# Control Principles

Block diagram practice

Block diagram

Transfer Function

Consists of Blocks

Can be reduced
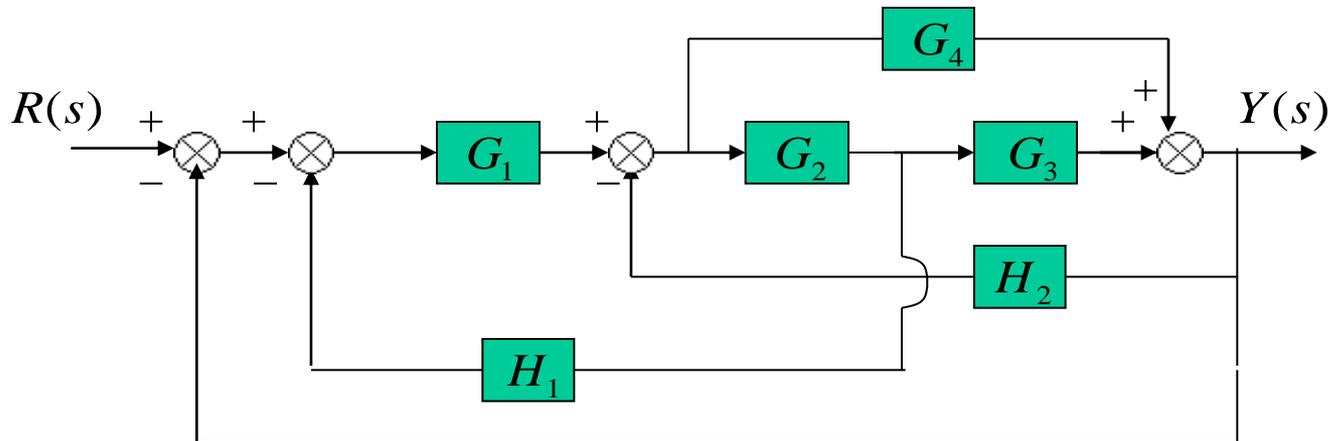
Reduction techniques

## 1. Combining blocks in cascade or in parallel
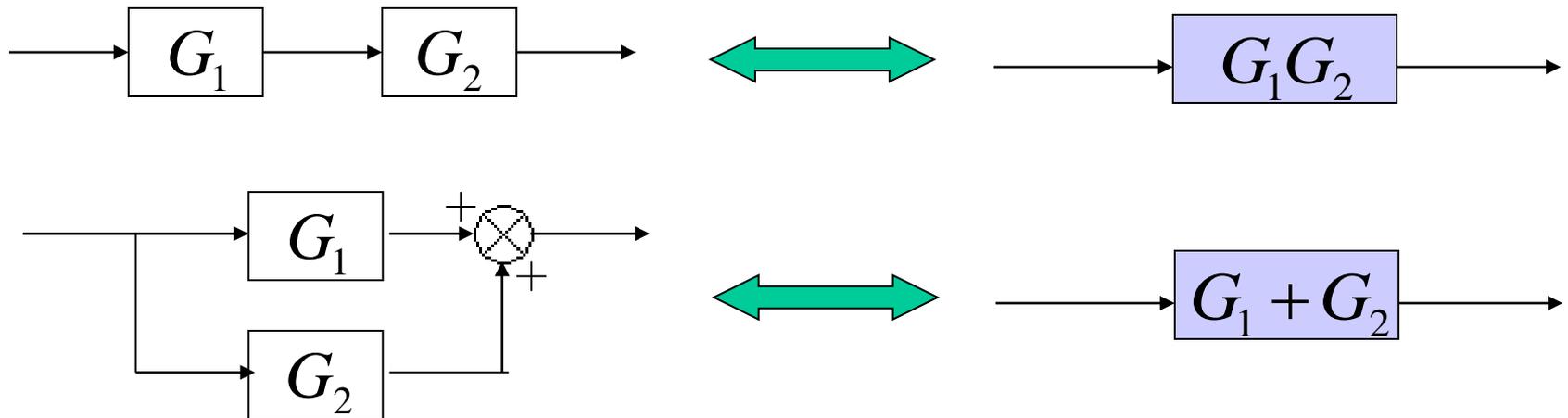
## 2. Moving a summing point behind a block
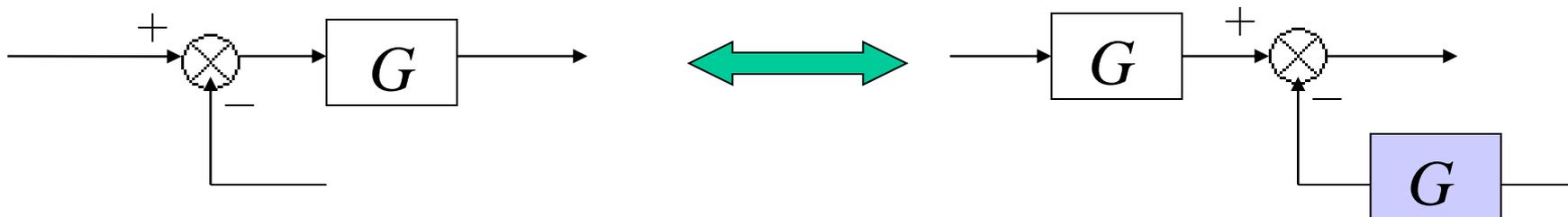
3. Moving a summing point ahead of a block
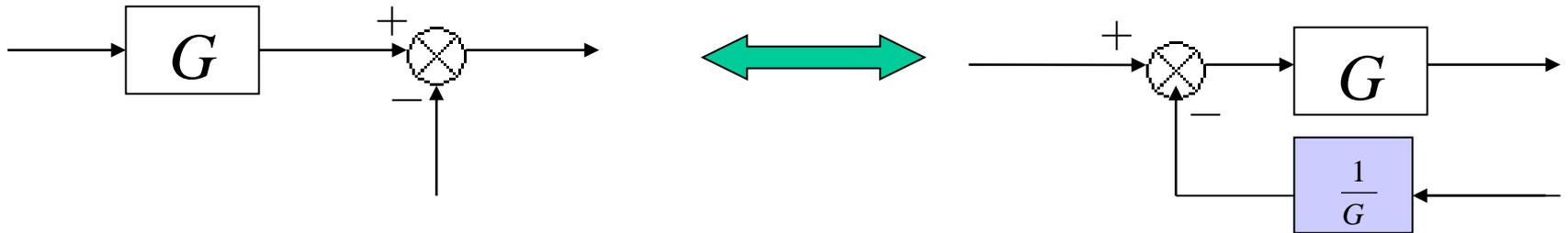

4. Moving a pickoff point behind a block
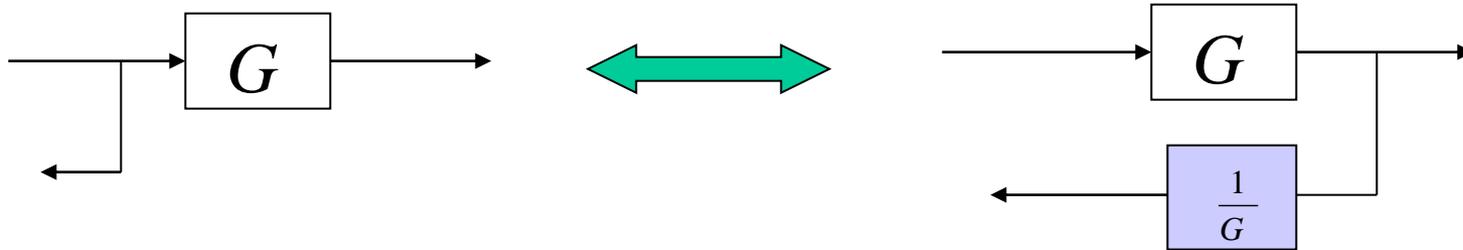
5. Moving a pickoff point ahead of a block
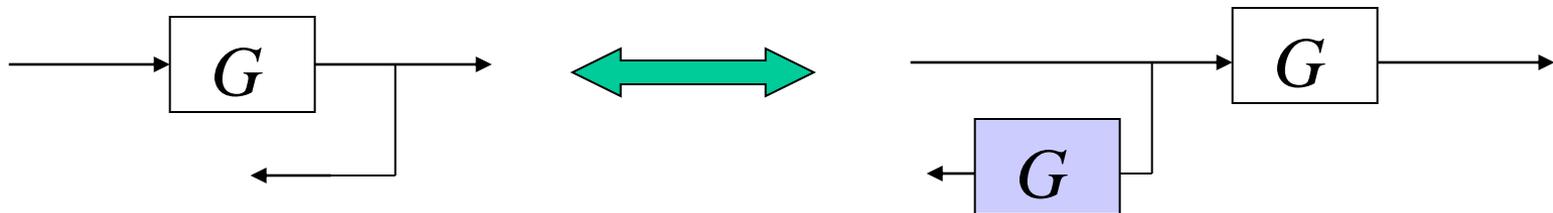
6. Eliminating a feedback loop

$$\frac{G}{1 \mp GH}$$

$$H = 1$$

$$\frac{G}{1 \mp G}$$
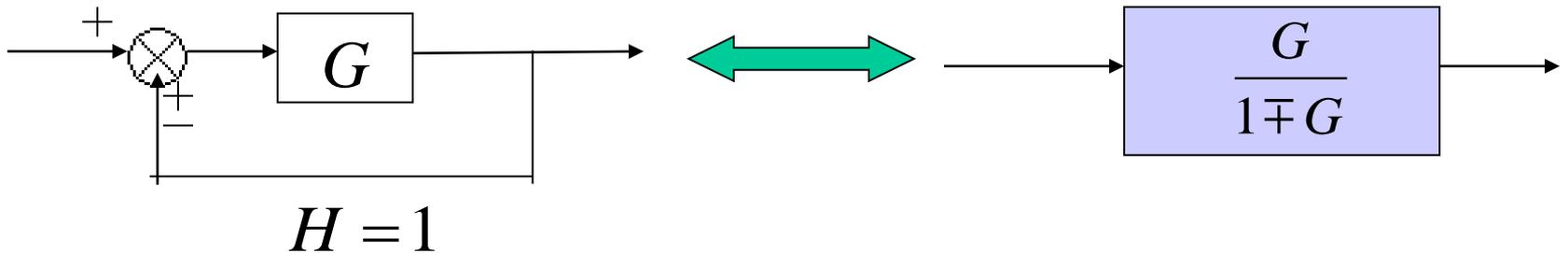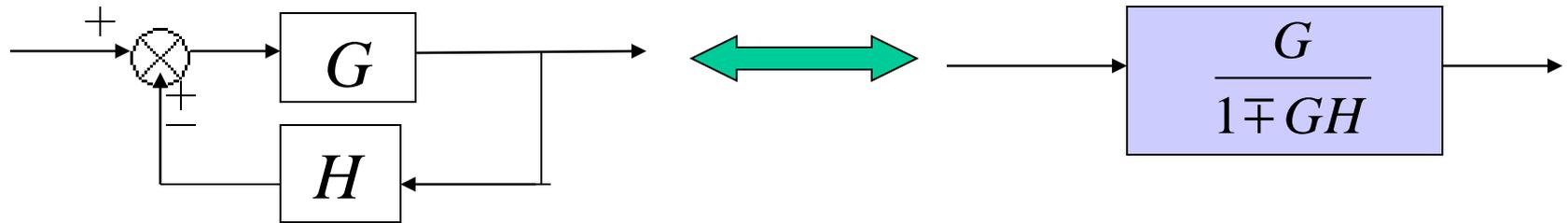
7. Swap with two neighboring summing points
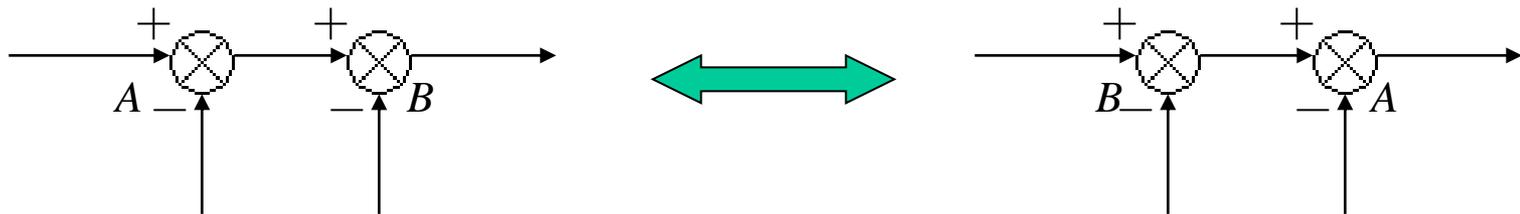
Example 1

Find the transfer function of the following block diagrams

(a)

I
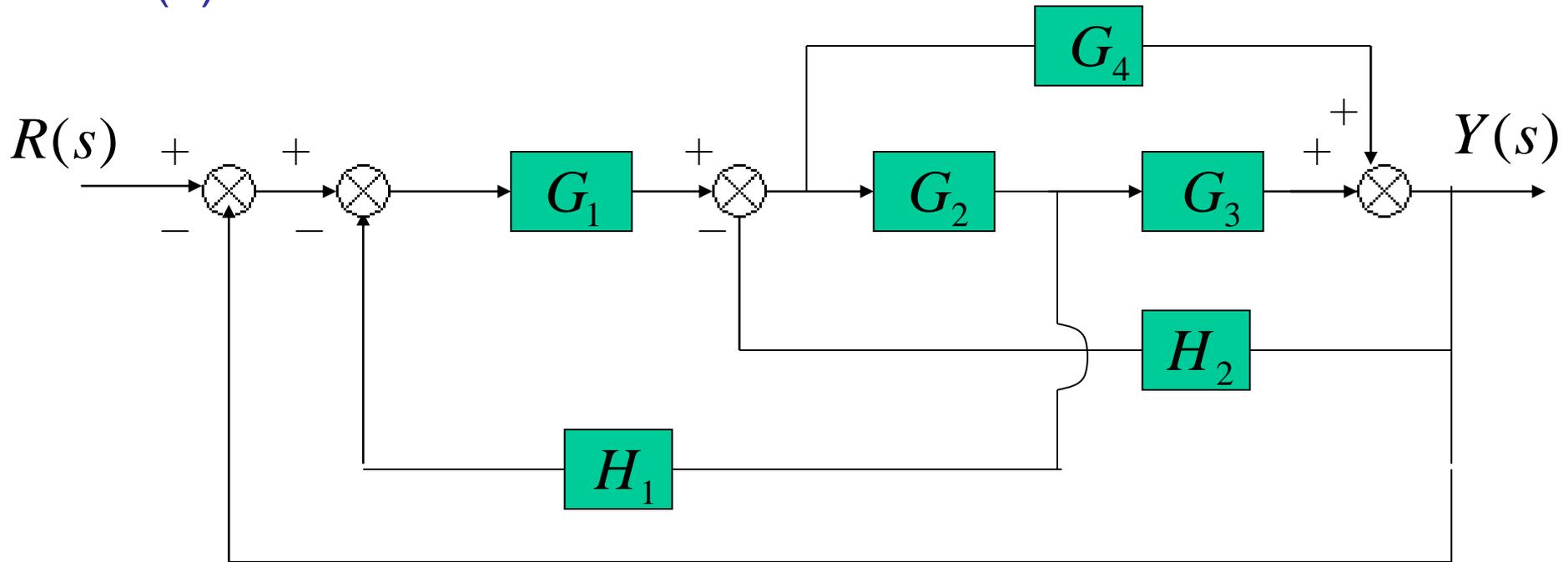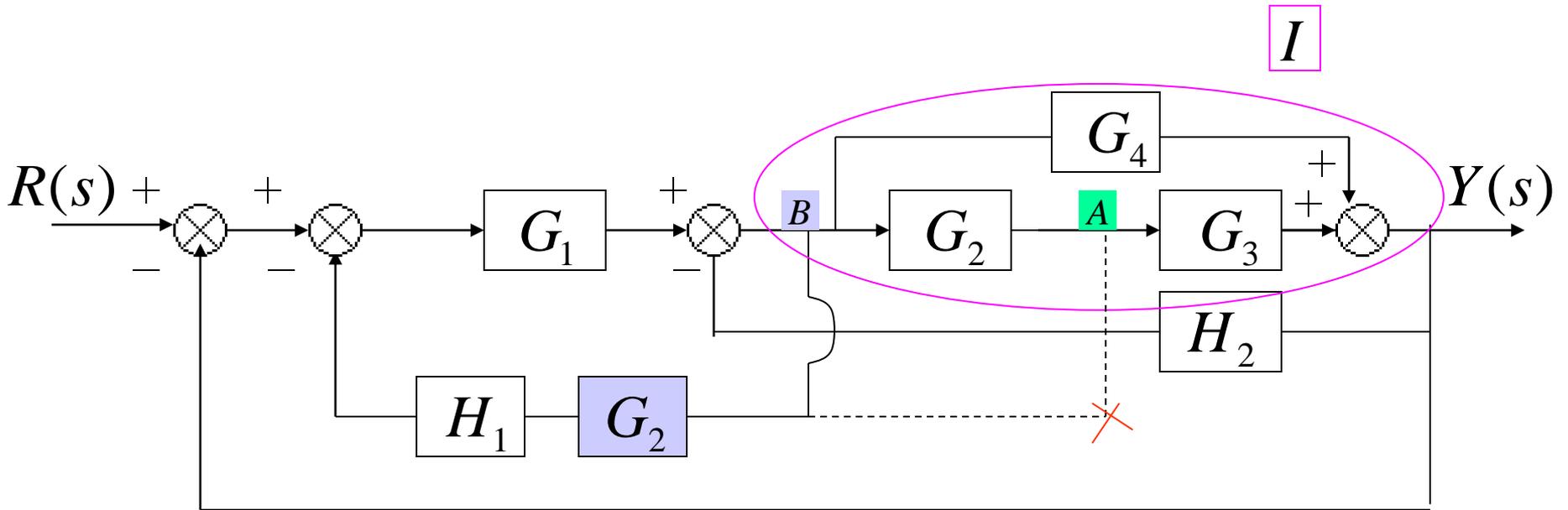
Solution:

1. Moving pickoff point A ahead of block $\boxed{G_2}$

2. Eliminate loop I & simplify

B ⟶ $\boxed{G_4 + G_2 G_3}$ ⟶

$R(s)$ $+$ $-$ $+$ $-$ G_1 $+$ $-$ B $G_4 + G_2 G_3$ $Y(s)$

$H_1 G_2$

H_2
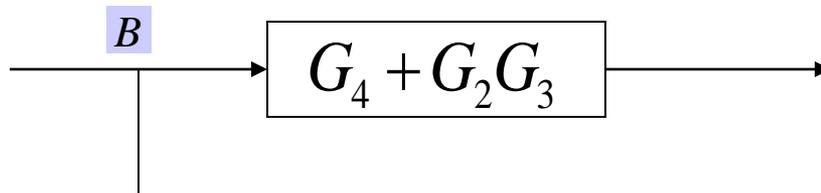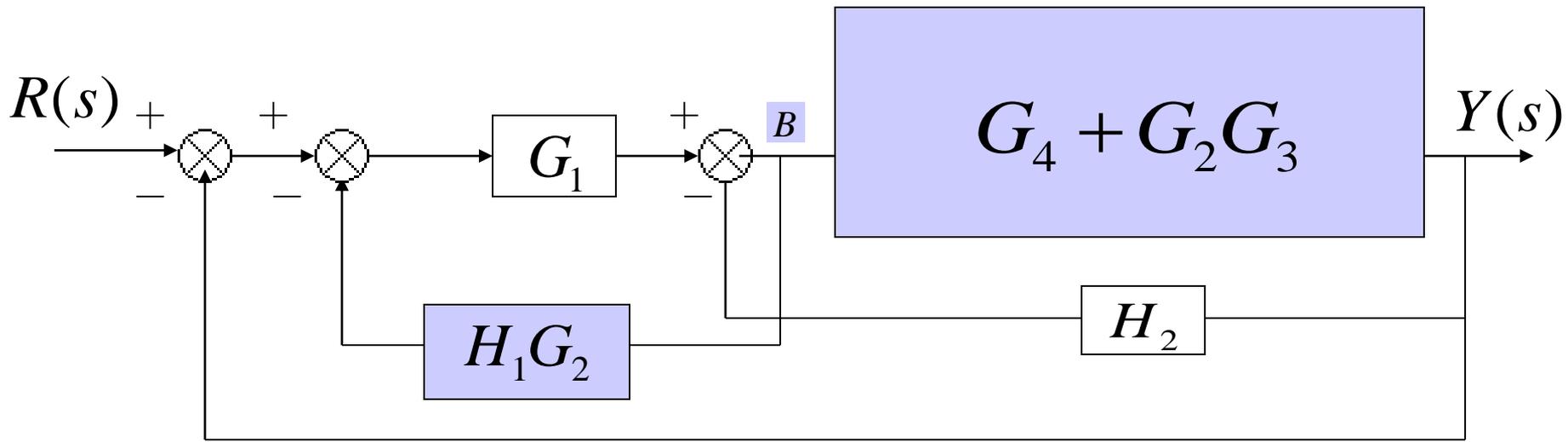
3. Moving pickoff point B behind block $\boxed{G_4 + G_2 G_3}$

II

$R(s)$ $+$ $-$ $+$ $-$ G_1 $+$ $-$ B $G_4 + G_2 G_3$ C $Y(s)$

H_2

$H_1 G_2$ $1/(G_4 + G_2 G_3)$

4. Eliminate loop III

$$R(s) \xrightarrow{+} \bigotimes \xrightarrow{+} \bigotimes \longrightarrow \boxed{G_1} \longrightarrow \boxed{\dfrac{G_4 + G_2 G_3}{1 + H_2(G_4 + G_2 G_3)}} \xrightarrow{c} Y(s)$$

$$\boxed{\dfrac{G_2 H_1}{G_4 + G_2 G_3}}$$
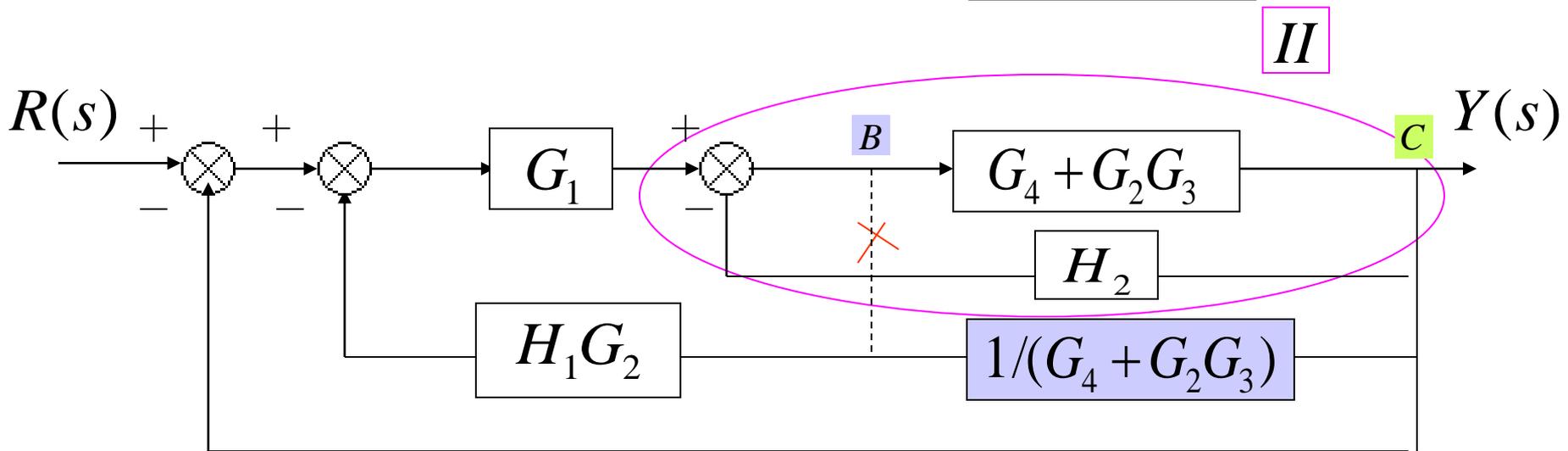
Using rule 6

$$R(s) \xrightarrow{+} \bigotimes \longrightarrow \boxed{\dfrac{G_1(G_4 + G_2 G_3)}{1 + G_1 G_2 H_1 + H_2(G_4 + G_2 G_3)}} \longrightarrow Y(s)$$
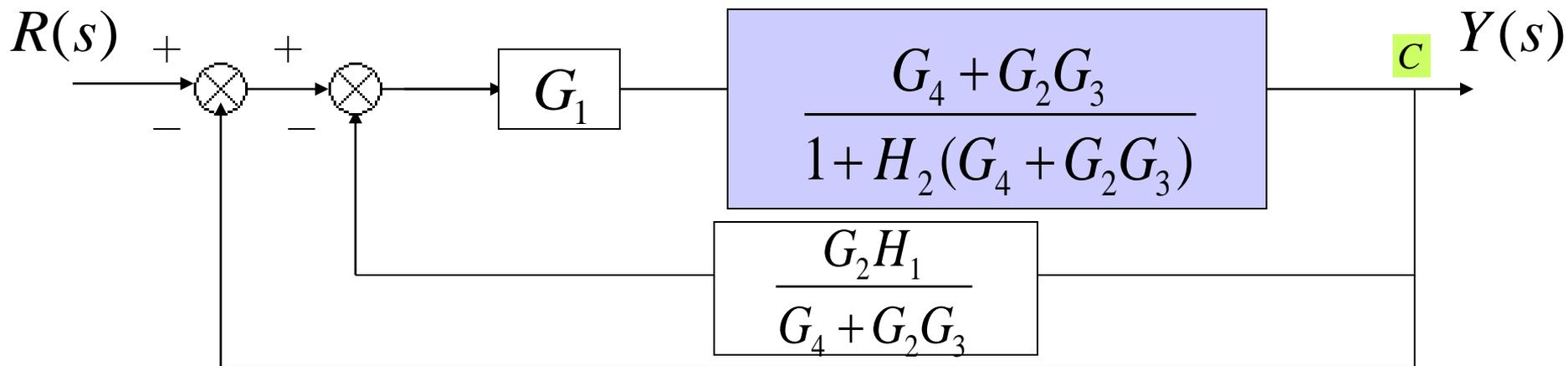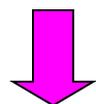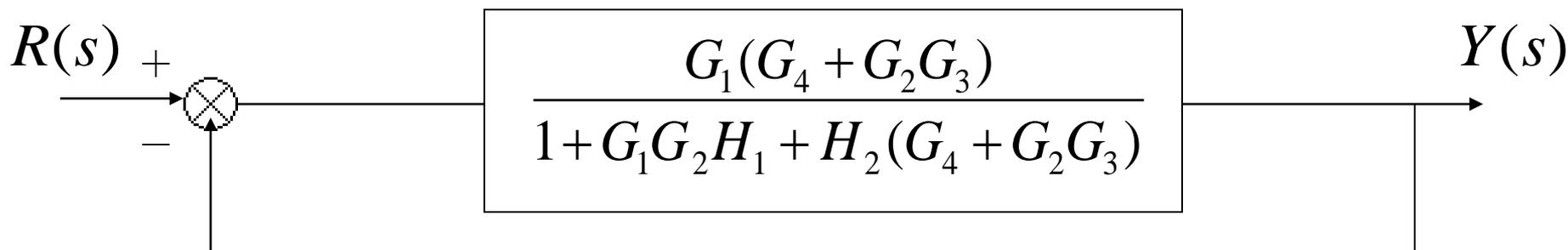
$$T(s) = \frac{Y(s)}{R(s)} = \frac{G_1(G_4 + G_2 G_3)}{1 + G_1 G_2 H_1 + H_2(G_4 + G_2 G_3) + G_1(G_4 + G_2 G_3)}$$

(b)
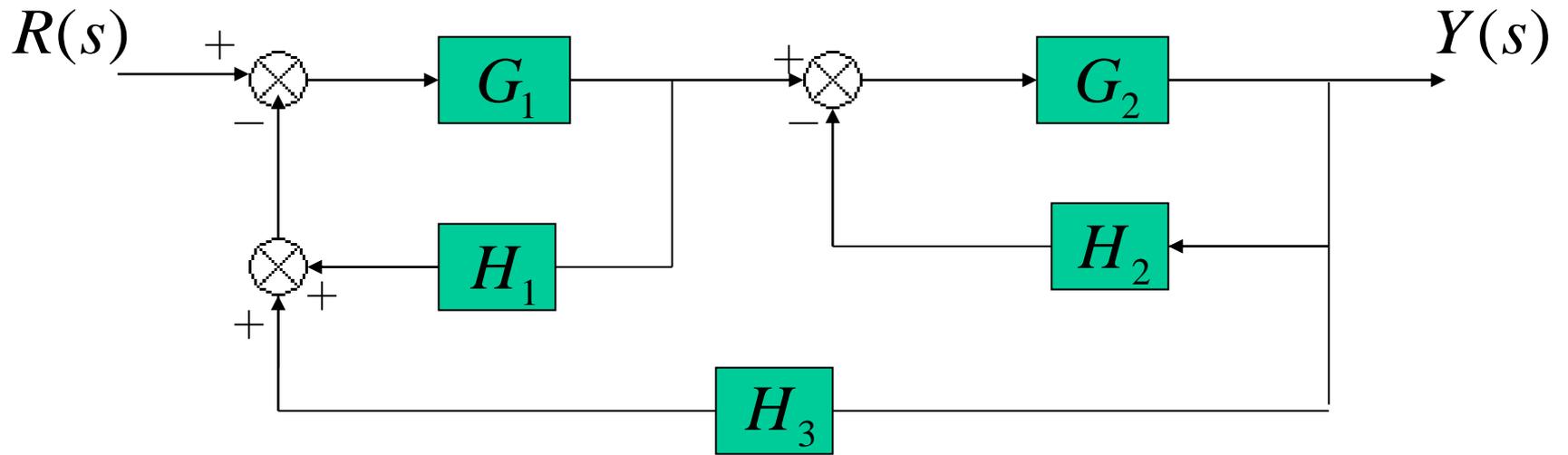
Solution:

1. Eliminate loop I

$R(s)$ $+$ $-$ $+$ G_1 A $$\frac{G_2}{1 + G_2 H_2}$$ B $Y(s)$

H_1

H_3
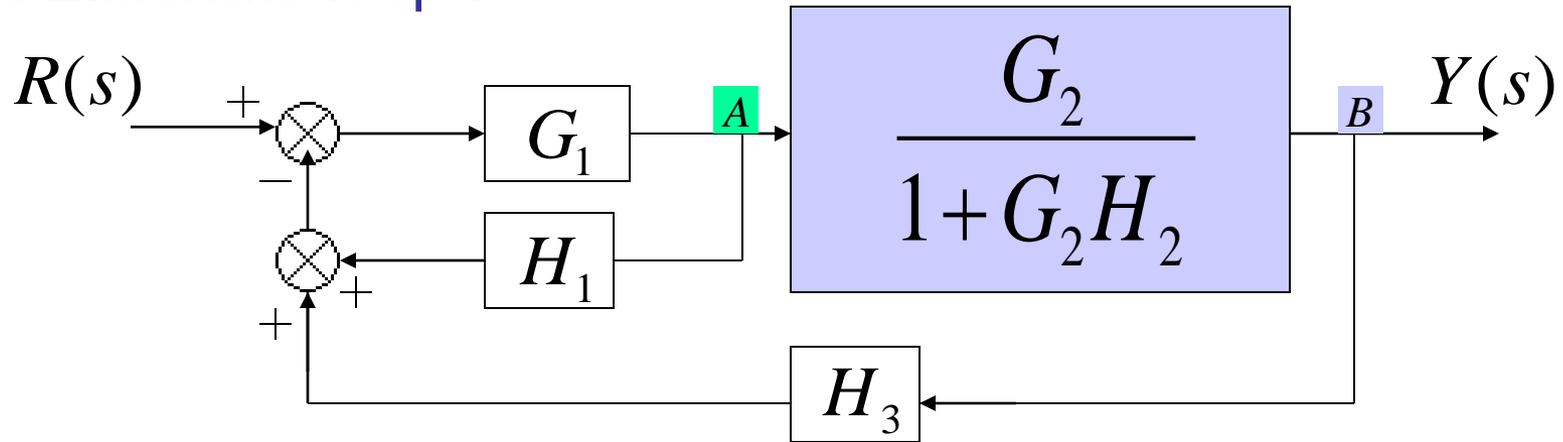
2. Moving pickoff point A behind block $\dfrac{G_2}{1 + G_2 H_2}$

$R(s)$ $+$ $-$ G_1 A $$\frac{G_2}{1 + G_2 H_2}$$ B $Y(s)$

$+$ $+$ H_1 $$\frac{1 + G_2 H_2}{G_2}$$ II

$$H_3 + H_1 \left(\frac{1 + G_2 H_2}{G_2} \right)$$

H_3

Not a feedback loop

3. Eliminate loop II

$R(s)$ $+$ $-$ $Y(s)$

$$\frac{G_1 G_2}{1 + G_2 H_2}$$

$$H_3 + \frac{H_1(1 + G_2 H_2)}{G_2}$$
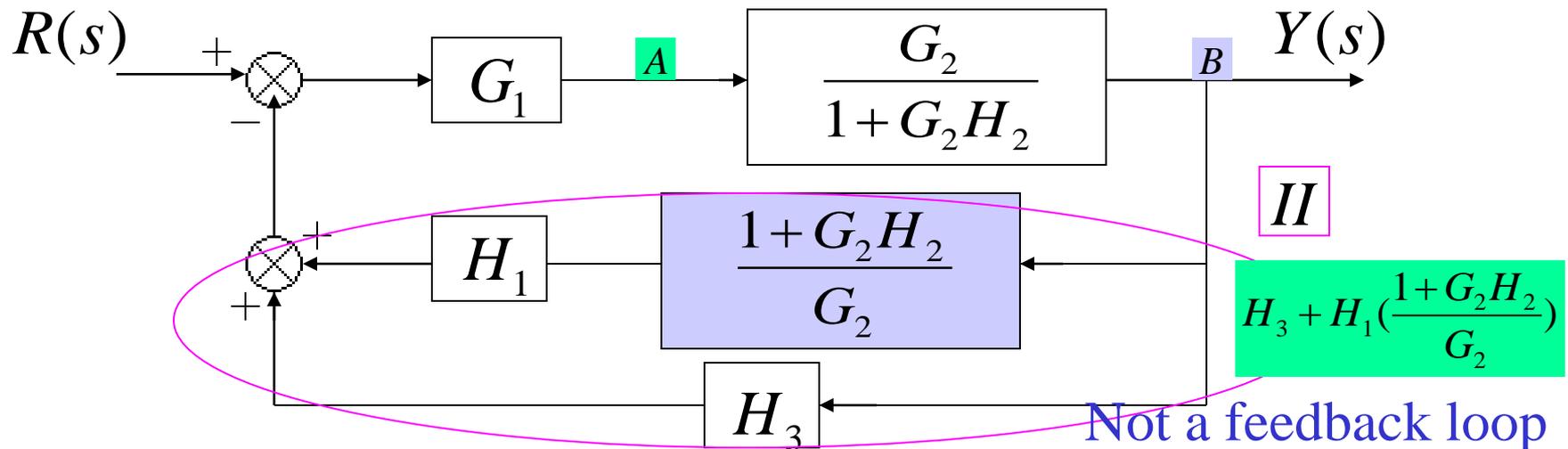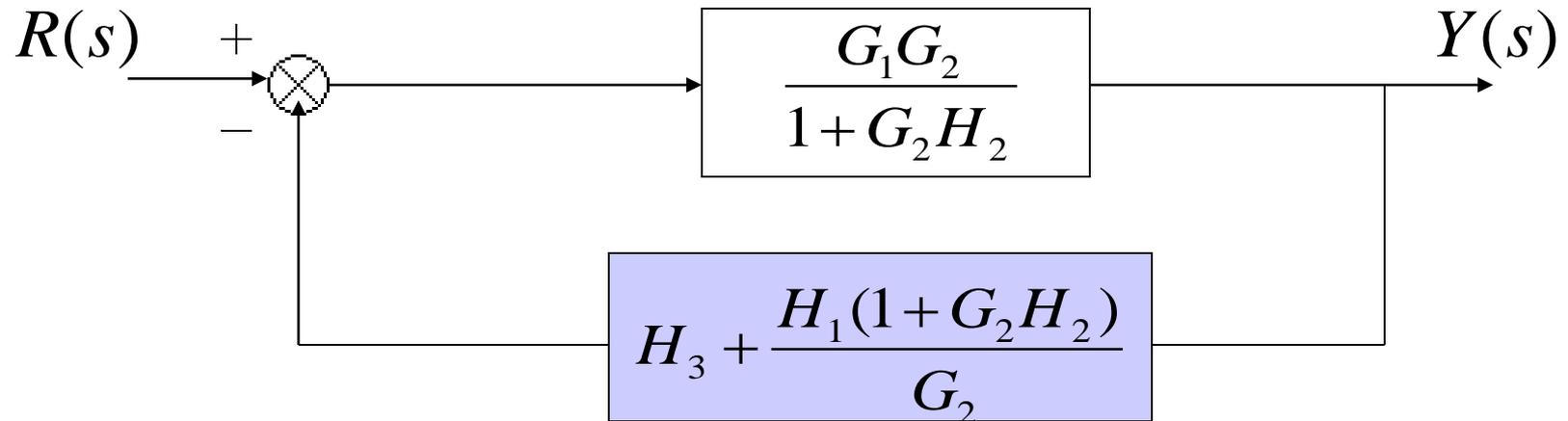
Using rule 6

$$T(s) = \frac{Y(s)}{R(s)} = \frac{G_1 G_2}{1 + G_2 H_2 + G_1 G_2 H_3 + G_1 H_1 + G_1 G_2 H_1 H_2}$$

(c)
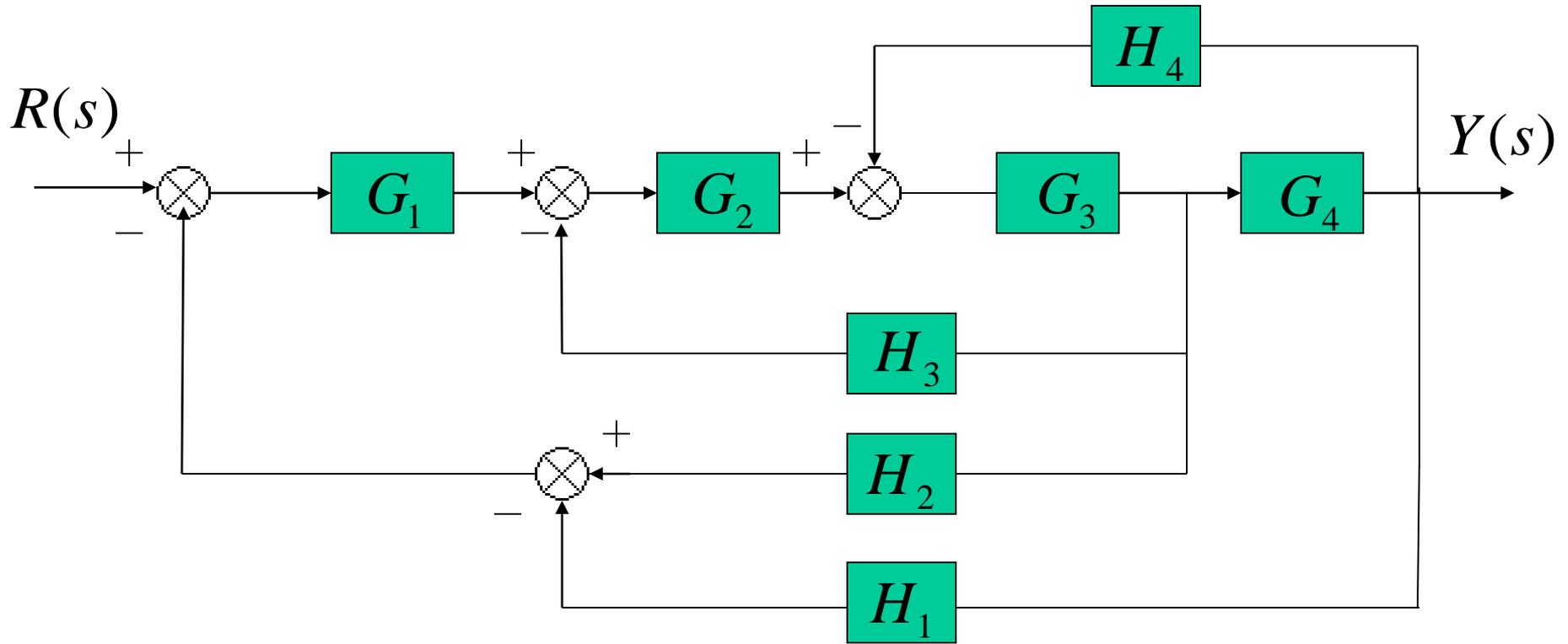
Solution:

## 1. Moving pickoff point A behind block G_4
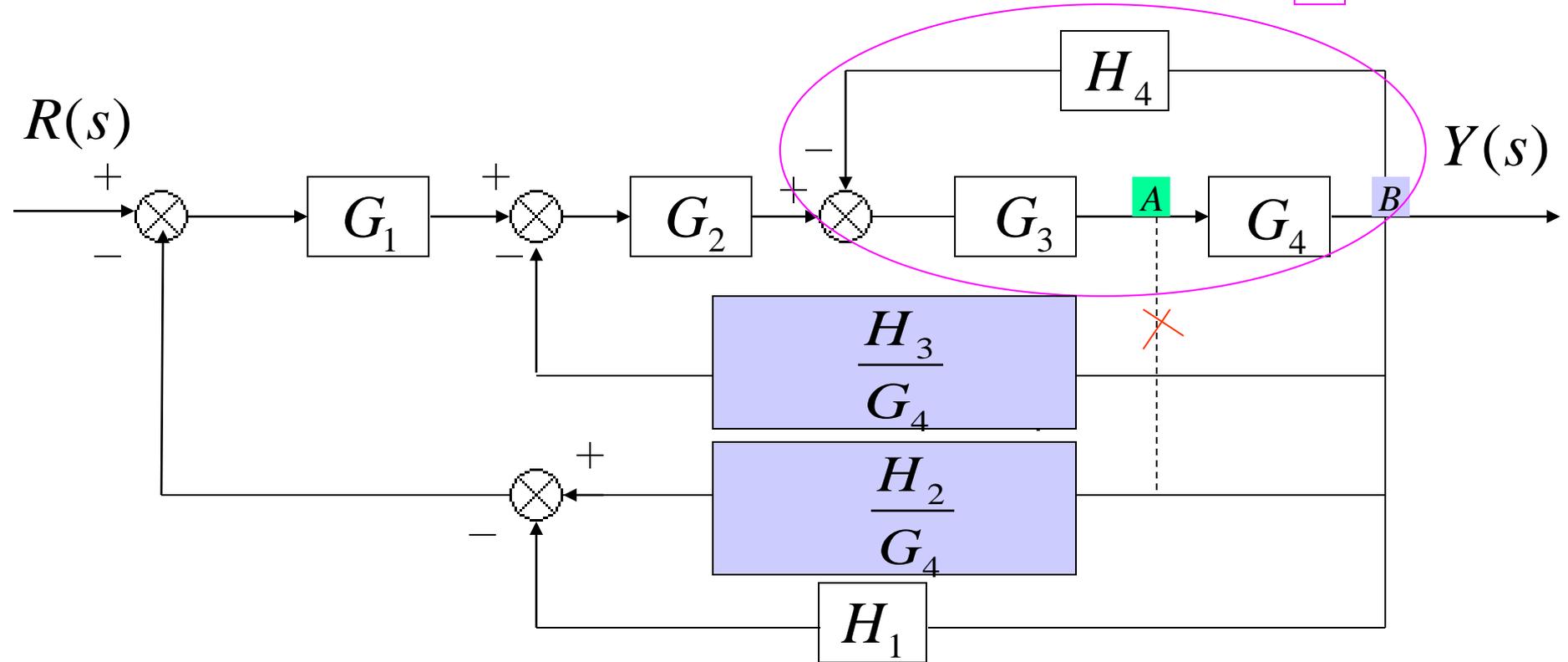
2. Eliminate loop I and Simplify

$R(s)$

$+$

$-$

G_1

$+$

$-$

$$\frac{G_2 G_3 G_4}{1 + G_3 G_4 H_4}$$

II

$Y(s)$

B

$$\frac{H_3}{G_4}$$

$+$

$-$

$$\frac{H_2}{G_4}$$

III

H_1
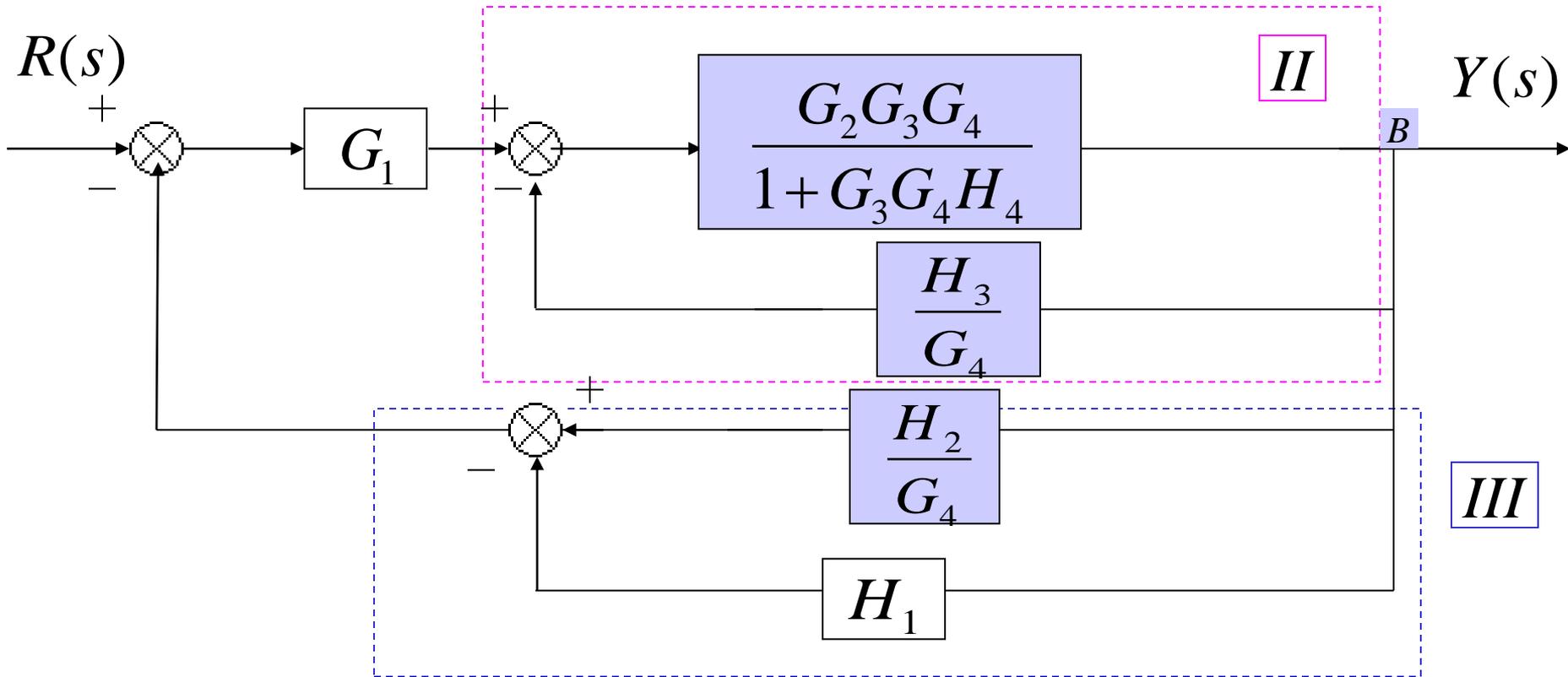
II ↻ feedback

$$\frac{G_2 G_3 G_4}{1 + G_3 G_4 H_4 + G_2 G_3 H_3}$$

III ↻ Not feedback

$$\frac{H_2 - G_4 H_1}{G_4}$$

3. Eliminate loop II & IIII

$$R(s) \xrightarrow{\;+\;} \bigotimes \xrightarrow{\;-\;} \boxed{\dfrac{G_1 G_2 G_3 G_4}{1 + G_3 G_4 H_4 + G_2 G_3 H_3}} \longrightarrow Y(s)$$

$$\boxed{\dfrac{H_2 - G_4 H_1}{G_4}}$$
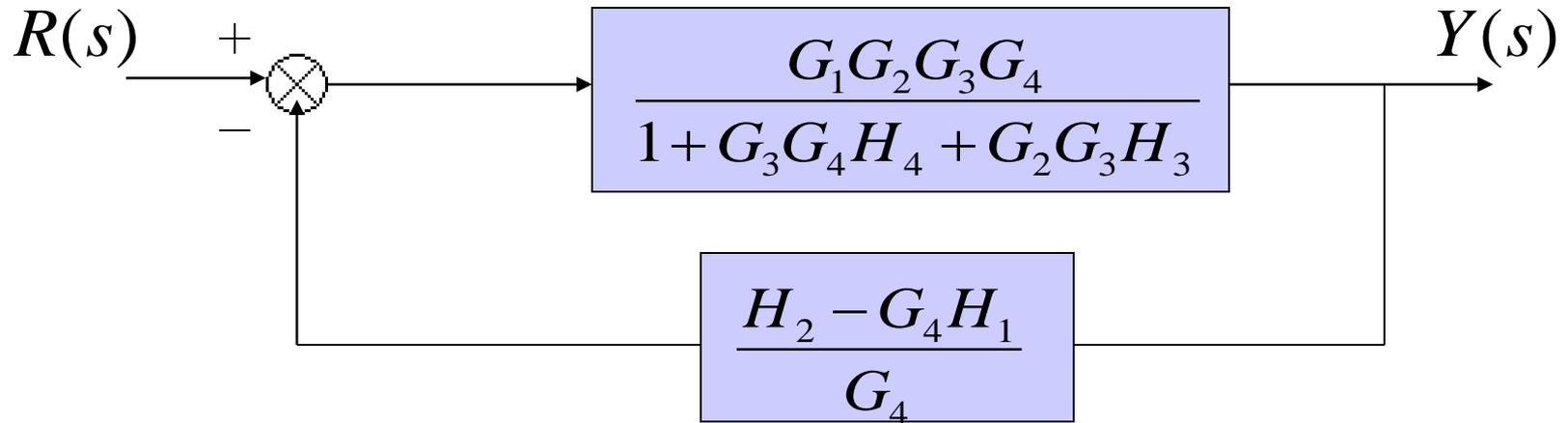
Using rule 6

$$T(s) = \frac{Y(s)}{R(s)} = \frac{G_1 G_2 G_3 G_4}{1 + G_2 G_3 H_3 + G_3 G_4 H_4 + G_1 G_2 G_3 H_2 - G_1 G_2 G_3 G_4 H_1}$$

(d)
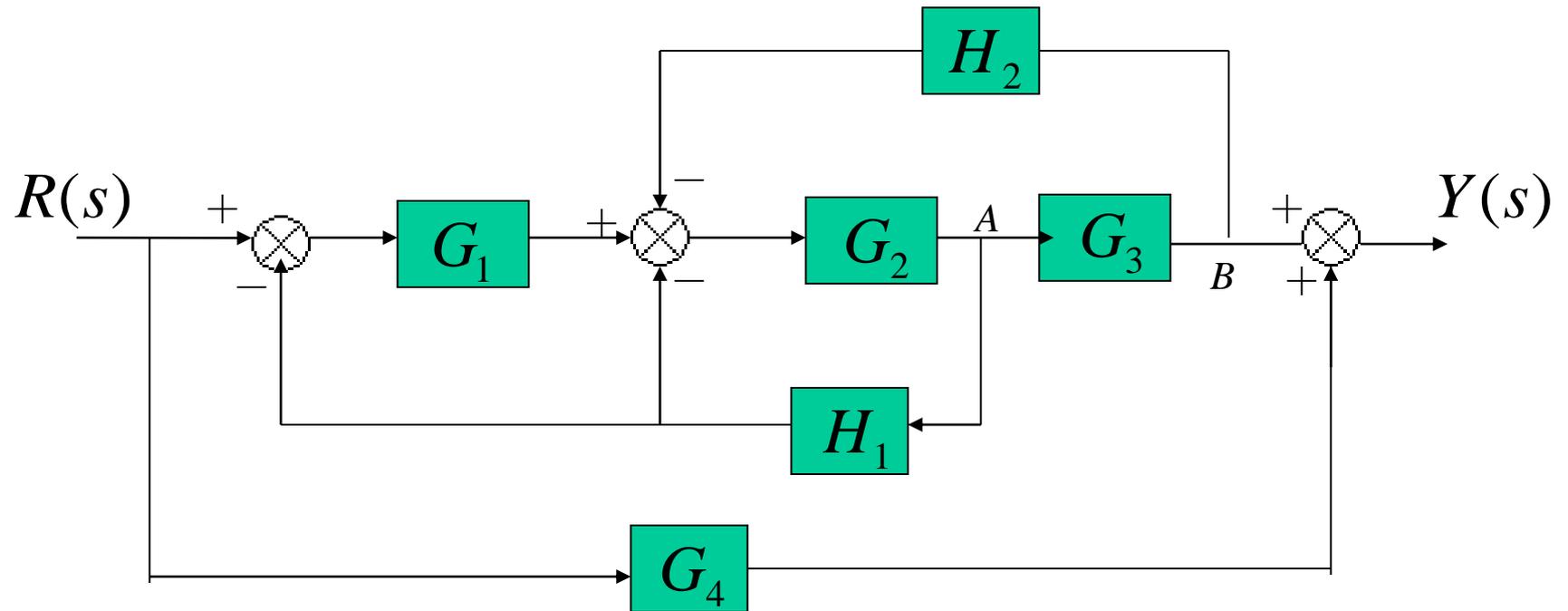
Solution:

1. Moving pickoff point A behind block G_3

I
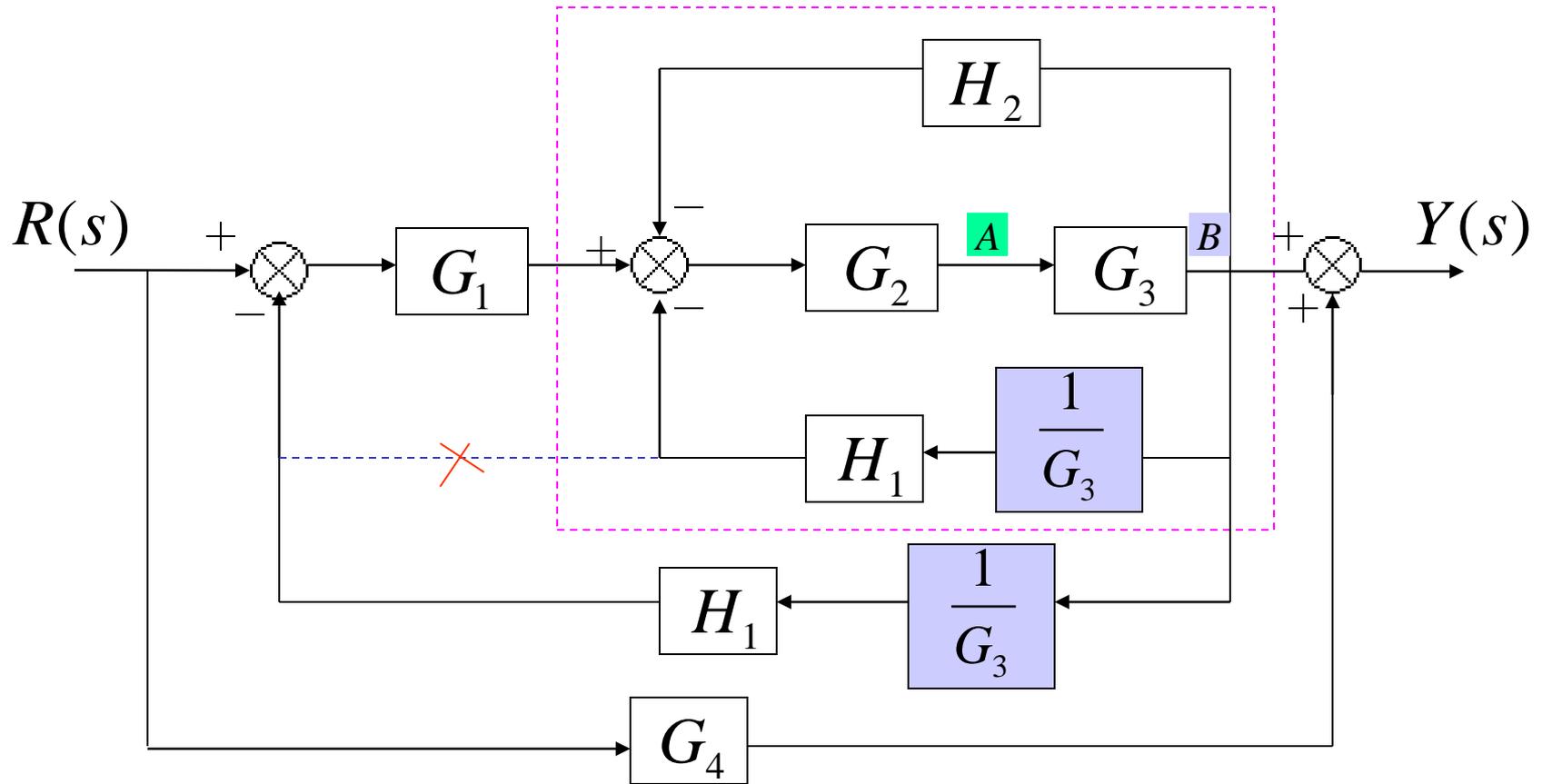
2. Eliminate loop I & Simplify

3. Eliminate loop II

$R(s)$ → $$\frac{G_1 G_2 G_3}{1 + G_2 H_1 + G_2 G_3 H_2 + G_1 G_2 H_1}$$ → $+$ $\otimes$ → $Y(s)$

G_4 $+$
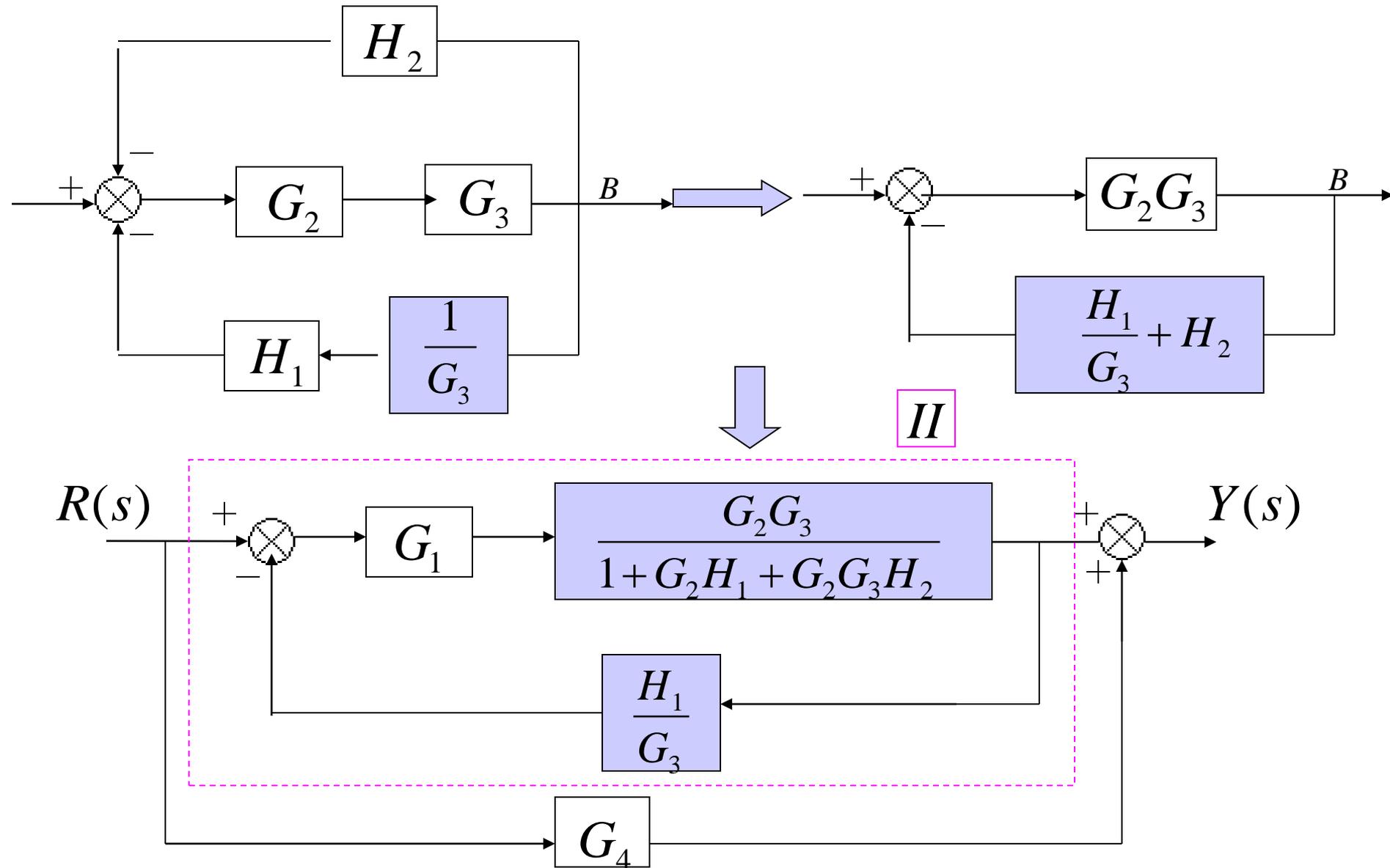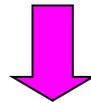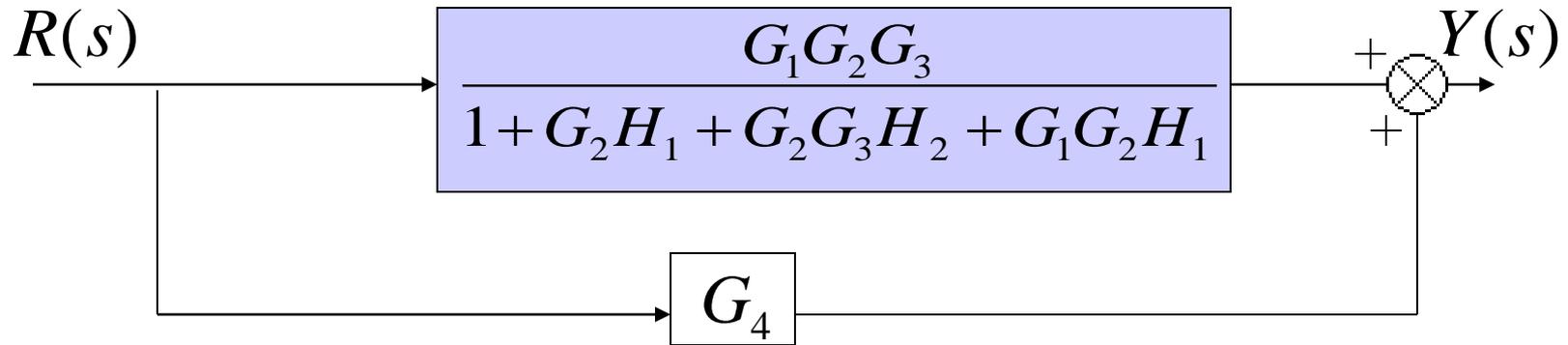
$$T(s) = \frac{Y(s)}{R(s)} = G_4 + \frac{G_1 G_2 G_3}{1 + G_2 H_1 + G_2 G_3 H_2 + G_1 G_2 H_1}$$

Example 2

Determine the effect of R and N on Y in the following diagram
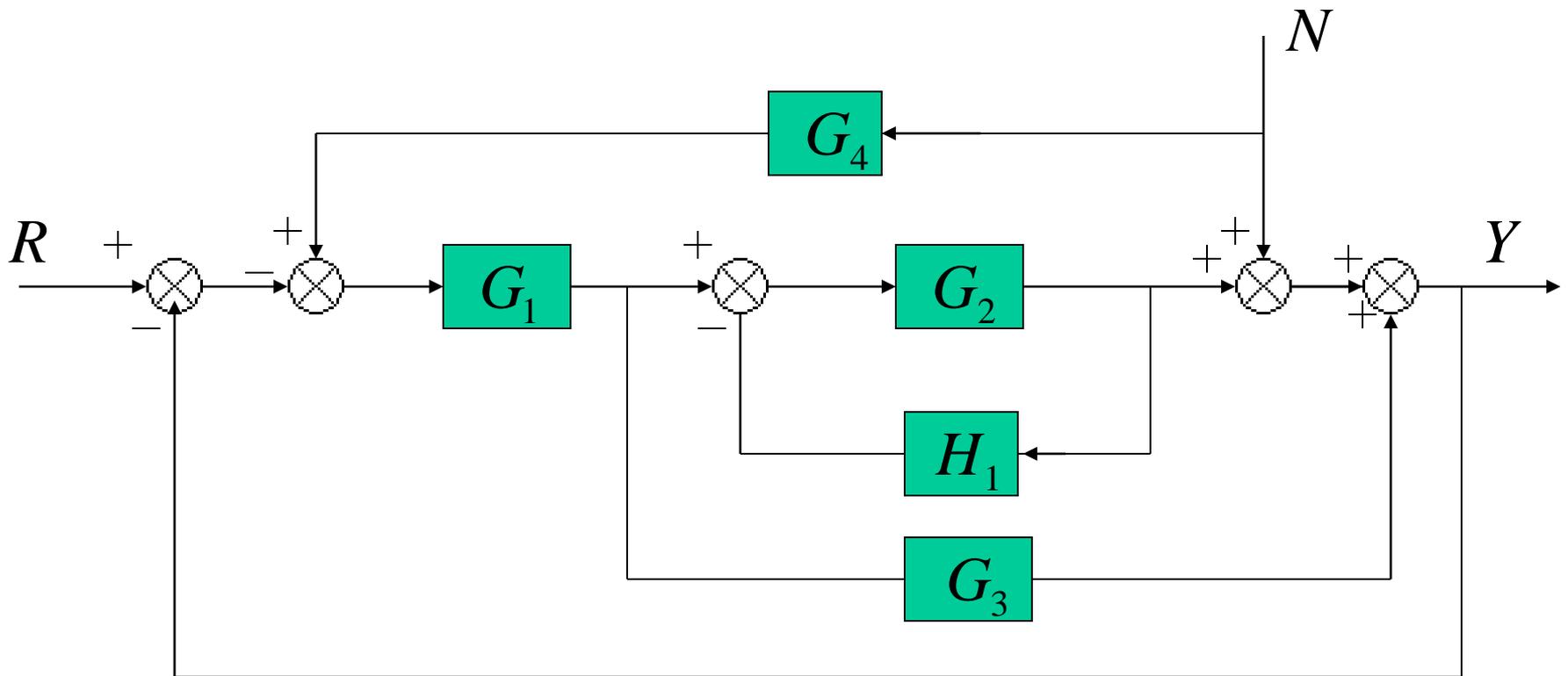
In this linear system, the output Y contains two parts, one part is related to R and the other is caused by N:

$$Y = Y_1 + Y_2 = T_1 R + T_2 N$$

If we set N=0, then we can get Y1:

$$Y_1 = Y_{N=0} = T_1 R$$

The same, we set R=0 and Y2 is also obtained:

$$Y_2 = Y_{R=0} = T_2 N$$

Thus, the output Y is given as follows:

$$Y = Y_1 + Y_2 = Y_{N=0} + Y_{R=0}$$

Solution:

1. Swap the summing points A and B
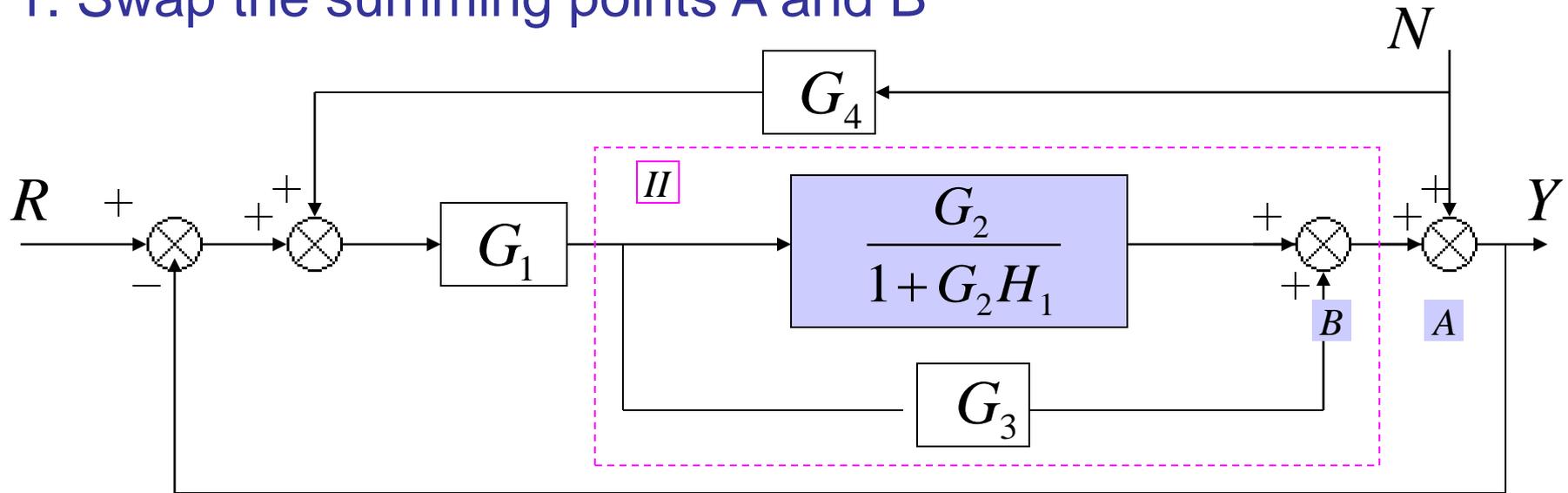
2. Eliminate loop II & simplify

Rewrite the diagram:

R + $+$ $+$ 0

$$G_1G_3 + \frac{G_1G_2}{1+G_2H_1}$$

$+$ 0 Y

3. Let N=0

R +

$$G_1G_3 + \frac{G_1G_2}{1+G_2H_1}$$

Y
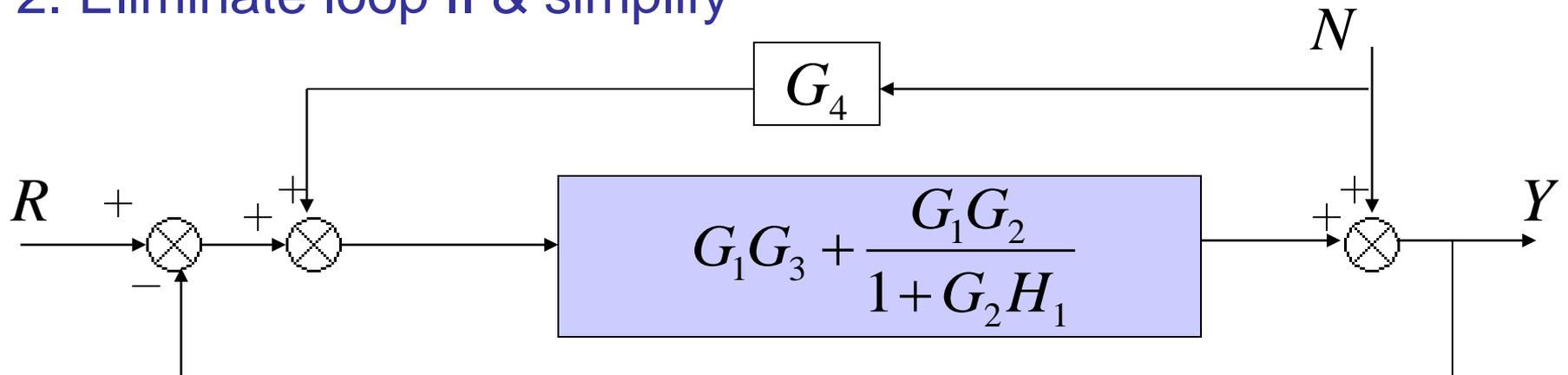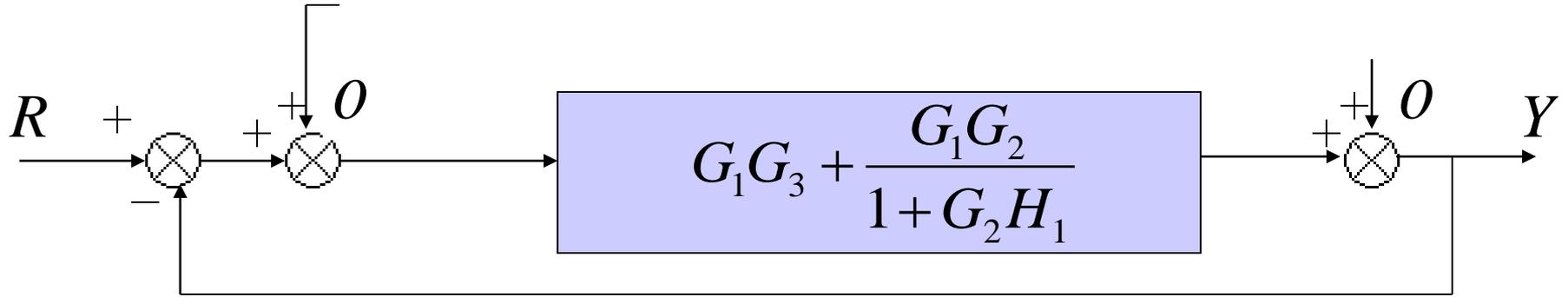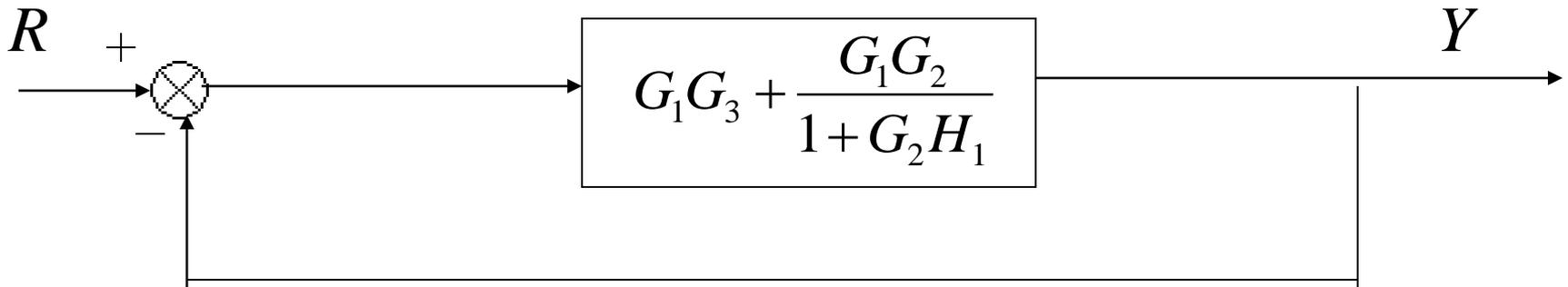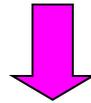
We can easily get Y_1

$$Y_1 = \frac{G_1G_2 + G_1G_3 + G_1G_2G_3H_1}{1+G_2H_1 + G_1G_2 + G_1G_3 + G_1G_2G_3H_1} R$$

4. Let R=0, we can get:


$$G_1G_3 + \frac{G_1G_2}{1+G_2H_1}$$

N G_4 M Y
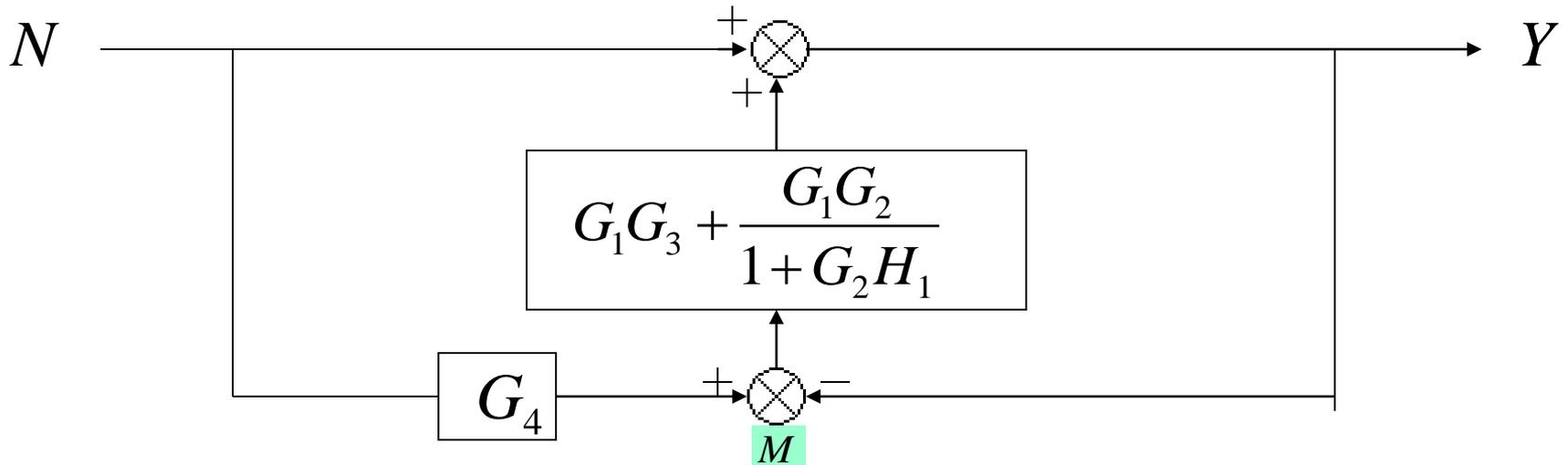
5. Break down the summing point M:

$$G_1G_3G_4 + \frac{G_1G_2G_4}{1+G_2H_1}$$

$$G_1G_3 + \frac{G_1G_2}{1+G_2H_1}$$

N Y

6. Eliminate above loops:
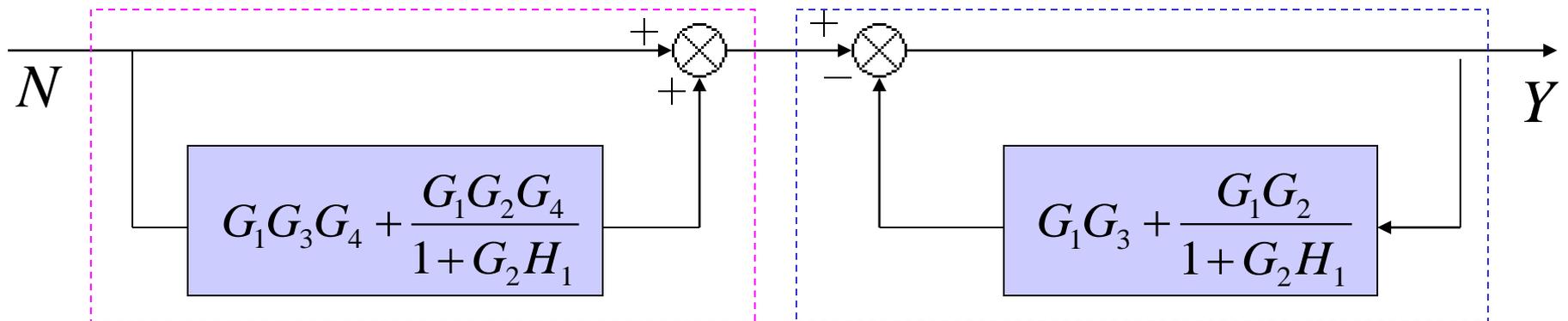
$$N \longrightarrow \boxed{1 + G_1 G_3 G_4 + \dfrac{G_1 G_2 G_4}{1 + G_2 H_1}} \longrightarrow \boxed{\dfrac{1}{1 + G_1 G_3 + \dfrac{G_1 G_2}{1 + G_2 H_1}}} \longrightarrow Y$$
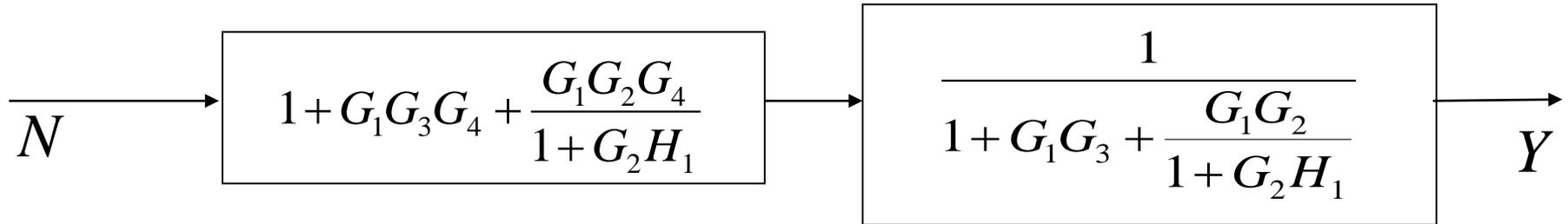
$$Y_2 = \frac{1 + G_2 H_1 + G_1 G_2 G_4 + G_1 G_3 G_4 + G_1 G_2 G_3 G_4 H_1}{1 + G_2 H_1 + G_1 G_2 + G_1 G_3 + G_1 G_2 G_3 H_1} N$$

7. According to the principle of superposition, Y_1 and Y_2 can be combined together, So:

$$Y = Y_1 + Y_2$$

$$= \frac{1}{1 + G_2 H_1 + G_1 G_2 + G_1 G_3 + G_1 G_2 G_3 H_1} [(G_1 G_2 + G_1 G_3 + G_1 G_2 G_3 H_1)R$$

$$+ (1 + G_2 H_1 + G_1 G_2 G_4 + G_1 G_3 G_4 + G_1 G_2 G_3 G_4 H_1)N]$$