Quidditch is a wizarding game played with two teams of 7 players each - two beaters, three chasers, a keeper and a seeker. The game is played with 3 different balls - two bludgers, three quaffles and a golden snitch. The bludgers are to be avoided, the quaffles are used for scoring, and the game is over when the golden snitch is caught by the seeker.

Consider the scenario when Fred and George (the Weasley twins), two beaters on the Gryffindor, are going after a bludger. Suppose that there are several obstacles in their way, such as the announcer's stand, the spectators' stands, and possibly other players from the opposing team. The table below shows the fraction of damage absorbed by each obstacle if one of the twins should fly through it.

Data for Quidditch Game problem

	Fraction of Contact Force Absorbed by Area (average)		
Area	**(Fred) 1**	**2 (George)**	**Restriction on Total Sustainable Contact**
1 (spectators' stand)	0.4	0.5	minimize
2 (announcer's stand)	0.3	0.1	□ 2.7
3 (opposing player)	0.5	0.5	= 6
4 (bludger)	0.6	0.4	≥ 6

Corner-point feasible/infeasible solutions.

Property: For an LP with *n* decision variables, each **corner-point solution** lies at the intersection of *n* constraint boundaries.

Definition: For any LP with *n* decision variables, two CPF solutions are **adjacent** to each other if they share $n - 1$ constraint boundaries. The two adjacent CPF solutions are connected by an **edge** of the feasible region.

Example:

The COMET TRADING CO.

$$
\begin{aligned}
\text{Max} \quad & Z = 30x_1 + 15x_2 \\
\text{s.t.} \quad & x_1 \leq 4 \\
& 2x_2 \leq 12 \\
& 3x_1 + 2x_2 \leq 18 \\
& x_1 \geq 0, \quad x_2 \geq 0
\end{aligned}
$$

Theorem: Consider any LP that possesses at least one optimal solution. If a CPF solution has no adjacent CPF solutions that are better (as measured by Z), then it must be an **optimal solution**.

Geometry of the Simplex Method

Initialization:	Choose the origin as the initial CPF solution
Step 1:	If current CPF solution is optimal, DONE.
Step 2:	Otherwise, consider all adjacent CPF solutions. Select the one that increases Z at a faster rate. Return to Step 1.

Apply the steps to the Comet Trading Co. example.

1. The simplex method focuses solely on CPF solutions

2. The simplex method is an iterative algorithm

3. Whenever possible, the Simplex Method starts with the *origin* as the initial CPF solution.

4. Only adjacent CPF solutions are considered at each iteration. Thus, the entire path to reach an optimal solution is along the *edges* of the feasible region.

5. The simplex method identifies the *rate of improvement* in Z that would be obtained by moving along the edge. Among the edges with *positive* improvement, it chooses to move along the one with the *largest* rate of improvement.

6. If none of the edges give a positive rate of improvement in Z, then the current CPF solution is optimal.

Augmented Form:

Slack variables	If a slack variable equals 0 in the current solution, then this solution lies on the constraint boundary for the corresponding functional constraint.
Augmented solution	A solution for the original variables, augmented by the corresponding values of the *slack variables*.
Basic solution	An augmented corner-point solution.
Basic feasible solution	An augmented CPF solution.

Basic Solution:

Variables are either **basic** or **non-basic**. The number of basic variables is equal to the number of constraints. Non-basic variables are set to 0. Values of basic variables are solutions to the system of equations.

If all basic variables are non-negative, solution is a **BFS**.

Two BFS are **adjacent** if all but one of their nonbasic variables are the same.