ME – VII SEM

Course Name- Mechanical Vibrations

U-I, Fundamentals of Vibrations

Oscillating Motions :

- The study of vibrations is concerned with the oscillating motion of elastic bodies and the force associated with them.
- All bodies possessing mass and elasticity are capable of vibrations.
- Most engineering machines and structures experience vibrations to some degree and their design generally requires consideration of their oscillatory motions.
- Oscillatory systems can be broadly characterized as linear or nonlinear.
- **Linear systems :**

  The principle of superposition holds

  - Mathematical technique available for their analysis are well developed.

FUNDAMENTALS OF VIBRATIONS

- **Nonlinear systems :**
  The principle of superposition doesn't hold
  - The technique for the analysis of the nonlinear systems are under development (or less well known) and difficult to apply.
- All systems tend to become nonlinear with increasing amplitudes of oscillations.
- There are two general classes of vibrations free and forced.
- ***Free vibration*** takes place when a system oscillates under the action of forces inherent in the system itself due to initial disturbance, and when the externally applied forces are absent.
- The system under free vibration will vibrate at *one or more* of its *natural frequencies*, which are *properties of the dynamical system*, established by its mass and stiffness distribution
- **Forced vibration** takes place under the excitation of external forces is called forced vibration.

FREE AND FORCED VIBRATIONS

- If excitation is harmonic, the system is forced to vibrate at ***excitation frequency*** . If the frequency of excitation coincide with one of the natural frequencies of the system, a condition of ***resonance*** is encountered and dangerously large oscillations may result, which results in failure of major structures, i.e., bridges, buildings, or airplane wings etc.
- Thus calculation of natural frequencies is of major importance in the study of vibrations.
- Because of friction & other resistances vibrating systems are subjected to ***damping*** to some degree due to dissipation of energy.
- Damping has very ***little effect on natural frequency*** of the system, and hence the calculations for natural frequencies are generally made on the basis of no damping.
- Damping is of great importance in ***limiting the amplitude*** of oscillation at resonance.

Degrees of Freedom (dof)

- The number of independent co-ordinates required to describe the motion of a system is termed as degrees of freedom.
- *For example*
  Particle - ***3 dof*** *(positions)*
  Rigid body ***-6 dof***
  *(**3**-positions and **3-**orientations)*
  Continuous elastic body *-* ***infinite dof***
  (three positions to each particle of the body).
- If part of such continuous elastic bodies may be assumed to be rigid (or lumped) and the system may be considered to be dynamically equivalent to one having finite dof (or lumped mass systems).
- Large number of vibration problems can be analyzed with sufficient accuracy by reducing the system to one having a few dof.

VIBRATION MEASUREMENT TERMINOLOGY

- **Peak value :** Indicates the maximum response of a vibrating part. It also places a limitation on the space requirement.
- **Average value :** Indicates a steady or static value (somewhat like the DC level of an electrical current) and it is defined as

$$\bar{x} = \lim_{T \to \infty} (1/T) \int_0^T x(t)\,dt \qquad (1.1)$$

where *x*(*t*) is the displacement, and *T* is the time span (for example time period)

- For a complete cycle of sine wave,

$$x(t) = A \sin \omega t : \bar{x} = \frac{1}{2\pi} \int_0^{2\pi} A \sin \omega t\,dt = \frac{A}{2\pi} \left[\frac{\cos \omega t}{\omega} \right]_0^{2\pi} = \frac{A}{2\pi\omega} [1.0 - 1.0] = 0 \qquad (1.2)$$

MEAN AND MEAN SQUARE VALUE

- For half cycle of the sine wave :

$$\bar{x} = 1/\pi \int_0^{\pi} A \sin \omega t dt = \frac{A}{\pi}\left[\frac{\cos \omega t}{\omega}\right]_0^{\pi} = \frac{A}{\pi\omega}\left[1-(-1)\right] = 2A/\pi = 0.637A$$

where *A* is the amplitude of the displacement.

- **Mean square value :** Square of the displacement generally is associated with the energy of the vibration for which the mean square value is a measure and is defined as

$$\langle x^2 \rangle = \lim_{T\to\infty} (1/T) \int_0^T x^2(t) dt$$

For a complete cycle of sine wave $x(t) = A \sin \omega t$ we have

$$\langle x^2 \rangle = \lim_{T\to\infty}\left(\frac{A^2}{T}\right)\int_0^T \frac{(1-\cos 2\omega t)}{2} dt = \lim_{T\to\infty}\left[\frac{A^2}{2} - \frac{A^2}{2T}\frac{\sin 2\omega t}{2\omega}\Big|_0^T\right] = \frac{A^2}{2} - \frac{A^2}{2}\lim_{T\to\infty}\left(\frac{\sin 2\omega T}{2\omega T}\right) = \frac{A^2}{2}$$

ROOT MEAN SQUARE VALUE (RMS)

- **Root mean square value (rms) :** This is the square root of the mean square value.
- For example **:** for a complete sine wave

$$x_{rms} = \left[\langle x^2 \rangle\right]^{\frac{1}{2}} = \left[\frac{A}{2}\right]^{\frac{1}{2}} = 0.707A$$

DECIBEL (D_B)

- **:** It is a unit of the relative measurement of the vibration and sound. It is defined in terms of a power ratio:

$$D_b = 10\log_{10}(p_1 / p_2)$$

where *p* is the power, since power is proportion to square of amplitude of vibrations or voltages, which is easily measurable, hence

$$D_b = 10\log_{10}(A_1 / A_2)^2 = 20\log_{10}(A_1 / A_2) \qquad (1.6)$$

- where *A* is the amplitude. For amplitude gain of 5, the decibel has a gain of

$$D_b = +20\log_{10} 5 = +14 \qquad (1.7)$$

- In vibrations *decibel* is used to express relative measured values of displacements, velocities and accelerations.

- $$D_b = 20\log_{10}(z / z_0) \qquad (1.8)$$

DECIBEL (D_B)

- where *z* is the quantity under consideration (e.g. displacement, velocity or acceleration), z_0 is the reference value (e.g. for velocity $v_0 = 10^{-8}\ m/\sec$ and acceleration $a_0 = 9.81 \times 10^{-6}\ m/\sec^2$) .
- For example $D_b = 20$ means 10 times the reference value (i.e. $D_b = 20 = 20\log_{10} 10$), and $40 D_b$ means 100 times the reference value (i.e. $D_b = 40\log_{10} 10 = 20\log_{10} 10^2$

VIBRATION TERMINOLOGY

Oscillatory Motion

- Repeat itself regularly for example pendulum of a wall clock
- Display irregularity for example earthquake
- **Periodic Motion :** This motion repeats at equal interval of time *T*.
- **Period of Oscillatory :** The time taken for one repetition is called period.
- **Frequency -** $f = \frac{1}{T}$ It is defined reciprocal of time period.
- The condition of periodic motion is

$$x(t+T) = x(t) \qquad (1.10)$$

where motion is designated by time function *x*(*t*) .

HARMONIC MOTION

Harmonic motion

- Simplest form of periodic motion is harmonic motion and it is called simple harmonic motion (SHM). It can be expressed as

$$x = A\sin 2\pi\frac{t}{T} \qquad (1.11)$$

where *A* is the amplitude of motion.

Harmonic motion is often represented by projection on line of a point that is moving on a circle at constant speed.

n, *t* is the time instant and *T* is the period of motion.

SIMPLE HARMONIC MOTION

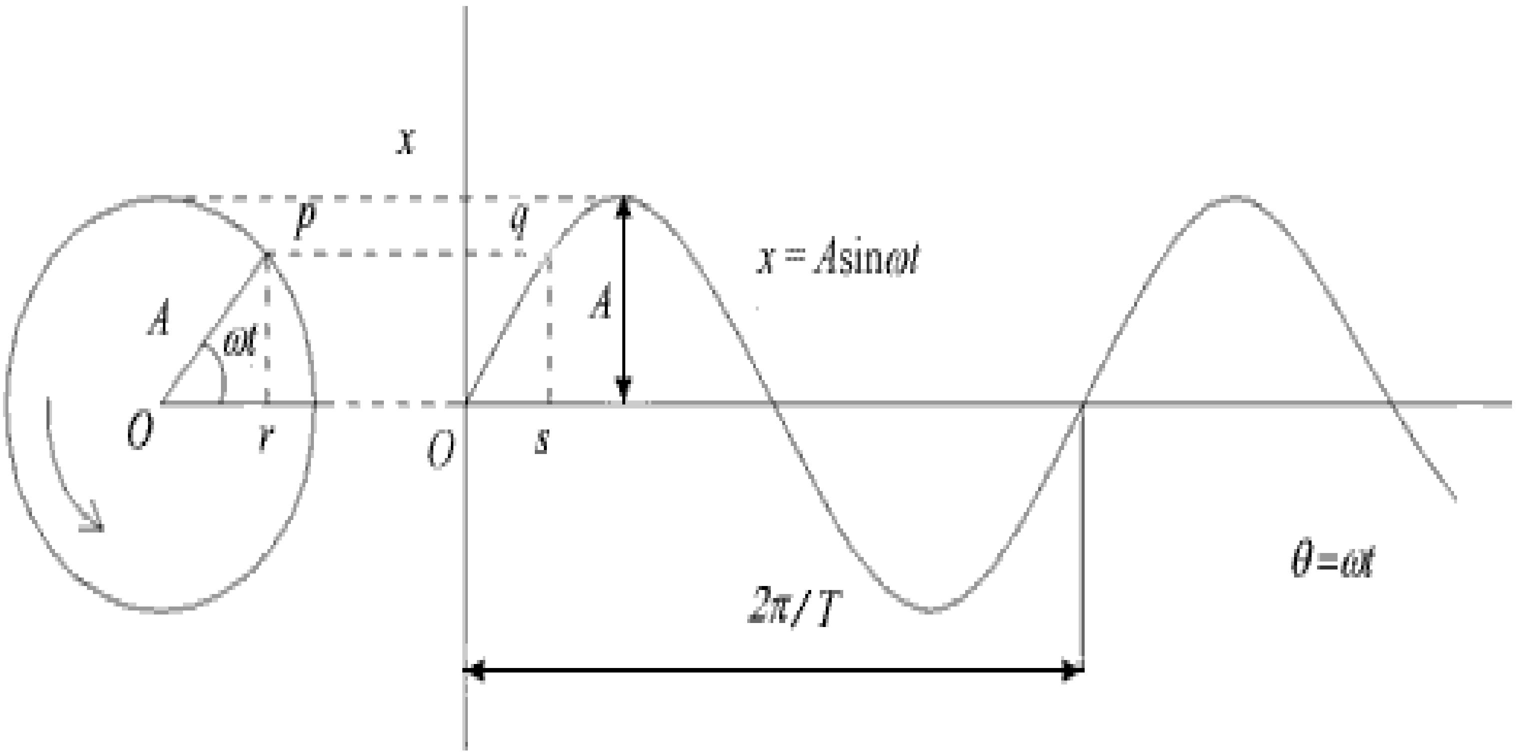


Figure 1.1: The Simple Harmonic Motion

SIMPLE HARMONIC MOTION

From Figure 1.1 , we have

$$x = pr = qs = A\sin\omega t$$

where *x* is the displacement and ω is the circular frequency in rad/sec.

$$\omega = \frac{2\pi}{T} = 2\pi f \qquad (1.12)$$

where *T* is the period (sec) and *f* is the frequency (cycle/sec) of the harmonic motion.

- The SHM repeats itself in 2π radians.
- Displacement can be expressed as $x = A\sin\omega t$ (1.13)

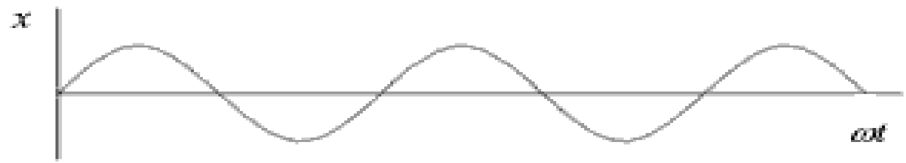
  velocity can be expressed as $\dot{x} = \omega A\cos\omega t = \omega A\sin(\omega t + \pi/2)$ (1.14)

  acceleration can be written as $\ddot{x} = -\omega^2 A\sin\omega t = \omega^2 A\sin(\omega t + \pi)$ (1.15)

DISPACEMENT,VELOCITY AND ACCELERATION

- Equations (1.12) to (1.14) are plotted in Figure 1.2

(a)



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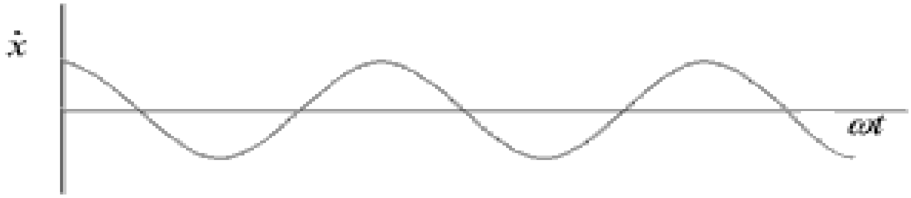

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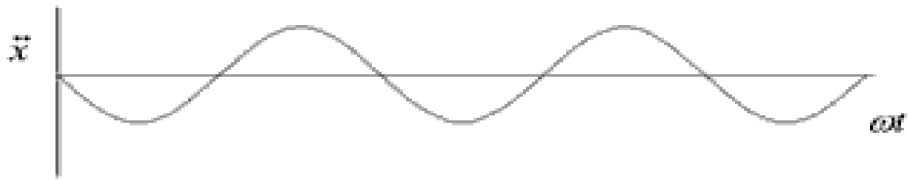


- Figure 1.2 : Variation of displacement, velocity and acceleration with nondimensional time.

SIMPLE HARMONIC MOTION

- It should be noted from equations (1.12-1.14) that when displacement is a SHM the velocity and acceleration are also harmonic motion with same frequency of oscillation (i.e. displacement). However, lead in phases occurs by 90^0 and 180^0 respectively with respect to the displacement as shown in Figure 1.2.
- From equations (1.12) and (1.14) we find

(1.15)

- In harmonic motion acce $\ddot{x} = -\omega^2 x$ oportional to the displacement and is directed towards the origin.
- The Newton's second law of motion states that the acceleration is proportional to the force. Hence for a spring (linear), we write

(1.16)

- where F_S is the $F_s = kx$ orce and k is the stiffness of the spring. It executes harmonic motion as force is proportional to the displacement. (animation)

EXPONENTIAL FORM

- **Exponential form :** From Euler's equation, we have

$$e^{j\theta} = \cos\theta + j\sin\theta \qquad (1.17)$$

- A rotating vector as shown in Figure 1.3 can be expressed as

$$z = Ae^{j\theta} = Ae^{j\omega t} \qquad (1.18)$$

- where *A* is the magnitude, θ is the orientation and *j* = $\sqrt{-1}$ the imaginary number. Equation (1.18) can also be written as

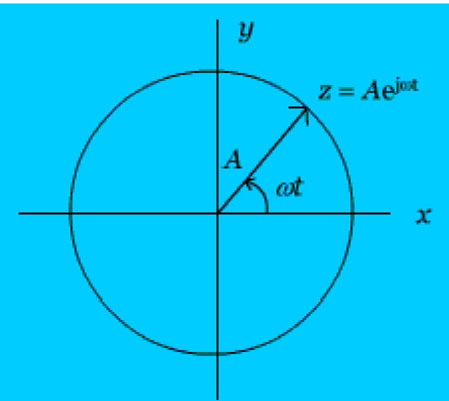
$$z = A\cos\omega t + jA\sin\omega t = x + jy \qquad (1.19)$$

- with

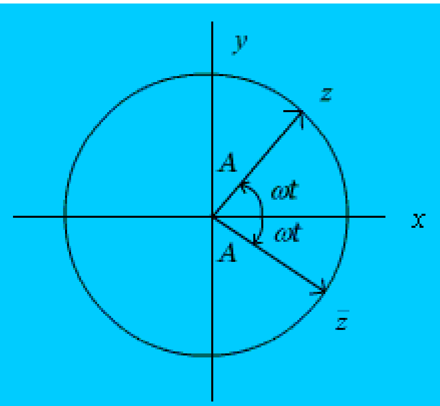
$$x = A\cos\omega t \quad \text{and} \quad y = A\sin\omega t \qquad (1.20)$$

where *z* is the complex sinusoid, *x* is the real component and *y* is the imaginary component.

ARGAND VECTOR DIAGRAM



Harmonic motion represented by a rotating vector
(Argand Vector Diagram)



Vector z and its conjugate $\bar{z}$

Differentiating equation (1.18) with respect to time gives

$$\dot{z} = j\omega A e^{j\omega t} \tag{1.21}$$

and

$$\ddot{z} = -\omega^2 A e^{j\omega t} = -\omega^2 z \tag{1.22}$$

From equation (1.19), we can write

$$x = \frac{1}{2}(z + \bar{z}) \tag{1.23}$$

where $\bar{z} = x - jy$

is the complex conjugate of *z* as shown Figure 1.4. We can also write

$$x = \mathrm{Re}(z) = \mathrm{Re}[Ae^{j\omega t}] = A\cos\omega t$$

where Re(*z*) is the real part of quantity *z*.

•The exponential form of the harmonic motion offers mathematical advantages over the trigonometric form.

FOURIER'S SERIES

- In this section, $f(x)$ denotes a function of the real variable x. This function is usually taken to be periodic, of period 2π, which is to say that $f(x + 2\pi) = f(x)$, for all real numbers x.
- **Fourier's formula for 2π-periodic functions using sines and cosines**
- For a periodic function $f(x)$ that is integrable on $[-\pi, \pi]$, the numbers

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)\,dx, \quad n \geq 0$$

and

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)\,dx, \quad n \geq 1$$

are called the Fourier coefficients of f. One introduces the *partial sums of the Fourier series* for f, often denoted by

$$(S_N f)(x) = \frac{a_0}{2} + \sum_{n=1}^{N} [a_n \cos(nx) + b_n \sin(nx)], \quad N \geq 0.$$

FOURIER'S SERIES

- The partial sums for f are trigonometric polynomials. One expects that the functions *SN* f approximate the function f, and that the approximation improves as *N* tends to infinity. The infinite sum

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \cos(nx) + b_n \sin(nx)]$$

- is called the **Fourier series** of f.

FOURIER'S SERIES

- The Fourier series does not always converge, and even when it does converge for a specific value $x0$ of x, the sum of the series at $x0$ may differ from the value $f(x0)$ of the function.
- It is one of the main questions in harmonic analysis to decide when Fourier series converge, and when the sum is equal to the original function.
- If a function is square-integrable on the interval $[-\pi, \pi]$, then the Fourier series converges to the function at *almost every* point.
- In engineering applications, the Fourier series is generally presumed to converge everywhere except at discontinuities, since the functions encountered in engineering are more well behaved than the ones that mathematicians can provide as counter-examples to this presumption.