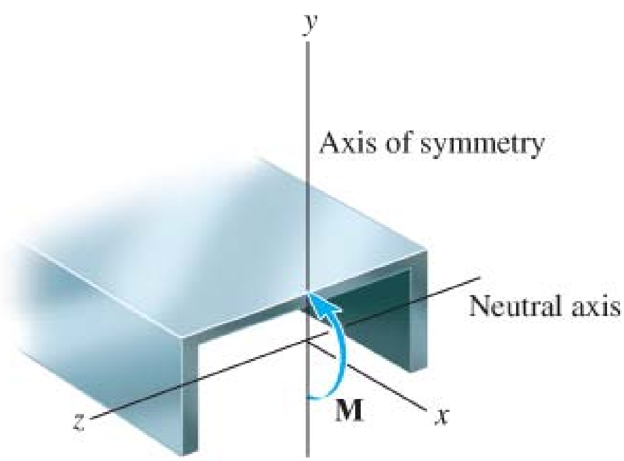
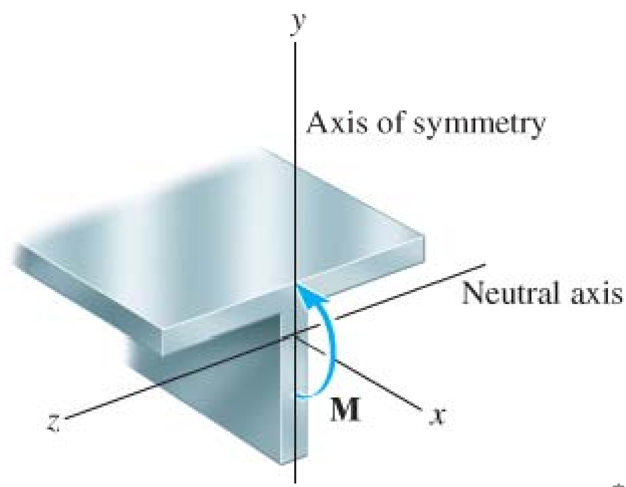
UNSYMMETRICAL BENDING

- A condition for flexure formula is the symmetric x-sectional area of beam about an axis perpendicular to neutral axis
- However, the flexure formula can also be applied either to a beam having x-sectional area of any shape OR to a beam having a resultant moment that acts in any direction





Moment applied along principal axis

- Consider a beam with unsymmetrical shape
- Establish coordinate system as per usual and that resultant moment **M** acts along +*z* axis
- Conditions:

1. Stress distribution acting over entire x-sectional area to be a zero force resultant,
2. Resultant internal moment about *y* axis to be zero
3. Resultant internal moment about *z* axis to be equal to **M**

Moment applied along principal axis

- Express the 3 conditions mathematically by considering force acting on differential element *dA* located at (0, *y*, *z*). Force is $dF = \sigma \, dA$, therefore

$F_R = F_y;$ $$0 = \int_A \sigma \, dA$$

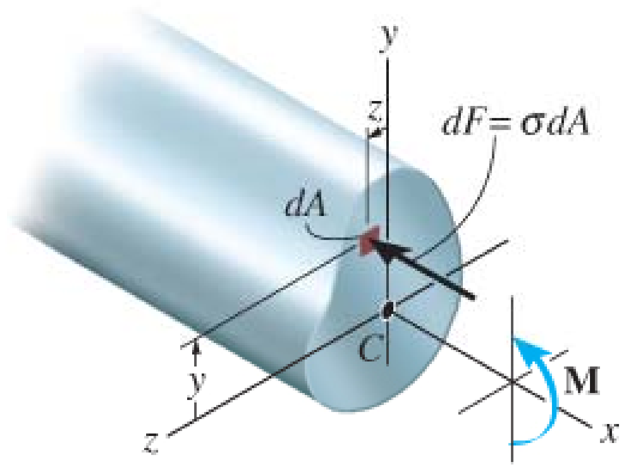
Equation 6-14

$(M_R)_y = \Sigma M_y;$ $$0 = \int_A z \, \sigma \, dA$$

Equation 6-15

$(M_R)_z = \Sigma M_z;$ $$0 = \int_A -y \, \sigma \, dA$$

Equation 6-16

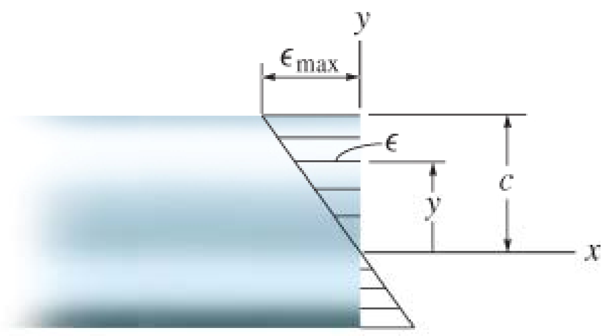


(a)

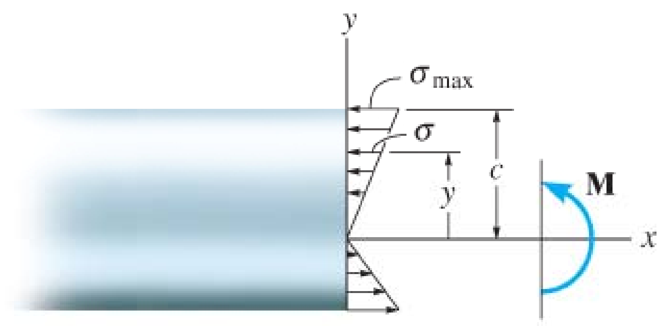
Moment applied along principal axis

- Eqn 6.14 is satisfied since *z* axis passes through *centroid* of x-sectional area
- If material has linear-elastic behavior, then we can substitute $\sigma = -(y/c)\sigma_{max}$ into Eqn 6-16 and after integration, we get

$$\int_A yz \, dA = 0$$



Normal strain distribution
(profile view)
(b)



Bending stress distribution
(profile view)
(c)

Moment applied along principal axis

$$\int_A yz \, dA = 0$$

- This integral is the product of inertia for the area. It will be zero if y and z axes are chosen as principal axes of inertia for the area.
- Thus, Eqns 6-14 to 6-16 will always be satisfied regardless of the direction of applied moment **M**

Moment arbitrarily applied

- If a member is loaded such that resultant internal moment does not act about one of the principal axes of x-section, resolve the moment into components directed along the principal axes
- Use flexure formula to determine normal stress caused by each moment component
- Use principle of superposition to determine resultant normal stress at the pt

Moment arbitrarily applied

- Resultant general normal stress at any pt on x-section is

$$\sigma = \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

Equation 6-17

σ = normal stress at the pt

y, z = coordinates of pt measured from x, y, z axes having origin at centroid of x-sectional area and forming a right-handed coordinate system

Moment arbitrarily applied

- Resultant general normal stress at any pt on x-section is

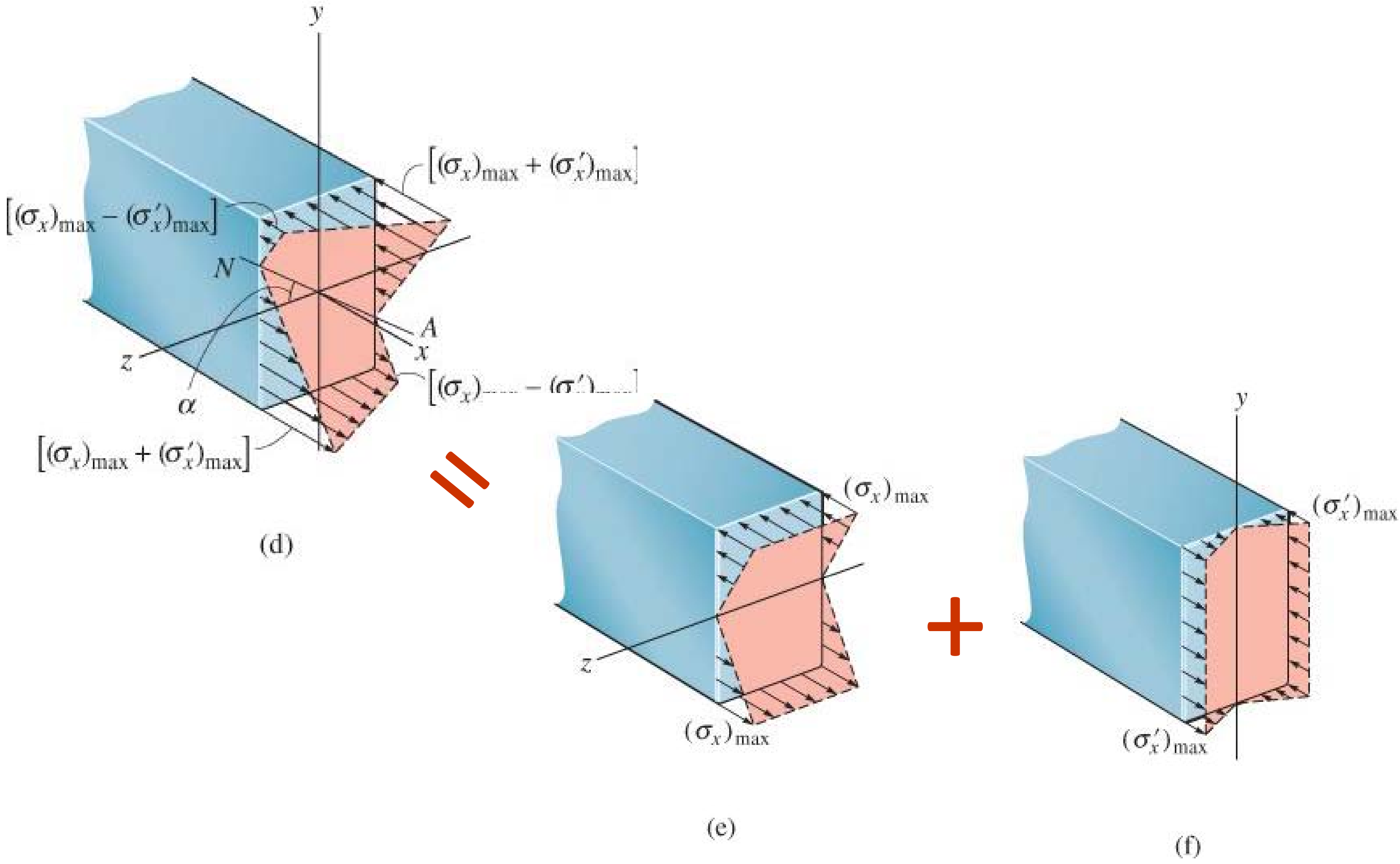
$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

Equation 6-17

M_y, M_z = resultant internal moment components along principal y and z axes. Positive if directed along $+y$ and $+z$ axes. Can also be stated as $M_y = M \sin \theta$ and $M_z = M \cos \theta$, where θ is measured positive from $+z$ axis toward $+y$ axis

I_y, I_z = principal moments of inertia computed about the y and z axes, respectively

Moment arbitrarily applied



Orientation of neutral axis

- Angle α of neutral axis can be determined by applying Eqn 6-17 with $\sigma = 0$, since no normal stress acts on neutral axis. Finally, we get

$$\tan \alpha = \frac{I_z}{I_y} \tan \theta$$

Equation 6-19

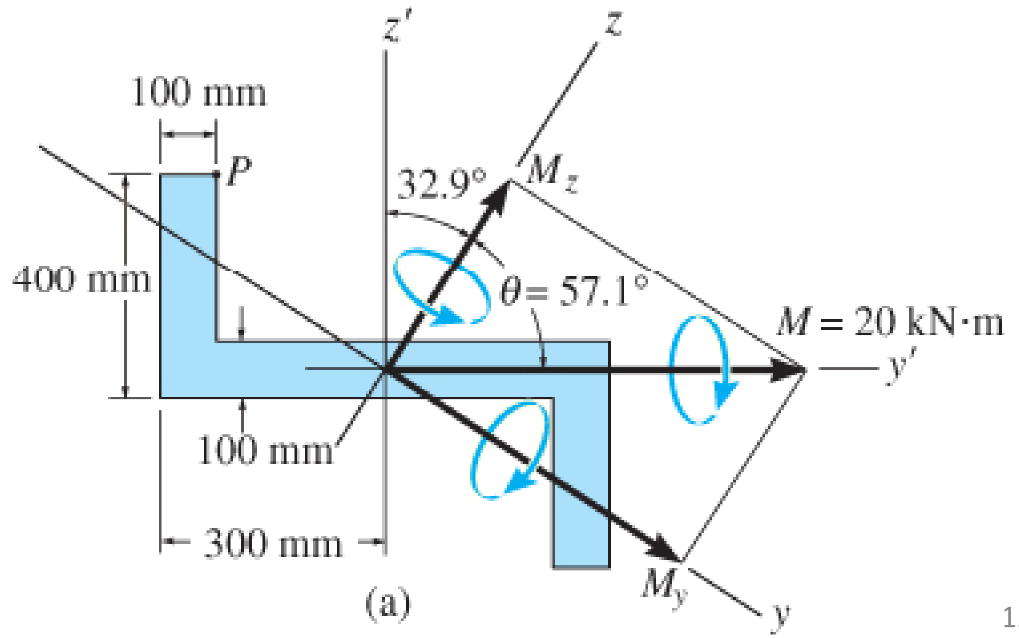
- For unsymmetrical bending, angle θ defining direction of moment **M** is not equal to angle α, angle defining inclination of neutral axis *unless* $I_z = I_y$.
- *Thus,* $\theta \leq \alpha \leq 90^{\circ}$

IMPORTANT

- Flexure formula applied only when bending occurs about axes that represent the principal axes of inertia for x-section
- These axes have their origin at centroid and are orientated along an axis of symmetry and perpendicular to it
- If moment applied about arbitrary axis, then resolve moment into components along each of the principal axes, and stress at a pt is determined by superposition of stress caused by each moment component.

Z-section shown is subjected to bending moment of M = 20 kN·m. Using methods from Appendix A, the principal axes y and z are oriented as shown such that they represent the maximum and minimum principal moments of inertia, $I_y = 0.960(10^{-3})\ m^4$ and $I_z = 7.54(10^{-3})\ m^4$ respectively.

Determine normal stress at point P and orientation of neutral axis.

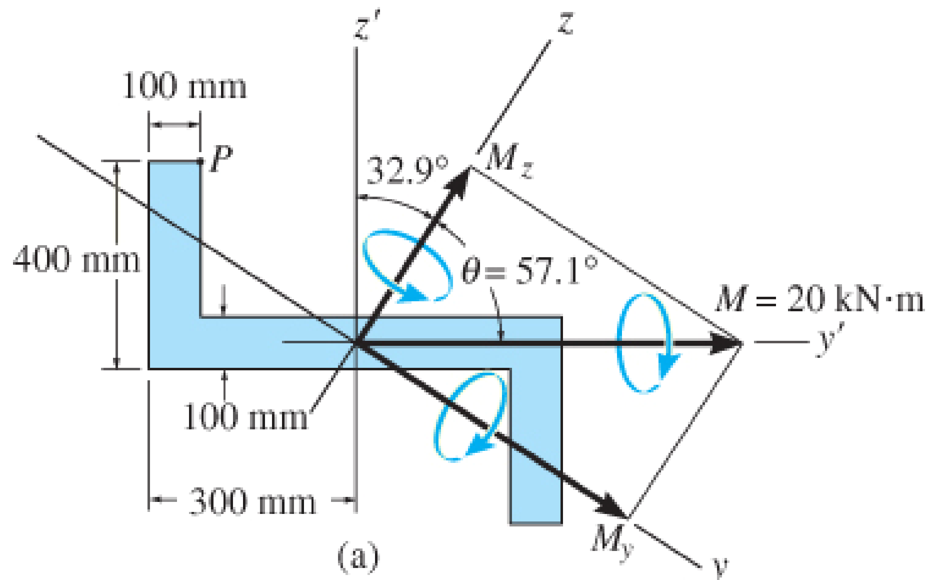


(a)

Internal moment components

To use Eqn 6-19, *z* axis needs to be principal axis for the maximum moment of inertia, as most of the area if located furthest away from this axis

$M_y = 20\ \text{kN·m}\ \sin 57.1^{\circ} = 16.79\ \text{kN·m}$
$M_z = 20\ \text{kN·m}\ \cos 57.1^{\circ} = 10.86\ \text{kN·m}$

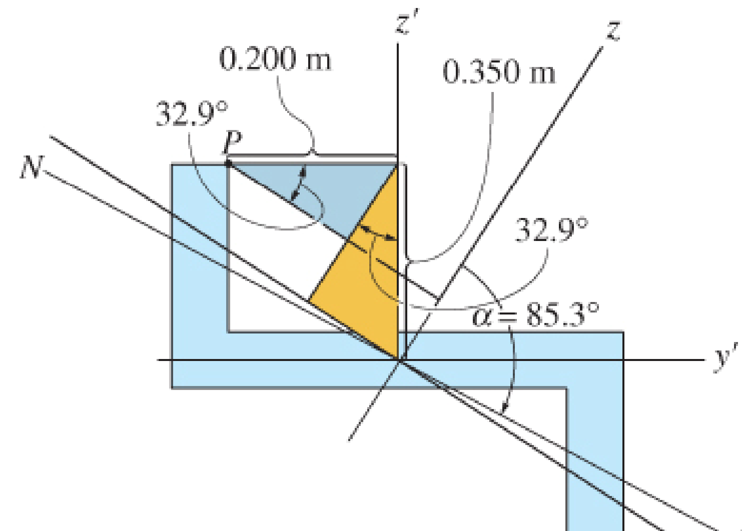


(a)

Bending stress

The *y* and *z* coordinates of *P* must be determined first. Note that *y′*, *z′* coordinates of *P* are (–0.2 m, 0.35 m). Using colored and shaded triangles from construction shown below,

$$y_P = -0.35 \sin 32.9^{\circ} - 0.2 \cos 32.9^{\circ} = 0.3580 \text{ m}$$
$$z_P = 0.35 \cos 32.9^{\circ} - 0.2 \sin 32.9^{\circ} = 0.1852 \text{ m}$$

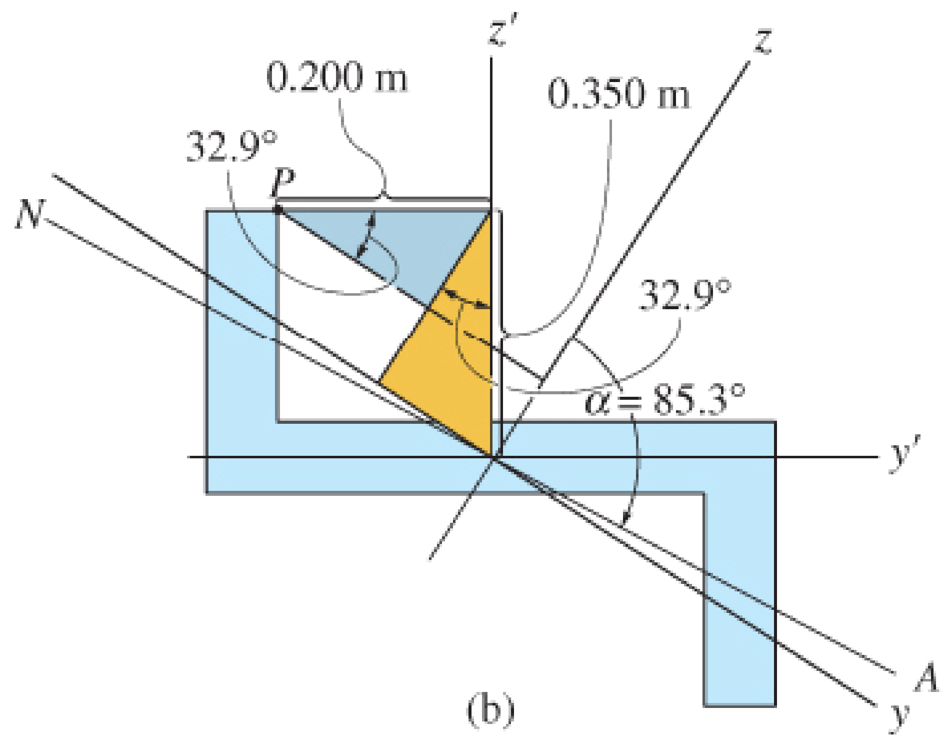


(b)

Bending stress

Applying Eqn 6-17, we have

$$\sigma = \frac{M_z y}{I_z} + \frac{M_y z}{I_y} = \ldots = 3.76 \text{ MPa}$$



(b)

Orientation of neutral axis

Angle $=57.1^{o}$ is shown, Thus,

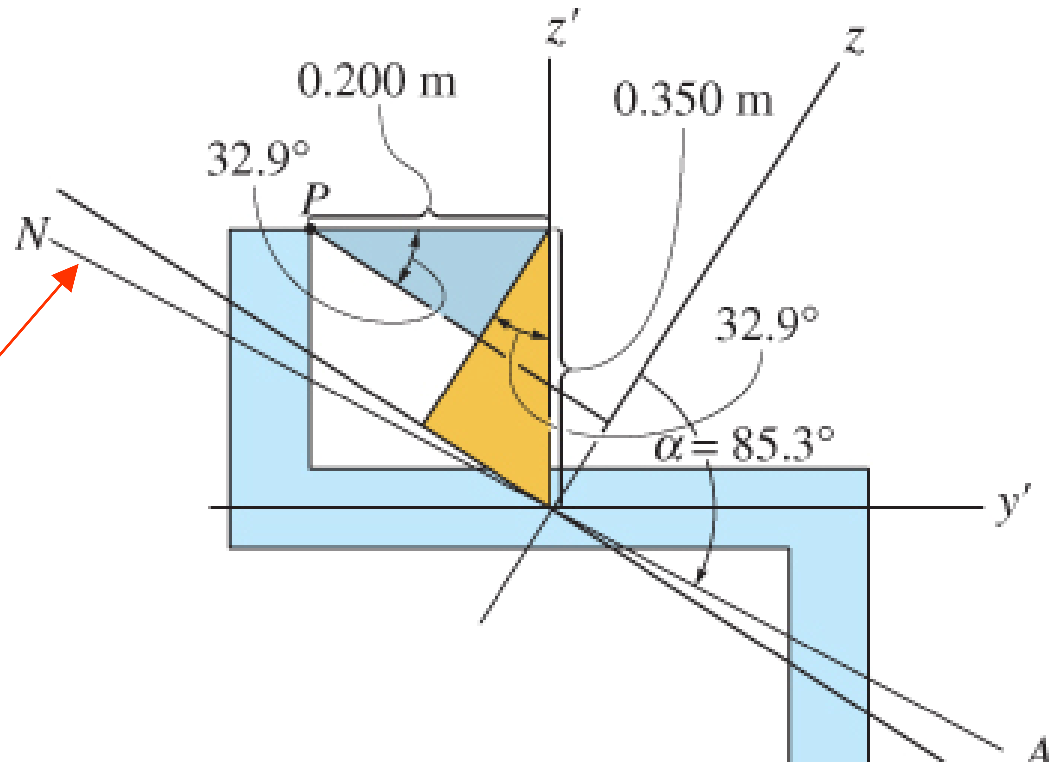
$$\tan \alpha = \underline{\tan \theta}$$

$$I_z$$

$$I_y$$

$$\alpha = 85.3^{o}$$

Neutral Axis



(b)