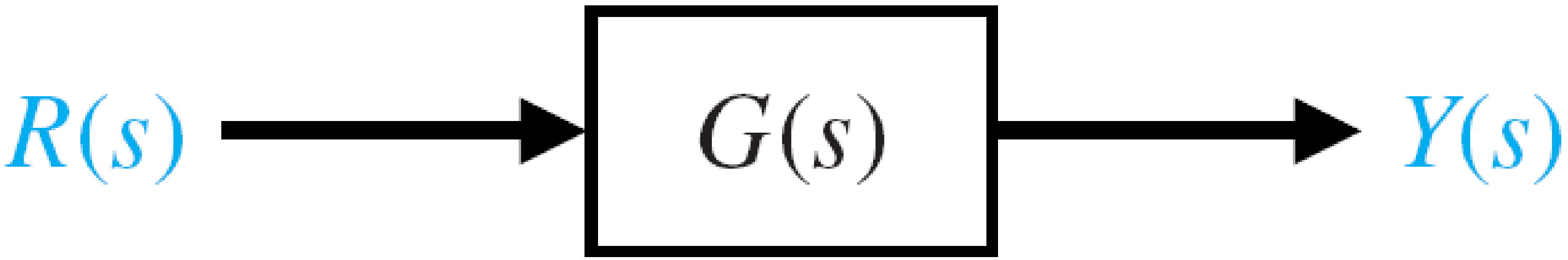
Steady-state response

- **If the steady-state response of the output does not agree with the steady-state of the input exactly, the system is said to have a steady-state error.**

- **It is a measure of system accuracy when a specific type of input is applied to a control system.**

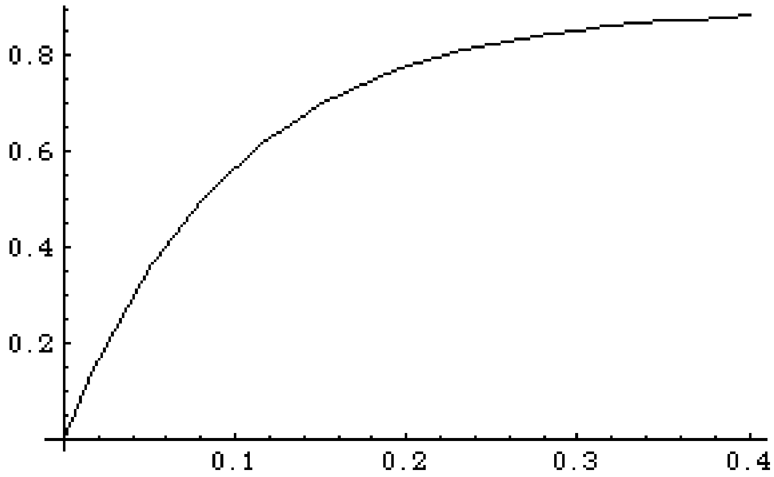


Y(s) = R(s) G(s)

Steady-state error

$T(s) = 9/(s + 10)$

$Y(s) = 9/s(s+10)$

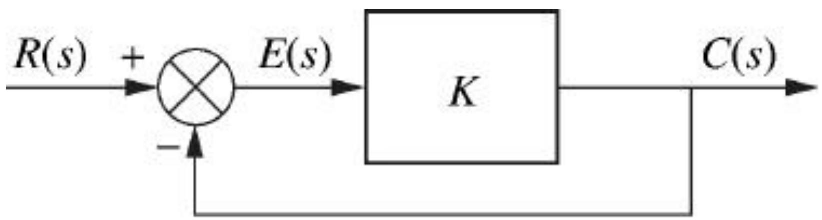


$y(t) = 0.9(1- e^{-10t})$

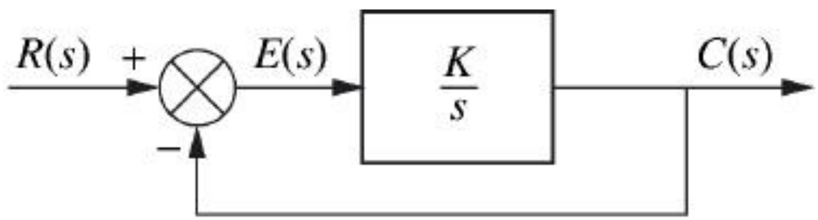
$y(\infty) = 0.9$

$E(s) = R(s) - Y(s)$

$e_{ss} = \lim_{s \to 0} s E(s) = 0.1$



(a)



(b)

$$E(s)=\frac{R(s)}{1+K}$$

$$e(t)=\frac{1}{1+K}$$

$$E(s)=\frac{R(s)}{1+\frac{K}{s}}$$

$$e(t)=e^{-K\,t}$$

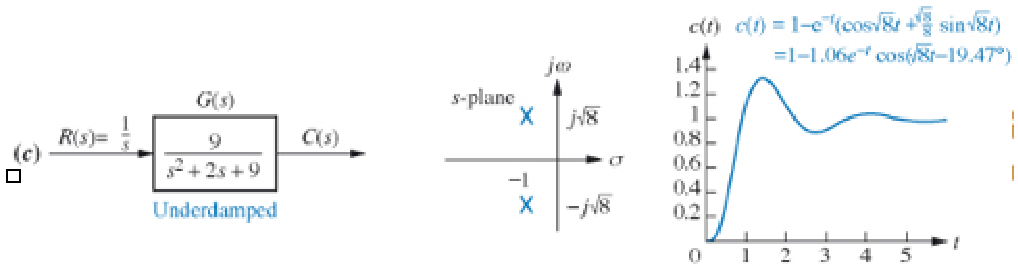
```
InverseLaplaceTransform[{1/(s + a), a/(s + a)}, s, t]
```

$$\left\{E^{-(a\,t)},\ \frac{a}{E^{a\,t}}\right\}$$

```
InverseLaplaceTransform[{1/s * 1/(s + a), 1/s * a/(s + a)}, s, t]
```

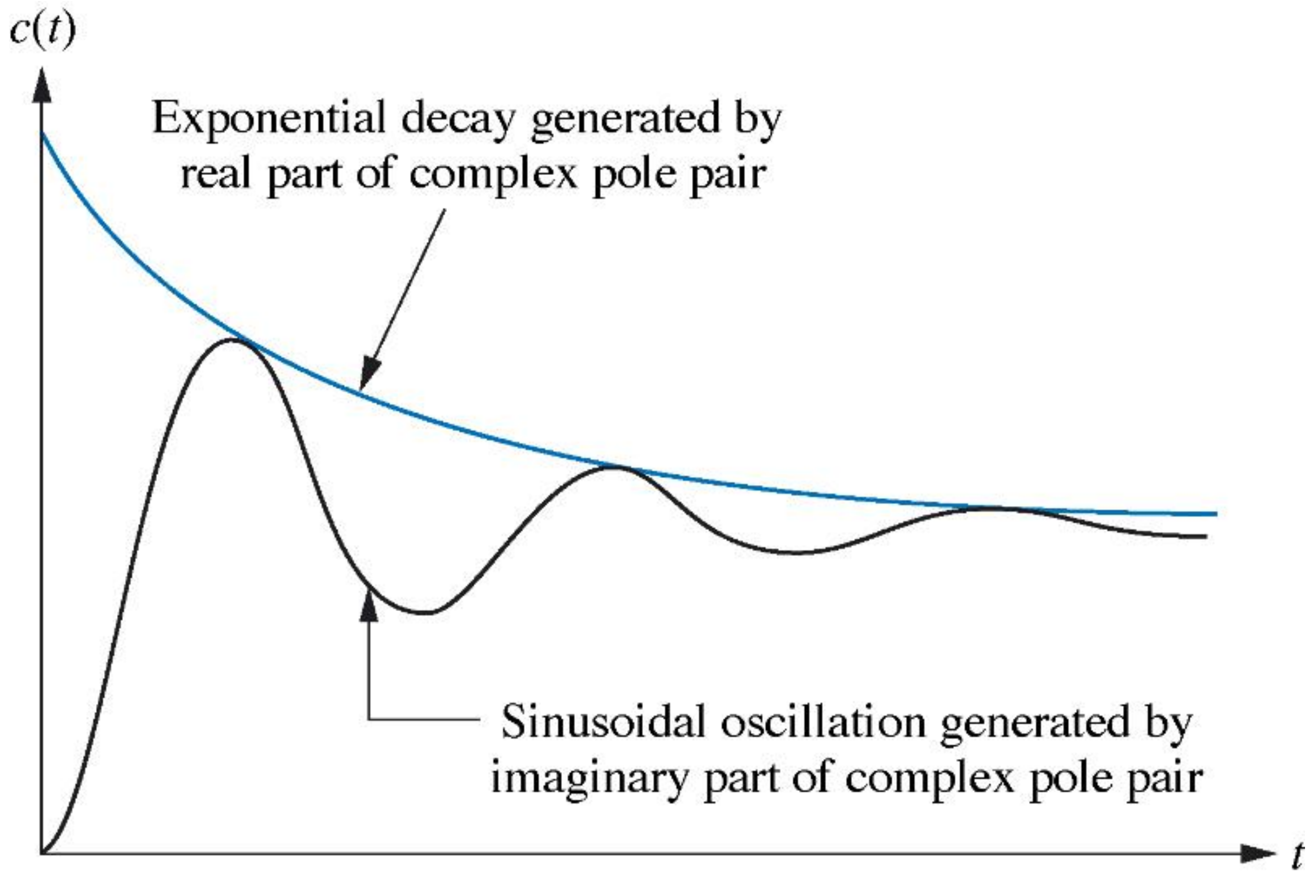
$$\left\{\frac{1 - E^{-(a\,t)}}{a},\ 1 - E^{-(a\,t)}\right\}$$

Underdamped



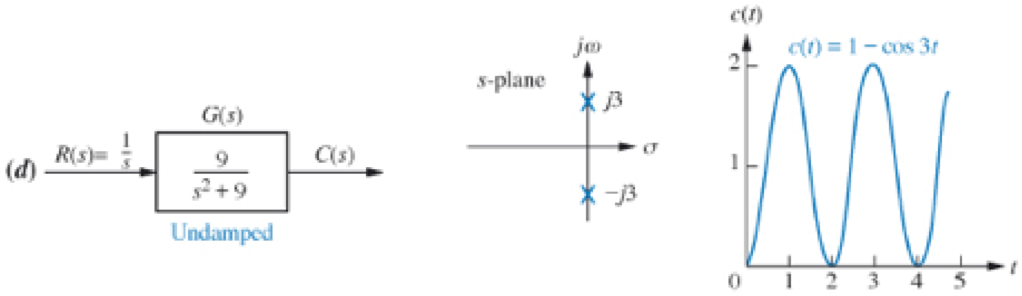
$$C(s)=\frac{9}{s(s^2+2s+9)}=\frac{9}{s(s+1-j\sqrt{8})(s+1+j\sqrt{8})}$$

$$c(t)=1-e^{-t}(\cos\sqrt{8}t+\frac{\sqrt{8}}{8}\sin\sqrt{8}t)$$



$$c(t) = 1 - e^{-t}\left(\cos\sqrt{8}t + \frac{\sqrt{8}}{8}\sin\sqrt{8}t\right)$$

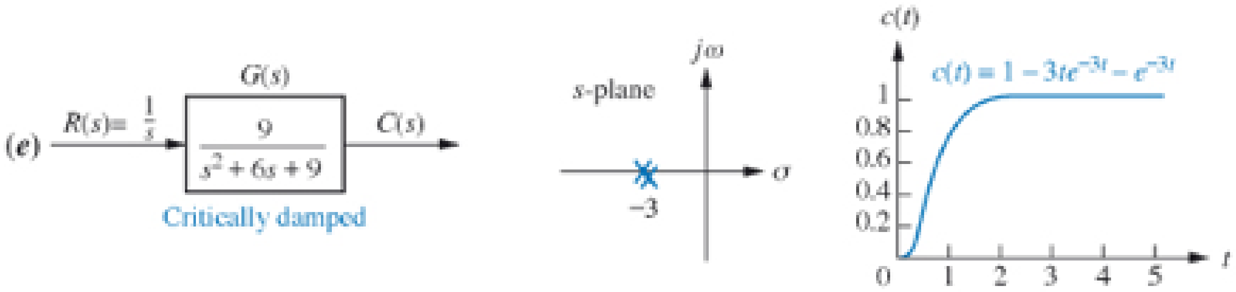
Undamped



$$C(s) = \frac{9}{s(s^2+9)}$$

$$c(t) = 1 - \cos 3t$$

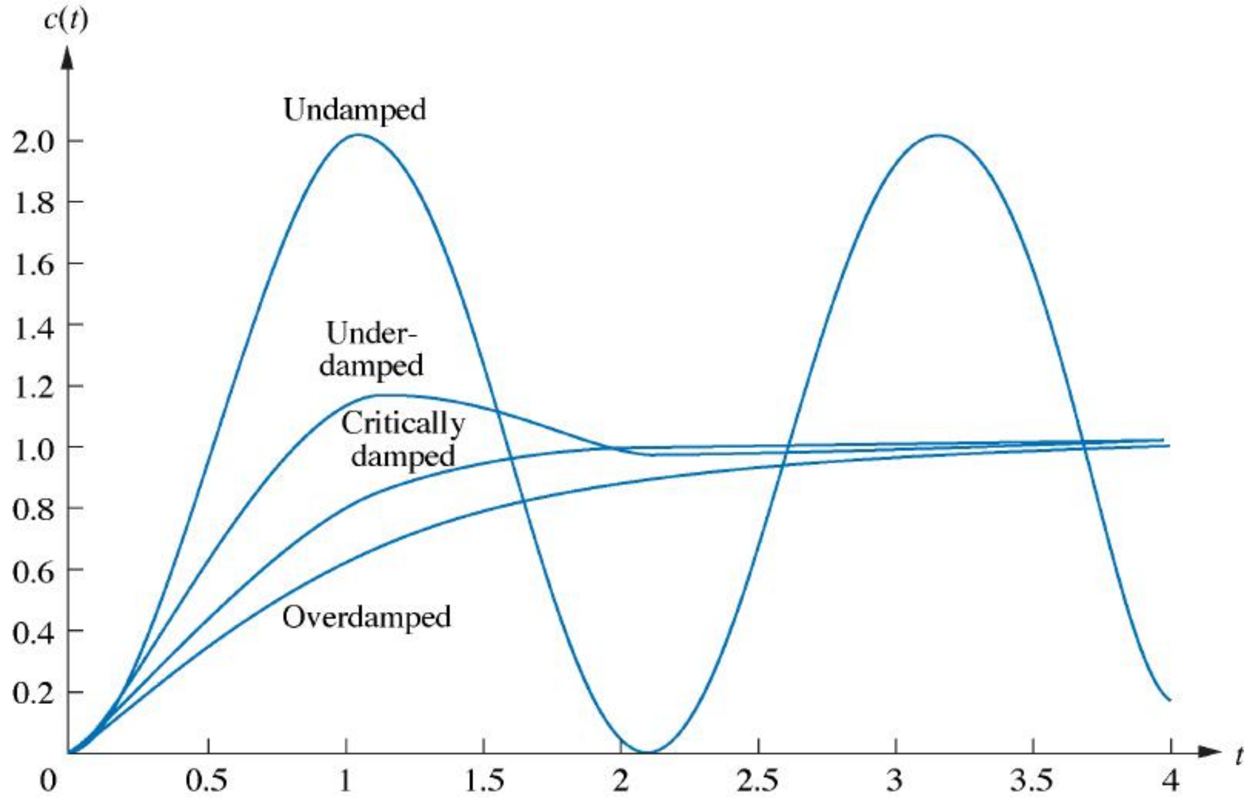
Critically damped



$$C(s) = \frac{9}{s(s^2+6s+9)} = \frac{9}{s(s+3)^2}$$

$$c(t) = 1 - 3te^{-3t} - e^{-3t}$$

Step response for second order system damping cases



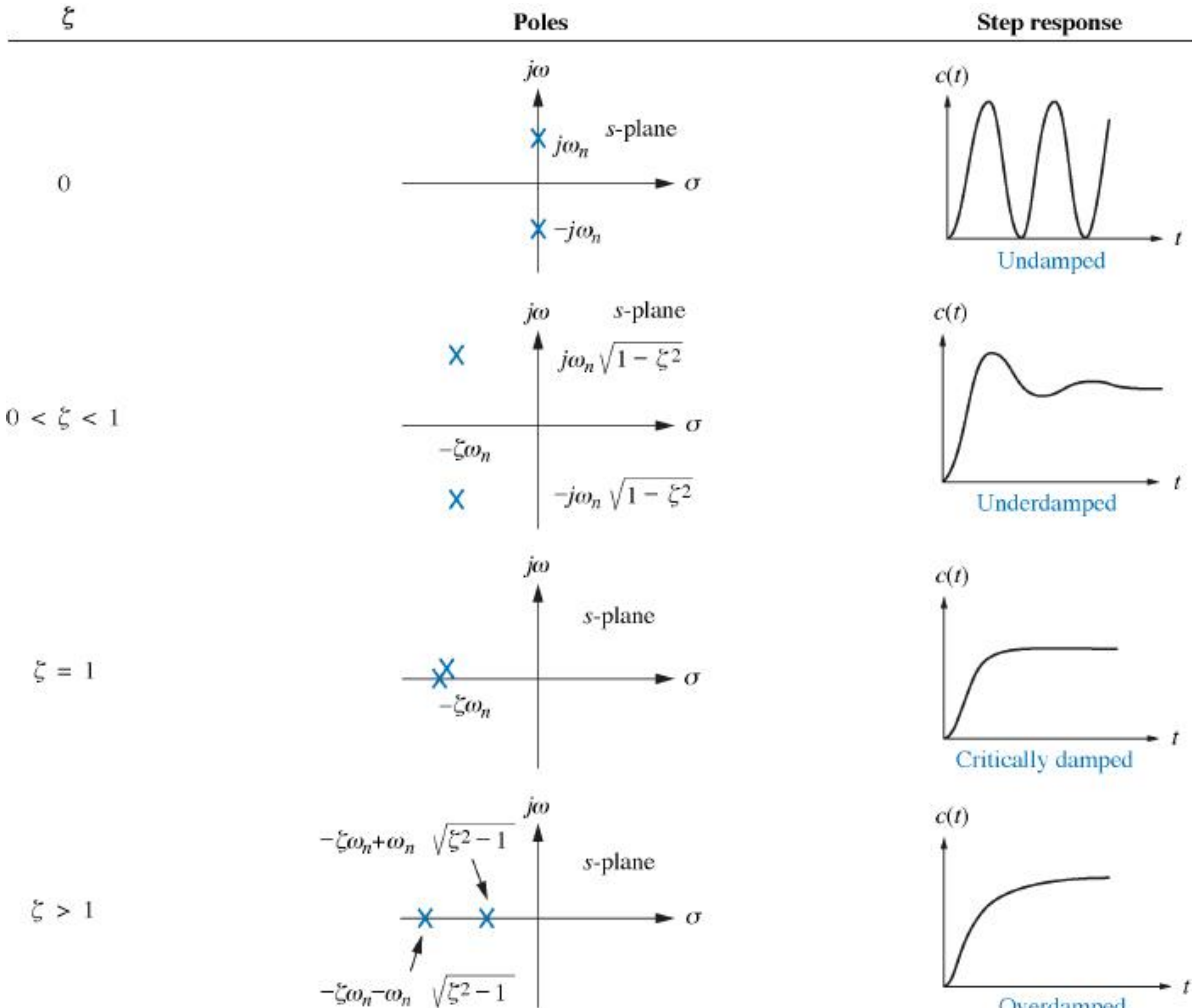

ζ
Poles
Step response
jω
s-plane
$j\omega_n$
σ
$-j\omega_n$
0
c(t)
t
Undamped
jω
s-plane
$j\omega_n\sqrt{1-\zeta^2}$
σ
$-\zeta\omega_n$
$-j\omega_n\sqrt{1-\zeta^2}$
$0 < \zeta < 1$
c(t)
t
Underdamped
jω
s-plane
σ
$-\zeta\omega_n$
$\zeta = 1$
c(t)
t
Critically damped
jω
$-\zeta\omega_n+\omega_n\sqrt{\zeta^2-1}$
s-plane
σ
$-\zeta\omega_n-\omega_n\sqrt{\zeta^2-1}$
$\zeta > 1$
c(t)
t
Overdamped

fig_04_11

Summary

- **Overdamped**

  Poles: Two real at $-\sigma_1, -\sigma_2$
- **Underdamped**

  Poles: Two complex at $-\sigma_d + j\omega_d, -\sigma_d - j\omega_d$
- **Undamped**

  Poles: Two imaginary at $+j\omega_1, -j\omega_1$
- **Critically damped**

  Poles: Two real at $-\sigma_1,$