

PERT:
Project **E**valuation
Review **&** **T**echnique

One of the shortcomings of CPM is the assumption that the durations of activities are deterministic, i.e., known with certainty.

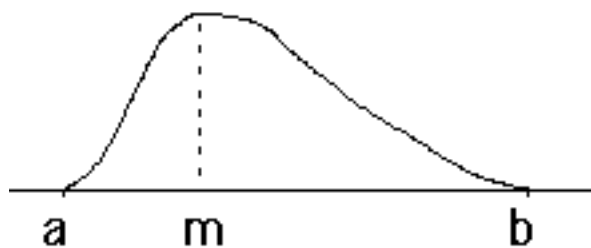
PERT assumes that the duration of each activity is a random variable with known mean and standard variation, and derives a probability distribution for the project completion time.

Assumptions of PERT:

- the duration of an activity is a random variable with BETA distribution
- the durations of the activities are statistically independent
- the critical path (computed assuming expected values of the durations) always requires a longer total time than any other path
- the Central Limit Theorem can be applied so that the sum of the durations of the activities on the critical path has approximately a NORMAL distribution

The BETA distribution

is **unimodal** with **finite endpoints**



$$\text{Mean: } \mu = \frac{a + 4m + b}{6}$$

Standard deviation:

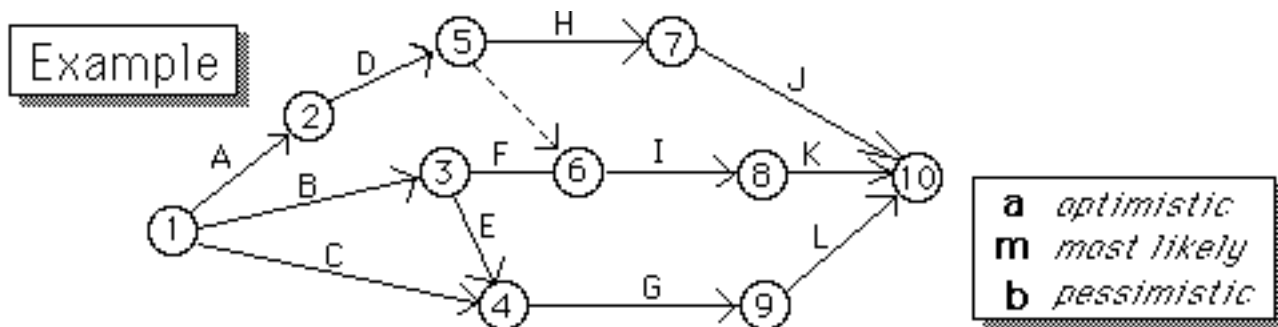
$$\sigma = \frac{b - a}{6}$$

(can be skewed either to right or left.)

The user provides estimates of the parameters for each activity:

- a: the optimistic estimate of completion time
- b: the pessimistic estimate of completion time
- m: estimate of the most likely completion time

From these parameters, the expected value and standard deviation of each activity's duration is computed.



Activity	i	j	a	m	b
A	1	2	3	5	8
B	1	3	5	6	10
C	1	4	6	8	12
D	2	5	4	6	12
E	3	4	5	10	16
F	3	6	3	4	6
G	4	9	7	11	15

Activity	i	j	a	m	b
(dummy)	5	6	0	0	0
H	5	7	2	6	10
I	6	8	1	2	4
J	7	10	11	13	16
K	8	10	4	8	15
L	9	10	8	12	16

Example

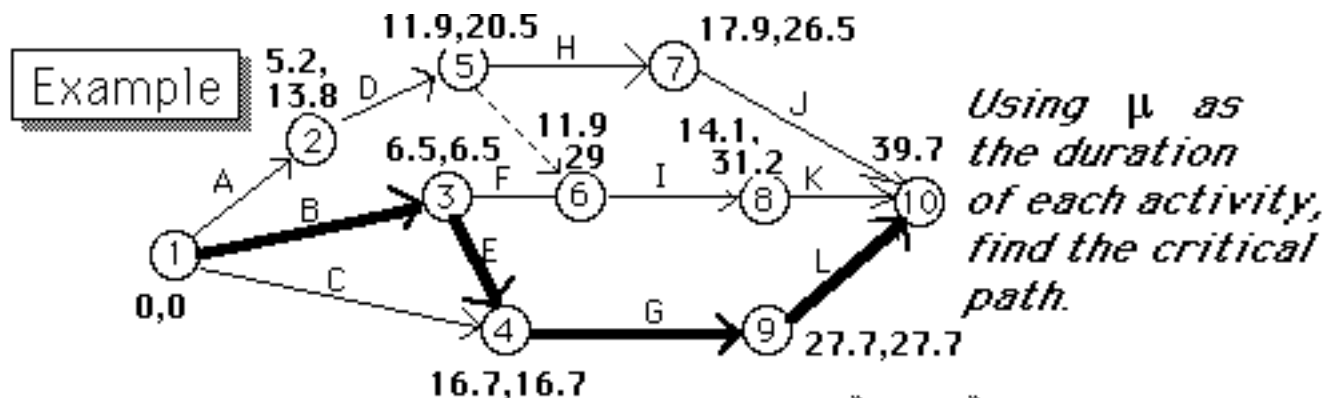
a <i>optimistic</i>
m <i>most likely</i>
b <i>pessimistic</i>

Calculate

$$\mu = \frac{a+4m+b}{6}$$

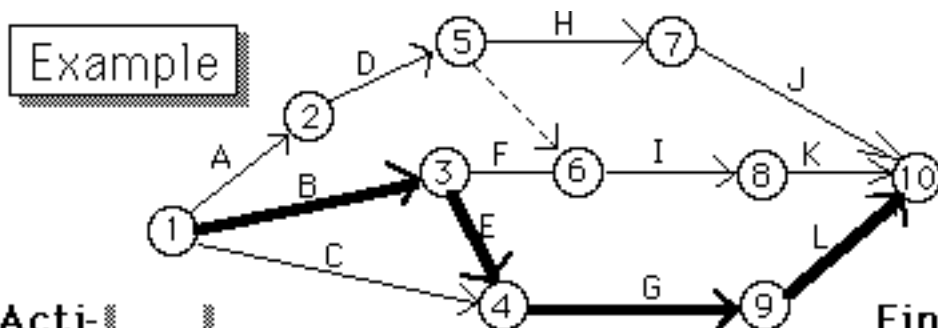
$$\sigma = \frac{b-a}{6}$$

Activity	i	j	a	m	b	μ	σ^2
A	1	2	3	5	8	5.2	0.69
B	1	3	5	6	10	6.5	0.69
C	1	4	6	8	12	8.3	1.00
D	2	5	4	6	12	6.7	1.78
E	3	4	5	10	16	10.2	3.36
F	3	6	3	4	6	4.2	0.25
G	4	9	7	11	15	11.0	1.78
(dummy)	5	6	0	0	0	0	0
H	5	7	2	6	10	6.0	1.78
I	6	8	1	2	4	2.2	0.25
J	7	10	11	13	16	13.2	0.69
K	8	10	4	8	15	8.5	3.36
L	9	10	8	12	16	12.0	1.78



Activity	i	j	μ
A	1	2	5.2
B	1	3	6.5 *** critical
C	1	4	8.3
D	2	5	6.7
E	3	4	10.2 *** critical
F	3	6	4.2
G	4	9	11.0 *** critical

Activity	i	j	μ
(dummy)	5	6	0
H	5	7	6.0
I	6	8	2.2
J	7	10	13.2
K	8	10	8.5
L	9	10	12.0 *** critical



Activity	i	j	μ	σ^2	
B	1	3	6.5	0.69	*** critical
E	3	4	10.2	3.36	*** critical
G	4	9	11.0	1.78	*** critical
L	9	10	12.0	1.78	*** critical
sum:			7.61		

$$\sigma_T = \sqrt{7.61}$$

$$= 2.759$$

Find the standard deviation of the sum of the durations on the critical path.

The completion time for the project is $N(39.7, 2.759)$

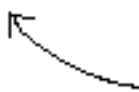
(Normal dist'n with mean 39.7 and std. deviation 2.759.)

The expected completion time of the project is 39.7 days.

What is the probability that it is completed within 42 days?

$$\begin{aligned} P\{T \leq 42\} &= P\left\{ \frac{T-39.7}{2.759} \leq \frac{42-39.7}{2.759} \right\} = P\{ \mathbf{X} \leq 0.8336 \} \\ &\doteq 79 \% \end{aligned}$$

*standard $N(0,1)$
random variable*



Assumptions of PERT:

- durations of activities are INDEPENDENT random variables with BETA distributions
- the critical path when durations are the mean values is ALWAYS the critical path
- the number of activities on the critical path is large enough to invoke the CENTRAL LIMIT THEOREM (*i.e., completion time has a NORMAL DISTRIBUTION*)