## Possible Glitches in the Simplex Solution:

What if there is a tie in the entering basic variable?
What if there is a tie for the leaving basic variable?
What if there is no leaving basic variable?
What if there are multiple optimal solutions?

| Iteration | Basic Var | Coeff |  |  |  |  |  | RHS |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Z | $\mathrm{x}_{1}$ | $\mathbf{x}_{2}$ | $\mathbf{x}_{3}$ | $\mathbf{x}_{4}$ | $\mathrm{x}_{5}$ |  |
| 0 | Z | 1 | -30 | -20 | 0 | 0 | 0 | 0 |
|  | $\mathrm{x}_{3}$ | 0 | 1 | 0 | 1 | 0 | 0 | 4 |
|  | $\mathrm{x}_{4}$ | 0 | 0 | 2 | 0 | 1 | 0 | 12 |
|  | $\mathrm{x}_{5}$ | 0 | 3 | 2 | 0 | 0 | 1 | 18 |
| 1 | Z | 1 |  |  |  |  |  |  |
|  | x | 0 |  |  |  |  |  |  |
|  | x | 0 |  |  |  |  |  |  |
|  | x | 0 |  |  |  |  |  |  |
| 2 | Z | 1 |  |  |  |  |  |  |
|  | x | 0 |  |  |  |  |  |  |
|  | x | 0 |  |  |  |  |  |  |
|  | x | 0 |  |  |  |  |  |  |
| 3 | Z | 1 |  |  |  |  |  |  |
|  | x | 0 |  |  |  |  |  |  |
|  | x | 0 |  |  |  |  |  |  |
|  | x | 0 |  |  |  |  |  |  |

Variations in Model Forms I: (artificial variables and the Big $M$ method)
Constraints to be satisfied at equality.

$$
\begin{array}{lr}
\text { Max } & \mathrm{Z}=30 \mathrm{x}_{1}+15 \mathrm{x}_{2} \\
\text { s.t. } & \mathrm{x}_{1} \quad \square 4 \\
& 2 \mathrm{x}_{2} \square 12 \\
& 3 \mathrm{x}_{1}+2 \mathrm{x}_{2}=18 \\
& \mathrm{x}_{1} \geq 0, \mathrm{x}_{2} \geq 0
\end{array}
$$

| Iteration | Basic <br> Var | Coeff |  |  |  |  |  | RHS |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Z | $\mathrm{x}_{1}$ | $\mathbf{x}_{2}$ | $\mathbf{x}_{3}$ | $\mathbf{x}_{4}$ | $\underline{\mathbf{x}}_{5}$ |  |
|  | Z | 1 |  |  | 0 | 0 |  |  |
|  | $\mathrm{x}_{3}$ | 0 | 1 | 0 | 1 | 0 | 0 | 4 |
|  | $\mathrm{x}_{4}$ | 0 | 0 | 2 | 0 | 1 | 0 | 12 |
|  | $\mathrm{x}_{5}$ | 0 | 3 | 2 | 0 | 0 | 1 | 18 |
| 0 | Z | 1 |  |  |  |  |  |  |
|  | x | 0 | 1 | 0 | 1 | 0 | 0 | 4 |
|  | x | 0 | 0 | 2 | 0 | 1 | 0 | 12 |
|  | x | 0 | 3 | 2 | 0 | 0 | 1 | 18 |
| 1 | Z | 1 |  |  |  |  |  |  |
|  | x | 0 |  |  |  |  |  |  |
|  | x | 0 |  |  |  |  |  |  |
|  | x | 0 |  |  |  |  |  |  |
| 2 | Z | 1 |  |  |  |  |  |  |
|  | x | 0 |  |  |  |  |  |  |
|  | x | 0 |  |  |  |  |  |  |
|  | x | 0 |  |  |  |  |  |  |

Variations in Model Forms II: Negative RHS.

Variations in Model Forms III: (surplus variable)
Data for Quidditch Game problem

|  | Fraction of Contact Force |  |  |
| :---: | :---: | :---: | :---: |
|  | Absorbed by Area (average) |  | Restriction on Total |
| Area | (Fred) 1 | 2 (George) | Sustainable Contact |
| 1 (spectators' stand) | 0.4 | 0.5 | minimize |
| 2 (announcer's stand) | 0.3 | 0.1 | $\square 2.7$ |
| 3 (opposing player) | 0.5 | 0.5 | $=6$ |
| 4 (bludger) | 0.6 | 0.4 | $\geq 6$ |

Constraints with opposite inequality signs.

Variations in Model Forms IV: Minimization problems.

| Iteration | Basic Var | Coeff |  |  |  |  |  |  | RHS |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Z | $\mathrm{x}_{1}$ | $\mathrm{x}_{2}$ | $\mathbf{x}_{3}$ | $\mathbf{x}_{4}$ | $\mathbf{x}_{5}$ | $\mathbf{x}_{6}$ |  |
|  | Z | -1 | 0.4 | 0.5 | 0 | M | 0 | M | 0 |
|  | $\mathrm{x}_{3}$ | 0 | 0.3 | 0.1 | 1 | 0 | 0 | 0 | 2.7 |
|  | $\mathrm{x}_{4}$ | 0 | 0.5 | 0.5 | 0 | 1 | 0 | 0 | 6 |
|  | $\mathrm{x}_{6}$ | 0 | 0.6 | 0.4 | 0 | 0 | -1 | 1 | 6 |
| 0 | Z | -1 |  |  |  |  |  |  |  |
|  | $\mathrm{X}_{3}$ | 0 | 0.3 | 0.1 | 1 | 0 | 0 | 0 | 2.7 |
|  | $\mathrm{x}_{4}$ | 0 | 0.5 | 0.5 | 0 | 1 | 0 | 0 | 6 |
|  | $\mathrm{x}_{6}$ | 0 | 0.6 | 0.4 | 0 | 0 | -1 | 1 | 6 |
| 1 | Z | -1 |  |  |  |  |  |  |  |
|  | x | 0 |  |  |  |  |  |  |  |
|  | x | 0 |  |  |  |  |  |  |  |
|  | x | 0 |  |  |  |  |  |  |  |
| 2 | Z | -1 |  |  |  |  |  |  |  |
|  | x | 0 |  |  |  |  |  |  |  |
|  | x | 0 |  |  |  |  |  |  |  |
|  | x | 0 |  |  |  |  |  |  |  |
| 3 | Z | -1 |  |  |  |  |  |  |  |
|  | x | 0 |  |  |  |  |  |  |  |
|  | x | 0 |  |  |  |  |  |  |  |
|  | x | 0 |  |  |  |  |  |  |  |

