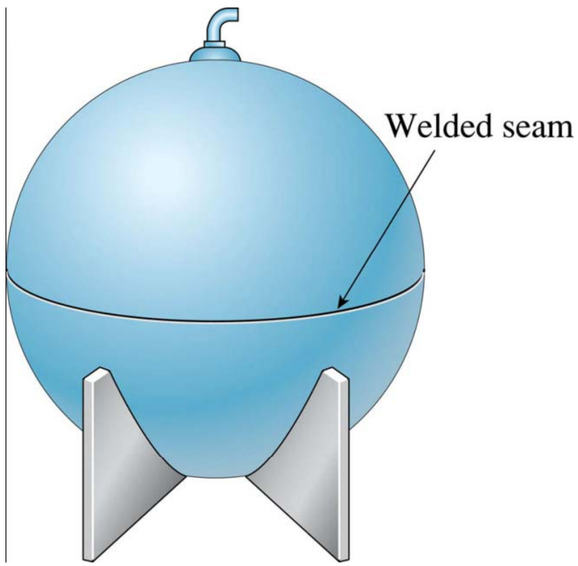
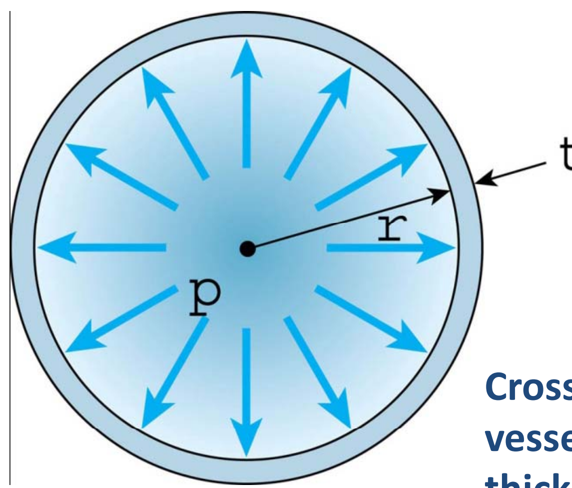
Spherical pressure vessels

- *Pressure vessels* are closed structures containing liquids or gases under pressure
- Examples include tanks, pipes, beverage containers and pressurized cabins in aircraft and space vehicles
- If pressure vessels have walls that are thin in comparison to their overall dimensions they are known as *shell structures*
- In this section we consider thin – walled (**r/t>10**) pressure vessels of spherical shape



Spherical pressure vessel



Cross section of spherical pressure vessel showing inner radius *r*, wall thickness *t*, and internal pressure *p*

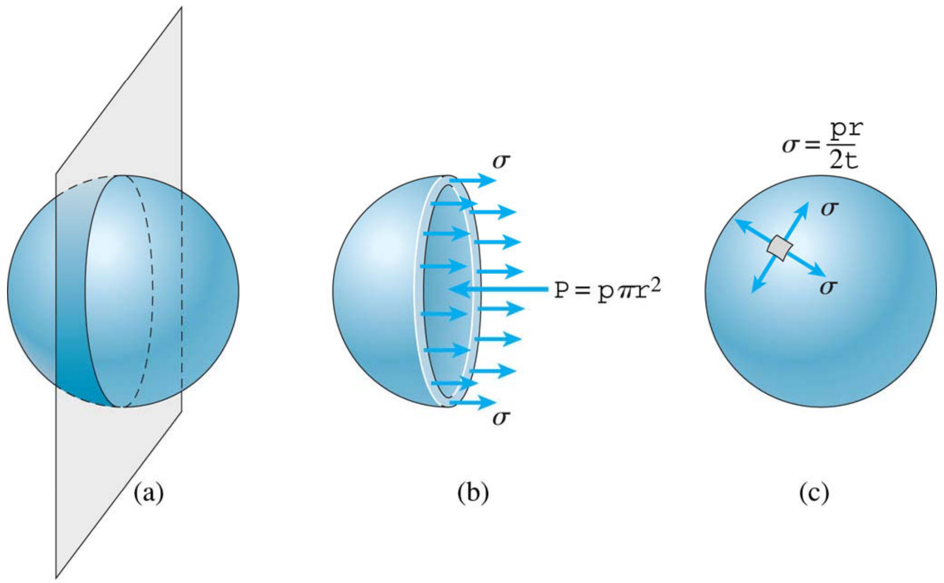
Spherical pressure vessels

- Formula for calculating the **tensile stresses in the wall of a spherical shell**

$$\sigma = \frac{pr}{2t}$$

- *The wall of a pressurized spherical vessel is subjected to uniform tensile stresses σ in all directions* (because of spherical symmetry)

- Stresses that act tangentially to the curved surface of a shell are known as *membrane stresses.* The name arises from the fact that these are the only stresses that exist in true membranes, such as thin polymer films, soap films etc



Tensile stresses *σ* in the wall of a spherical pressure vessel

Stresses at the outer surface

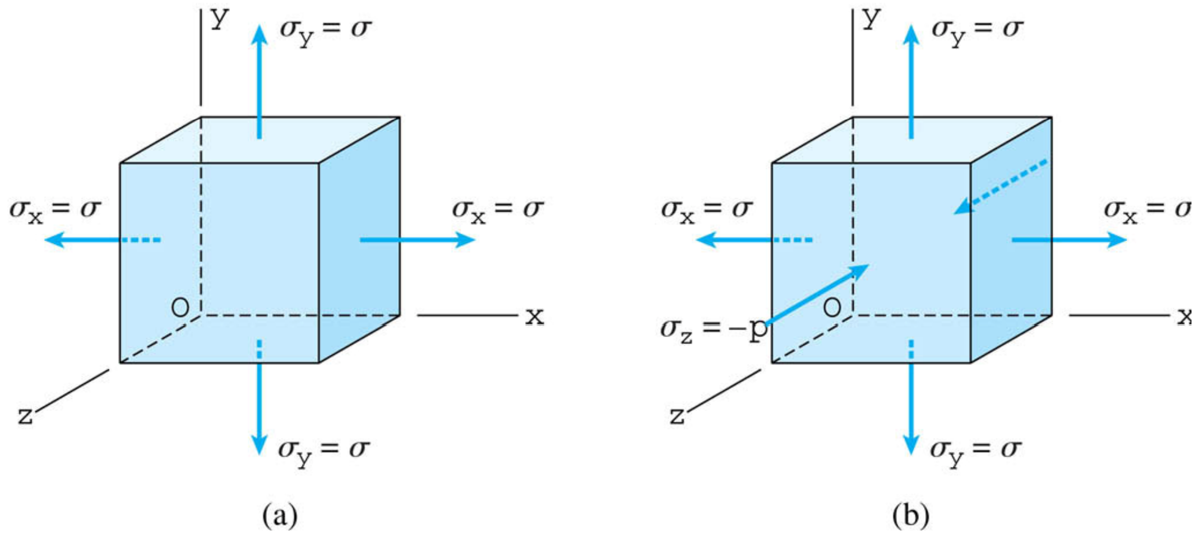


FIG. 8-4 **Stresses in a spherical pressure vessel at (a) the outer surface and (b) the inner surface**

- Element in fig 8-4(a) is in biaxial stress. No in-plane shear stresses acting on the thin element

- *Every plane is a principal plane and every direction is a principal direction*

$$\sigma_1 = \sigma_2 = \frac{pr}{2t}, \sigma_3 = 0$$

Principal stresses for the element

$$\tau_{max} = \frac{\sigma}{2} = \frac{pr}{4t}$$

Out of plane maximum shear stresses

General comments

1. The wall thickness must be small in comparison to the outer dimensions. Ratio r/t should be greater than 10
2. The internal pressure must exceed the external pressure in order to avoid inward buckling
3. We consider the effects of internal pressure only. The effects of external loads, reactions, the weight of the contents and the weight of the structure are not considered
4. The formulas derived in this section are valid throughout the wall of the vessel *except near points of stress concentration*

Cylindrical pressure vessels

- Cylindrical pressure vessels with a circular cross-section are found in industrial settings (compressed air tanks, rocket motors), in homes (fire extinguishers, spray cans) and in the countryside (propane tanks, grain silos)
- Pressurized pipes are also classified as cylindrical pressure vessels

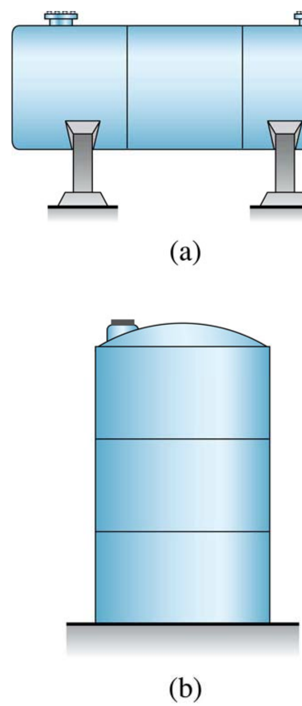


FIG. 8-6
Cylindrical pressure vessels with circular cross sections

Circumferential and longitudinal stress

- The stress σ_1 is called the *circumferential stress* or the *hoop stress*
- The stress σ_2 is called the *longitudinal stress* or the *axial stress*

$$\sigma_1 = \frac{pr}{t}$$

$$\sigma_2 = \frac{pr}{2t}$$

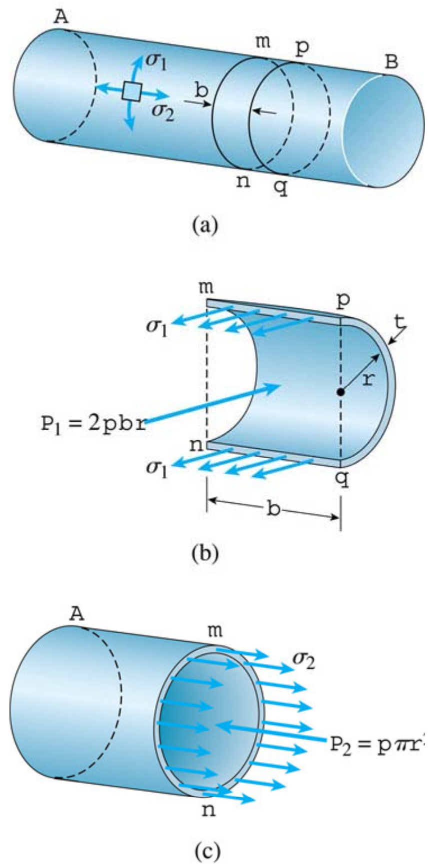


FIG. 8-7
Stresses in a circular cylindrical pressure vessel

Stresses at the outer surface

- The principal stresses σ_1 and σ_2 at the outer surface of the cylindrical vessel are shown on the stress element in fig 8-8(a) where $\sigma_3 = 0$ (i.e. biaxial stress state)

- The maximum out of plane shear stresses is;

$$\tau_{max} = \frac{\sigma_1}{2} = \frac{pr}{2t}$$

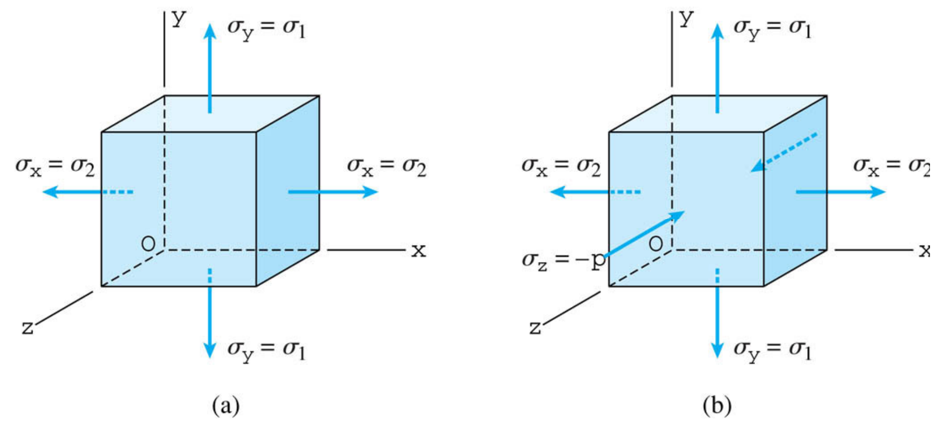


FIG. 8-8 **Stresses in a circular cylindrical pressure vessel at (a) the outer surface and (b) the inner surface**