Combined Loadings & Thin-Walled vessels

This chapter serves as a review of the stress-analysis that has been developed in the previous chapters regarding axial load, torsion, bending and shear. The solution to problems where several of these loads occur simultaneously will be studied. Prior to this, the stresses in thin-walled vessels will be analyzed.

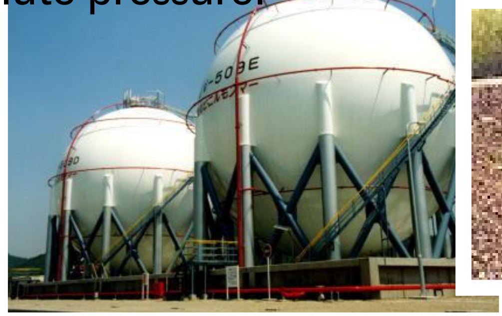
What is a thin walled pressure vessel?

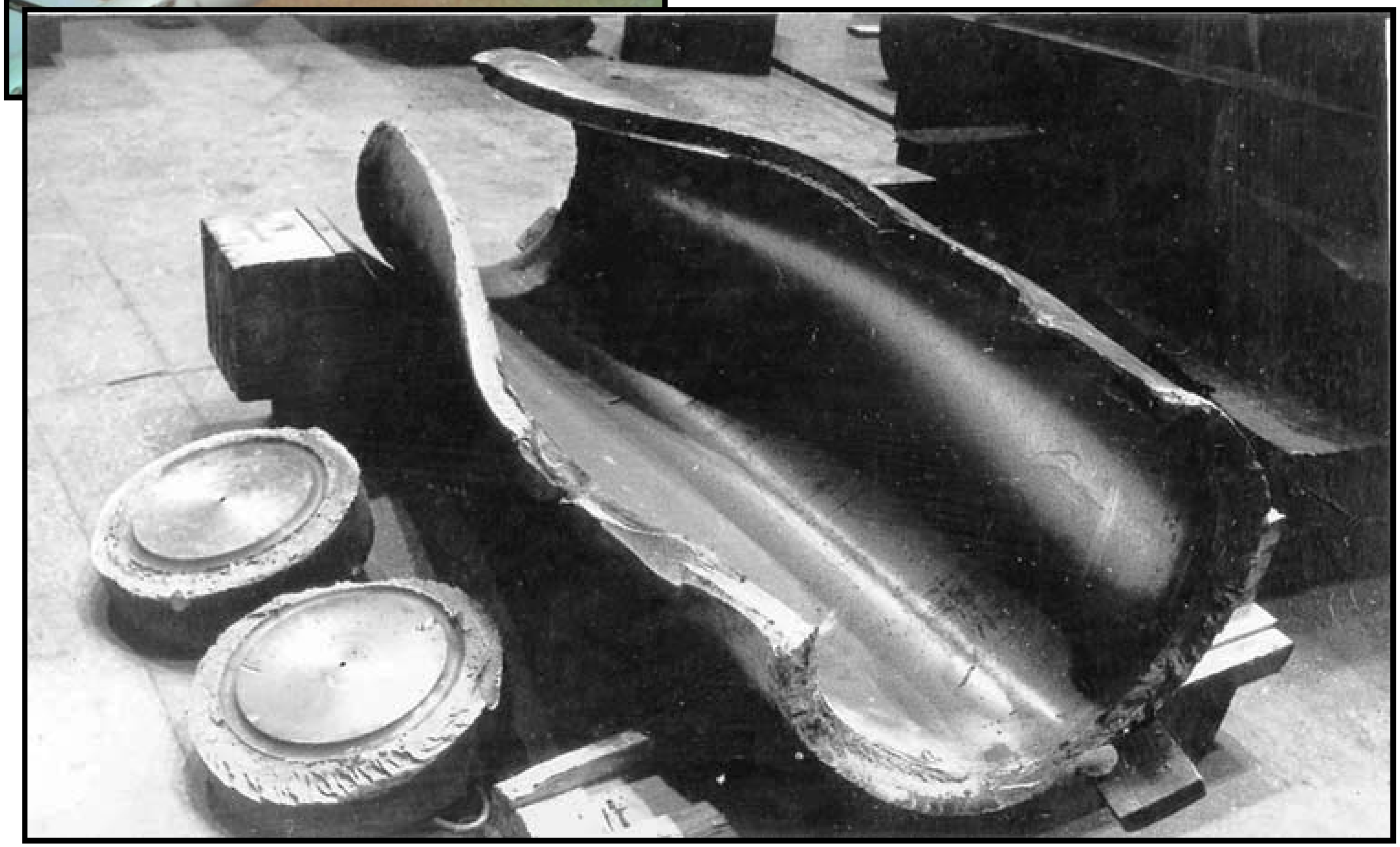
Cylindrical or spherical vessel that has a radius to wall thickness ratio of 10 or more.

$$r/t \geq 10$$

It may be assumed for the sake of simplifying the analysis of stresses in the wall of these vessels that the stress distribution throughout its thickness remains uniform or constant.

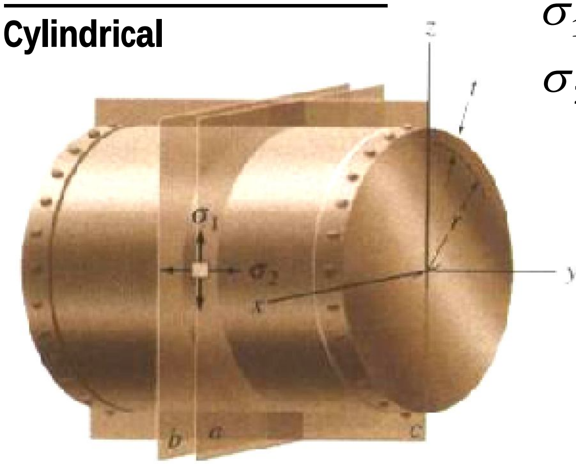
Also assumed is that the pressure referred to is the *gauge* pressure (above atmospheric pressure) and not the absolute pressure.





Thin-Walled Vessels

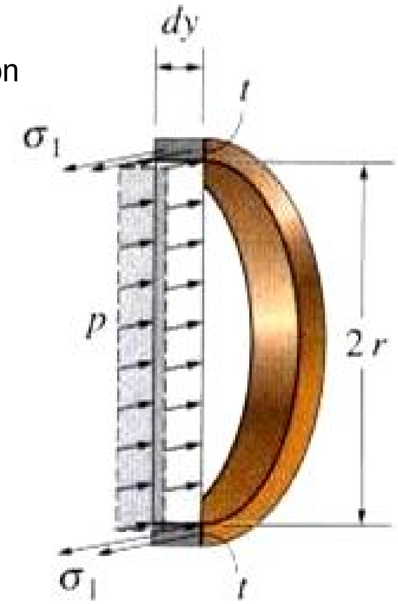
Cylindrical



σ_1 Normal Stress in the hoop direction

σ_2 Longitudinal or axial direction

Both exert tension on the material
To compute the values of each use equilibrium of a section of the tank

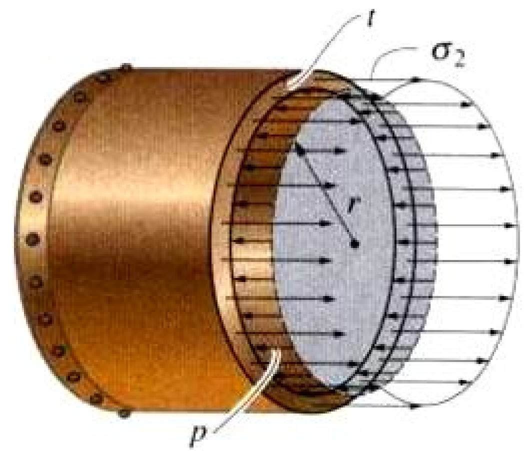


$$\Sigma F_x = 0; \qquad 2[\sigma_1(t\,dy)] - p(2r\,dy) = 0$$

$$\boxed{\sigma_1 = \frac{pr}{t}}$$

$$\Sigma F_y = 0; \qquad \sigma_2(2\pi r t) - p(\pi r^2) = 0$$

$$\boxed{\sigma_2 = \frac{pr}{2t}}$$



In the above equations,

σ_1, σ_2 = the normal stress in the hoop and longitudinal directions, respectively. Each is assumed to be *constant* throughout the wall of the cylinder, and each subjects the material to tension

p = the internal gauge pressure developed by the contained gas or fluid

r = the inner radius of the cylinder

t = the thickness of the wall ($r/t \geq 10$)

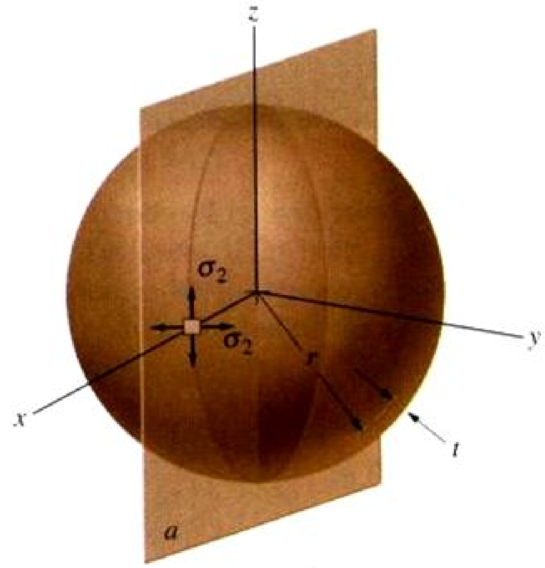
Thin-Walled Vessels

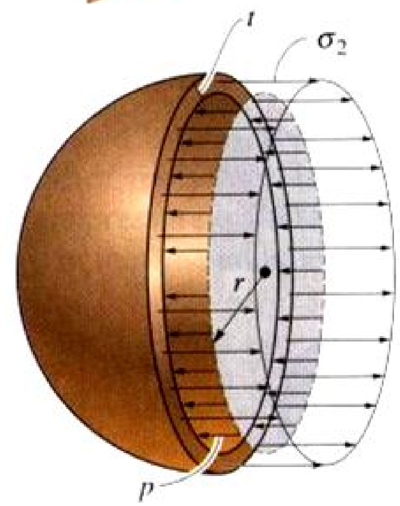
Spherical

Likewise using equilibrium we can analyse spherical vessels using equilibrium as shown:

$$\sigma_2(2\pi r t) - p(\pi r^2) = 0$$

$$\boxed{\sigma_2 = \frac{pr}{2t}}$$





The result is the same as longitudinal stress of cylinders (the lesser of the two). This is true no matter what orientation you are considering (when the weight of the contents is neglected as is usually the case).

Radial Stress

Another stress component in thin walled vessels is *radial* stress σ_3 .

Radial stress acts along a radial line (outwards) and varies in magnitude from pressure ‘p’ at the interior surface to zero on the outer surface of the vessel wall – it is neglected due to its relatively small value compared to σ_1 σ_2

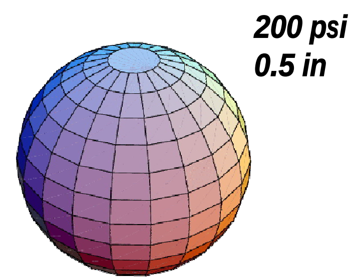
Thin-Walled vessels- Problem Solving

Space shuttle fuel tank in the woods after crash

8-1. A spherical gas tank has an inner radius of $r = 1.5$ m. If it is subjected to an internal pressure of $p = 300$ kPa, determine its required thickness if the maximum normal stress is not to exceed 12 MPa.

$$\sigma_{allow} = \frac{pr}{2t} \qquad 12(10^6) = \frac{300(10^3)(1.5)}{2t} \qquad t = 0.0188\,m = 18.8\,mm$$

8-2. A pressurized spherical tank is to be made of 0.5-in.-thick steel. If it is subjected to an internal pressure of $p = 200$ psi, determine its outer radius if the maximum normal stress is not to exceed 15 ksi.

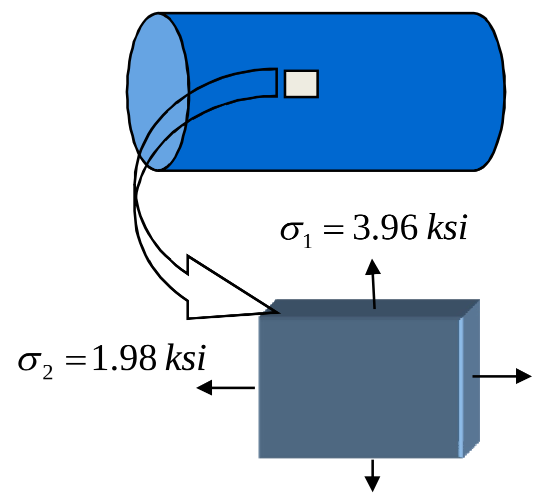


$$\sigma_{allow} = \frac{pr}{2t} \qquad 15(10^6) = \frac{300(r_i)}{2(0.5)}$$

$$r_i = 75\,in$$

$$r_o = 75\,in + 0.5\,in = 75.5\,in$$

8-3. The tank of a cylindrical air compressor is subjected to an internal pressure of 90 psi. If the internal diameter of the tank is 22 in., and the wall thickness is 0.25 in., determine the stress components acting at a point. Draw a volume element of the material at this point, and show the results on the element.



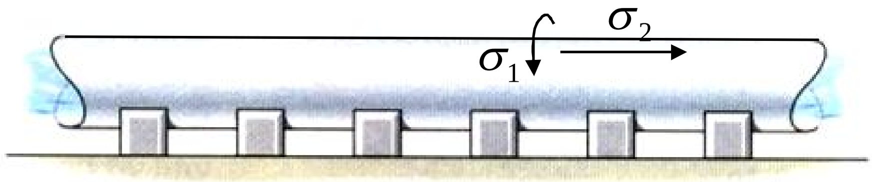
$$\sigma_1 = \frac{pr}{t} = \frac{90(11)}{0.25} = 3960\,psi = 3.96\,ksi$$

$$\sigma_2 = \frac{pr}{2t} = \frac{90(11)}{2(0.25)} = 1980\,psi = 1.98\,ksi$$

Thin-Walled vessels- Problem Solving

8-6. The open-ended polyvinyl chloride pipe has an inner diameter of 4 in. and thickness of 0.2 in. If it carries flowing water at 60 psi pressure, determine the state of stress in the walls of the pipe.

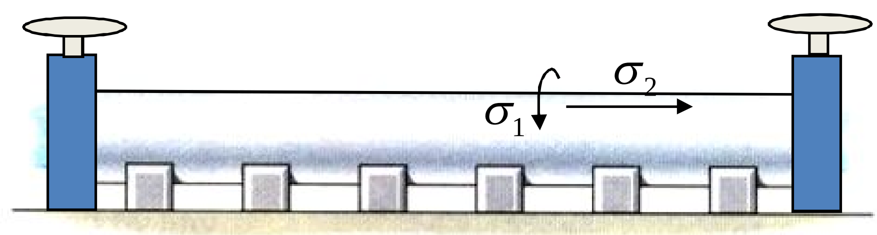
$$\sigma_1 = \frac{pr}{t} = \frac{60(2)}{0.2} = 600\ psi$$



$$\sigma_2 = 0$$ (Open Ends)

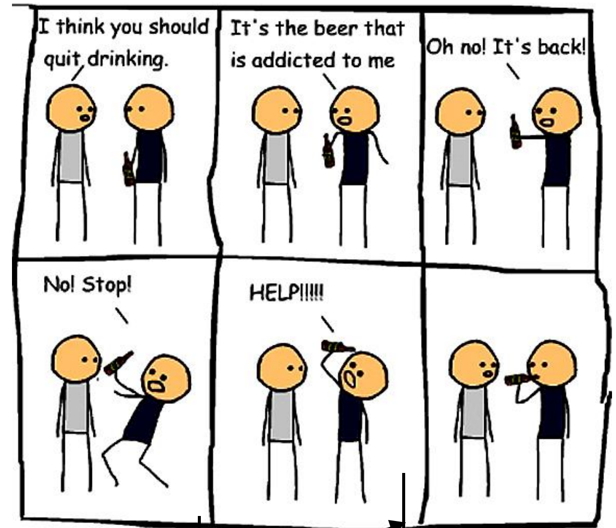
8-7. If the flow of water within the pipe in Prob. 8–6 is stopped due to the closing of a valve, determine the state of stress in the walls of the pipe. Neglect the weight of the water. Assume the supports only exert vertical forces on the pipe.

$$\sigma_1 = \frac{pr}{t} = \frac{60(2)}{0.2} = 600\ psi$$

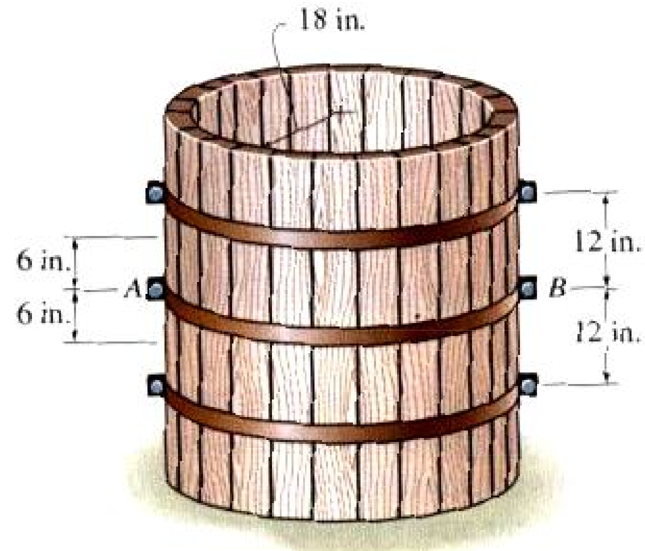


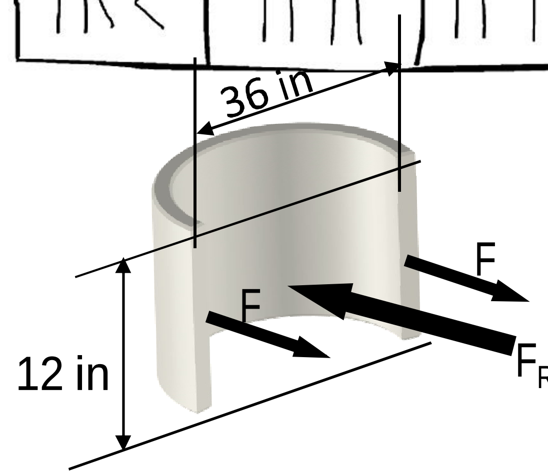
$$\sigma_2 = \frac{pr}{2t} = \frac{60(2)}{2(0.2)} = 300\ psi$$

Thin-Walled vessels- Problem Solving



8-11. The staves or vertical members of the wooden tank are held together using semicircular hoops having a thickness of 0.5 in. and a width of 2 in. Determine the normal stress in hoop AB if the tank is subjected to an internal gauge pressure of 2 psi and this loading is transmitted directly to the hoops. Also, if 0.25-in.-diameter bolts are used to connect each hoop together, determine the tensile stress in each bolt at A and B. Assume hoop AB supports the pressure loading within a 12-in. length of the tank as shown.





$$F_R = 2(36)(12) = 864\ lb$$

$$\sum F = 0; \quad 864 - 2F = 0; \quad F = 432\ lb$$

Stress in the metal hoop:

$$\sigma_h = \frac{F}{A_h} = \frac{432}{0.5(2)} = 432\ psi$$

Stress in the bolts:

$$\sigma_b = \frac{F}{A_b} = \frac{432}{\pi / 4(0.25)} = 8801\ psi$$

Combined Loadings

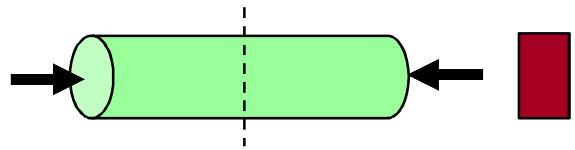
We have learned to determine the stress distribution in a structural element or member subjected to either:

Axial Force

Bending Force

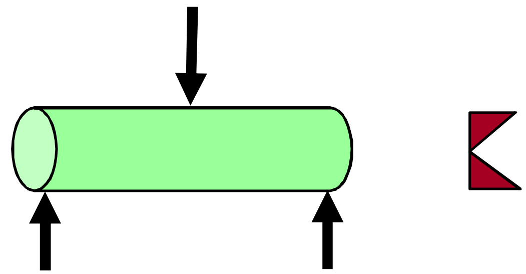
Shear Moment

Torsional Moments.



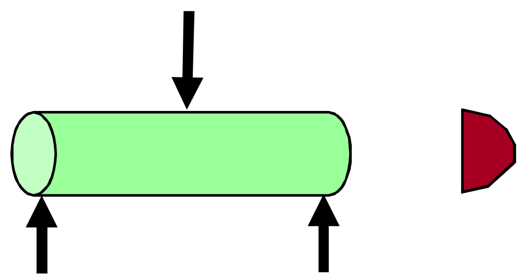
$$\sigma_{normal} = \frac{P}{A}$$

Uniform Normal stress due to axial load



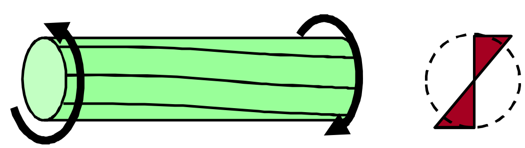
$$\sigma = \frac{My}{I}$$

Stress due to flexure



$$\tau = \frac{VQ}{It}$$

Shear stress due to Bending



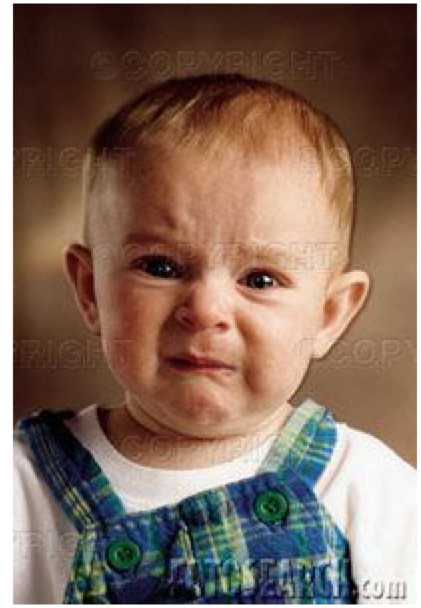
$$\tau = \frac{T\rho}{J}$$

Shear stress due to torque

Combined Loadings

Unfortunately for the students of 4312, it is rare that a structural member is subject to only one of these loading conditions, usually a member is subject to several loadings *simultaneously*.

The method of Superposition can be used most often to determine the resultant stress distribution caused by the loads; ie. determine the stress distribution due to each loading then superimpose them to get the resultant. Loads and stresses must be linearly related and members cannot undergo significant geometric change due to loading in order for superposition to



PROCEDURE FOR ANALYSIS

The following procedure provides a general means for establishing the normal and shear stress components at a point in a member when the member is subjected to several different types of loadings simultaneously. It is assumed that the material is homogeneous and behaves in a linear-elastic manner. Also, Saint-Venant's principle requires that the point where the stress is to be determined is far removed from any discontinuities in the cross section or points of applied load.

Combined Loadings Procedure for Analysis

Step 1

Internal Loading.

- Section the member perpendicular to its axis at the point where the stress is to be determined and obtain the resultant internal normal and shear force components and the bending and torsional moment components.
- The force components should act through the *centroid* of the cross section, and the moment components should be computed about *centroidal axes*, which represent the principal axes of inertia for the cross section.

Step 2

Average Normal Stress.

- Compute the stress component associated with *each* internal loading. For each case, represent the effect either as a distribution of stress acting over the entire cross-sectional area, or show the stress on an element of the material located at a specified point on the cross section.

Combined Loadings Procedure for Analysis

Step 3

Normal Force.

The internal normal force is developed by a uniform normal-stress distribution determined from $\sigma = P/A$.

Step 4

Shear Force.

The internal shear force in a member that is subjected to bending is developed by a shear-stress distribution determined from the shear formula, $\tau = VQ/It$. Special care, however, must be exercised when applying this equation, as noted in Sec. 7.3.

Step 5

Bending Moment.

For *straight members* the internal bending moment is developed by a normal-stress distribution that varies linearly from zero at the neutral axis to a maximum at the outer boundary of the member. The stress distribution is determined from the flexure formula, $\sigma = -My/I$. If the member is *curved*, the stress distribution is nonlinear and is determined from $\sigma = My/[Ae(R - y)]$.

Combined Loadings Procedure for Analysis

Step 6

Torsional Moment.

For circular shafts and tubes the internal torsional moment is developed by a shear-stress distribution that varies linearly from the central axis of the shaft to a maximum at the shaft's outer boundary. The shear-stress distribution is determined from the torsional formula, $\tau = T\rho/J$. If the member is a closed thin-walled tube, use $\tau = T/2A_m t$.

Step 7

Thin-Walled Pressure Vessels.

If the vessel is a thin-walled cylinder, the internal pressure p will cause a biaxial state of stress in the material such that the hoop or circumferential stress component is $\sigma_1 = pr/t$ and the longitudinal stress component is $\sigma_2 = pr/2t$. If the vessel is a thin-walled sphere, then the biaxial state of stress is represented by two equivalent components, each having a magnitude of $\sigma_2 = pr/2t$.

Combined Loadings Procedure for Analysis

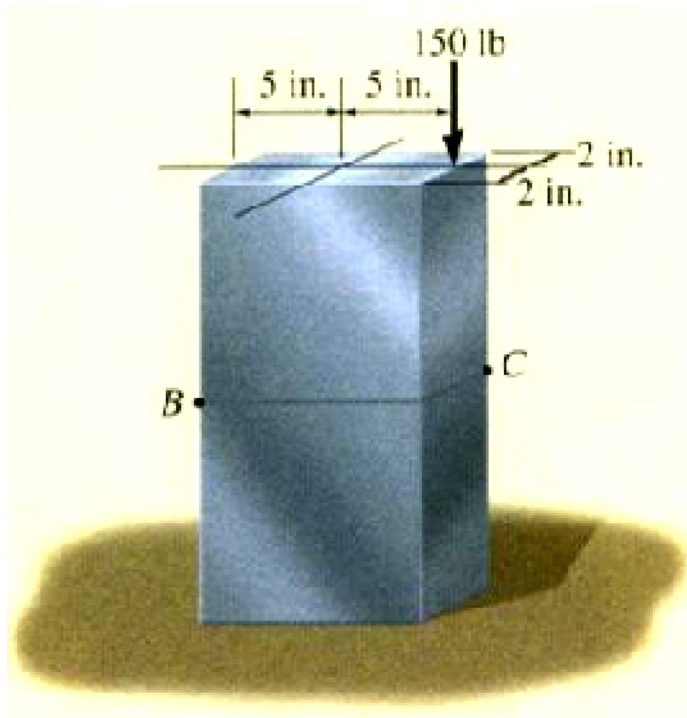
Step 8

Superposition.

- Once the normal and shear stress components for each loading have been calculated, use the principle of superposition and determine the resultant normal and shear stress components.
- Represent the results on an element of material located at the point, or show the results as a distribution of stress acting over the member's cross-sectional area.

Combined Loadings
Example

A force of 150 lb is applied to the edge of the member shown in Fig. 8–3*a*. Neglect the weight of the member and determine the state of stress at points *B* and *C*.



Combined Loadings Example

Solution

Internal Loadings. The member is sectioned through B and C. For equilibrium at the section there must be an axial force of 150 lb acting through the centroid and a bending moment of 750 lb · in. about the centroidal or principal axis, Fig. 8–3*b*.

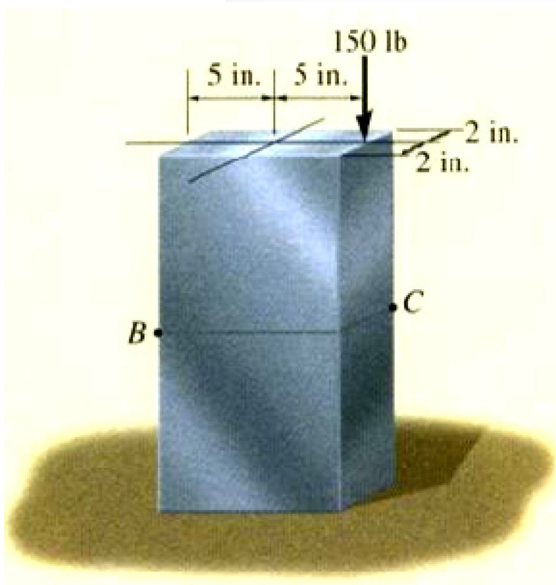
Stress Components.

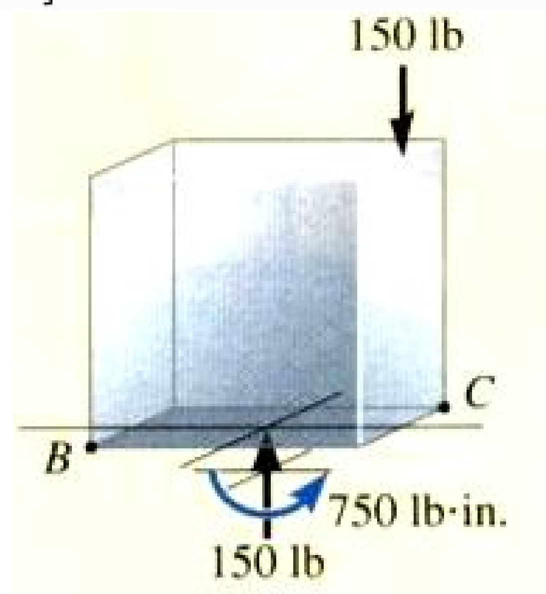
Normal Force. The uniform normal-stress distribution due to the normal force is shown in Fig. 8–3*c*. Here

$$\sigma = \frac{P}{A} = \frac{150 \text{ lb}}{(10 \text{ in.})(4 \text{ in.})} = 3.75 \text{ psi}$$

Bending Moment. The normal-stress distribution due to the bending moment is shown in Fig. 8–3*d*. The maximum stress is

$$\sigma_{\max} = \frac{Mc}{I} = \frac{750 \text{ lb} \cdot \text{in.}(5 \text{ in.})}{\left[\frac{1}{12}(4 \text{ in.})(10 \text{ in.})^3\right]} = 11.25 \text{ psi}$$





Combined Loadings
Example

Superposition. If the above normal-stress distributions are added algebraically, the resultant stress distribution is shown in Fig. 8–3*e*. Although it is not needed here, the location of the line of zero stress can be determined by proportional triangles; i.e.,

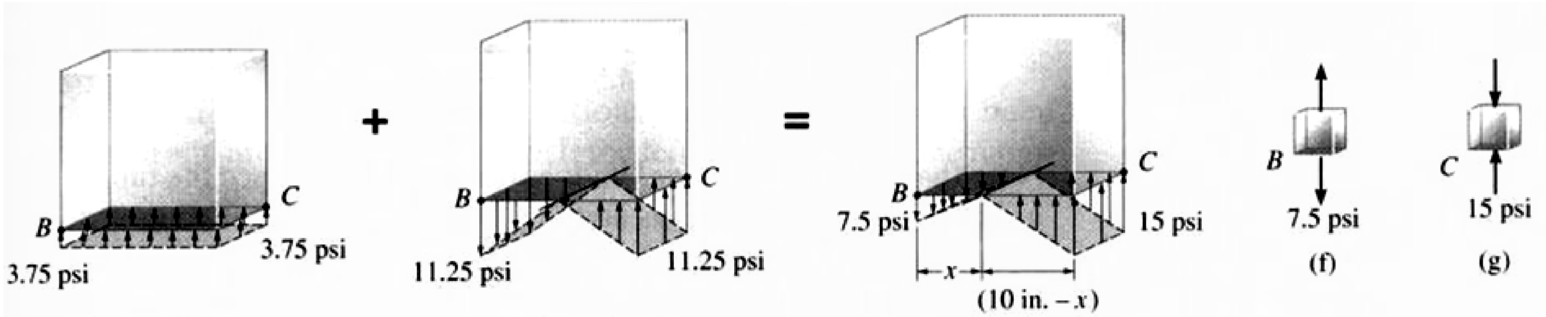
$$\frac{7.5\text{ psi}}{x} = \frac{15\text{ psi}}{(10\text{ in.} - x)}$$

$$x = 3.33\text{ in.}$$

Elements of material at B and C are subjected only to normal or *uniaxial stress* as shown in Fig. 8–3*f* and 8–3*g*. Hence,

$$\sigma_B = 7.5\text{ psi} \quad \text{(tension)} \qquad \textit{Ans.}$$

$$\sigma_C = 15\text{ psi} \quad \text{(compression)} \qquad \textit{Ans.}$$



Normal Force (c) Bending Moment (d) Combined Loading (e) (f) (g)