Combined Loadings

Thin-Walled Pressure Vessels

Stress caused by Combined Loadings

Introduction

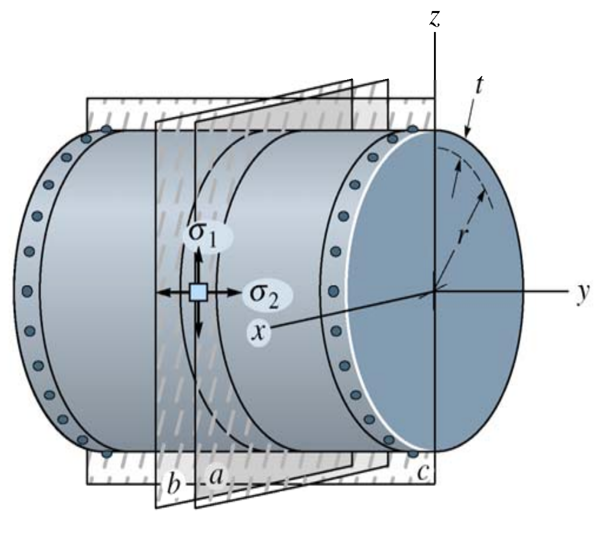
- There are numerous applications that require the use of containers for storage or transmission of gasses and fluids under high pressure.
- Two types of pressure vessels are thin-walled and thick-walled.
- The distinctions is based on the Hoop stress over the thickness.
  - Constant – thin-walled, r/t>10
  - Not constant – thick walled

Thin-Walled Pressure Vessels

- The most commonly used types of thin-walled pressure vessels are cylindrical and spherical.
- We will develop the stress equations for the case of cylindrical thin-walled pressure vessels and then extrapolate to spherical.

Cylindrical Vessel

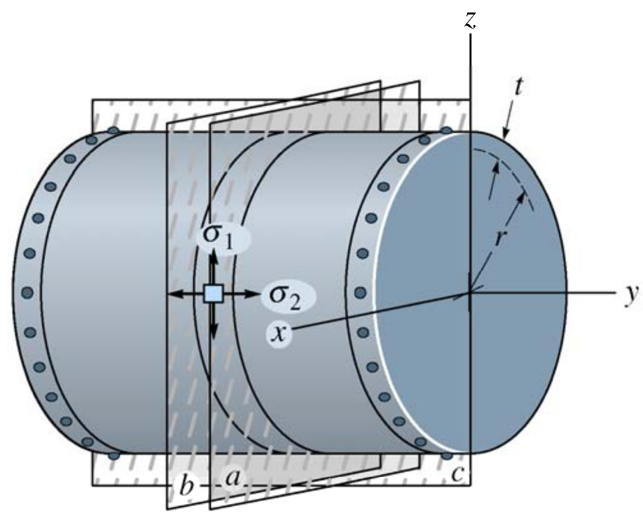
- Consider the thin-walled cylindrical vessel shown, which is subjected to internal fluid pressure p_1 and assumed to have closed ends.
- A result of this pressure, the circumference of the vessel will expand, causing tensile stress σ_1 tangent to the circumference called Hoop Stress.



(a)

Cylindrical Vessel

- Also, because of this internal pressure, longitudinal fibers in the vessel will tend to stretch, creating a tensile stress σ_2 called longitudinal or axial stress.
- These stresses are shown on a plane stress element of the outside surface.
  - One set of the elements planes are parallel to the axis of the cylinder and the other set perpendicular to this axis.



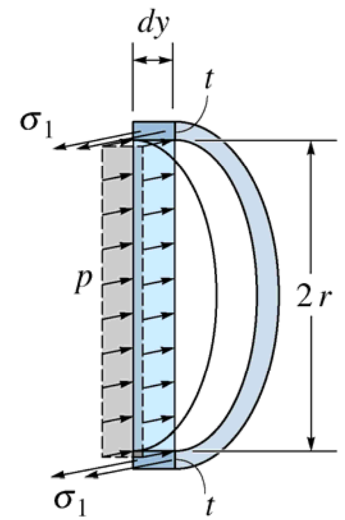
(a)

Cylindrical Vessel

- These stresses are applied to the surface of the vessel, and therefore do not experience shear stress.
- If the stress element were taken on the inner surface of the cylinder, it would be treated as a 3 dimensional element.

Cylindrical Vessel

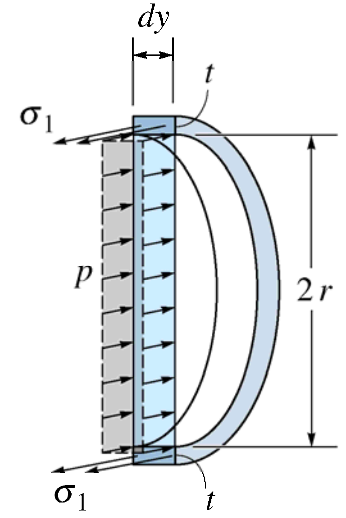
- Consider the FBD of a small portion of the cylinder as shown in 8-1b.
- We have sliced the cylinder into two halves along the x-z plane, then isolated a small segment from one of the two halves a distance dy apart.



(b)

Cylindrical Vessel

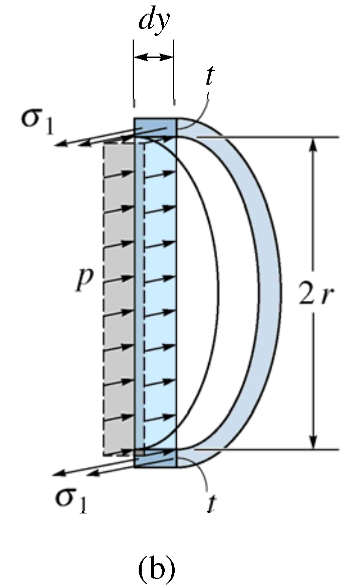
- The FBD is in equilibrium in the x-direction
  - Under the action of the pressure p
  - Under the action of the Hoop stress σ_1
  - Both uniformly distributed over tdy.



(b)

Cylindrical Vessel

- The total force produced in the x direction by the internal fluid pressure p, is the product of p and the projected area 2rdy.
- The resultant force produced by the Hoop stress σ_1, is the product of the σ_1 and the area tdy.
- Note there are two $\sigma_1$tdy acting on the FBD, one on the top and one on the bottom.



Cylindrical Vessel

- Then:

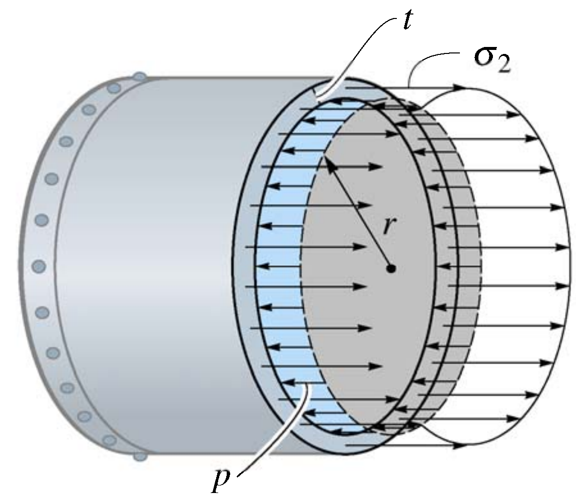
$$\sum F_x = 0 \Rightarrow 2\sigma_1(tdy) - p(2rdy) = 0$$

- Solving for the Hoop stress yields:

$$\sigma_1 = \text{tangential stress} = \frac{pr}{t}$$

Cylindrical Vessel

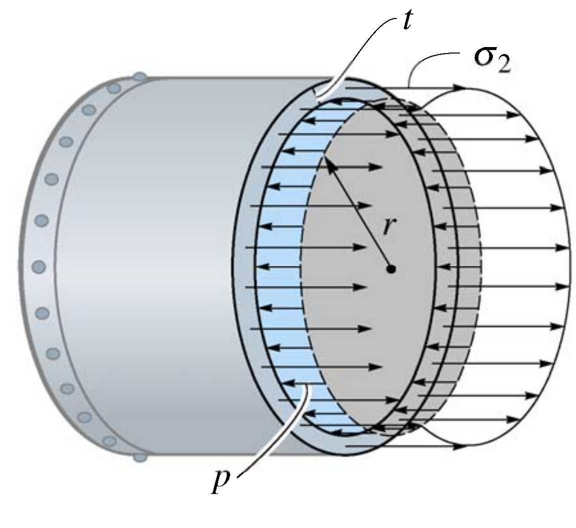
- The longitudinal stress σ_y or σ_1 is then obtained from the FBD of 8-1c.
- This FBD drawn by cutting the cylinder into two parts perpendicular to its axis and isolating one side.



(c)

Cylindrical Vessel

- Is in equilibrium in the y direction under the action of the fluid pressure p and stress σ_1
  - Which is assumed uniformly distributed over the annular area.



(c)

Cylindrical Vessel

- Then: (mean radius≈r (inner radius) (thin walled))

$$\sum F_y = 0 \Rightarrow \sigma_Z \left(2\pi rt\right) - p\left(\pi r^2\right) = 0$$

- Solving for the axial stress is:

$$\sigma_y = \sigma_2 = \frac{pr}{2t}$$

Cylindrical Vessel

- This says that the longitudinal stress is exactly ½ of the circumferential stress.
- These equations were derived on the basis of equilibrium and do not depend on the material being linearly elastic.
- The stresses calculated apply only away from the cylinder ends.
  - The constraint imposed by the cylinder ends complicates the stress state.

Cylindrical Vessel

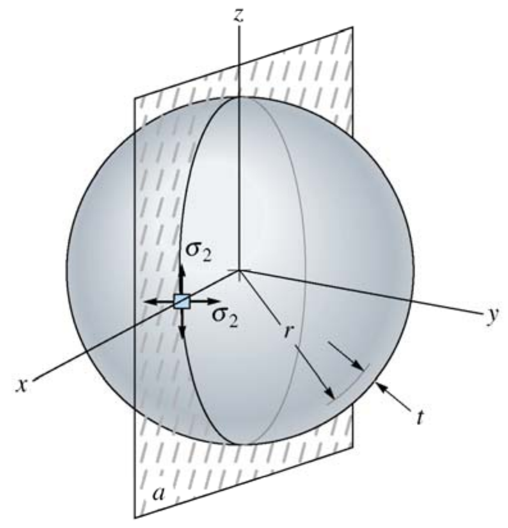
- Note that the pressure p is the the gage pressure
  - Difference between the total internal pressure and the external atmospheric pressure.
  - If the internal and external pressure are the same, no stresses are developed in the wall of the vessel.
  - Only the excess of internal pressure over external pressure has any effect on these stresses.

Spherical Vessels

- A sphere is the theoretically ideal shape for a vessel that resists internal pressure.
  - Think about a soap bubble.
- To determine the stresses in a spherical vessel, it is cut on the vertical axis and isolated from the other side.

Spherical Vessels

- Because of the symmetry of the vessel and its loadings figure b, the tensile stress is uniform around the circumference.
- We also know, because it is thin walled, that the stress is uniformly distributed across the thickness.
  - The accuracy becomes greater as the wall becomes thinner, and worse as the wall becomes thicker.



(a)

Spherical Vessels

- Equilibrium of forces in the horizontal direction yields equation 3.

$$\sigma_2 = \frac{pr}{2t}$$

- It is obvious from symmetry of a spherical shell that we would get the same equation for tensile stresses when the sphere is cut through the center in any direction.

The wall of a pressurized spherical vessel is subjected to uniform tensile stresses in all directions.

Spherical Vessels

- Stresses that act tangentially to the curved surface of a shell are called Membrane Stresses.
- They are called this because these are the only stresses that exist in true membranes, like soap films.

Comments

- Pressure vessels usually have openings in their walls (inlet and outlets for the fluid).
- They have fittings and supports that exert forces on the shell.
- These features result in non-uniformities in the stress distribution (stress concentrations).
  - These stress concentrations cannot be analyzed with the elementary formulas we have learned.
  - Other factors include corrosion, impact, temperature, etc.

Limitations on Thin-shell theory

- r/t ratio ≥ 10
- Internal pressure>external pressure (no inward buckling)
- Formulas apply only to the effects of internal pressure (no external loads, etc)
- Formulas valid throughout the wall of the vessel except near the points of stress concentrations.

Combined Loadings

- We have analyzed structural members subjected to a single type of loading.
  - Axially loaded bars
  - Shafts in torsion
  - Beams in bending
  - Pressure vessels
- For each type of loading we developed methods for finding stresses, strains, and deformations.

Combined Loadings

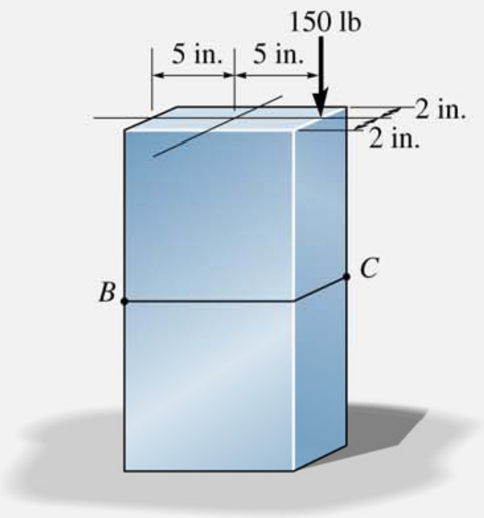
- In systems the members are required to resist more than one kind of loading.
  - Bending moment & axial forces
  - Pressure vessel supported as a beam
  - Shaft in torsion with a bending load
- These are called Combined Loadings.
- They occur in machines, buildings, vehicles, tools,etc.

Combined Loadings

- Structural members subjected to combined loadings can be analyzed by superposition.
- This is possible only under certain conditions, such as:
  - No interaction between various loads (the stresses & strains due to one load must not be affected by the presence of the other loads).
  - All rules as applied to loadings when equations were derived.

Combined Loadings Method of Analysis

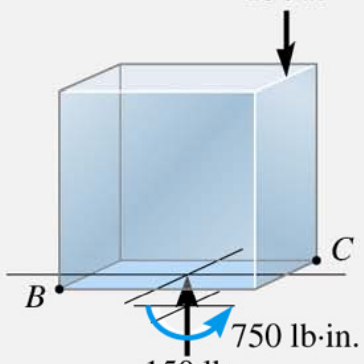
- Internal Loading
- Average Normal Stress
- Normal Force
- Shear Force
- Bending Moment
- Torsional Moment
- Thin-Walled Pressure Vessels
- Superposition



(a)

EXAMPLE 8–2

A force of 150 lb is applied to the edge of the member shown in Fig. 8–3*a*. Neglect the weight of the member and determine the state of stress at points *B* and *C*.



(b)

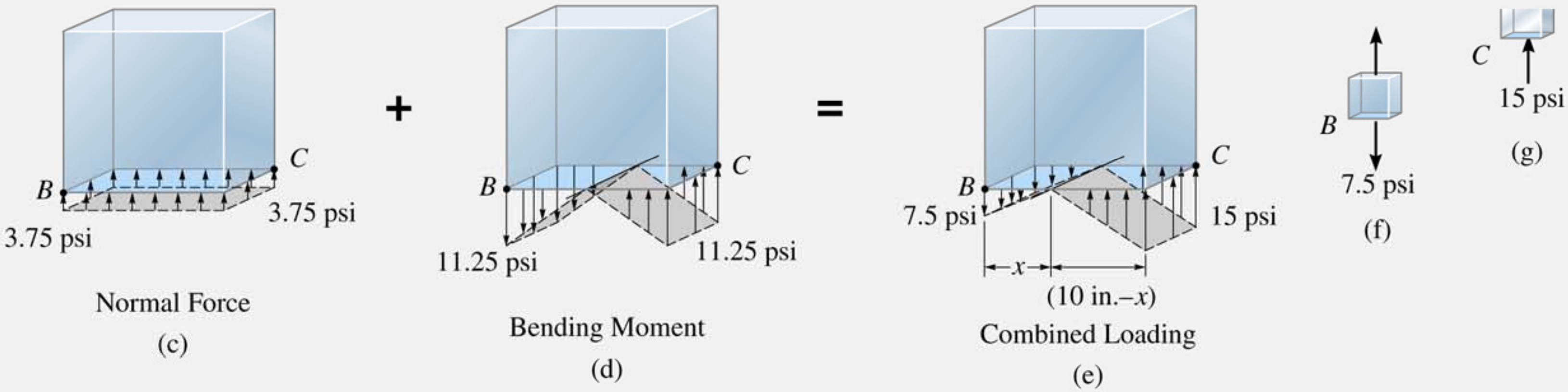
SOLUTION

Internal Loadings. The member is sectioned through *B* and *C*. For equilibrium at the section there must be an axial force of 150 lb acting through the centroid and a bending moment of 750 lb · in. about the centroidal or principal axis, Fig. 8–3*b*.

Stress Components.

Normal Force. The uniform normal-stress distribution due to the normal force is shown in Fig. 8–3*c*. Here

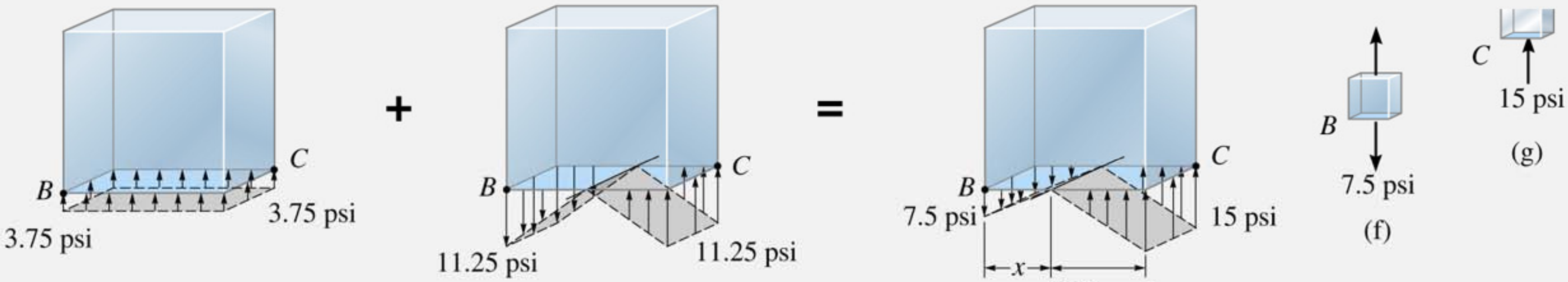
$$\sigma = \frac{P}{A} = \frac{150\text{ lb}}{(10\text{ in.})(4\text{ in.})} = 3.75\text{ psi}$$



Stress Components.

BENDING MOMENT. The normal-stress distribution due to the bending moment is shown in Fig. 8–3*d*. The maximum stress is

$$\sigma_{max} = \frac{Mc}{I} = \frac{750 \text{ lb} \cdot \text{in.}(5 \text{ in.})}{[\frac{1}{12}(4 \text{ in.})(10 \text{ in.})^3]} = 11.25 \text{ psi}$$



Normal Force (c) Bending Moment (d) Combined Loading (e) (f) (g)

Stress Components.

Superposition. If the above normal-stress distributions are added algebraically, the resultant stress distribution is shown in Fig. 8–3*e*. Although it is not needed here, the location of the line of zero stress can be determined by proportional triangles; i.e.,

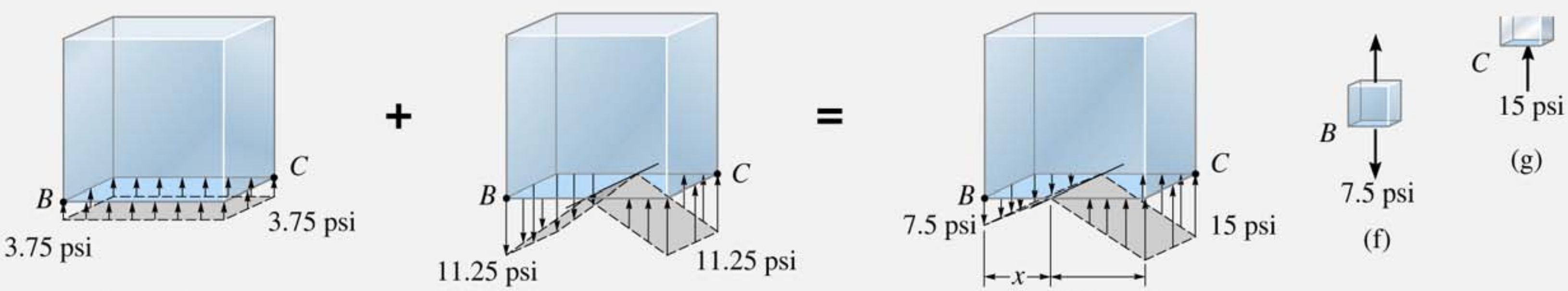
$$\frac{7.5 \text{ psi}}{x} = \frac{15 \text{ psi}}{(10 \text{ in.} - x)}$$

$$x = 3.33 \text{ in.}$$

Elements of material at B and C are subjected only to normal or *uniaxial stress* as shown in Fig. 8–3*f* and 8–3*g*. Hence,

$$\sigma_B = 7.5 \text{ psi} \quad \text{(tension)}$$ ***Ans.***

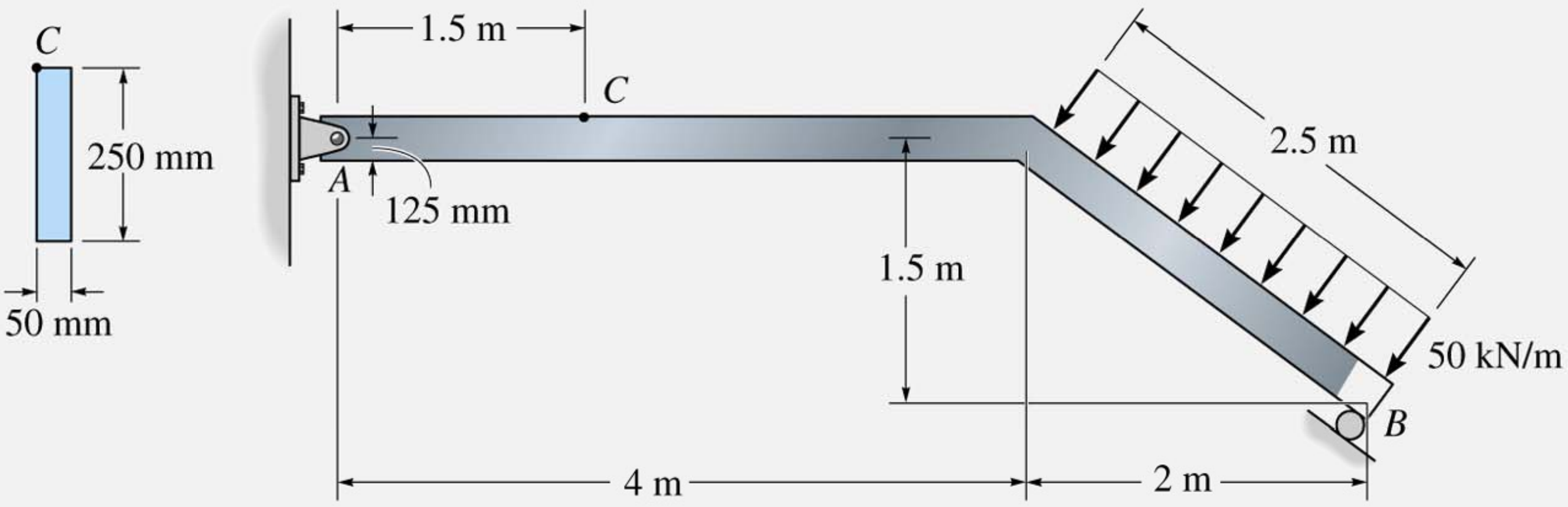
$$\sigma_C = 15 \text{ psi} \quad \text{(compression)}$$ ***Ans.***



Normal Force (c) Bending Moment (d) Combined Loading (e) (f) (g)

EXAMPLE 8-4

The member shown in Fig. 8–5*a* has a rectangular cross section. Determine the state of stress that the loading produces at point *C*.

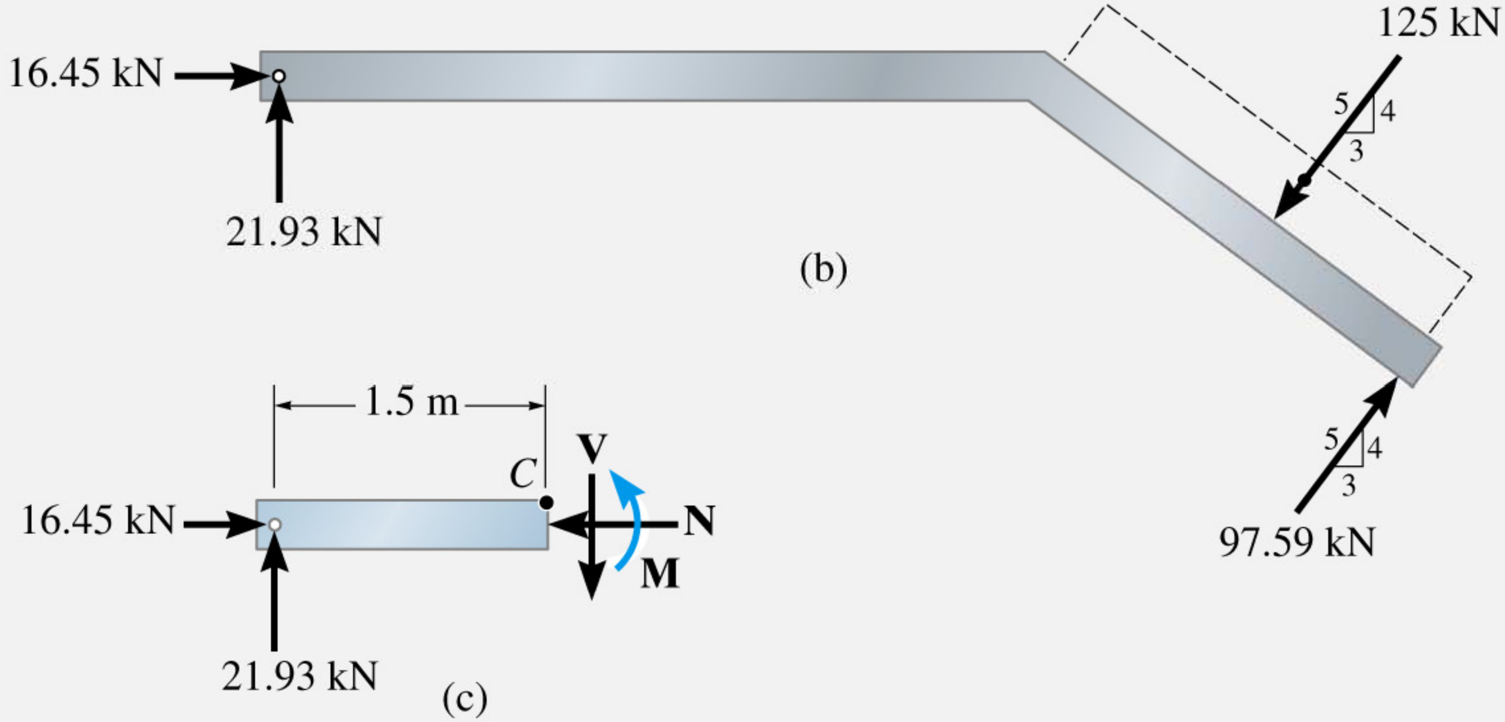


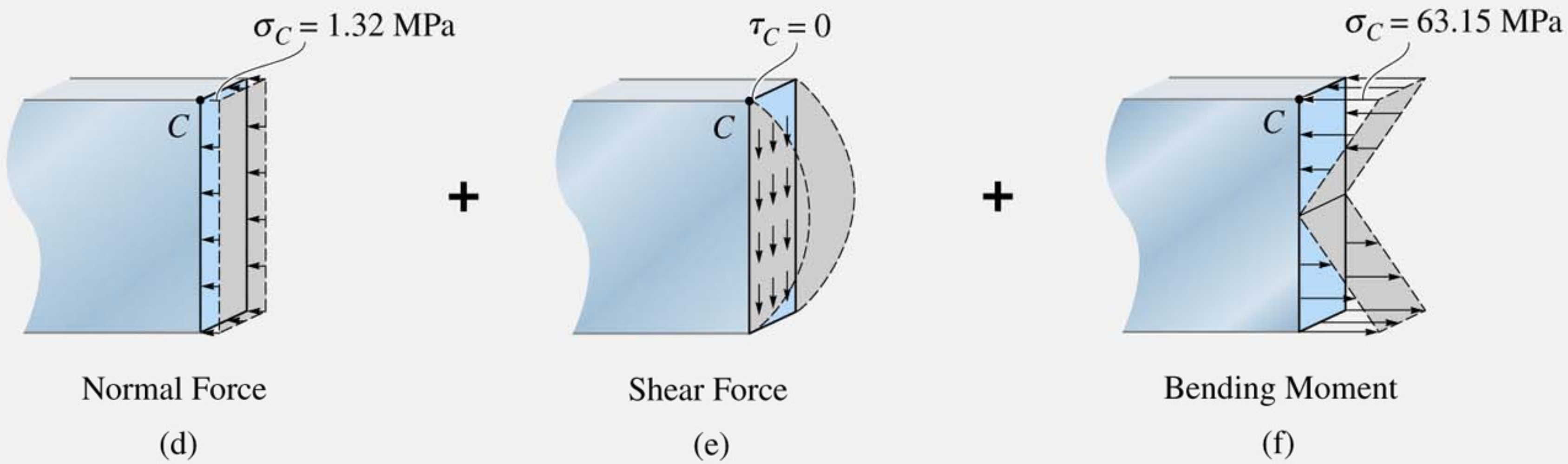
(a)

SOLUTION

Internal Loadings. The support reactions on the member have been determined and are shown in Fig. 8–5*b*. If the left segment *AC* of the member is considered, Fig. 8–5*c*, the resultant internal loadings at the section consist of a normal force, a shear force, and a bending moment. Solving,

$$N = 16.45\ \text{kN} \qquad V = 21.93\ \text{kN} \qquad M = 32.89\ \text{kN} \cdot \text{m}$$

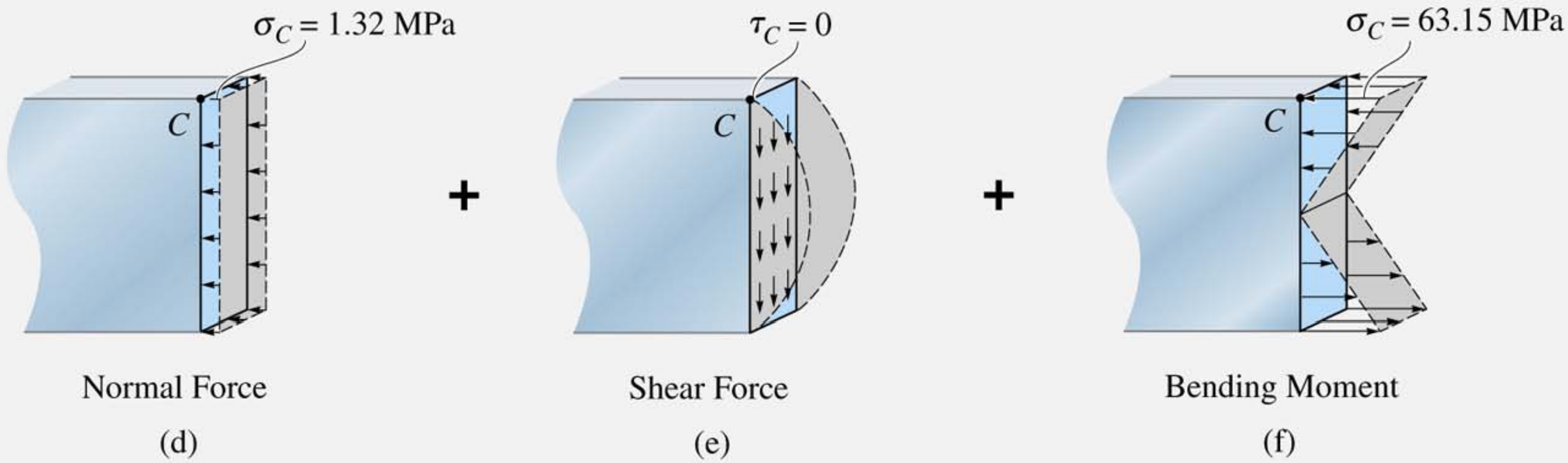




Stress Components.

NORMAL FORCE. The uniform normal-stress distribution acting over the cross section is produced by the normal force, Fig. 8–5*d*. At point *C*,

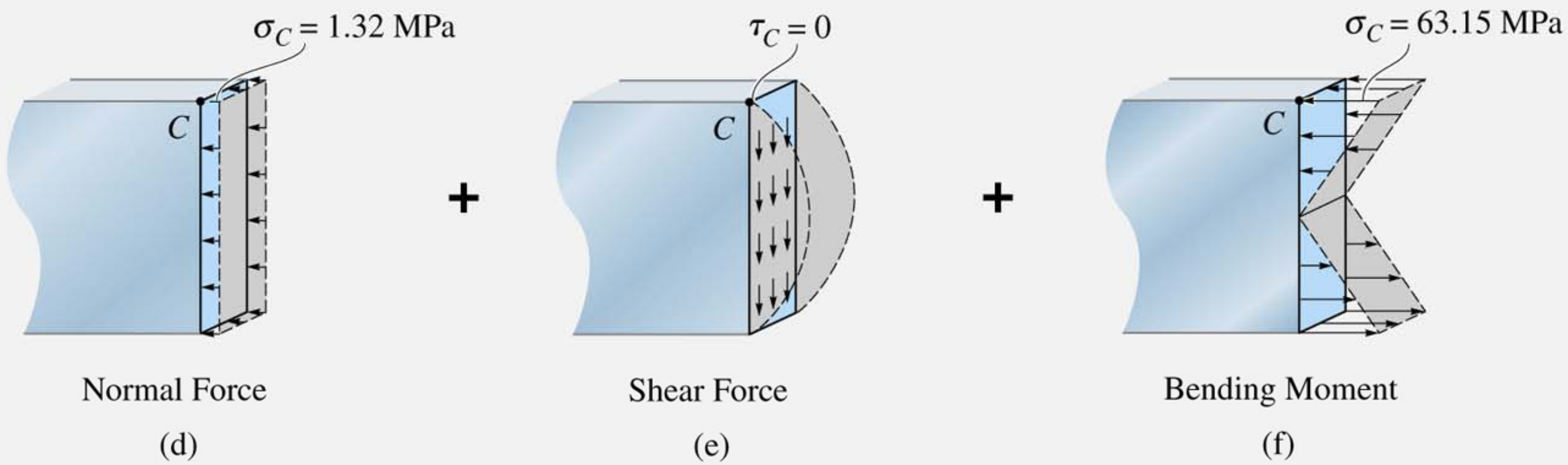
$$\sigma_C = \frac{P}{A} = \frac{16.45\text{ kN}}{(0.050\text{ m})(0.250\text{ m})} = 1.32\text{ MPa}$$



Stress Components.

SHEAR FORCE. Here the area $A' = 0$, since point C is located at the top of the member. Thus $Q = \bar{y}'A' = 0$ and for C, Fig. 8–5e, the shear stress

$$\tau_C = 0$$



Normal Force

(d)

Shear Force

(e)

Bending Moment

(f)

Stress Components.

Bending moment. Point C is located at $y = c = 125$ mm from the neutral axis, so the normal stress at C, Fig. 8–5*f*, is

$$\sigma_C = \frac{Mc}{I} = \frac{(32.89\ \text{kN} \cdot \text{m})(0.125\ \text{m})}{[\frac{1}{12}\,(0.050\ \text{m})(0.250)^3]} = 63.15\ \text{MPa}$$

Superposition. The shear stress is zero. Adding the normal stresses determined above gives a compressive stress at C having a value of

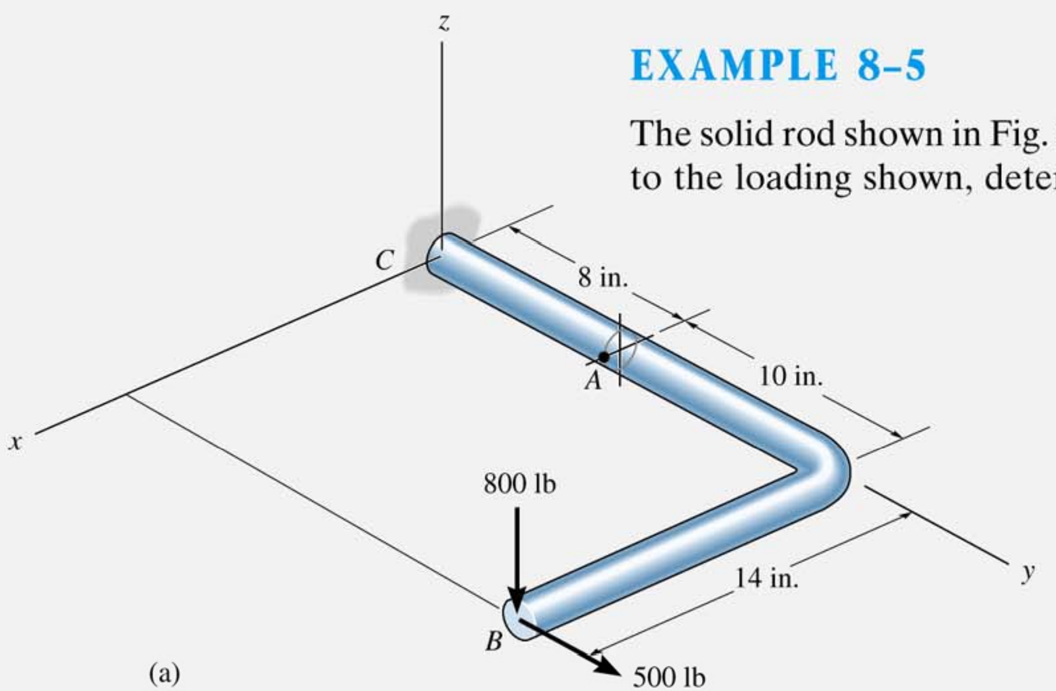
$$\sigma_C = 1.32 \text{ MPa} + 63.15 \text{ MPa} = 64.5 \text{ MPa} \qquad \textbf{\textit{Ans.}}$$

(g)

This result, acting on an element at C, is shown in Fig. 8–5*g*.

EXAMPLE 8-5

The solid rod shown in Fig. 8–6*a* has a radius of 0.75 in. If it is subjected to the loading shown, determine the state of stress at point *A*.



(a)

SOLUTION

Internal Loadings. The rod is sectioned through point *A*. Using the free-body diagram of segment *AB*, Fig. 8–6*b*, the resultant internal loadings can be determined from the six equations of equilibrium. Verify these results. The normal force (500 lb) and shear force (800 lb) must act through the centroid of the cross section and the bending-moment components (8000 lb · in. and 7000 lb · in.) are applied about centroidal (principal) axes. In order to better "visualize" the stress distributions due to each of these loadings, we will consider the *equal but opposite resultants* acting on *AC*, Fig. 8–6*c*.

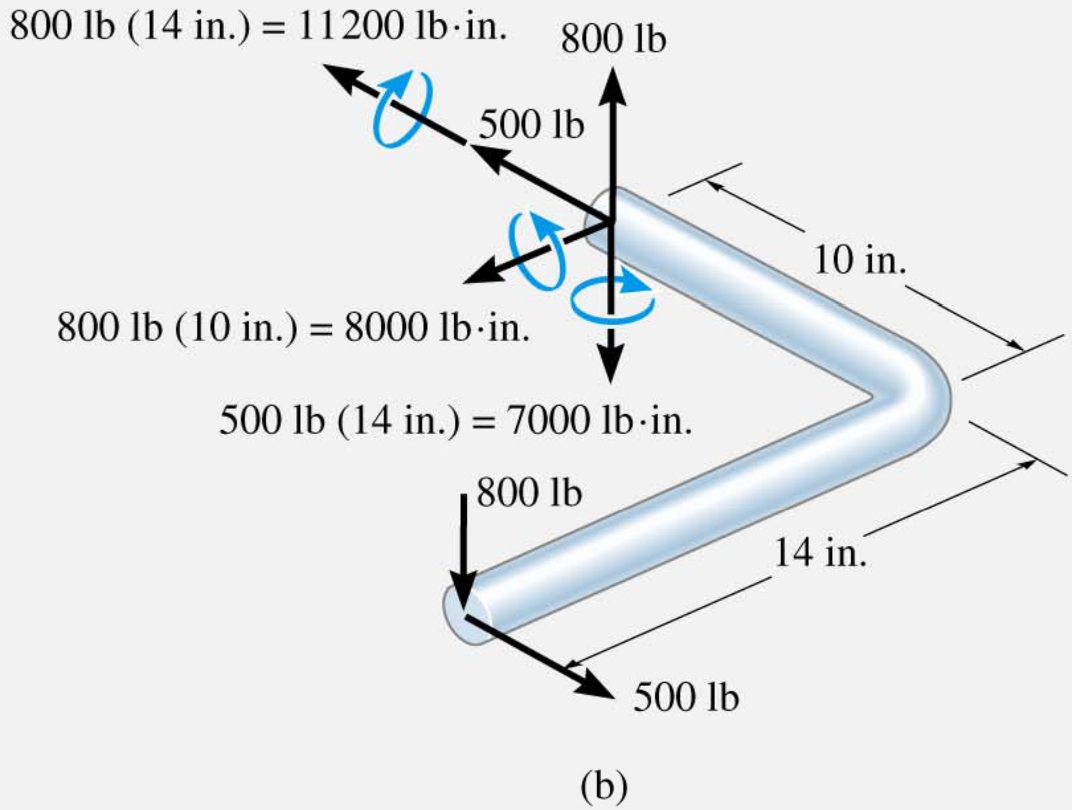
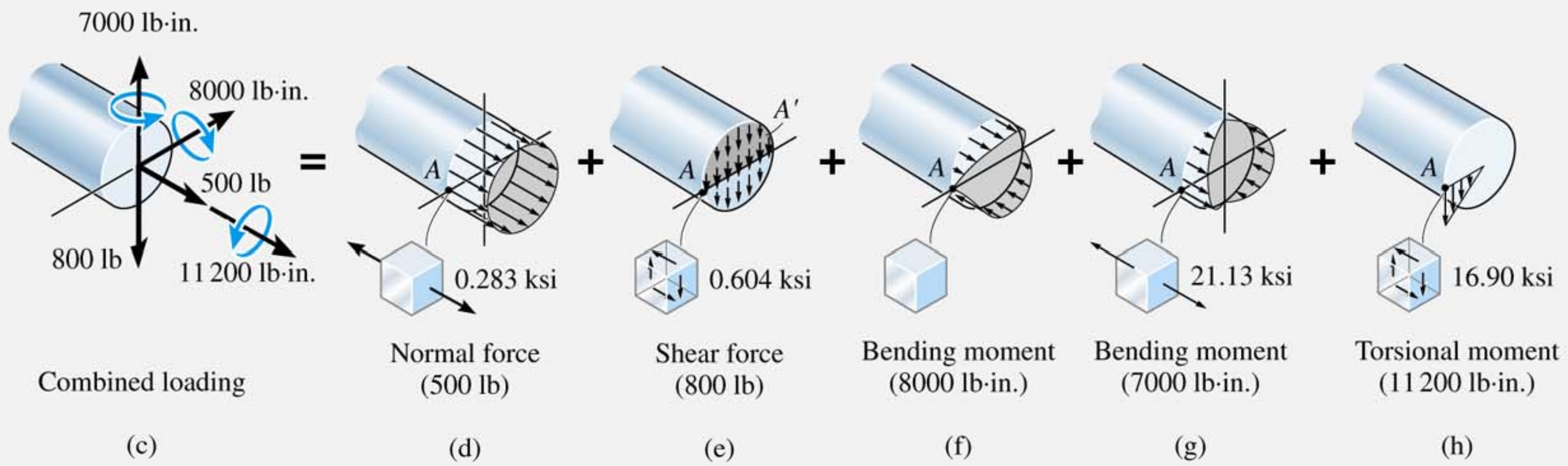

800 lb (14 in.) = 11200 lb·in.
800 lb
500 lb
10 in.
800 lb (10 in.) = 8000 lb·in.
500 lb (14 in.) = 7000 lb·in.
800 lb
14 in.
500 lb
(b)

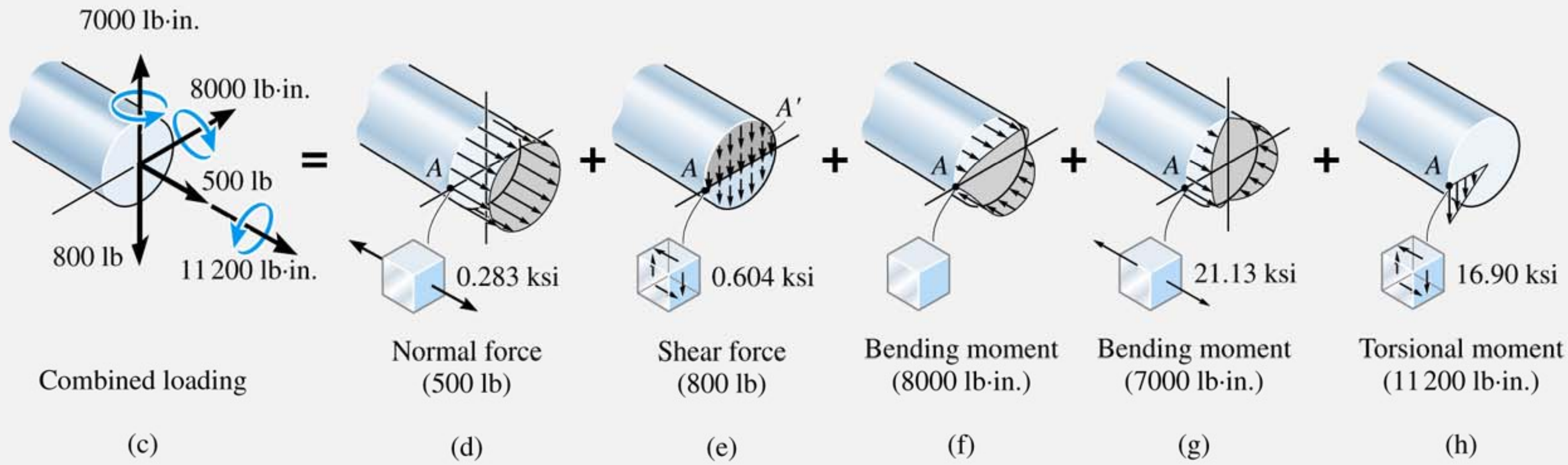
Fig. 8–6



Stress Components.

NORMAL FORCE. The normal-stress distribution is shown in Fig. 8–6*d*. For point *A*, we have

$$\sigma_A = \frac{P}{A} = \frac{500 \text{ lb}}{\pi(0.75 \text{ in.})^2} = 283 \text{ psi} = 0.283 \text{ ksi}$$



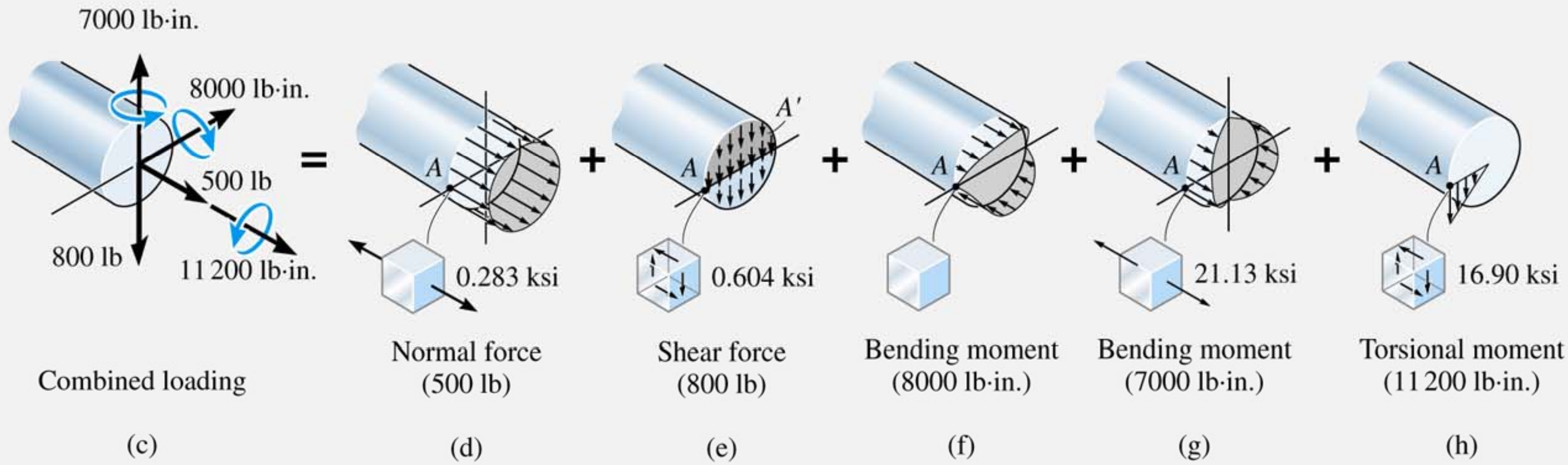
Stress Components.

Shear force. The shear-stress distribution is shown in Fig. 8–6*e*. For point *A*, *Q* is determined from the shaded *semicircular* area. Using the table on the inside front cover, we have

$$Q = \bar{y}'A' = \frac{4(0.75\text{ in.})}{3\pi}\left[\frac{1}{2}\pi(0.75\text{ in.})^2\right] = 0.2813\text{ in}^3$$

so that

$$\tau_A = \frac{VQ}{It} = \frac{800\text{ lb}(0.2813\text{ in}^3)}{[\frac{1}{4}\pi(0.75\text{ in.})^4]2(0.75\text{ in.})} = 604\text{ psi} = 0.604\text{ ksi}$$



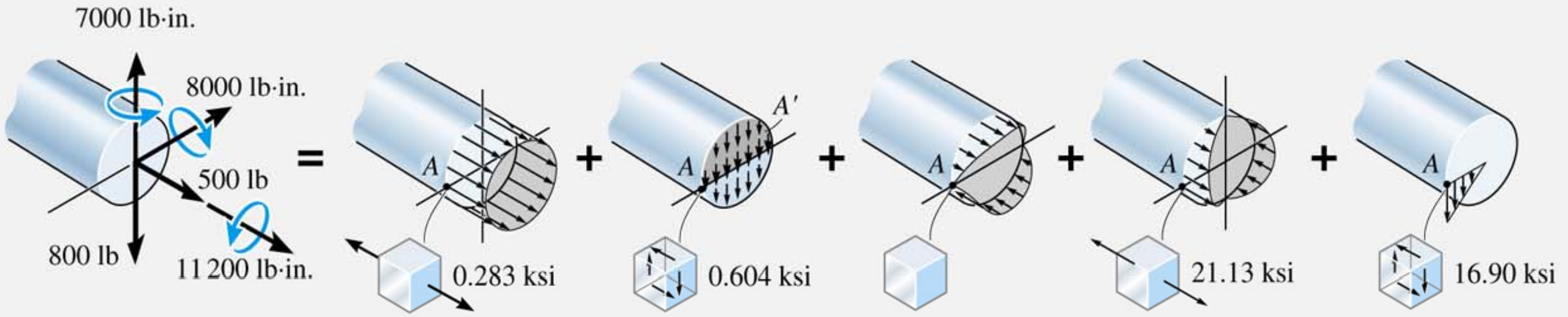
Stress Components.

Bending moments. For the 8000-lb · in. component, point A lies on the neutral axis, Fig. 8–6*f*, so the normal stress is

$$\sigma_A = 0$$

For the 7000-lb · in. moment, $c = 0.75$ in., so the normal stress at point A, Fig. 8–6*g*, is

$$\sigma_A = \frac{Mc}{I} = \frac{7000 \text{ lb} \cdot \text{in.}(0.75 \text{ in.})}{[\frac{1}{4}\pi(0.75 \text{ in.})^4]} = 21\ 126 \text{ psi} = 21.13 \text{ ksi}$$



Combined loading (c)

Normal force (500 lb) (d)

Shear force (800 lb) (e)

Bending moment (8000 lb·in.) (f)

Bending moment (7000 lb·in.) (g)

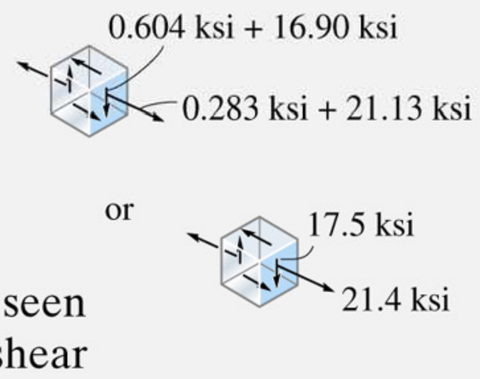
Torsional moment (11 200 lb·in.) (h)

Stress Components.

Torsional moment. At point A, $\rho_A = c = 0.75$ in., Fig. 8–6h. Thus the shear stress is

$$\tau_A = \frac{Tc}{J} = \frac{11\ 200 \text{ lb} \cdot \text{in.}(0.75 \text{ in.})}{[\frac{1}{2}\pi(0.75 \text{ in.})^4]} = 16\ 901 \text{ psi} = 16.90 \text{ ksi}$$

Superposition. When the above results are superimposed, it is seen that an element of material at A is subjected to both normal and shear stress components, Fig. 8–6i.



(i)