Getting an Initial Basic Feasible Solution

The simplex method assumes that you have an initial tableau with a basic feasible solution.

In the LP problems solved by the simplex method thus far, we have used as the initial basic variables the objective (-Z) and the slack variables.

What if the LP has no slack variables which we can use for this purpose?

- ⇨ Introducing "Artificial" Variables
- ⇨ Eliminating Artificial Variables

If an equation contains a slack variable (and if the RHS is ≥ 0!), the slack variable may be used as the basic variable in that row.

Otherwise,

If necessary, multiply both sides by -1 to get a nonnegative RHS

Then add an "artificial" variable which will be eventually forced to zero, and use this new variable as the initial basic variable for this row.

Example

$$4X_1 + X_2 \geq 20$$

$$\Rightarrow 4X_1 + X_2 - S = 20$$

If the variable **S** were used as the basic variable for this equation, we would obtain an **infeasible** solution, $S = -20$.

Therefore, add an **artificial** variable (**a**):

$$\Rightarrow 4X_1 + X_2 - S + \mathbf{a} = 20$$

Letting the variable a be basic in this equation, we obtain a "pseudo-feasible" solution with $\mathbf{a} = 20$.

Example:

Maximize $z = 3X_1 + 2X_2 - X_3 + 4X_4$

subject to

$$\begin{cases} -X_1 + X_2 - 4X_3 + 2X_4 \geq 4 \\ 3X_1 + X_2 - 2X_3 \leq 6 \\ X_2 - X_4 = -1 \\ -X_1 + X_2 - X_3 = 0 \end{cases}$$

$X_j \geq 0, \ j=1,2,3,4$

First modify the 3rd constraint so as to have rhs≥0:

$$-X_2 + X_4 = 1$$

Next, add slack & subtract surplus variables to convert inequalities to equations:

$$\begin{array}{ll}
\text{Maximize} & z = 3X_1 + 2X_2 - X_3 + 4X_4 \\
\text{subject to} & -X_1 + X_2 - 4X_3 + 2X_4 - S_1 = 4 \\
 & 3X_1 + X_2 - 2X_3 + S_2 = 6 \\
 & -X_2 + X_4 = 1 \\
 & -X_1 + X_2 - X_3 = 0 \\
 & X_j \geq 0,\ j=1,2,3,4; \quad S_i \geq 0,\ i=1,2
\end{array}$$

surplus slack

$$\begin{array}{llr}
\text{Maximize} & z = 3X_1 + 2X_2 - X_3 + 4X_4 & \\
\text{subject to} & -X_1 + X_2 - 4X_3 + 2X_4 - S_1 & = 4 \\
 & 3X_1 + X_2 - 2X_3 \qquad\qquad + S_2 & = 6 \\
 & \quad - X_2 + X_4 & = 1 \\
 & -X_1 + X_2 - X_3 & = 0
\end{array}$$

$$X_j \geq 0,\ j=1,2,3,4; \quad S_i \geq 0,\ i=1,2$$

We can use $(-Z)$ and S_2 as basic variables in the objective row and the second constraint. The other constraints need **artificial** variables!

$$
\begin{array}{ll}
\text{Max} & z = 3X_1 + 2X_2 - X_3 + 4X_4 \\
\text{s.t.} & -X_1 + X_2 - 4X_3 + 2X_4 - S_1 + a_1 = 4 \\
 & 3X_1 + X_2 - 2X_3 + S_2 = 6 \\
 & -X_2 + X_4 + a_3 = 1 \\
 & -X_1 + X_2 - X_3 + a_4 = 0 \\
 & X_j \geq 0,\ j=1,2,3,4;\ \ S_i \geq 0,\ i=1,2;\ a_i \geq 0,\ i=1,3,4
\end{array}
$$

artificial variables

$$
\begin{array}{llr}
\text{Max} & z = 3X_1 + 2X_2 - X_3 + 4X_4 & \\
\text{s.t.} & -X_1 + X_2 - 4X_3 + 2X_4 - S_1 \qquad + a_1 & = 4 \\
 & 3X_1 + X_2 - 2X_3 \qquad\qquad + S_2 & = 6 \\
 & \quad - X_2 + X_4 \qquad\qquad\qquad + a_3 & = 1 \\
 & -X_1 + X_2 - X_3 \qquad\qquad\qquad\qquad + a_4 & = 0 \\
\end{array}
$$

$$X_j \geq 0,\ j=1,2,3,4;\ \ S_i \geq 0,\ i=1,2;\ a_i \geq 0,\ i=1,3,4$$

We can now use (−Z), a_1, S_2, a_3, and a_4 as the basic variables in the respective equations, getting a "pseudo-feasible" solution with **basic** variables

$$Z = 0,\ a_1 = 4,\ S_2 = 6,\ a_3 = 1,\ a_4 = 0$$

and nonbasic variables = zero

Use of Single Artificial Variable

It is possible to obtain a "pseudo-feasible" basic solution using only a ***single*** artificial variable!

In this method, before adding the artificial variable, pivot an arbitrary variable into the basis in each constraint row (e.g., the slack or surplus variable for that row, if there is one.) The result, in general, is a basic ***infeasible*** solution (with one or more basic variables negative).

Example

$$
\begin{array}{ll}
\text{Minimize} & -2X_1 + 2X_2 + X_3 + X_4 \\
\text{s.t.} & X_1 + 2X_2 + X_3 + X_4 \leq 2 \\
 & \left\{ \begin{array}{l} X_1 - X_2 + X_3 + 5X_4 \geq 4 \\ 2X_1 - X_2 + X_3 \geq 2 \end{array} \right. \\
 & X_i \geq 0, i=1,2,3,4
\end{array}
$$

note the "greater-than" inequalities

Convert all rows to equations:

slack

$$
\begin{array}{llr}
\text{Minimize } -Z - 2X_1 + 2X_2 + X_3 + X_4 & & = 0 \\
\text{s.t.} \quad X_1 + 2X_2 + X_3 + X_4 + S_1 & & = 2 \\
X_1 - X_2 + X_3 + 5X_4 & - S_2 & = 4 \\
2X_1 - X_2 + X_3 & - S_3 & = 2 \\
X_j \geq 0,\ j = 1,2,3,4,\ S_i \geq 0,\ i = 1,2,3 & &
\end{array}
$$

surplus

tableau

-Z	X_1	X_2	X_3	X_4	S_1	S_2	S_3	rhs
1	¯2	2	1	1	0	0	0	0
0	1	2	1	1	1	0	0	2
0	1	¯1	1	5	0	¯1	0	4
0	2	¯1	1	0	0	0	¯1	2

$-Z$	X_1	X_2	X_3	X_4	S_1	S_2	S_3	rhs
1	-2	2	1	1	0	0	0	0
0	1	2	1	1	1	0	0	2
0	1	-1	1	5	0	-1	0	4
0	2	-1	1	0	0	0	-1	2

Choose an initial basis (not necessarily feasible!)

Pivot variables into the basis:

$-Z$	X_1	X_2	X_3	X_4	S_1	S_2	S_3	rhs
1	-2	2	1	1	0	0	0	0
0	1	2	1	1	1	0	0	2
0	-1	1	-1	-5	0	1	0	-4
0	-2	1	-1	0	0	0	1	-2

Infeasible! (2 basic variables are negative)

$-Z$	X_1	X_2	X_3	X_4	S_1	S_2	S_3	rhs
1	-2	2	1	1	0	0	0	0
0	1	2	1	1	1	0	0	2
0	-1	1	-1	-5	0	1	0	-4
0	-2	1	-1	0	0	0	1	-2

For every row having an infeasible basic variable, insert $-A$, where A is the artificial variable:

$-Z$	X_1	X_2	X_3	X_4	S_1	S_2	S_3	A	rhs
1	-2	2	1	1	0	0	0	0	0
0	1	2	1	1	1	0	0	0	2
0	-1	1	-1	-5	0	1	0	-1	-4
0	-2	1	-1	0	0	0	1	-1	-2

artificial variable

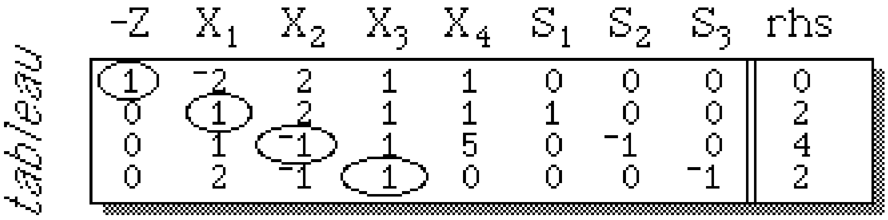
-Z	X_1	X_2	X_3	X_4	S_1	S_2	S_3	A	rhs
1	-2	2	1	1	0	0	0	0	0
0	1	2	1	1	1	0	0	0	2
0	-1	1	-1	-5	0	1	0	-1	-4
0	-2	1	-1	0	0	0	1	-1	-2

Pivot the artificial variable into the basis in the row with maximum infeasibility (most negative right-hand-side)

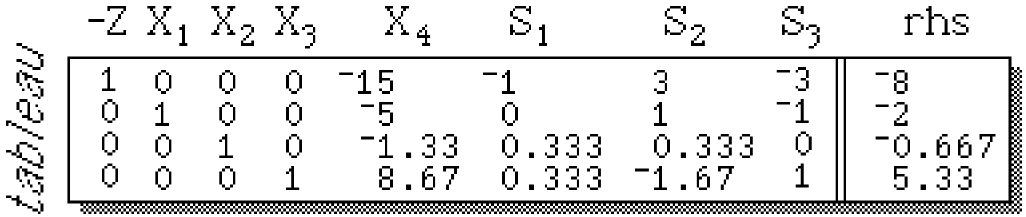
-Z	X_1	X_2	X_3	X_4	S_1	S_2	S_3	A	rhs
1	-2	2	1	1	0	0	0	0	0
0	1	2	1	1	1	0	0	0	2
0	1	-1	1	5	0	-1	0	1	4
0	-1	0	0	5	0	-1	1	0	2

The resulting tableau gives a "pseudo-feasible" basic solution!

The choice of initial basic variables, except for −Z, is arbitrary:



-Z	X_1	X_2	X_3	X_4	S_1	S_2	S_3	rhs
1	¯2	2	1	1	0	0	0	0
0	1	2	1	1	1	0	0	2
0	1	¯1	1	5	0	¯1	0	4
0	2	¯1	1	0	0	0	¯1	2



-Z	X_1	X_2	X_3	X_4	S_1	S_2	S_3	rhs
1	0	0	0	¯15	¯1	3	¯3	¯8
0	1	0	0	¯5	0	1	¯1	¯2
0	0	1	0	¯1.33	0.333	0.333	0	¯0.667
0	0	0	1	8.67	0.333	¯1.67	1	5.33

tableau

-Z	X_1	X_2	X_3	X_4	S_1	S_2	S_3	rhs
1	0	0	0	¯15	¯1	3	¯3	¯8
0	1	0	0	¯5	0	1	¯1	¯2
0	0	1	0	¯1.33	0.333	0.333	0	¯0.667
0	0	0	1	8.67	0.333	¯1.67	1	5.33

Subtract artificial variable in rows 2&3:

-Z	X_1	X_2	X_3	X_4	S_1	S_2	S_3	A	rhs
1	0	0	0	¯15	¯1	3	¯3	0	¯8
0	1	0	0	¯5	0	1	¯1	¯1	¯2
0	0	1	0	¯1.33	0.333	0.333	0	¯1	¯0.667
0	0	0	1	8.67	0.333	¯1.67	1	0	5.33

-Z	X_1	X_2	X_3	X_4	S_1	S_2	S_3	A	rhs
1	0	0	0	¯15	¯1	3	¯3	0	¯8
0	1	0	0	¯5	0	1	¯1	¯1	¯2
0	0	1	0	¯1.33	0.333	0.333	0	¯1	¯0.667
0	0	0	1	8.67	0.333	¯1.67	1	0	5.33

Pivot in constraint row with most infeasibility

⬇

-Z	X_1	X_2	X_3	X_4	S_1	S_2	S_3	A	rhs
1	0	0	0	¯15	¯1	3	¯3	0	¯8
0	¯1	0	0	5	0	¯1	1	1	2
0	¯1	1	0	3.67	0.333	¯0.667	1	0	1.33
0	0	0	1	8.67	0.333	¯1.67	1	0	5.33

Right-hand-sides of constraints now non-negative!

Example *Note that all constraints are inequalities*

$$
\begin{array}{lrcl}
\text{Maximize} & \multicolumn{3}{l}{-30X_1 + 4X_2 + 2X_3 - 7X_4 - 8X_5 - 9X_6} \\
\text{s.t.} & X_1 - X_2 + X_4 - X_5 + 2X_6 & = & 1 \\
 & X_2 - X_4 + X_5 + X_6 & = & -4 \\
 & X_2 + X_3 + X_4 + 2X_5 - 2X_6 & = & -4
\end{array}
$$

$$X_j \geq 0,\ j=1,2,3,4,5,6$$

$-Z$	X_1	X_2	X_3	X_4	X_5	X_6	rhs
1	-30	4	2	-7	-8	-9	0
0	1	-1	0	1	-1	2	1
0	0	1	0	-1	1	1	-4
0	0	1	1	1	2	-2	-4

-Z	X_1	X_2	X_3	X_4	X_5	X_6	rhs
1	-30	4	2	-7	-8	-9	0
0	1	-1	0	1	-1	2	1
0	0	1	0	-1	1	1	-4
0	0	1	1	1	2	-2	-4

-Z	X_1	X_2	X_3	X_4	X_5	X_6	rhs
1	0	0	0	-7	-14	83	-74
0	1	0	0	0	0	3	-3
0	0	1	0	-1	1	1	-4
0	0	0	1	2	1	-3	0

We arbitrarily select variables X , X , & X for basic variables in the 3 constraint rows, and pivot them into the basis

-Z	X_1	X_2	X_3	X_4	X_5	X_6	rhs
1	0	0	0	¯7	¯14	83	¯74
0	1	0	0	0	0	3	¯3
0	0	1	0	¯1	1	1	¯4
0	0	0	1	2	1	¯3	0

Subtract the artificial variable from rows 2 & 3 (which have infeasibilities)

⬇

-Z	X_1	X_2	X_3	X_4	X_5	X_6	A	rhs
1	0	0	0	¯7	¯14	83	0	¯74
0	1	0	0	0	0	3	¯1	¯3
0	0	1	0	¯1	1	1	¯1	¯4
0	0	0	1	2	1	¯3	0	0

-Z	X_1	X_2	X_3	X_4	X_5	X_6	A	rhs
1	0	0	0	¯7	¯14	83	0	¯74
0	1	0	0	0	0	3	¯1	¯3
0	0	1	0	¯1	1	1	¯1	¯4
0	0	0	1	2	1	¯3	0	0

Pivot in the constraint row with maximum infeasibility

⇓

-Z	X_1	X_2	X_3	X_4	X_5	X_6	A	rhs
1	0	0	0	¯7	¯14	83	0	¯74
0	1	¯1	0	1	¯1	2	0	1
0	0	¯1	0	1	¯1	¯1	1	4
0	0	0	1	2	1	¯3	0	0

The resulting tableau is "pseudo-feasible", with nonnegative rhs.

Forcing Artificial Variables from the Solution:

- ⇨ "Big-M" method
- ⇨ Two-Phase method

"Big-M" Method

To eliminate an artificial variable from the solution, we can attach a very high cost (**M**) to the variable if we are minimizing, or a very large penalty (**-M**) if we are maximizing the objective. If M is sufficiently large and if there is a feasible solution of the LP, then the artificial variable(s) will be zero in the optimal solution.

"Big-M" Method

Drawbacks:

- we don't know *a priori* how large M should be.
- using very large values for M may lead to numerical difficulties (round-off, etc.) in a computer implementation of the simplex method.

Example revisited:

a_1, a_3, & a_4 are artificial variables:

$$
\begin{array}{llr}
\text{Max} & z= 3X_1 + 2X_2 - X_3 + 4X_4 \qquad -Ma_1 - Ma_3 - Ma_4 & \\
\text{s.t.} & -X_1 + X_2 - 4X_3 + 2X_4 - S_1 \quad + a_1 & = 4 \\
 & 3X_1 + X_2 - 2X_3 \qquad + S_2 & = 6 \\
 & - X_2 + X_4 \qquad + a_3 & = 1 \\
 & -X_1 + X_2 - X_3 \qquad + a_4 & = 0 \\
\end{array}
$$

$X_j \geq 0$, j=1,2,3,4; $S_i \geq 0$, i=1,2; $a_i \geq 0$, i=1,3,4

$-Ma_i$ for i=1,3,4 is added to the objective, where M is some large number.

Two-Phase Method

While in the "Big-M" method, we simultaneously consider the original objective and the objective of eliminating the artificial variables, in this method we **first** eliminate the artificial variables (**Phase One**) and **then** optimize our original objective function (**Phase Two**).

Two-Phase Method

Example:

Maximize $z = 3X_1 + 2X_2 - X_3 + 4X_4$

subject to

$$\begin{cases} -X_1 + X_2 - 4X_3 + 2X_4 \geq 4 \\ 3X_1 + X_2 - 2X_3 \leq 6 \\ X_2 - X_4 = -1 \\ -X_1 + X_2 - X_3 = 0 \end{cases}$$

$$X_j \geq 0,\ j=1,2,3,4$$

$$
\begin{array}{ll}
\text{Max} & z = 3X_1 + 2X_2 - X_3 + 4X_4 \\
\text{s.t.} & -X_1 + X_2 - 4X_3 + 2X_4 - S_1 + a_1 = 4 \\
 & 3X_1 + X_2 - 2X_3 + S_2 = 6 \\
 & - X_2 + X_4 + a_3 = 1 \\
 & -X_1 + X_2 - X_3 + a_4 = 0 \\
\end{array}
$$

$$X_j \geq 0,\ j=1,2,3,4;\ \ S_i \geq 0,\ i=1,2;\ a_i \geq 0,\ i=1,3,4$$

We can use (−Z), a_1, S_2, a_3, and a_4 as basic variables initially.

We need to find a basic solution which has

$$a_1 = a_3 = a_4 = 0,$$

so we introduce a new ("Phase One") objective:

$$\begin{array}{lll}
\text{Min} & w = & a_1 + a_3 + a_4 \\
 & -z + 3X_1 + 2X_2 - X_3 + 4X_4 & = 0 \\
\text{s.t.} & -X_1 + X_2 - 4X_3 + 2X_4 - S_1 + a_1 & = 4 \\
 & 3X_1 + X_2 - 2X_3 + S_2 & = 6 \\
 & - X_2 + X_4 + a_3 & = 1 \\
 & -X_1 + X_2 - X_3 + a_4 & = 0
\end{array}$$

$X_j \geq 0,\ j=1,2,3,4;\ \ S_i \geq 0,\ i=1,2;\ a_i \geq 0,\ i=1,3,4$

After "Phase One" is completed, i.e., all artificial variables are removed from the basis, then we discard the Phase One objective and the artificial variables, and use the current basic solution as the initial basic feasible solution for "Phase Two", which optimizes the original objective.

-Z	X_1	X_2	X_3	X_4	S_1	S_2	rhs
1	3	2	¯1	4	0	0	0
0	¯1	1	¯4	2	¯1	0	4
0	3	1	¯2	0	0	1	6
0	0	¯1	0	1	0	0	1
0	¯1	1	¯1	0	0	0	0

We add Phase-1 row with –W and artificial variables:

⇓

-W	-Z	X_1	X_2	X_3	X_4	S_1	S_2	a_1	a_3	a_4	rhs
1	0	0	0	0	0	0	0	1	1	1	0
0	1	3	2	¯1	4	0	0	0	0	0	0
0	0	¯1	1	¯4	2	¯1	0	1	0	0	4
0	0	3	1	¯2	0	0	1	0	0	0	6
0	0	0	¯1	0	1	0	0	0	1	0	1
0	0	¯1	1	¯1	0	0	0	0	0	1	0

−W	−Z	X_1	X_2	X_3	X_4	S_1	S_2	a_1	a_3	a_4	rhs
1	0	0	0	0	0	0	0	1	1	1	0
0	1	3	2	¯1	4	0	0	0	0	0	0
0	0	¯1	1	¯4	2	¯1	0	1	0	0	4
0	0	3	1	¯2	0	0	1	0	0	0	6
0	0	0	¯1	0	1	0	0	0	1	0	1
0	0	¯1	1	¯1	0	0	0	0	0	1	0

We choose −W, −Z, a_1 , S_2, a_3 , and a_4 as the initial variables

⇓

−W	−Z	X_1	X_2	X_3	X_4	S_1	S_2	a_1	a_3	a_4	rhs
1	0	2	¯1	5	¯3	1	0	0	0	0	¯5
0	1	3	2	¯1	4	0	0	0	0	0	0
0	0	¯1	1	¯4	2	¯1	0	1	0	0	4
0	0	3	1	¯2	0	0	1	0	0	0	6
0	0	0	¯1	0	1	0	0	0	1	0	1
0	0	¯1	1	¯1	0	0	0	0	0	1	0

(Pivoting is required to eliminate the artificial variables from first row)

-W	-Z	X_1	X_2	X_3	X_4	S_1	S_2	a_1	a_3	a_4	rhs
1	0	2	¯1	5	¯3	1	0	0	0	0	¯5
0	1	3	2	¯1	4	0	0	0	0	0	0
0	0	¯1	1	¯4	2	¯1	0	1	0	0	4
0	0	3	1	¯2	0	0	1	0	0	0	6
0	0	0	¯1	0	1	0	0	0	1	0	1
0	0	¯1	1	¯1	0	0	0	0	0	1	0

⇓

-W	-Z	X_1	X_2	X_3	X_4	S_1	S_2	a_1	a_3	a_4	rhs
1	0	1	0	4	¯3	1	0	0	0	1	¯5
0	1	5	0	1	4	0	0	0	0	¯2	0
0	0	0	0	¯3	2	¯1	0	1	0	¯1	4
0	0	4	0	¯1	0	0	1	0	0	¯1	6
0	0	¯1	0	¯1	1	0	0	0	1	1	1
0	0	¯1	1	¯1	0	0	0	0	0	1	0

Since we are minimizing W, we select X_2 or X_4 to enter the basis.

Let's choose X_2.

(Note that minimum ratio in test is zero in last row.)

-W	-Z	X_1	X_2	X_3	X_4	S_1	S_2	a_1	a_3	a_4	rhs
1	0	1	0	4	¯3	1	0	0	0	1	¯5
0	1	5	0	1	4	0	0	0	0	¯2	0
0	0	0	0	¯3	2	¯1	0	1	0	¯1	4
0	0	4	0	¯1	0	0	1	0	0	¯1	6
0	0	¯1	0	¯1	①	0	0	0	1	1	1
0	0	¯1	1	¯1	0	0	0	0	0	1	0

Next we enter X_4 into the basis, replacing a_3

⇓

-W	-Z	X_1	X_2	X_3	X_4	S_1	S_2	a_1	a_3	a_4	rhs
1	0	¯2	0	1	0	1	0	0	3	4	¯2
0	1	9	0	5	0	0	0	0	¯4	¯6	¯4
0	0	2	0	¯1	0	¯1	0	1	¯2	¯3	2
0	0	4	0	¯1	0	0	1	0	0	¯1	6
0	0	¯1	0	¯1	1	0	0	0	1	1	1
0	0	¯1	1	¯1	0	0	0	0	0	1	0

-W	-Z	X_1	X_2	X_3	X_4	S_1	S_2	a_1	a_3	a_4	rhs
1	0	¯2	0	1	0	1	0	0	3	4	¯2
0	1	9	0	5	0	0	0	0	¯4	¯6	¯4
0	0	2	0	¯1	0	¯1	0	1	¯2	¯3	2
0	0	4	0	¯1	0	0	1	0	0	¯1	6
0	0	¯1	0	¯1	1	0	0	0	1	1	1
0	0	¯1	1	¯1	0	0	0	0	0	1	0

Finally, X_1
enters the
basis,
replacing a_1

⇓

-W	-Z	X_1	X_2	X_3	X_4	S_1	S_2	a_1	a_3	a_4	rhs
1	0	0	0	0	0	0	0	1	1	1	0
0	1	0	0	9.5	0	4.5	0	¯4.5	5	7.5	¯13
0	0	1	0	¯0.5	0	¯0.5	0	0.5	¯1	¯1.5	1
0	0	0	0	1	0	2	1	¯2	4	5	2
0	0	0	0	¯1.5	1	¯0.5	0	0.5	0	¯0.5	2
0	0	0	1	¯1.5	0	¯0.5	0	0.5	¯1	¯0.5	1

All
artificial
variables
are now
nonbasic
(=zero)!

$-W$	$-Z$	X_1	X_2	X_3	X_4	S_1	S_2	a_1	a_3	a_4	rhs
1	0	0	0	0	0	0	0	1	1	1	0
0	1	0	0	9.5	0	4.5	0	−4.5	5	7.5	−13
0	0	1	0	−0.5	0	−0.5	0	0.5	−1	−1.5	1
0	0	0	0	1	0	2	1	−2	4	5	2
0	0	0	0	−1.5	1	−0.5	0	0.5	0	−0.5	2
0	0	0	1	−1.5	0	−0.5	0	0.5	−1	−0.5	1

⇓

$-Z$	X_1	X_2	X_3	X_4	S_1	S_2	rhs
1	0	0	9.5	0	4.5	0	−13
0	1	0	−0.5	0	−0.5	0	1
0	0	0	1	0	2	1	2
0	0	0	−1.5	1	−0.5	0	2
0	0	1	−1.5	0	−0.5	0	1

We can now drop the Phase One objective row and the artificial variables from the tableau

-Z	X_1	X_2	X_3	X_4	S_1	S_2	rhs
1	0	0	9.5	0	4.5	0	¯13
0	1	0	¯0.5	0	¯0.5	0	1
0	0	0	①	0	2	1	2
0	0	0	¯1.5	1	¯0.5	0	2
0	0	1	¯1.5	0	¯0.5	0	1

⇓

-Z	X_1	X_2	X_3	X_4	S_1	S_2	rhs
1	0	0	0	0	¯14.5	¯9.5	¯32
0	1	0	0	0	0.5	0.5	2
0	0	0	1	0	2	1	2
0	0	0	0	1	2.5	1.5	5
0	0	1	0	0	2.5	1.5	4

We now have an initial basic feasible solution for the original problem.
We begin "Phase Two", which optimizes the original objective.

-Z	X_1	X_2	X_3	X_4	S_1	S_2	rhs
1	0	0	0	0	¯14.5	¯9.5	¯32
0	1	0	0	0	0.5	0.5	2
0	0	0	1	0	2	1	2
0	0	0	0	1	2.5	1.5	5
0	0	1	0	0	2.5	1.5	4

This is the optimal tableau for Phase Two, i.e., the optimal solution is

$$\begin{cases} Z = 32, \\ X_1 = 2,\ X_2 = 4,\ X_3 = 2,\ X_4 = 5 \\ S_1 = S_2 = 0 \end{cases}$$