Example: Water Resource in Central London

Getting initial solution: Norwest Corner rule

Vogel's approximation method Russel's approximation method

	1	2	3	4	5	supp	$\mathbf{u}_{\mathrm{i}}$
1	16	16	13	22	17	50	
2	14	14	13	19	15	60	
3	19	19	20	23	M	50	
4(D)	M	0	M	0	0	50	
dem	30	20	70	30	60		
$\mathbf{v}_{\mathrm{j}}$							

## The Transportation Simplex Method

Initialization: Construct an initial BFS by one of the procedure outlined in previous lecture.

Optimality Test: Derive  $u_i$  and  $v_j$  by selecting the row having the largest number of allocations, setting its  $u_i = 0$ , then solving the set of equations  $c_{ij} = u_i + v_j$  for each (i,j) such that  $x_{ij}$  is basic. If  $c_{ij} - u_i - v_j \ge 0$  for every (i,j) such that  $x_{ij}$  is nonbasic, then the current solution is optimal.

1

- 1. Determine the entering basic variable: Select the nonbasic variable  $x_{ij}$  having the largest (in absolute terms) negative value of  $c_{ij} u_i v_j$ .
- 2. Determine the leaving basic variable: Identify the chain reaction required to retain feasibility when the entering basic variable is increased. Fromt eh donor cells, select the basic variable having the *smallest* value.
- 3. Determine the new BFS: Add the value of the leaving basic variable to the allocation for each recipient cell. Subtract this value from the allocation for each donor cell.

For basic  $x_{ij}$ ,  $c_{ij} = u_i + v_j$ . Solve for  $u_i$  and  $v_j$ .

		1	2			3		4		5	supp	$\mathbf{u}_{\mathrm{i}}$
1	16		16		13	40	22		17	10	50	
2	14	30	14		13	30	19		15		60	
3	19	0	19	20	20		23	30	M		50	
4(D)	M		0		M		0		0	50	50	
dem	3	30	20	)	7	70	;	30	(	30		
$\mathbf{v}_{\mathrm{j}}$												

	1	2	3	4	5	supp	$\mathbf{u}_{\mathrm{i}}$
1	16	16	13	22	17	50	
2	14	14	13	19	15	60	
3	19	19	20	23	M	50	
4(D)	M	0	M	0	0	50	
dem	30	20	70	30	60		
$\mathbf{v}_{\mathrm{j}}$							

	1	2	3	4	5	supp	$\mathbf{u}_{\mathrm{i}}$
1	16	16	13	22	17	50	
2	14	14	13	19	15	60	
3	19	19	20	23	M	50	
4(D)	М	0	M	0	0	50	
dem	30 20		70	30	60		
$\mathbf{v}_{\mathrm{j}}$							

	1	2	3	4	5	supp	$\mathbf{u}_{\mathrm{i}}$
1	16	16	13	22	17	50	
2	14	14	13	19	15	60	
3	19	19	20	23	М	50	
4(D)	М	0	M	0	0	50	
dem	30 20		70	30	60		
$\mathbf{v}_{\mathrm{j}}$							