- Sensitivity

System sensitivity

System sensitivity is the ratio of the change in the system transfer function to the change of a process transfer function (or parameter) for a small incremental change.

3.2 Sensitivity of system to parameter variations

System are time-varying in its nature because of inevitable uncertainties such as changing environment , aging , and other factors that affect a control process.All these uncertainties in open-loop system will result in inaccurate output or low performance. However, a closed-loop system can overcome this disadvantage.

Continue

A primary advantage of a closed-loop feedback control system is its ability to reduce the system's sensitivity to parameter variation.

Sensitivity analysis → Robust control

Effect of parameter variations

If process $G(s)$ is change as $G(s)+\Delta G(s)$

- Open-loop system

$$\Delta Y(s)=\Delta G(s)R(s)$$

- Closed-loop system

$$\Delta Y(s)=\frac{\Delta G(s)}{(1+GH)(1+GH+\Delta GH)}R(s)$$

$$=\frac{\Delta G(s)}{(1+GH)^2}R(s)$$

continue

In the limit, for small incremental changes, last formula is

$$S = \frac{\partial T(s)/T(s)}{\partial G(s)/G(s)} = \frac{\partial InT}{\partial InG}$$

SENSITIVITY

- Measure of the effectiveness of feedback in reducing the influence of the variations (changing environment) on system performance.
- It gives an assessment of the system performance as affected due to parameter variation.

EFFECT OF TRANSFER FUNCTION PARAMETER VARIATIONS IN AN OPEN LOOP CONTROL SYSTEM

$$M(s) = \frac{C(s)}{R(S)} = G(S) \rightarrow (1)$$

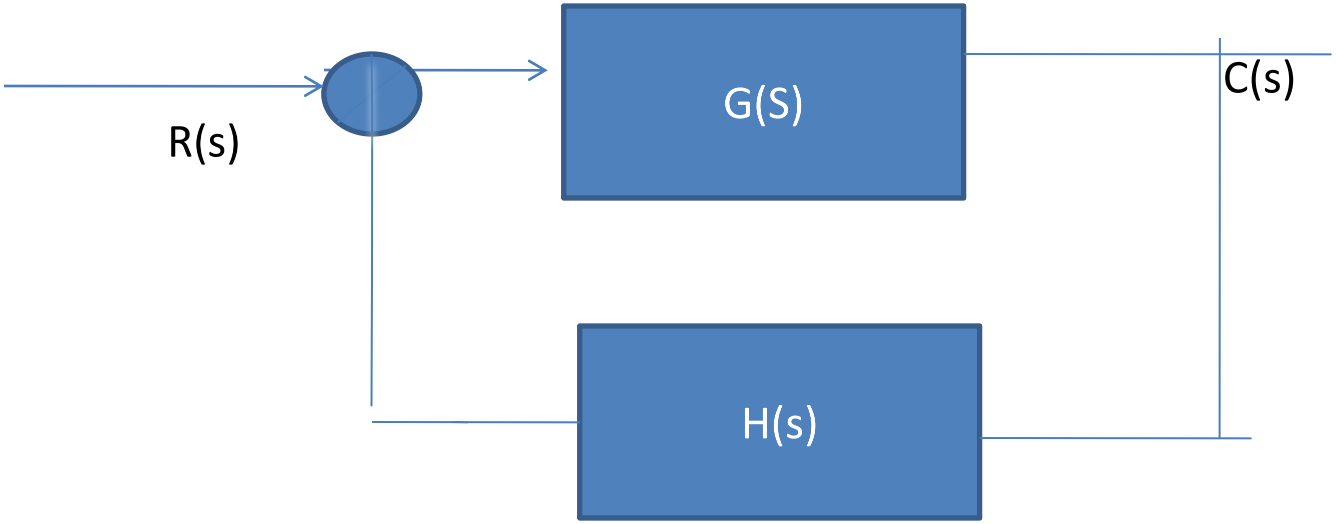
$$C(S) + \Delta \mathrm{C(S)} = [\mathrm{G(S)} + \Delta \mathrm{G(S)}]\, \mathrm{R(S)}$$

$$C(S) + \Delta \mathrm{C(S)} = \mathrm{G(S)R(S)} + \Delta \mathrm{G(S)R(S)} \qquad (2)$$

$$PUT\ EQN.1\, \mathrm{EQN.2}$$

$$\Delta \mathrm{C(S)} = \Delta \mathrm{G(S)R(S)}$$

EFFECT OF TRANSFER FUNCTION PARAMETER VARIATIONS IN AN CLOSED LOOP CONTROL SYSTEM



$$M(s) = \frac{C(s)}{R(S)} = \frac{G(s)}{1+G(S)H(S)} \rightarrow (1)$$

$$C(S) + \Delta \mathrm{C(S)} = \frac{[\mathrm{G(S)} + \Delta \mathrm{G(S)}]}{1+[G(S)H(S) + \Delta \mathrm{G(S)H(S)}]} \mathrm{R(S)}$$

$$= \frac{[\mathrm{G(S)R(S)}\ ]}{1+[G(S)H(S) + \Delta \mathrm{G(S)H(S)}]} + \frac{\Delta \mathrm{G(S)R(S)}}{1+[G(S)H(S) + \Delta \mathrm{G(S)H(S)}}$$

$$NEGLECT\ \Delta \mathrm{G(S)\ AS}\, \Delta \mathrm{G(S)} << \mathrm{G(S)}$$

$$\therefore \Delta \mathrm{G(S)H(S)\ CAN\ BE\ NEGLECTED}$$

$$= \frac{[\mathrm{G(S)R(S)}\ ]}{1+[G(S)H(S)]} + \frac{\Delta \mathrm{G(S)R(S)}}{1+[G(S)H(S) + \Delta \mathrm{G(S)H(S)}}$$

$$PUT\ EQN.1\, \mathrm{EQN.3}$$

$$\Delta \mathrm{C(S)} = \frac{\Delta \mathrm{G(S)}}{1+G(S)H(S)} \mathrm{R(S)}$$

-IT IS CONCLUDED THAT DUE TO FEEDBACK THE VARIATION IN O/P CAUSED BY THE CHANGE IN THE FWD PATH TRANSFER FUNCTION IS REDUCED BY A FACTOR OF

1+ G(s)H(s) IN A CLOSED LOOP .

-O/P VARIATIONS MORE SENSITIVE IN OPEN LOOP SYSTEM

$K = PARAMETER\ VARIATION$ OF ELEMENT SUCH AS GAIN OR FEEDBACK

A = VARIABLE IN CONTROL SYSTEM WHICH CHANGES ITS VALUE, O/P

$$SENSITIVITY = \frac{\%CHANGE\ IN\ A}{\%CHANGE\ IN\ K}$$

$$\mathbf{S}_K^A = \frac{\partial A / A}{\partial K / K}$$

SENSITIVITY SHOULD BE KEPT MINIMUM

SENSITIVITY OF OVERALL TRANSFER FUNCTION M(s) W.R.T. FWD PATH T.F. G(s)

$$S_G^M = \frac{\partial M(S)/M(s)}{\partial G(S)/G(s)}$$

$$\text{OPEN LOOP CONTROLSYSTEM}$$

$$M(S) = \frac{C(S)}{R(S)} = G(s)$$

$$\frac{M(S)}{G(S)} = 1$$

$$DIFFERENTIATING\ M(s)\ \text{W.R.T. G(s)}$$

$$S_G^M = \frac{G(S)}{M(S)} \cdot \frac{\partial M(S)}{\partial G(S)} = 1$$

SENSITIVITY OF OVERALL TRANSFER FUNCTION M(s) W.R.T. FWD PATH T.F. G(s)

CLOSED LOOP CONTROL SYSTEM

$$M(S) = \frac{C(S)}{R(S)} = \frac{G(s)}{1 + G(s)H(s)}$$

DIFFERENTIATING $M(s)$ W.R.T. G(s)

$$\frac{\partial M(S)}{\partial G(S)} =$$

$$S_G^M = \frac{G(S)}{M(S)} \cdot \frac{\partial M(S)}{\partial G(S)} = \frac{1}{1 + G(S)H(S)}$$

SENSITIVITY OF OVERALL TRANSFER FUNCTION W.R.T. FWD PATH T.F. IN CASE OF CLOSED LOOP SYSTEM IS REDUCED BY 1+G(S)H(S) AS COMPARED TO OPEN LOOP SYSTEM

$$M(S) = \frac{G(S)}{1+G(s)H(s)}$$

$DIFFERENTIATING$ W.R.T. G(S)

Example of sensitivity

- Feedback amplifier

- Goal: Reduce the sensitivity to parameters variation, that is enhance the robustness to change in amplifier gain.