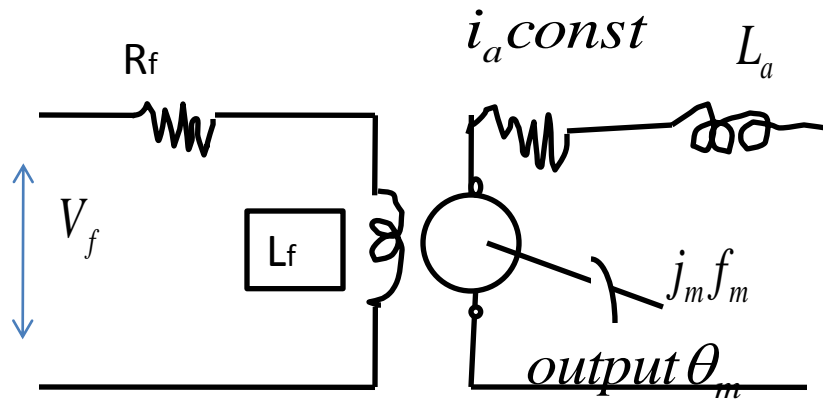


SERVO MOTOR

FIELD CONTROLLED D.C.MOTOR



J_m moment of inertia
 F_m coefficient of friction
 θ_m angular shift
 ω_m angular velocity
 T_m motor torque
 K_f is motor torque constant

Field controlled d.c. motor

$$T_M \propto i_f$$

$$T_M = K_f i_f$$

$$V_f = R_f i_f + L_f \frac{di_f}{dt}$$

$$T_M = K_f I_f$$

$$T_M = J_m \frac{d^2 \theta_m}{dt^2} + f_m \frac{d \theta_m}{dt}$$

$$T_M = J_m \frac{d\omega_m}{dt} + f_m \omega_m$$

$$V_f(s) = R_f I_f(s) + sL_f I_f(s)$$

$$T_M = K_f I_f(s)$$

$$T_M = s^2 J_m \theta_m(s) + s f_m \theta_m(s)$$

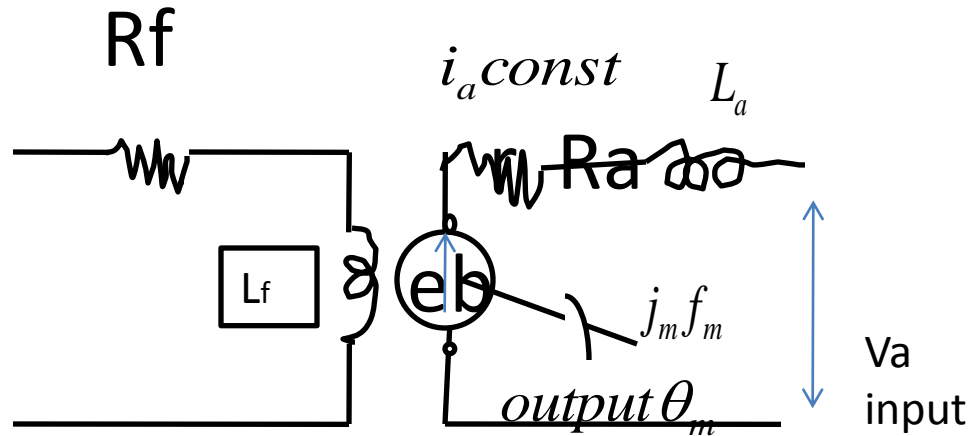
$$T_M = s J_m \omega_m(s) + f_m \omega_m(s)$$

$$\frac{\theta_m(s)}{V_f(s)} = \frac{K_f}{s(R_f + sL_f)(sJ_m + f_m)}$$

$$\frac{\omega_m(s)}{V_f(s)} = \frac{K_f}{(R_f + sL_f)(sJ_m + f_m)}$$

SERVO MOTOR

Armature CONTROLLED D.C.MOTOR



Armature controlled DC motor

$$T_M \propto i_a$$

$$T_M = K_T i_a$$

K_T is the motor torque

$$e_b \propto \omega_m \because \omega_m = \frac{d\theta}{dt}$$

$$e_b = K_b \frac{d\theta_m}{dt}$$

K_b is back emf

$$V_a - e_b = R_a i_a + L_a \frac{di_a}{dt}$$

$$V_a(s) - E_b(s) = R_a I_a(s) + sL_a I_a(s)$$

$$e_b = K_b \frac{d\theta_m}{dt}$$

$$E_b(s) = sK_b \theta_m$$

$$e_b = K_b \omega_m$$

$$E_b(s) = K_b \omega_m(s)$$

$$T_M = K_T i_a$$

$$T_M(s) = K_T I_a(s)$$

$$T_M = J_m \frac{d^2\theta_m}{dt^2} + f_m \frac{d\theta_m}{dt} \quad T_M(s) = s^2 J_m \theta_m(s) + s f_m \theta_m(s)$$

$$T_M(s) = sJ_m \omega_m(s) + f_m \omega_m(s)$$

$$T_M = J_m \frac{d\omega_m}{dt} + f_m \omega_m$$

$$G(s) = \frac{K_T}{s(R_a + sL_a)(sJ_m + f_m)}$$

$$\frac{\theta_m(s)}{V_a(s)} = \frac{\frac{K_T}{s(R_a + sL_a)(sJ_m + f_m)}}{1 + \frac{K_T}{s(R_a + sL_a)(sJ_m + f_m)} sK_b}$$

$$\frac{\theta_m(s)}{V_a(s)} = \frac{K_T}{s(R_a + sL_a)(sJ_m + f_m) + sK_b K_T}$$

If the armature inductance is neglected

$$\frac{\theta_m(s)}{V_a(s)} = \frac{K_T}{sR_a(sJ_m + f_m) + sK_bK_T}$$

$$\frac{\theta_m(s)}{V_a(s)} = \frac{K_T}{s(sR_aJ_m + f_mR_a + K_bK_T)}$$

$$\frac{\theta_m(s)}{V_a(s)} = \frac{\left[\frac{K_T}{(f_mR_a + K_bK_T)} \right]}{s \left[\frac{sR_aJ_m}{f_mR_a + K_bK_T} + 1 \right]}$$

$$\frac{\theta_m(s)}{V_a(s)} = \frac{K_m}{s(1 + sT_m)}$$

$$\text{where } K_m = \frac{K_T}{(f_mR_a + K_bK_T)}$$

$$T_m = \frac{R_aJ_m}{(f_mR_a + K_bK_T)}$$

$$\omega_m(s) = s \theta_m(s)$$

$$\frac{\omega_m(s)}{V_a(s)} = \frac{K_m}{(1 + sT_m)}$$

Relation between torque and back e.m.f constant K_b

$$T_M \omega_m = e_b \dot{i}_a$$

$$T_M = K_T \dot{i}_a$$

$$e_b = K_b \omega_m$$

$$K_T \dot{i}_a \omega_m = K_b \omega_m \dot{i}_a$$

$$K_T = K_b$$