UNSYMMETRICAL BENDING

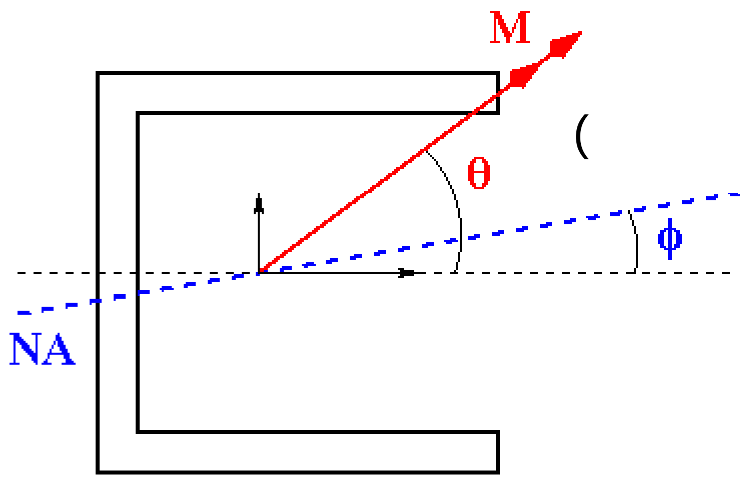
The general bending stress equation for **elastic, homogeneous** beams is given as

$$\sigma_z = -\frac{\left(M_y I_x - M_x I_{xy}\right)}{I_x I_y - I_{xy}^2}\, x - \frac{\left(M_x I_y - M_y I_{xy}\right)}{I_x I_y - I_{xy}^2}\, y \qquad \text{(II.1)}$$

where Mx and My are the bending moments about the x and y centroidal axes, respectively. Ix and Iy are the second moments of area (also known as moments of inertia) about the x and y axes, respectively, and Ixy is the product of inertia. Using this equation it would be possible to calculate the bending stress at any point on the beam cross section regardless of moment orientation or cross-sectional shape. Note that Mx, My, Ix, Iy, and Ixy are all unique for a given **section** along the length of the beam. In other words, they will not change from one point to another on the cross section. However, the x and y variables shown in the equation correspond to the coordinates of a point on the cross section at which the stress is to be determined.

Neutral Axis:

- When a homogeneous beam is subjected to elastic bending, the neutral axis (NA) will pass through the centroid of its cross section, but the orientation of the NA depends on the orientation of the moment vector and the cross sectional shape of the beam.
- When the loading is unsymmetrical (at an angle) as seen in the figure below, the NA will also be at some angle - **NOT** necessarily the same angle as the bending moment.



- Realizing that at any poin ... , we can use the general bending stress equation to find its orientation. Setting the stress to zero and solving for the slope y/x gives

$$\tan\phi = -\frac{M_y I_x - M_x I_{xy}}{M_x I_y - M_y I_{xy}}$$

SHEAR FLOW AND SHEAR CEN

Restrictions:

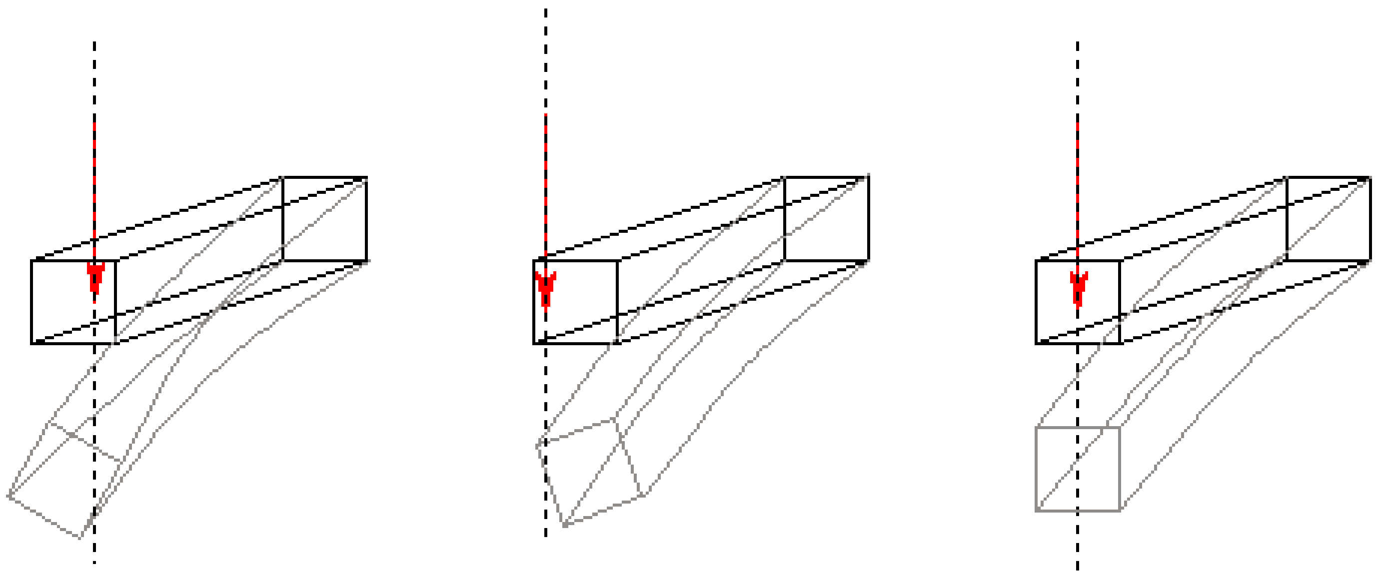
1. **Shear stress at every point in the beam must be less than the elastic limit of the material in shear.**
2. **Normal stress at every point in the beam must be less than the elastic limit of the material in tension and in compression.**
3. **Beam's cross section must contain at least one axis of symmetry.**
4. **The applied transverse (or lateral) force(s) at every point on the beam must pass through the elastic axis of the beam. Recall that elastic axis is a line connecting cross-sectional shear centers of the beam. Since shear center always falls on the cross-sectional axis of symmetry, to assure the previous statement is satisfied, at every point the transverse force is applied along the cross-sectional axis of symmetry.**
5. **The length of the beam must be much longer than its cross sectional dimensions.**
6. **The beam's cross section must be uniform along its length.**

Shear Center

If the line of action of the force passes through the **Shear Center** of the beam section, then the beam will only bend without any twist. Otherwise, twist will accompany bending.

The shear center is in fact the *centroid of the internal shear force system.* Depending on the beam's cross-sectional shape along its length, the location of shear center may vary from section to section. A line connecting all the shear centers is called the **elastic axis** of the beam. When a beam is under the action of a more general lateral load system, then to prevent the beam from twisting, the load must be centered along the elastic axis of the beam.

Shear Center

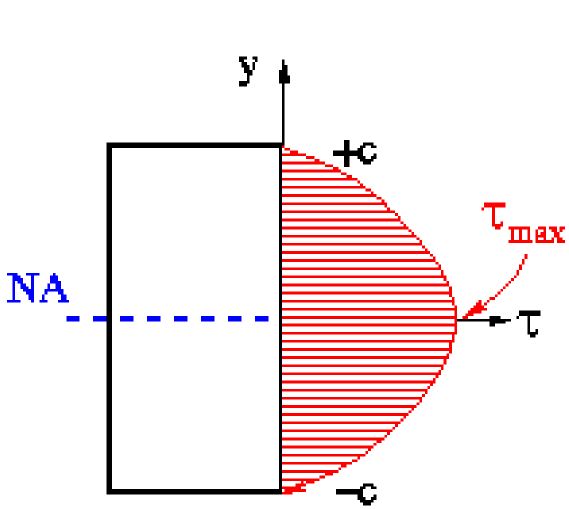


- The two following points facilitate the determination of the shear center location.

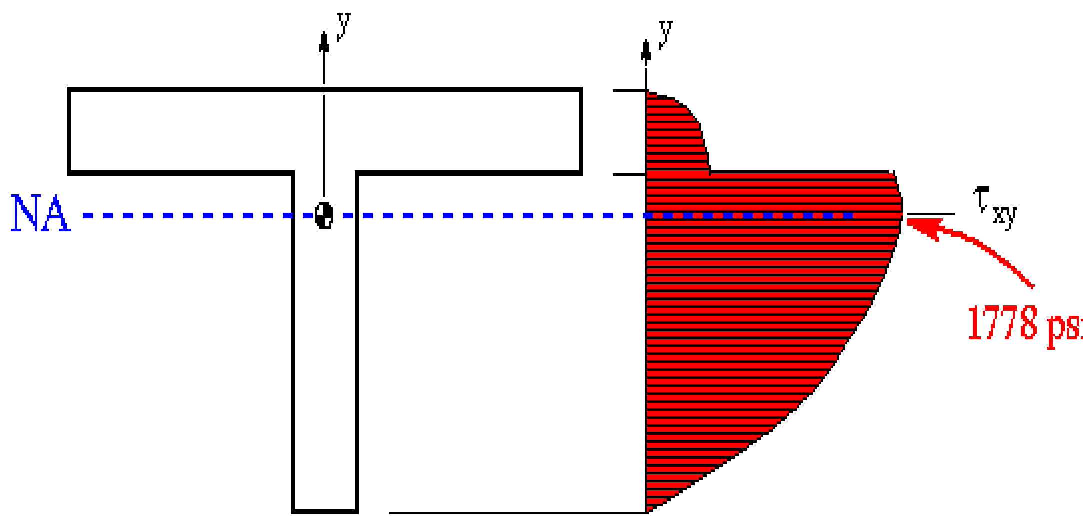
1. The shear center always falls on a cross-sectional axis of symmetry.
2. If the cross section contains two axes of symmetry, then the shear center is located at their intersection. Notice that this is the only case where shear center and centroid coincide.

SHEAR STRESS DISTRIBUTION

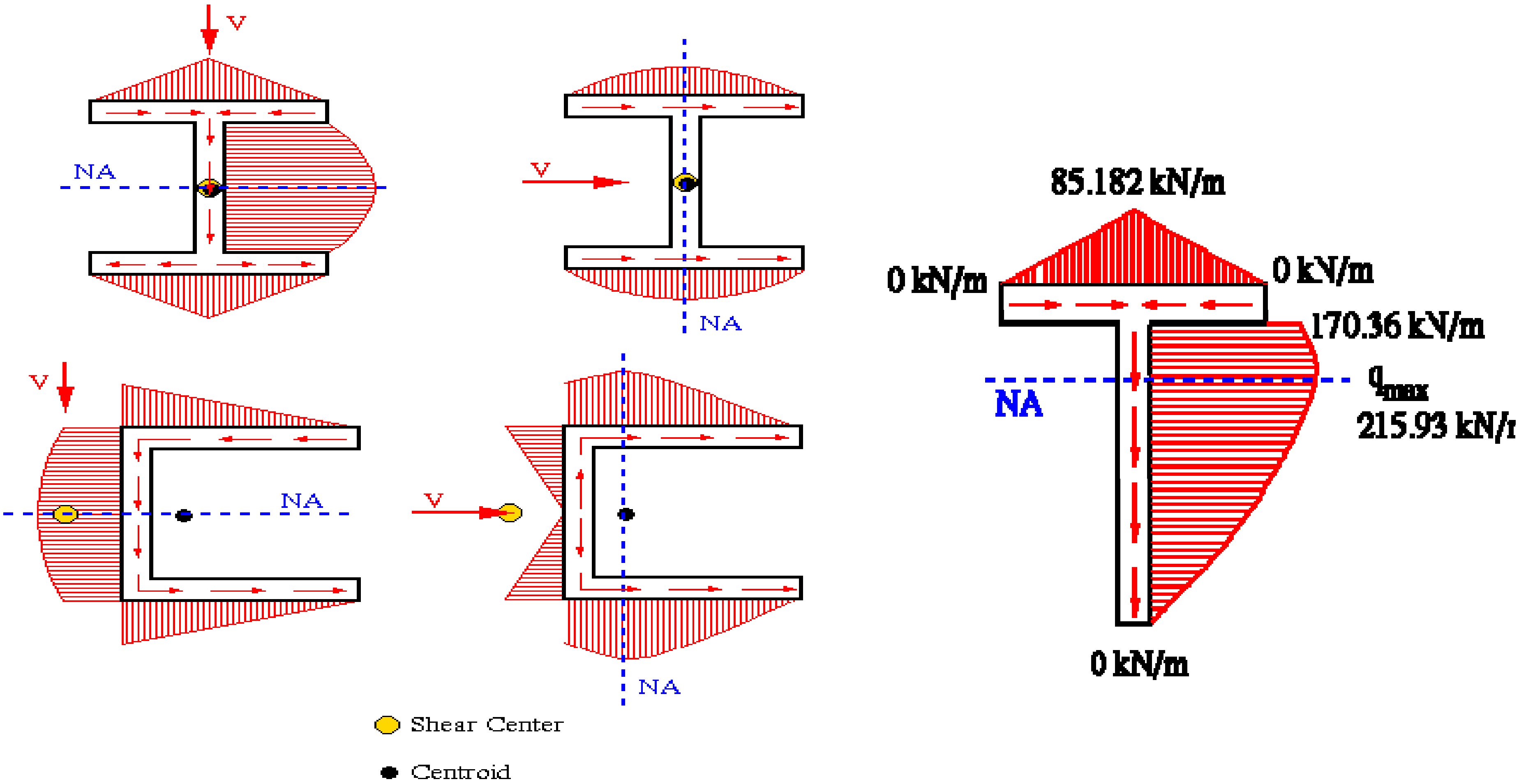
RECTANGLE



T-SECTION



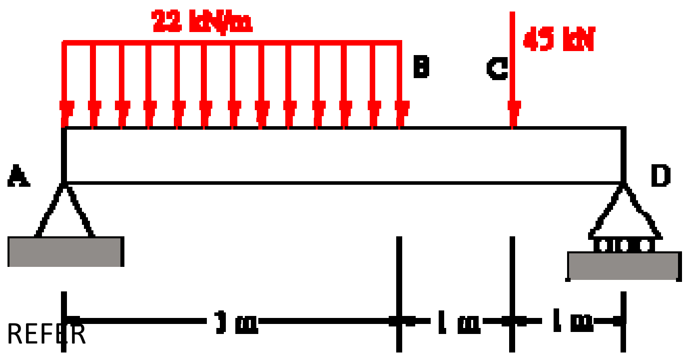
SHEAR FLOW DISTRIBUTION

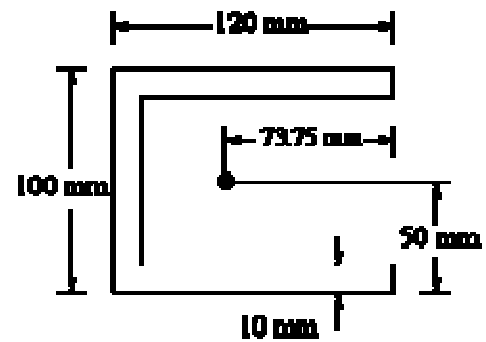


EXAMPLES

- For the beam and loading shown, determine:

(a) the location and magnitude of the maximum transverse shear force 'Vmax',
(b) the shear flow 'q' distribution due the 'Vmax',
(c) the 'x' coordinate of the shear center measured from the centroid,
(d) the maximun shear stress and its location on the cross section.
Stresses induced by the load do not exceed the elastic limits of the material.
NOTE: In this problem the applied transverse shear force passes through the centroid of the cross section, and not its shear center.





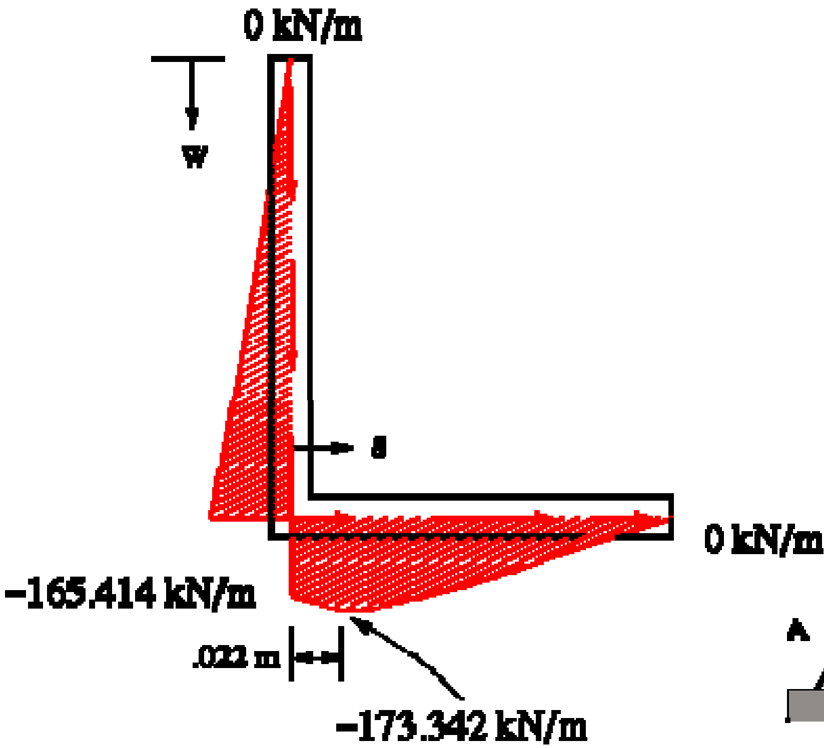
FOR ANSWER REFER
http://www.ae.msstate.edu/~masoud/Teaching/exp/A14.7_ex3.html

Shear Flow Analysis for Unsymmetric Beams

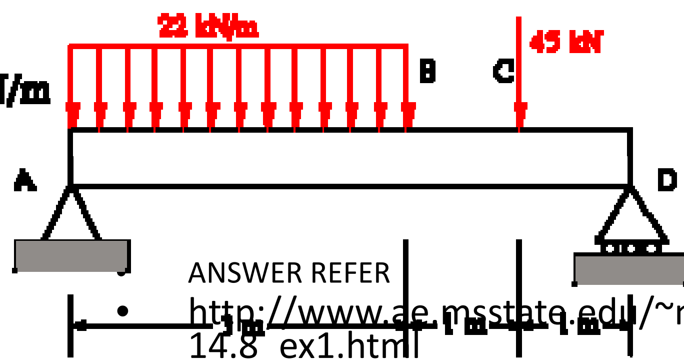
- SHEAR FOR EQUATION FOR UNSUMMETRIC SECTION IS

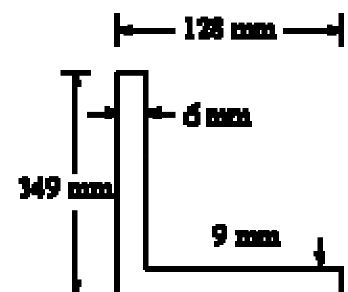
$$q = -\left(\frac{I_x V_x - I_{xy} V_y}{I_x I_y - I_{xy}^2}\right) \Sigma xA - \left(\frac{I_y V_y - I_{xy} V_x}{I_x I_y - I_{xy}^2}\right) \Sigma yA$$

SHEAR FLOW DISTRIBUTION



- For the beam and loading shown, determine:
- (a) the location and magnitude of the maximum transverse shear force,
- (b) the shear flow 'q' distribution due to 'Vmax',
- (c) the 'x' coordinate of the shear center measured from the centroid of the cross section.
- Stresses induced by the load do not exceed the elastic limits of the material. The transverse shear force is applied through the shear center at every section of the beam. Also, the length of each member is measured to the middle of the adjacent member.





- ANSWER REFER
- http://www.ae.msstate.edu/~masoud/Teaching/exp/A14.8_ex1.html

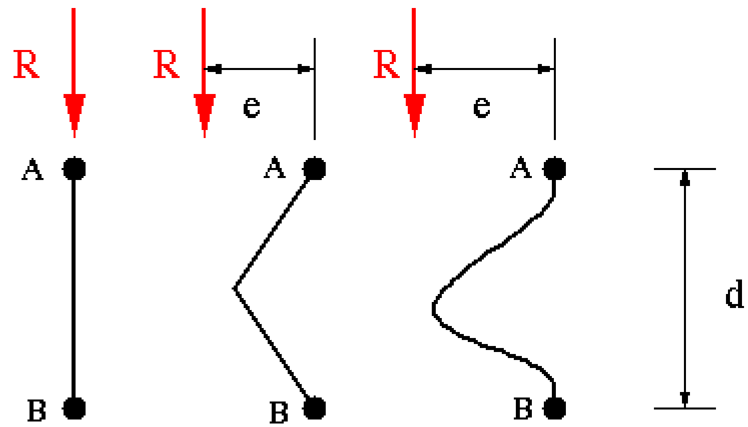
Beams with Constant Shear Flow Webs

Assumptions:

1. Calculations of **centroid, symmetry, moments of area and moments of inertia** are based totally on the **areas and distribution** of beam stiffeners.
2. A web does not change the shear flow between two adjacent stiffeners and as such would be in the state of constant shear flow.
3. The stiffeners carry the entire bending-induced normal stresses, while the web(s) carry the entire shear flow and corresponding shear stresses.

Analysis

- Let's begin with a simplest thin-walled stiffened beam. This means a beam with two stiffeners and a web. Such a beam can only support a transverse force that is parallel to a straight line drawn through the centroids of two stiffeners. Examples of such a beam are shown below. In these three beams, the value of shear flow would be equal although the webs have different shapes.



- The reason the shear flows are equal is that the distance b
'd' in all cases, and the applied force is shown to be equal t
can be determined by the following relationship

$$R = qd$$