CE 384 STRUCTURAL ANALYSIS I

Indeterminate Structures – Slope-Deflection Method

Introduction

- *Slope-deflection method* is the first of the two classical methods presented in this course. This method considers the deflection as the primary unknowns, while the redundant forces were used in the force method.
- In the slope-deflection method, the relationship is established between moments at the ends of the members and the corresponding rotations and displacements.

The *basic assumption* used in the slope-deflection method is that a typical member can flex but the shear and axial deformation are negligible. It is no different from that used with the force method.

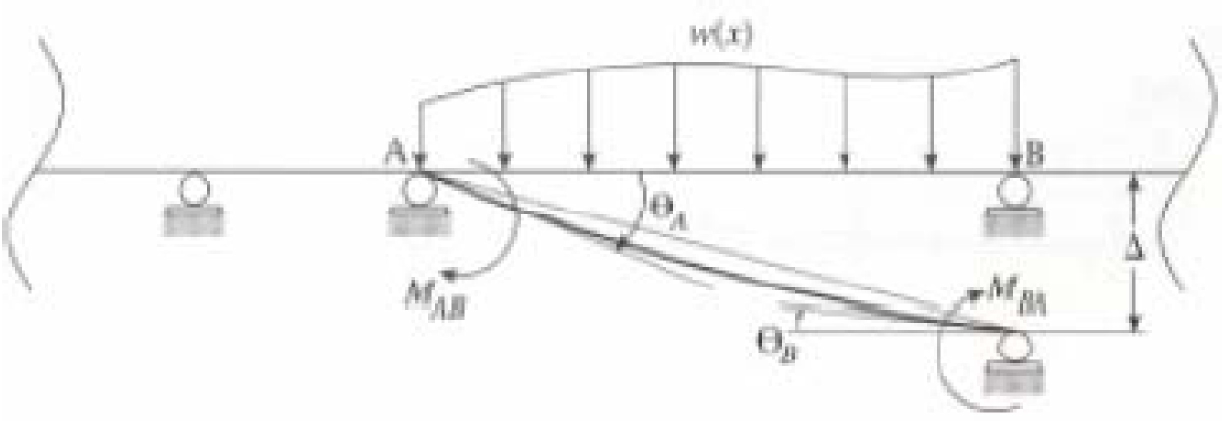
• *Kinematically indeterminate* structures versus statically indeterminate structures:

- **Sign convention**: **All clockwise internal moments and end rotation are positive.**

Basic Idea of Slope Deflection Method

The basic idea of the slope deflection method is to write the equilibrium equations for each node in terms of the deflections and rotations. Solve for the generalized displacements. Using moment displacement relations, moments are then known. The structure is thus reduced to a determinate structure.

Fundamental Slope-Deflection Equations:



$$\begin{cases} M_{AB} = \dfrac{2EI}{L}\left[2\theta_A + \theta_B - 3\dfrac{\Delta}{L}\right] + FEM_{AB} \\[2em] M_{BA} = \dfrac{2EI}{L}\left[2\theta_B + \theta_A - 3\dfrac{\Delta}{L}\right] + FEM_{BA} \end{cases}$$

Analysis of Beams – Slope-Deflection Method (General Procedure)

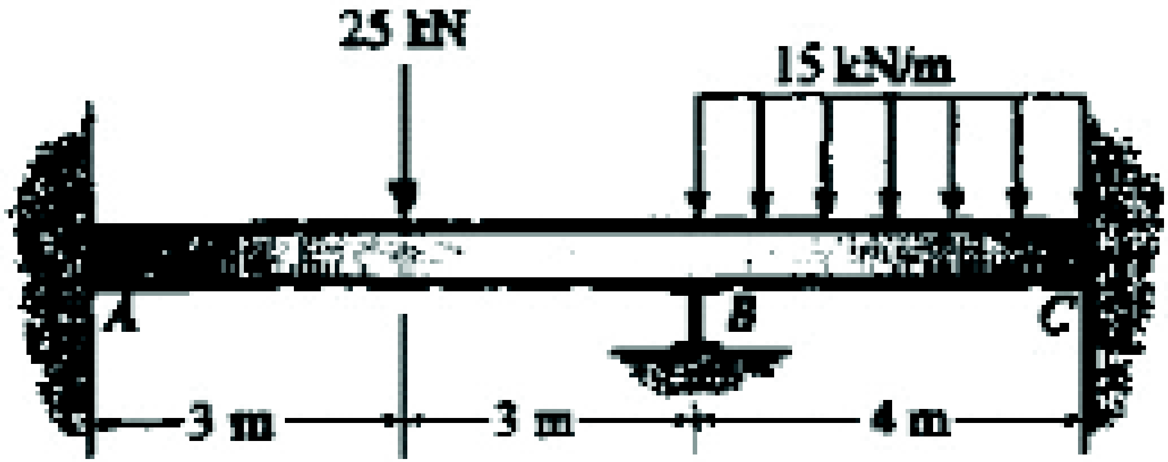
- ***Step 1*:** Scan the beam and identify the number of (a) segments and (b) kinematic unknowns. A segment is the portion of the beam between two nodes. Kinematic unknowns are those rotations and displacements that are not zero and must be computed. The support or end conditions of the beam will help answer the question.

Step 2**:** For each segment, generate the two governing equations.Check the end conditions to see whether one of the end rotations is zero or not (it is not possible for both the end rotations and other deflection components to be zero). If there are no element loads, the FEM term is zero. If there are one or more element loads, use the appropriate formula to compute the FEM for each element load and then sum all the FEMs. If one end of the segment displace relative to the other, compute the chord rotation; otherwise it is zero.
Step 3**:** For each kinematic unknown, generate an equilibrium condition using the free-body diagram.

Step 4: Solve for all unknowns by combining all the equations from steps 2 and 3. Now the equations are entirely in terms of the kinematic unknowns.

Step 5: Compute the support reactions with appropriate FBDs.

10–1. Determine the moments at the supports *A* and *C*, then draw the moment diagram. Assume joint *B* is a roller. *EI* is constant.



Prob. 10–1

$$M_N = 2E\left(\frac{I}{L}\right)(2\theta_N + \theta_F - 3\psi) + (\mathrm{FEM})_N$$

$$M_{AB} = \frac{2EI}{6}(0 + \theta_B) - \frac{(25)(6)}{8}$$

$$M_{BA} = \frac{2EI}{6}(2\theta_B) + \frac{(25)(6)}{8}$$

$$M_{BC} = \frac{2EI}{4}(2\theta_B) - \frac{(15)(4)^2}{12}$$

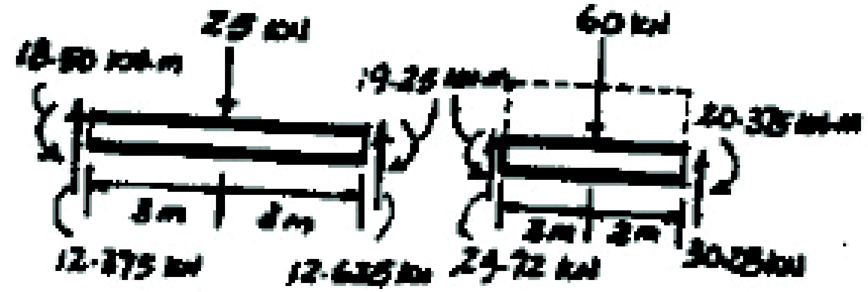
$$M_{CB} = \frac{2EI}{4}(\theta_B) + \frac{15(4)^2}{12}$$

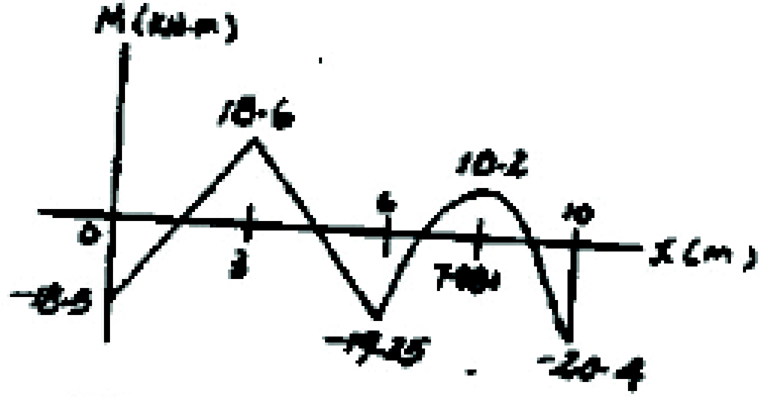
Equilibrium

$$M_{BA} + M_{BC} = 0$$

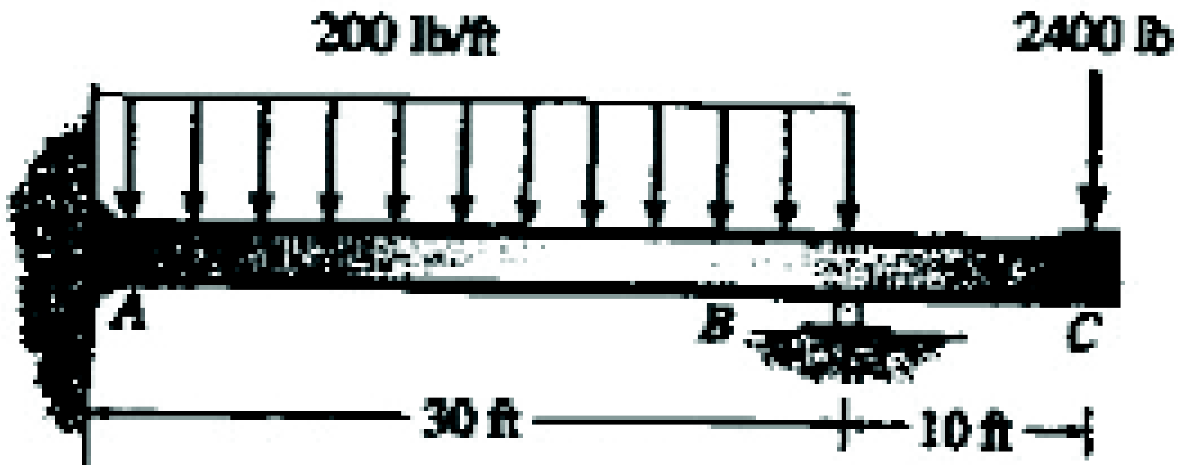
$$\frac{2EI}{6}(2\theta_B) + \frac{25(6)}{8} + \frac{2EI}{4}(2\theta_B) - \frac{15(4)^2}{12} = 0$$

$$\theta_B = \frac{0.75}{EI}$$





10–3. Determine the moments at A and B, then draw the moment diagram for the beam. EI is constant.



$$FEM_{AB} = \frac{1}{12}(w)(L^2) = \frac{1}{12}(200)(30^2) = 15 \text{ k}\cdot\text{ft}$$

$$M_{AB} = \frac{2EI}{30}(0 + \theta_B - 0) - 15$$

$$M_{BA} = \frac{2EI}{30}(2\theta_B + 0 - 0) + 15$$

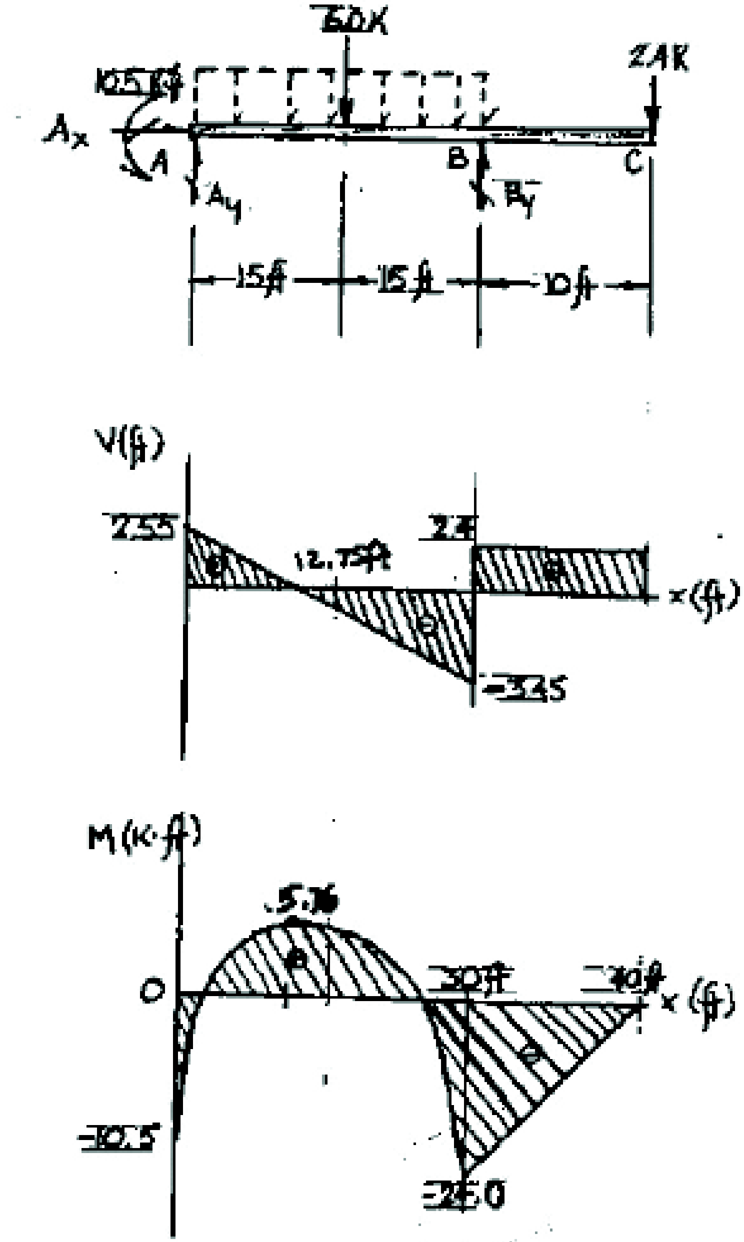
$$\Sigma M_B = 0; \qquad M_{BA} + 2400\,(10)$$

Solving,

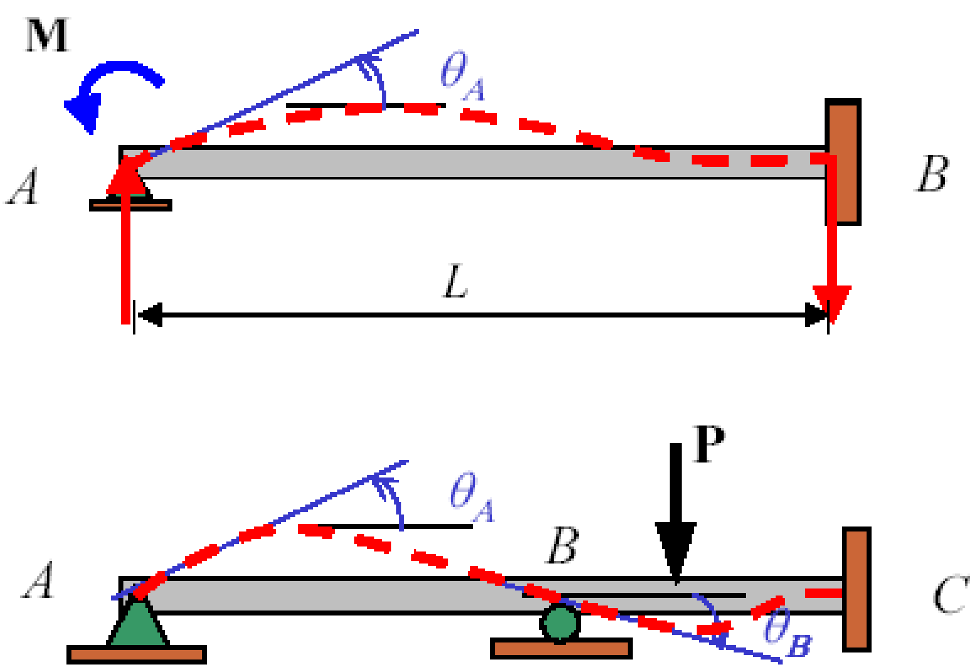
$$\theta_B = \frac{67.5}{EI}$$

$$M_{AB} = -10.5 \text{ k}\cdot\text{ft} \qquad \text{Ans}$$

$$M_{BA} = 24 \text{ k}\cdot\text{ft} \qquad \text{Ans}$$



Degrees of Freedom

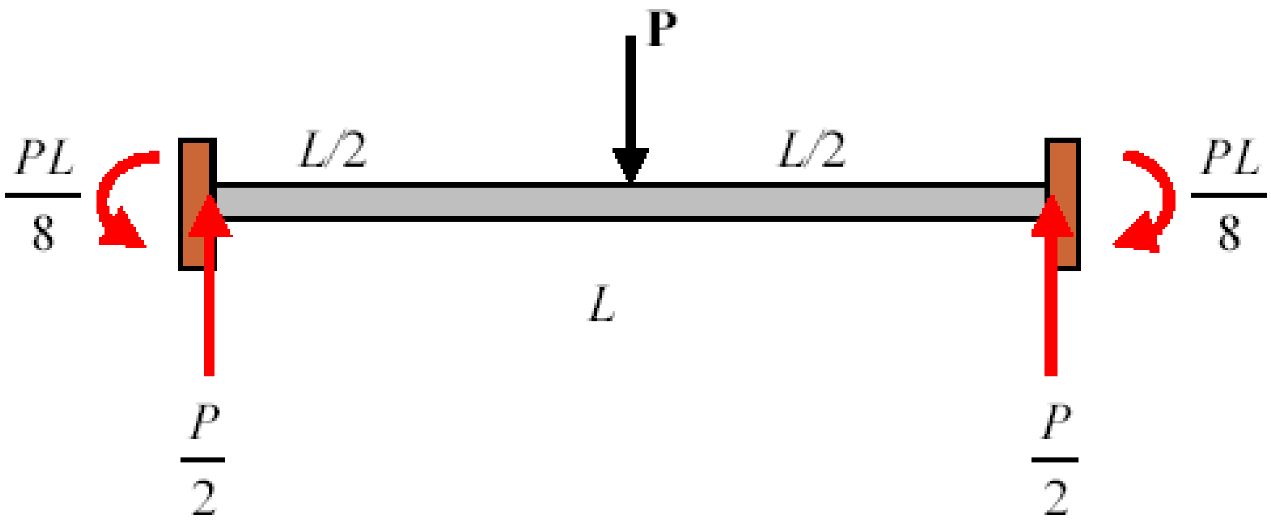


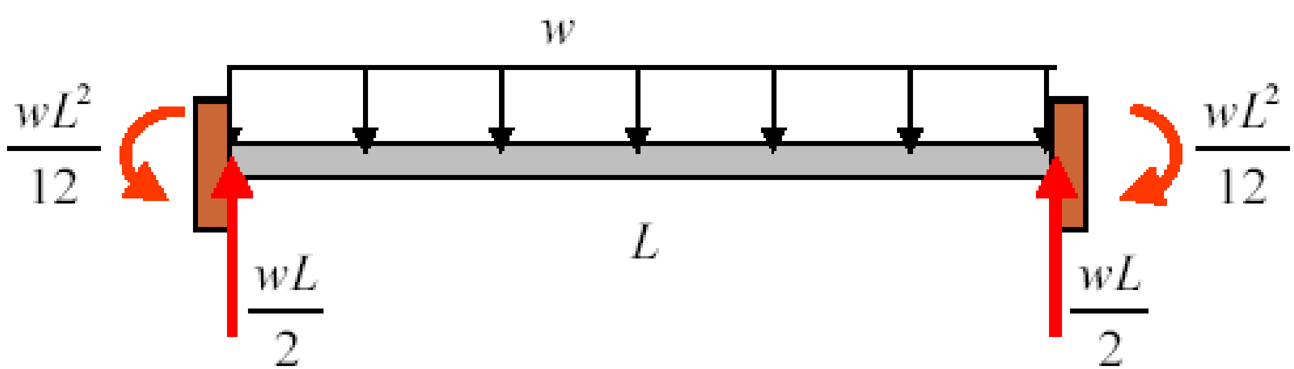
1 DOF: θ_A

2 DOF: θ_A, θ_B

Fixed-End Forces

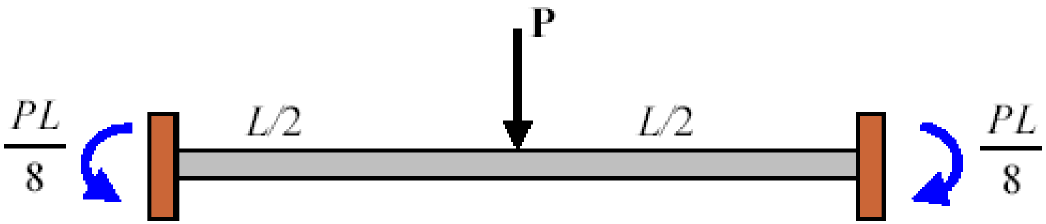
- Fixed-End Moments: Loads

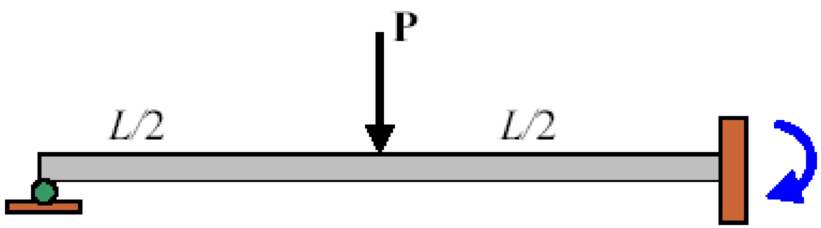




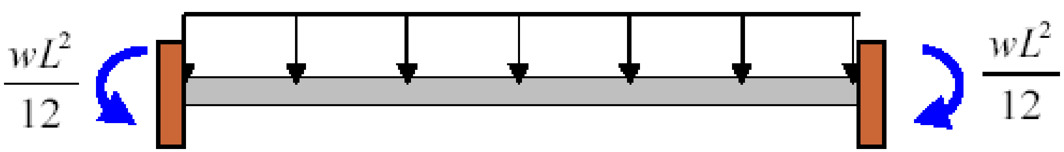
Fixed-End Moments

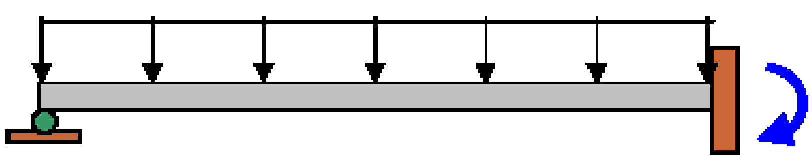
- **Fixed-End Moments: Loads**





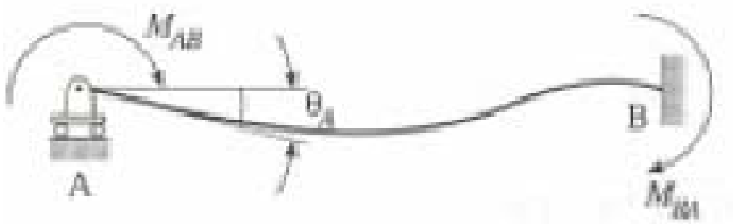
$$\frac{PL}{8} + \left(\frac{1}{2}\right)\left[-\left(-\frac{PL}{8}\right)\right] = \frac{3PL}{16}$$





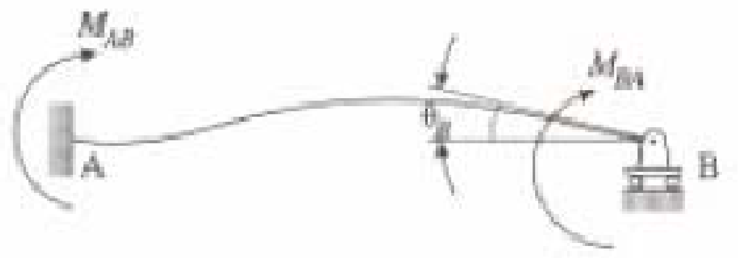
$$\frac{wL^2}{12} + \left(\frac{1}{2}\right)\left[-\left(-\frac{wL^2}{12}\right)\right] = \frac{wL^2}{8}$$

Case B: rotation at A



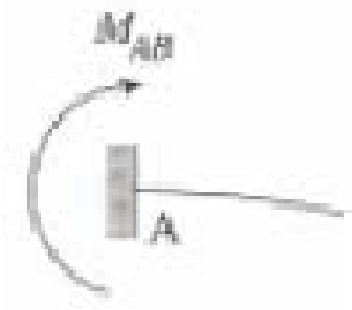
$$M_{AB} = \frac{4EI}{L}\theta_A; \quad M_{BA} = \frac{2EI}{L}\theta_A$$

Case C: rotation at B

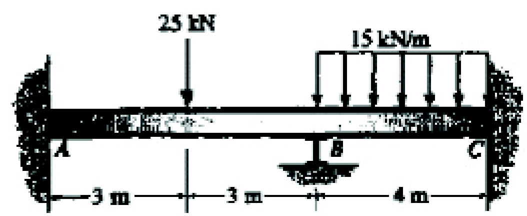


$$M_{AB} = \frac{2EI}{L}\theta_B; \quad M_{BA} = \frac{4EI}{L}\theta_B$$

Case D: displacement of end B related to end A



10–1. Determine the moments at the supports A and C, then draw the moment diagram. Assume joint B is a roller. EI is constant.

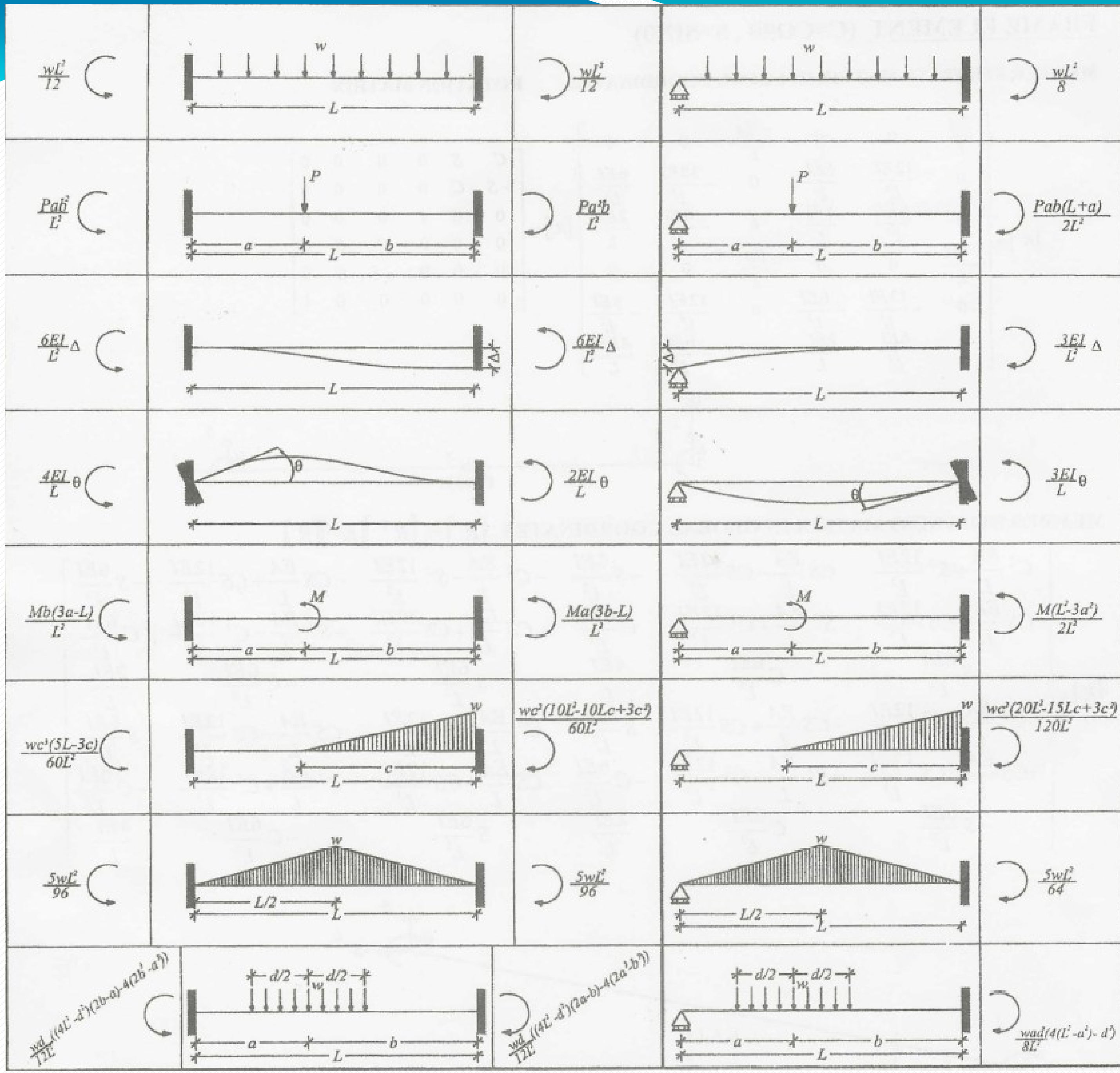


Prob. 10–1

$$M_{AB} = -\frac{6}{L}$$

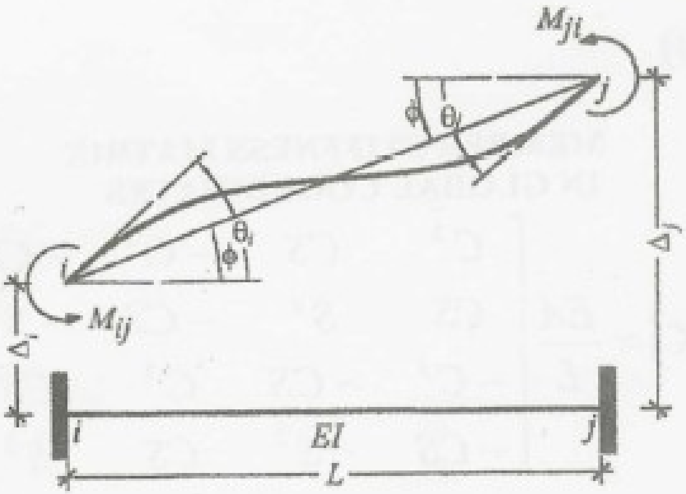
L

Example



$\frac{wL^2}{12}$
w
L
$\frac{wL^2}{12}$
w
L
$\frac{wL^2}{8}$
$\frac{Pab^2}{L^2}$
P
a
b
L
$\frac{Pa^2b}{L^2}$
P
a
b
L
$\frac{Pab(L+a)}{2L^2}$
$\frac{6EI}{L^2}\Delta$
Δ
L
$\frac{6EI}{L^2}\Delta$
Δ
L
$\frac{3EI}{L^2}\Delta$
$\frac{4EI}{L}\theta$
θ
L
$\frac{2EI}{L}\theta$
θ
L
$\frac{3EI}{L}\theta$
$\frac{Mb(3a-L)}{L^2}$
M
a
b
L
$\frac{Ma(3b-L)}{L^2}$
M
a
b
L
$\frac{M(L^2-3a^2)}{2L^2}$
$\frac{wc^3(5L-3c)}{60L^2}$
w
c
L
$\frac{wc^2(10L^2-10Lc+3c^2)}{60L^2}$
w
c
L
$\frac{wc^2(20L^2-15Lc+3c^2)}{120L^2}$
$\frac{5wL^2}{96}$
w
L/2
L
$\frac{5wL^2}{96}$
w
L/2
L
$\frac{5wL^2}{64}$
$\frac{wd}{12L^2}((4L^2-d^2)(2b-a)-4(2b^3-a^3))$
d/2
d/2
w
a
b
L
$\frac{wd}{12L^2}((4L^2-d^2)(2a-b)-4(2a^3-b^3))$
d/2
d/2
w
a
b
L
$\frac{wad(4(L^2-a^2)-d^2)}{8L^2}$

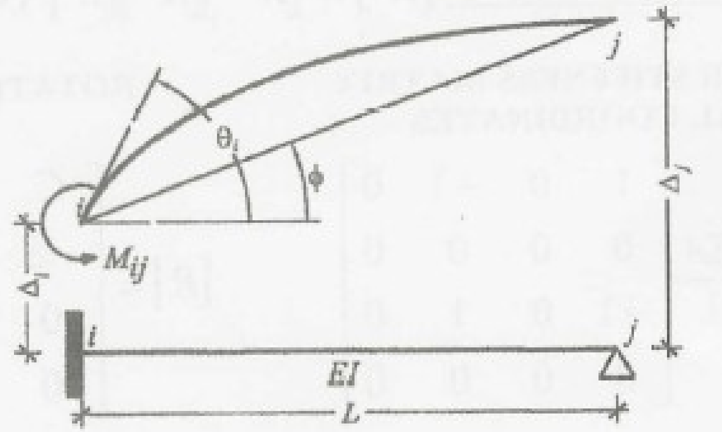
SLOPE DEFLECTION EQUATIONS



$$M_{ij} = \frac{2EI}{L}(2\theta_i + \theta_j - 3\phi) + M^F_{ij}$$

$$M_{ji} = \frac{2EI}{L}(\theta_i + 2\theta_j - 3\phi) + M^F_{ji}$$

$$\phi = \frac{(\Delta_j - \Delta_i)}{L}$$

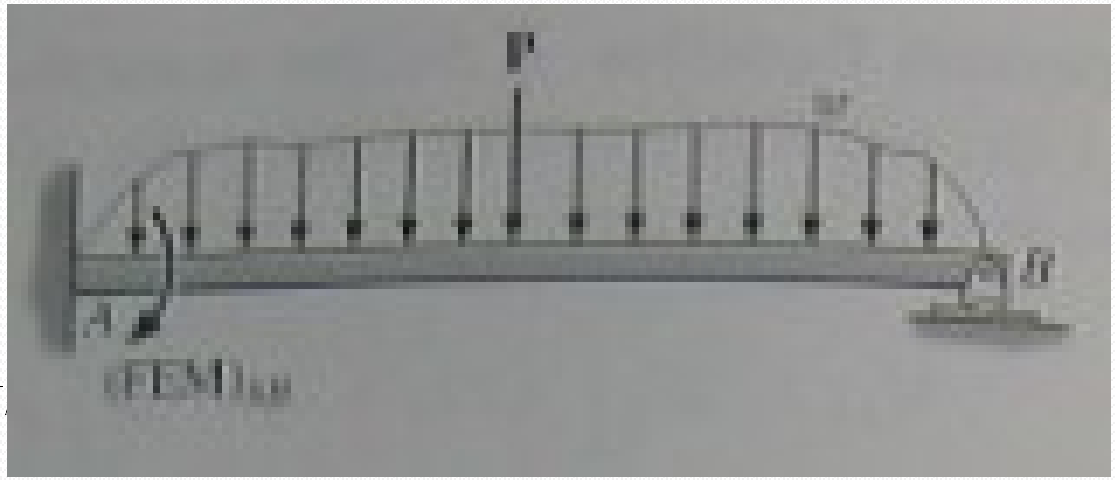


$$M_{ij} = \frac{3EI}{L}(\theta_i - \phi) + M^F_{ij}$$

$$M_{ji} = 0$$

$$\phi = \frac{(\Delta_j - \Delta_i)}{L}$$

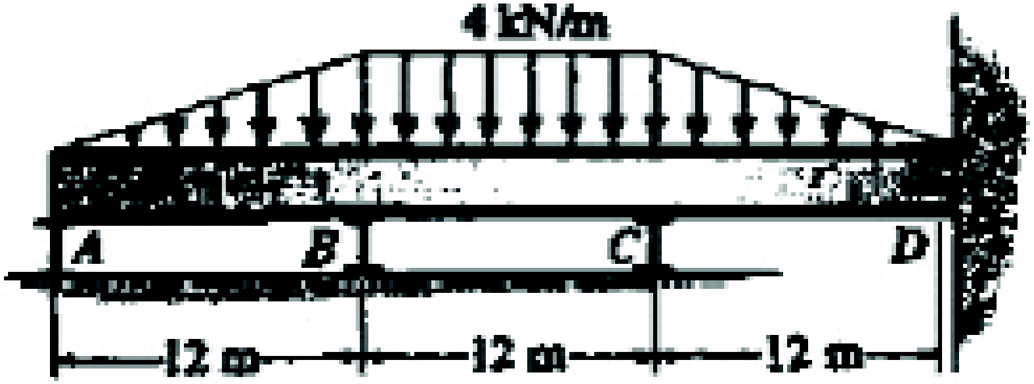
Pin-Supported End Span



M

$$M_N = 3Ek(\theta_N - \Psi) + (FEM)_N$$

*10–4. Determine the moments at B and C, then draw the moment diagram. Assume A, B, and C are rollers and D is pinned. EI is constant.



$$M_N = 3E\left(\frac{I}{L}\right)(\theta_N - \psi) + (\mathrm{FEM})_N$$

$$M_{BA} = \frac{3EI}{12}(\theta_B) + \frac{(4)(12)^2}{15}$$

$$M_N = 2E\left(\frac{I}{L}\right)(2\theta_N + \theta_F - 3\psi) + (\mathrm{FEM})_N$$

$$M_{BC} = \frac{2EI}{12}(2\theta_B + \theta_C) - \frac{(4)(12)^2}{12}$$

$$M_{CB} = \frac{2EI}{12}(2\theta_C + \theta_B) + \frac{4(12)^2}{12}$$

$$M_N = 3E\left(\frac{I}{L}\right)(\theta_N - \psi) + (\mathrm{FEM})_N$$

$$M_{CD} = \frac{3EI}{12}(\theta_C) - \frac{4(12)^2}{15}$$

Equations

$$M_{BA} + M_{BC} = 0$$

$$M_{CB} + M_{CD} = 0$$

$$\frac{3EI}{12}(\theta_B) + \frac{(4)(12)^2}{15} + \frac{2EI}{12}(2\theta_B + \theta_C) - \frac{4(12)^2}{12} = 0$$

$$0.5833\theta_B + 0.1667\theta_C = 9.60$$

$$\frac{2EI}{12}(2\theta_C + \theta_B) + \frac{4(12)^2}{12} + \frac{3EI}{12}(\theta_C) - \frac{4(12)^2}{15} = 0$$

$$0.5833\theta_C + 0.1667\theta_B = -9.60$$

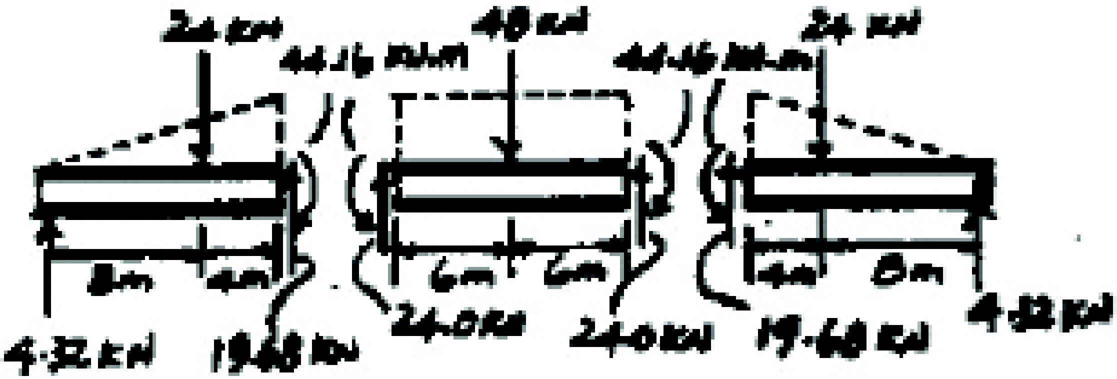
$$\theta_B = -\theta_C = \frac{\pm 3.40}{EI}$$

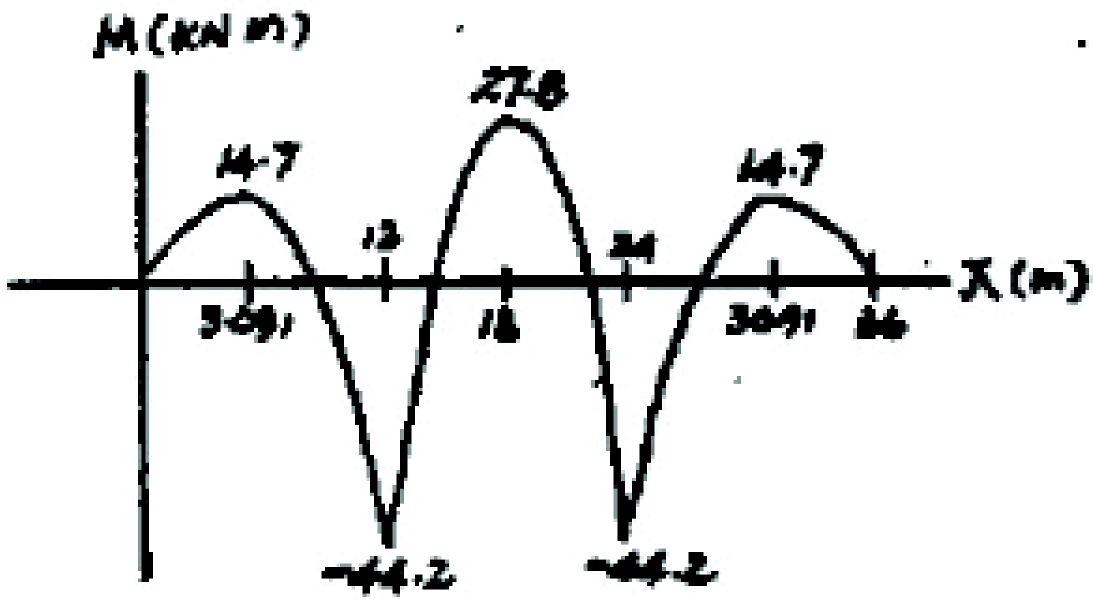
$M_{BA} = 44.2$ k·ft Ans

$M_{BC} = -44.2$ k·ft Ans

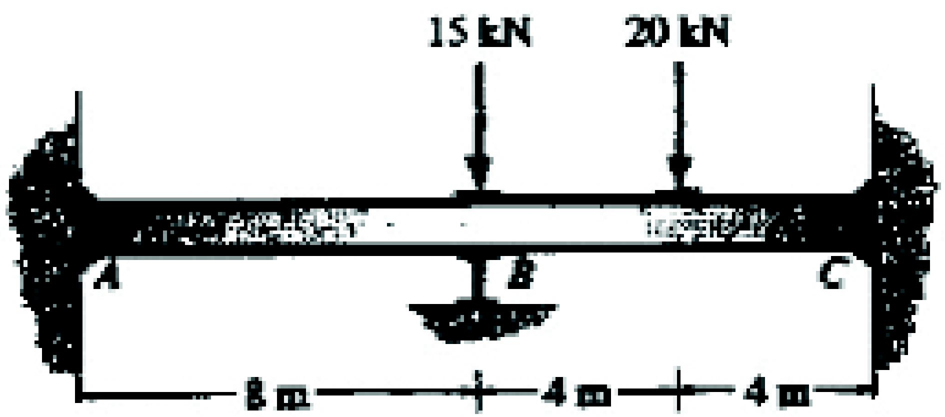
$M_{CB} = 44.2$ k·ft Ans

$M_{CD} = -44.2$ k·ft Ans





10–5. Determine the moment at *B*, then draw the moment diagram for the beam. Assume the supports at *A* and *C* are pinned. *EI* is constant.



(A nad C are not pin)

$$M_{AB} = \frac{2EI}{8}(0 + \theta_B - 0) + 0 = \frac{EI}{4}\theta_B$$

$$M_{BA} = \frac{2EI}{8}(2\theta_B + 0 - 0) + 0 = \frac{EI}{2}\theta_B$$

$$M_{BC} = \frac{2EI}{8}(2\,\theta_B + 0 - 0) - 20 = \frac{EI}{2}\theta_B - 20$$

$$M_{CB} = \frac{2EI}{8}(0 + \theta_B - 0) + 20 = \frac{EI}{4}\theta_B + 20$$

$$M_{BA} + M_{BC} = 0$$

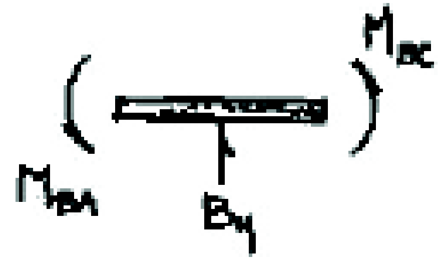
$$\theta_B = \frac{20}{EI}$$

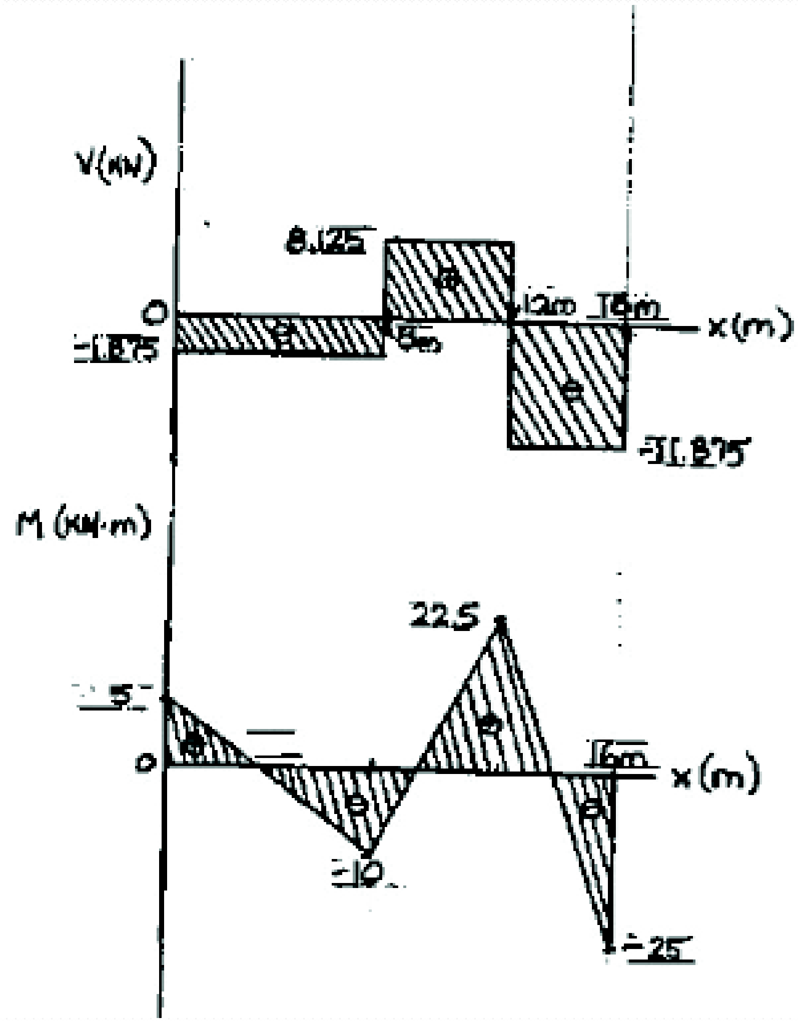
$$M_{AB} = 5 \text{ kN}\cdot\text{m}$$

$$M_{BA} = 10 \text{ kN}\cdot\text{m}$$

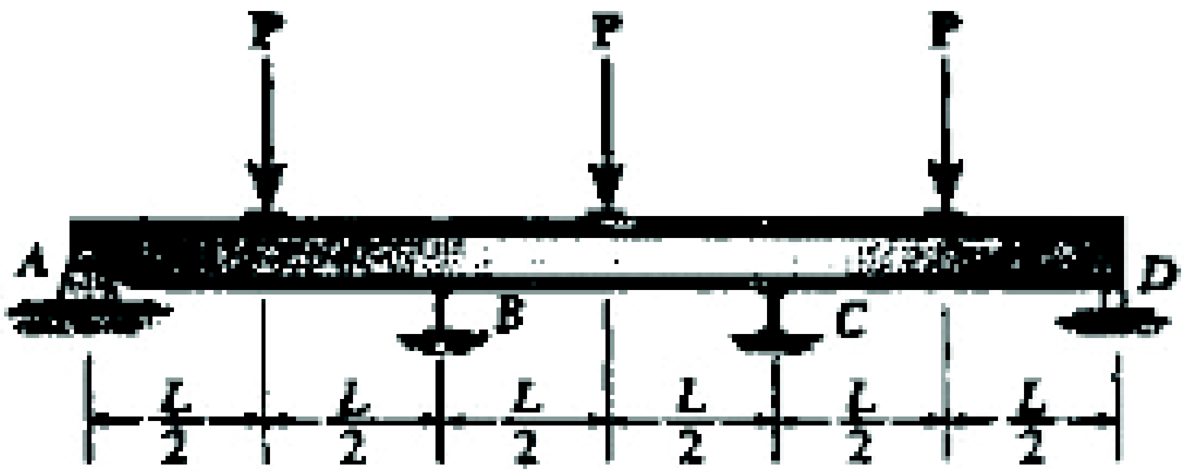
$$M_{BC} = -10 \text{ kN}\cdot\text{m}$$

$$M_{CB} = 25 \text{ kN}\cdot\text{m}$$





10–17. The continuous beam supports the three concentrated loads. Determine the maximum moment in the beam and then draw the moment diagram. EI is constant.



$$M_N = 3E\frac{I}{L}(\theta_N - \psi) + (\text{FEM})_N$$

$$M_{BA} = \frac{3EI}{L}(\theta_B - 0) + \frac{3PL}{16}$$

$$M_N = 2E\frac{I}{L}(2\theta_N + \theta_F - 3\psi) + (\text{FE}$$

$$M_{BC} = \frac{2EI}{L}(2\theta_B + \theta_C - 0) - \frac{PL}{8}$$

$$M_{CB} = \frac{2EI}{L}(2\theta_C + \theta_B - 0) + \frac{PL}{8}$$

$$M_N = 3E\frac{I}{L}(\theta_N - \psi) + (\text{FEM})_N$$

$$M_{CD} = \frac{3EI}{L}(\theta_C - 0) - \frac{3PL}{16}$$

Equilibrium :

$$M_{BA} + M_{BC} = 0$$

$$M_{CB} + M_{CD} = 0$$

$$\frac{3EI}{L}\theta_B + \frac{3PL}{16} + \frac{4EI}{L}\theta_B + \frac{2EI}{L}\theta_C - \frac{PL}{8} = 0$$

$$2\theta_C + 7\theta_B = -\frac{PL^2}{16EI} \qquad (1)$$

$$\frac{2EI}{L}(2\theta_C + \theta_B) + \frac{PL}{8} + \frac{3EI}{L}\theta_C - \frac{3PL}{16} = 0$$

$$7\theta_C + 2\theta_B = \frac{PL^2}{16EI} \qquad (2)$$

Solving Eqs. 1 – 2 :

$$\theta_B = \frac{-PL^2}{80EI}$$

$$\theta_C = \frac{PL^2}{80EI}$$

Thus,

$$M_{BA} = \frac{3PL}{20}$$

$$M_{BC} = \frac{-3PL}{20}$$

$$M_{CB} = \frac{3PL}{20}$$

$$M_{CD} = -\frac{3PL}{20}$$

$+\Sigma M_B = 0;\qquad -A_y(L) + P\left(\frac{L}{2}\right) - \frac{3PL}{20} = 0$

$$A_y = \frac{7}{20}P$$

$+\Sigma M_C = 0;\qquad -V_{BR}(L) + P\left(\frac{L}{2}\right) + \frac{3PL}{20} - \frac{3PL}{20} = 0$

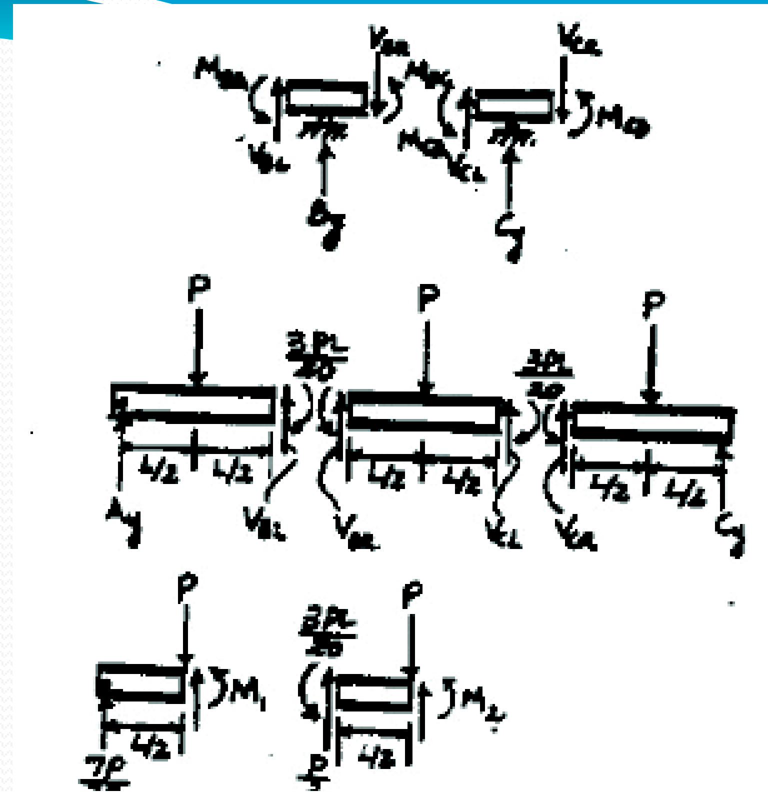
$$V_{BR} = \frac{PL}{2}$$

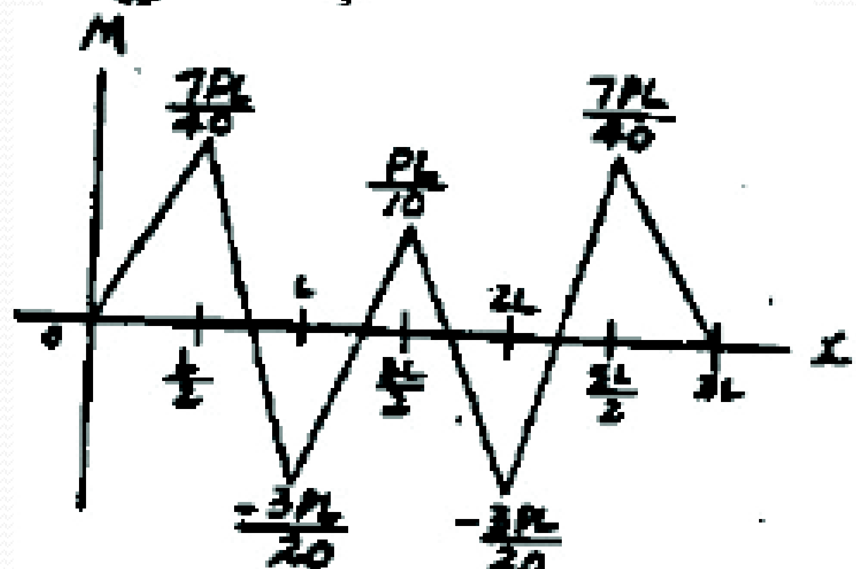
$+\Sigma M = 0;\qquad M_1 - \frac{7}{20}P\left(\frac{L}{2}\right) = 0$

$M_1 = M_{max} = \frac{7}{40}PL$ Ans

$+\Sigma M = 0;\qquad \frac{3PL}{20} - \frac{P}{2}\left(\frac{L}{2}\right) + M_2 = 0$

$$M_2 = \frac{1}{10}PL$$





Analysis of Frames without Sidesway – Slope-Deflection Method

The analysis of frames via the slope-deflection method can also be carried out systematically by applying the two governing equations of beams.

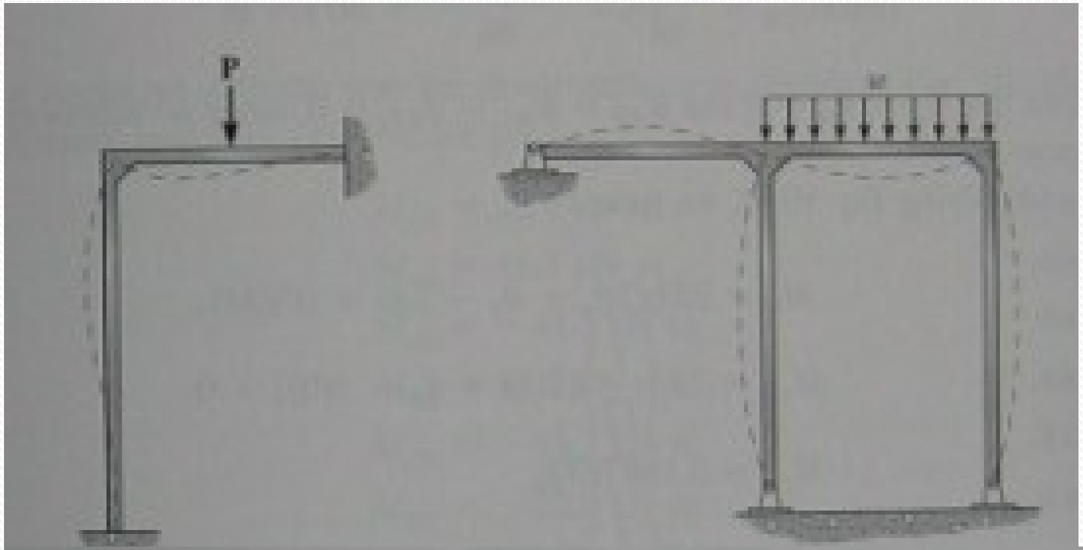
A sidesway will not occur if

- (a) the frame geometry and loading are symmetric, and
- (b) sidesway is prevented due to supports.

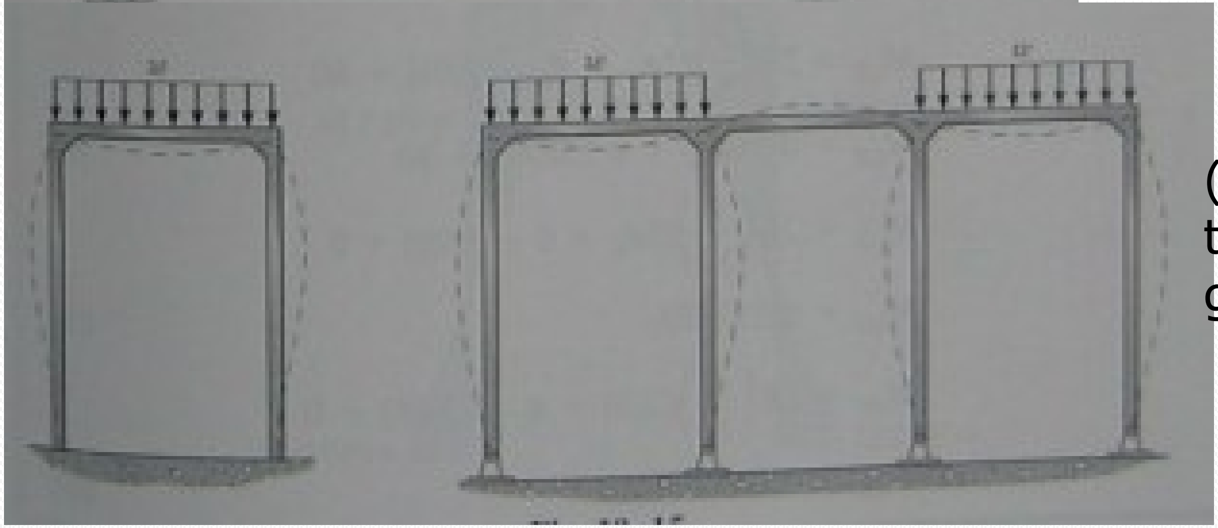
A sidesway will occur if

- (a) the frame geometry and loading are unsymmetrical, and
- (b) sidesway is not prevented due to supports.

Analysis of frames: No Sideway

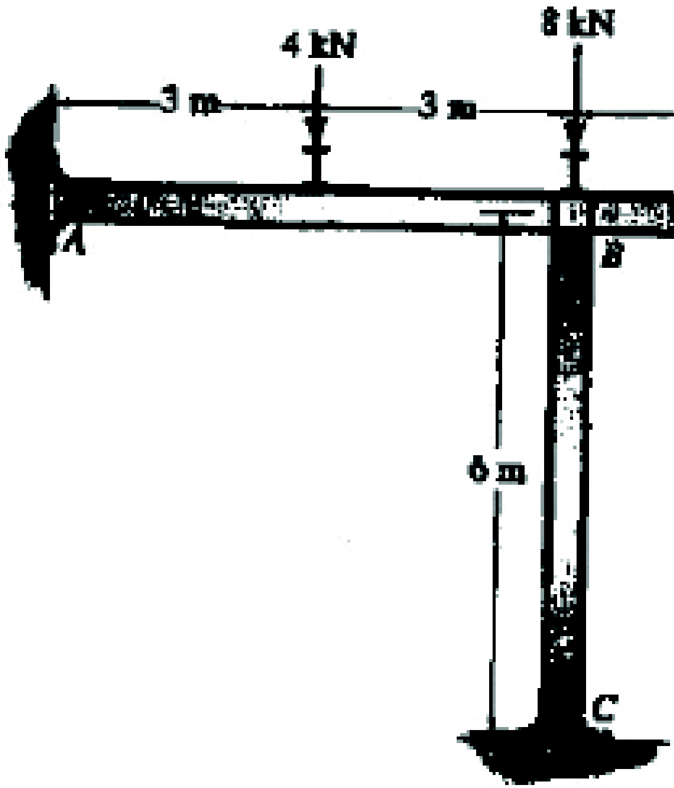


(İt is properly restrained)

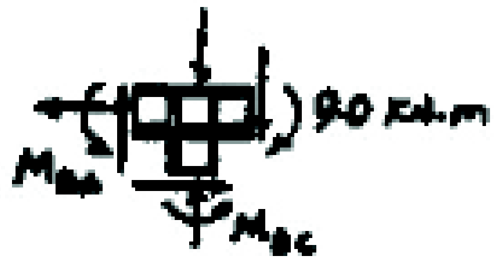


(Symmetric with respect to both loading and geometry)

*10–16. Determine the moments at the ends of each member of the frame. The supports at A and C and joint B are fixed connected. EI is constant.



$$M_N = 2E\left(\frac{I}{L}\right)(2\theta_N + \theta_F - 3\psi) + (FEM)_N$$



$$M_{AB} = \frac{2EI}{6}(0 + \theta_B) - \frac{(4)(6)}{8}$$

$$M_{BA} = \frac{2EI}{6}(2\theta_B) + \frac{4(6)}{8}$$

$$M_{BC} = \frac{2EI}{6}(2\theta_B)$$

$$M_{CB} = \frac{2EI}{6}(\theta_B)$$

Equilibrium

$$M_{BA} + M_{BC} - 9 = 0$$

$$\frac{2EI}{6}(2\theta_B) + \frac{4(6)}{8} + \frac{2EI}{6}(2\theta_B) - 9 = 0$$

$$\theta_B = \frac{4.5}{EI}$$

Thus,

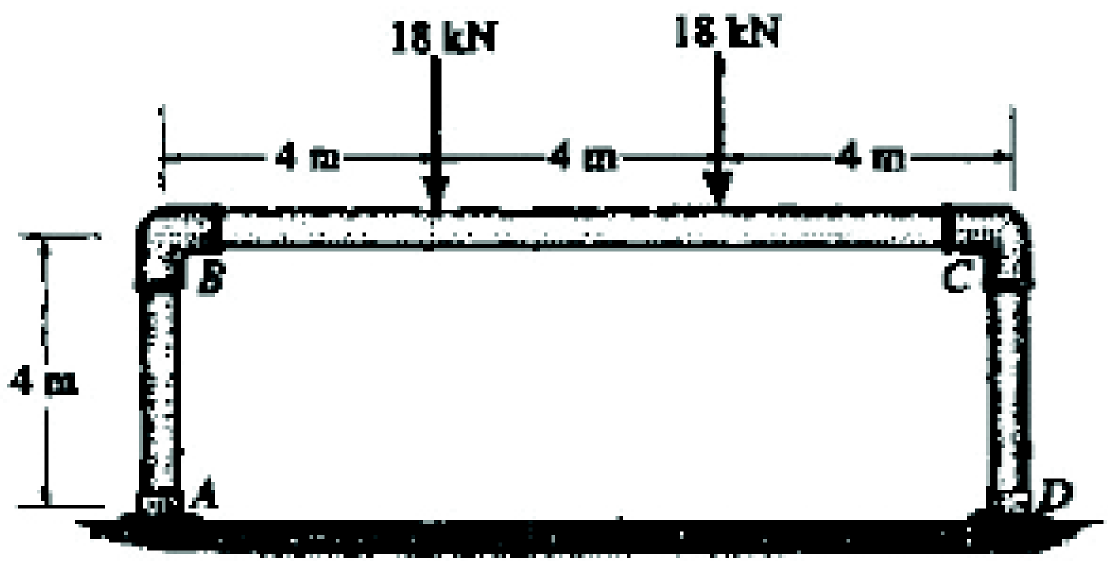
$$M_{AB} = \frac{2EI}{6}\left(0 + \frac{4.5}{EI}\right) - \frac{4(6)}{8} = -1.50 \text{ kN}\cdot\text{m}$$

$$M_{CB} = \frac{2EI}{6}\left(\frac{4.5}{EI}\right) = 1.50 \text{ kN}\cdot\text{m}$$

$$M_{BA} = \frac{2EI}{6}\left(2\frac{4.5}{EI}\right) - \frac{4(6)}{8} = 6.00 \text{ kN}\cdot\text{m}$$

$$M_{BC} = \frac{2EI}{6}\left(2\left(\frac{4.5}{EI}\right)\right) = 3.00 \text{ kN}\cdot\text{m}$$

10–19. The frame is made from pipe that is fixed connected. If it supports the loading shown, determine the moments developed at each of the joints and supports. EI is constant.



$$M_N = 2E\left(\frac{I}{L}\right)(2\theta_N + \theta_F - 3\psi) + (\mathrm{FEM})_N$$

$$M_{AB} = \frac{2EI}{4}(0 + \theta_B) + 0$$

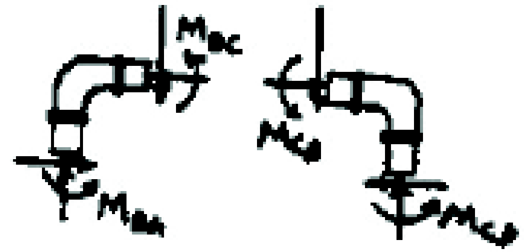
$$M_{BA} = \frac{2EI}{4}(2\theta_B + 0) + 0$$

$$M_{BC} = \frac{2EI}{12}(2\theta_B + \theta_C) - \frac{2(18)(12)}{9}$$

$$M_{CB} = \frac{2EI}{12}(2\theta_C + \theta_B) + \frac{2(18)(12)}{9}$$

$$M_{CD} = \frac{2EI}{4}(2\theta_C + 0) + 0$$

$$M_{DC} = \frac{2EI}{4}(0 + \theta_C) + 0$$



Equilibrium

$$M_{BA} + M_{BC} = 0$$

$$\frac{2EI}{4}(2\theta_B) + \frac{2EI}{12}(2\theta_B + \theta_C) - 48 = 0$$

$$1.333\theta_B + 0.1667\theta_C = \frac{48}{EI} \qquad (1)$$

$$M_{CB} + M_{CD} = 0$$

$$\frac{2EI}{12}(2\theta_C + \theta_B) + 48 + \frac{2EI}{4}(2\theta_C) = 0$$

$$1.333\theta_C + 0.1667\theta_B = -\frac{48}{EI} \qquad (2)$$

Solving Eqs. (1) and (2)

$$\theta_B = -\theta_C = \frac{41.143}{EI}$$

$M_{AB} = 20.6$ kN·m

$M_{BA} = 41.1$ kN·m

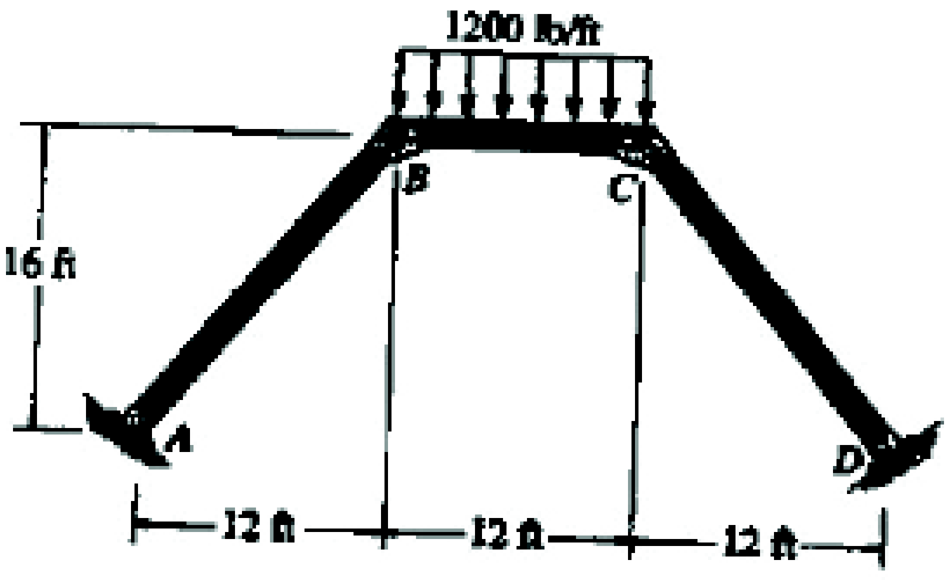
$M_{BC} = -41.1$ kN·m

$M_{CB} = 41.1$ kN·m

$M_{CD} = -41.1$ kN·m

$M_{DC} = -20.6$ kN·m

10–18. Determine the moments at each joint and support of the battered-column frame. The joints and supports are fixed connected. *EI* is constant.



$$M_N = 2E\left(\frac{I}{L}\right)(2\theta_N + \theta_F - 3\psi) + (FEM)_N$$

$$M_{AB} = \frac{2EI}{20}(0 + \theta_B) + 0$$

$$M_{BA} = \frac{2EI}{20}(2\theta_B + 0) + 0$$

$$M_{BC} = \frac{2EI}{12}(2\theta_B + \theta_C) - \frac{1.2(12)}{12}$$

$$M_{CB} = \frac{2EI}{12}(2\theta_C + \theta_B) + \frac{1.2(12)}{12}$$

$$M_{CD} = \frac{2EI}{20}(2\theta_C + 0) + 0$$

$$M_{DC} = \frac{2EI}{20}(0 + \theta_C) + 0$$

Equilibrium

$$M_{BA} + M_{BC} = 0$$

$$\frac{2EI}{20}(2\theta_B) + \frac{2EI}{12}(2\theta_B + \theta_C) - 14.4 = 0$$

$$0.5333\theta_B + 0.1667\theta_C = \frac{14.4}{EI} \qquad (1)$$

$$M_{CB} + M_{CD} = 0$$

$$\frac{2EI}{12}(2\theta_C + \theta_B) + 14.4 + \frac{2}{20}(2\theta_C) = 0$$

$$0.5333\theta_C + 0.1667\theta_B = \frac{-14.4}{EI} \qquad (2)$$

Solving Eqs. 1-2:

$$\theta_B = -\theta_C = \frac{39.27}{EI}$$

$M_{AB} = 3.93$ k·ft **Ans**

$M_{BA} = 7.85$ k·ft **Ans**

$M_{BC} = -7.85$ k·ft **Ans**

$M_{CB} = 7.85$ k·ft **Ans**

$M_{CD} = -7.85$ k·ft **Ans**

$M_{DC} = -3.93$ k·ft **Ans**

Analysis of frames: Sideway

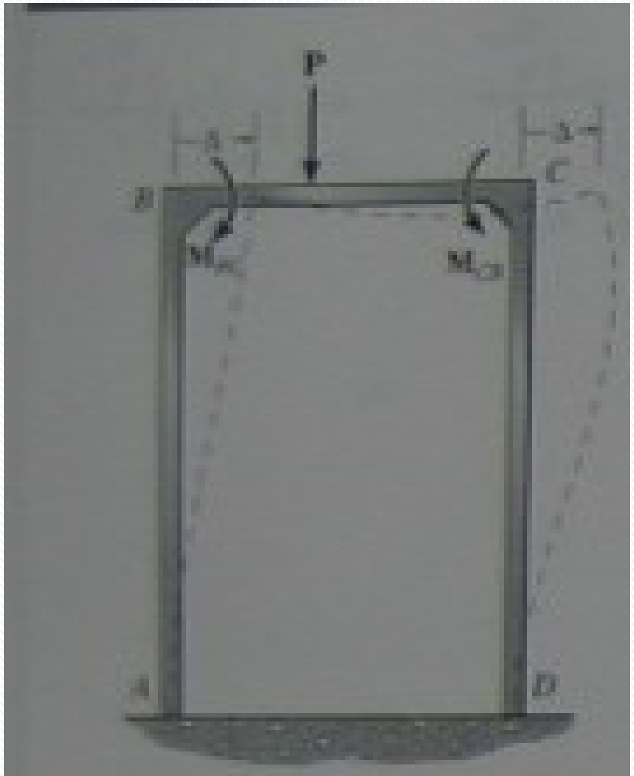
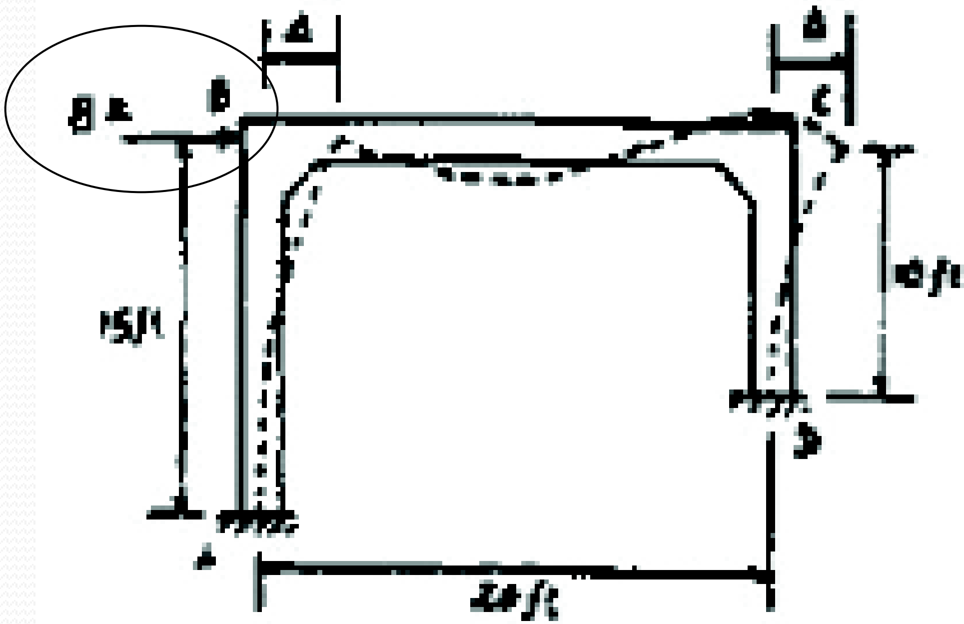


Fig. 10–18

A frame will sideway, or be displaced to the side, when it or the loading action on it is nonsymmetric

*10–20. Determine the moments at each joint and fixed support, then draw the moment diagram. *EI* is constant.



$$(FEM)_{AB} = (FEM)_{BA} = 0$$

$$(FEM)_{BC} = (FEM)_{CB} = 0$$

$$(FEM)_{CD} = (FEM)_{DC} = 0$$

$$\psi_{AB} = \frac{2}{3}\psi_{DC}$$

$$M_{NF} = 2E\left(\frac{I}{L}\right)(2\theta_N + \theta_F - 3\psi) + (\text{FEM})_{NF}$$

$$M_{AB} = 2E\left(\frac{I}{15}\right)\left(2(0) + \theta_B - 3\left(\frac{2}{3}\right)\psi_{DC}\right) + 0$$

$$M_{AB} = 0.1333EI\theta_B - 0.2667EI\psi_{DC} \qquad (1)$$

$$M_{BA} = 2E\left(\frac{I}{15}\right)\left(2\theta_B + 0 - 3\left(\frac{2}{3}\right)\psi_{DC}\right) + 0$$

$$M_{BA} = 0.2667EI\theta_B - 0.2667EI\psi_{DC} \qquad (2)$$

$$M_{BC} = 2E\left(\frac{I}{20}\right)(2\theta_B + \theta_C - 3(0)) + 0$$

$$M_{BC} = 0.2EI\theta_B + 0.1EI\theta_C \qquad (3)$$

$$M_{CB} = 2E\left(\frac{I}{20}\right)(2\theta_C + \theta_B - 3(0)) + 0$$

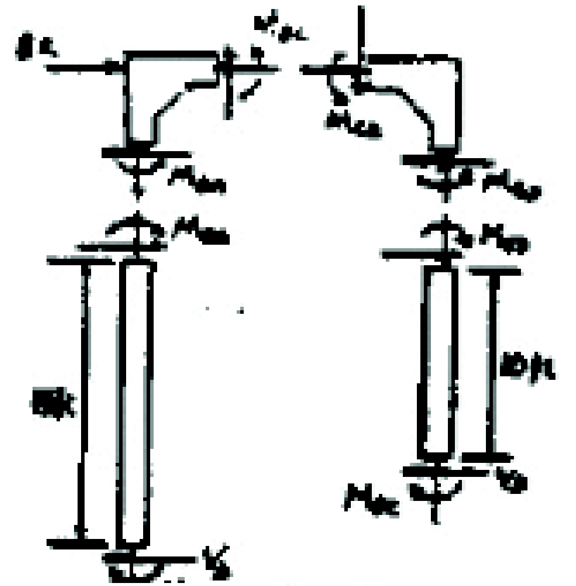
$$M_{CB} = 0.2EI\theta_C + 0.1EI\theta_B \qquad (4)$$

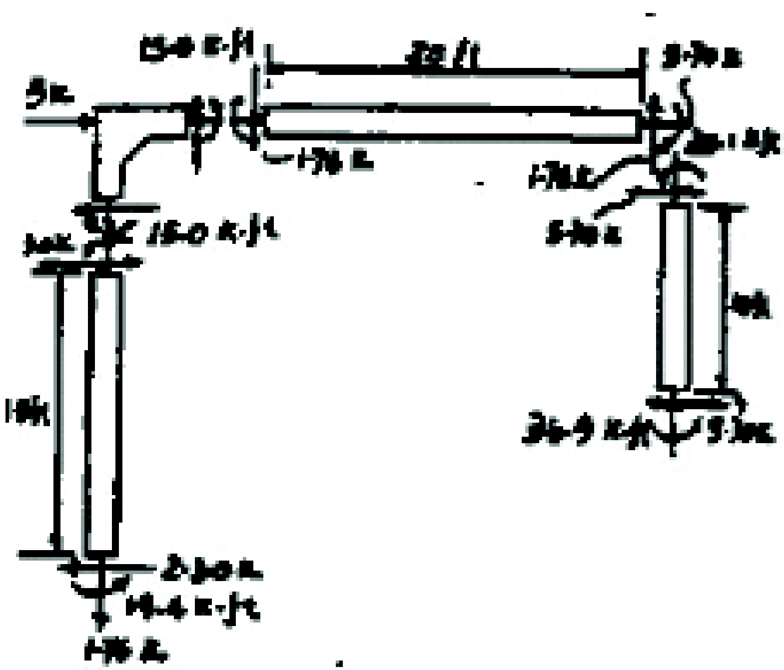
$$M_{CD} = 2E\left(\frac{I}{10}\right)(2\theta_C + 0 - 3\psi_{DC}) + 0$$

$$M_{CD} = 0.4EI\theta_C - 0.6EI\psi_{DC} \qquad (5)$$

$$M_{DC} = 2E\left(\frac{I}{10}\right)(2(0) + \theta_C - 3\psi_{DC}) + 0$$

$$M_{DC} = 0.2EI\theta_C - 0.6EI\psi_{DC} \qquad (6)$$





Equilibrium

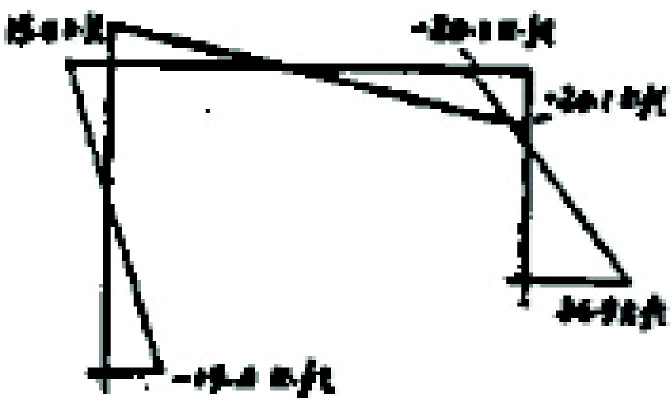
$$M_{BA} + M_{BC} = 0 \qquad (7)$$

$$M_{CB} + M_{CD} = 0 \qquad (8)$$

$$V_A + V_D - 8 = 0$$

$$-\frac{(M_{AB} + M_{BA})}{15} - \frac{(M_{CD} + M_{DC})}{10} - 8 = 0$$

$$2M_{AB} + 2M_{BA} + 3M_{CD} + 3M_{DC} = -240 \qquad (9)$$



Solving these equations :

$$\theta_B = \frac{33.149}{EI}, \qquad \theta_C = \frac{83.978}{EI}$$

$$\psi_{DC} = \frac{89.503}{EI}$$

$M_{AB} = -19.4$ k·ft Ans

$M_{BA} = -15.0$ k·ft Ans

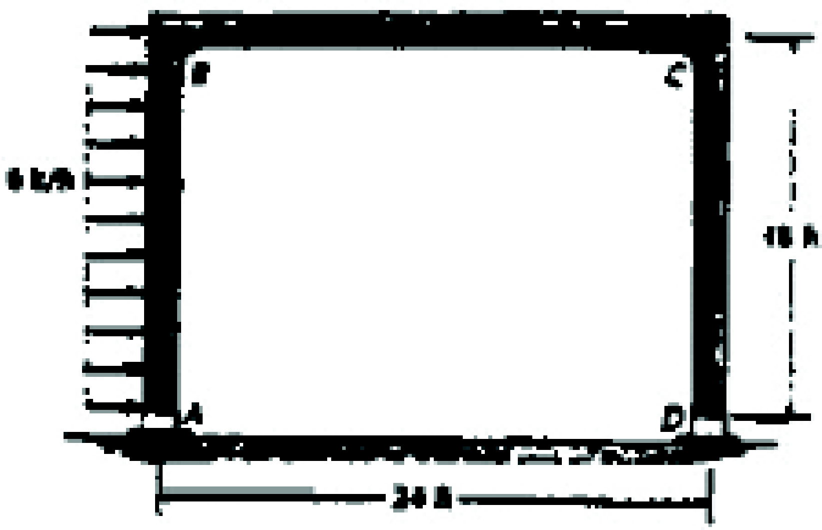
$M_{BC} = 15.0$ k·ft Ans

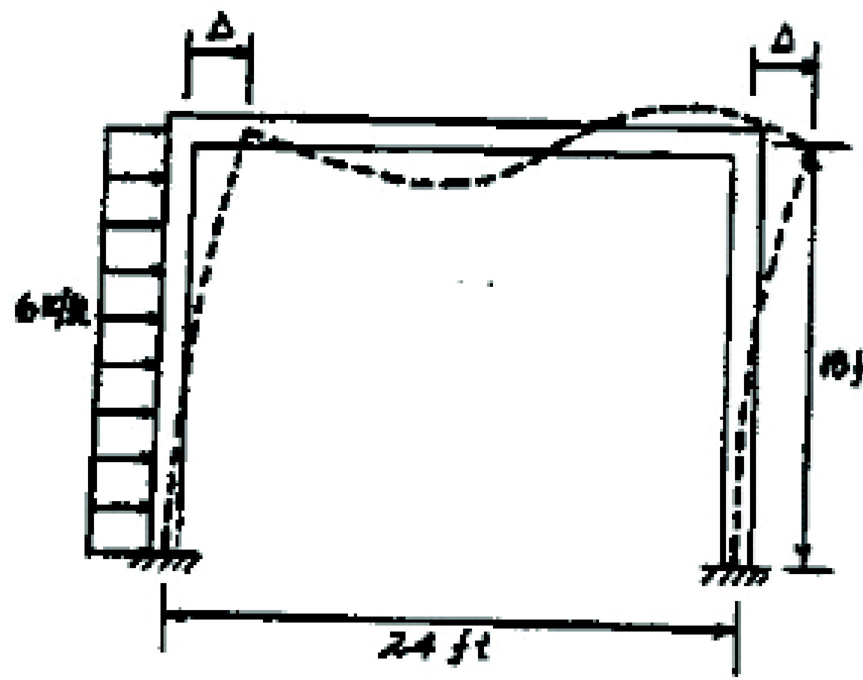
$M_{CB} = 20.1$ k·ft Ans

$M_{CD} = -20.1$ k·ft Ans

$M_{DC} = -36.9$ k·ft Ans

10–21. Determine the moments at each joint and support. There are fixed connections at *B* and *C* and fixed supports at *A* and *D*. *EI* is constant.





$$(FEM)_{AB} = \frac{-6(18)^2}{12} = -162 \text{ k}\cdot\text{ft}$$

$$(FEM)_{BA} = 162 \text{ k}\cdot\text{ft}$$

$$(FEM)_{BC} = (FEM)_{CB} = 0$$

$$(FEM)_{CD} = (FEM)_{DC} = 0$$

$$\psi_{AB} = \psi_{DC}$$

$$M_N = 2E\left(\frac{I}{L}\right)(2\theta_N + \theta_F - 3\psi) + (FEM)_N$$

$$M_{AB} = 2E\left(\frac{I}{18}\right)(2(0) + \theta_B - 3\psi_{AB}) - 162$$

$$M_{AB} = 0.1111EI\theta_B - 0.3333EI\psi_{AB} - 162 \qquad (1)$$

$$M_{BA} = 2E\left(\frac{I}{18}\right)(2\theta_B + 0 - 3\psi_{AB}) + 162$$

$$M_{BA} = 0.2222EI\theta_B - 0.333EI\psi_{AB} + 162 \qquad (2)$$

$$M_{BC} = 2E\left(\frac{I}{24}\right)(2\theta_B + \theta_C - 3(0)) + 0$$

$$M_{BC} = 0.1667EI\theta_B + 0.08333EI\theta_C \qquad (3)$$

$$M_{CB} = 2E\left(\frac{I}{24}\right)(2\theta_C + \theta_B - 3(0)) + 0$$

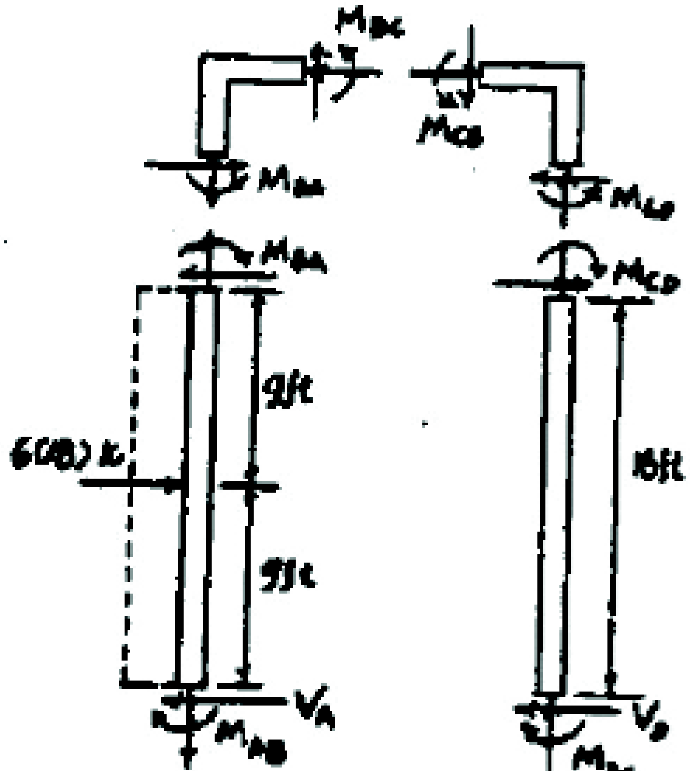
$$M_{CB} = 0.1667EI\theta_B + 0.08333EI\theta_C \qquad (4)$$

$$M_{CD} = 2E\left(\frac{I}{18}\right)(2\theta_C + 0 - 3\psi_{AB}) + 0$$

$$M_{CD} = 0.2222EI\theta_C - 0.3333EI\psi_{AB} \qquad (5)$$

$$M_{DC} = 2E\left(\frac{I}{18}\right)(2(0) + \theta_C - 3\psi_{AB}) + 0$$

$$M_{DC} = 0.1111EI\theta_C - 0.3333EI\psi_{AB} \qquad (6)$$



Equilibrium

$$M_{BA} + M_{BC} = 0 \qquad (7)$$

$$M_{CD} + M_{CB} = 0 \qquad (8)$$

$$V_A + V_D - 6(18) = 0$$

$$-\frac{(M_{AB} + M_{BA} - 972)}{18} - \frac{(M_{CD} + M_{DC})}{18} - 108 = 0$$

$$M_{AB} + M_{BA} + M_{CD} + M_{DC} = -972 \qquad (9)$$

Solving these equations :

$$\theta_B = \frac{265.09}{EI}, \qquad \theta_C = \frac{795.27}{EI}$$

$$\psi_{AB} = \frac{994.09}{EI}$$

$M_{AB} = -464$ k·ft Ans

$M_{BA} = -110$ k·ft Ans

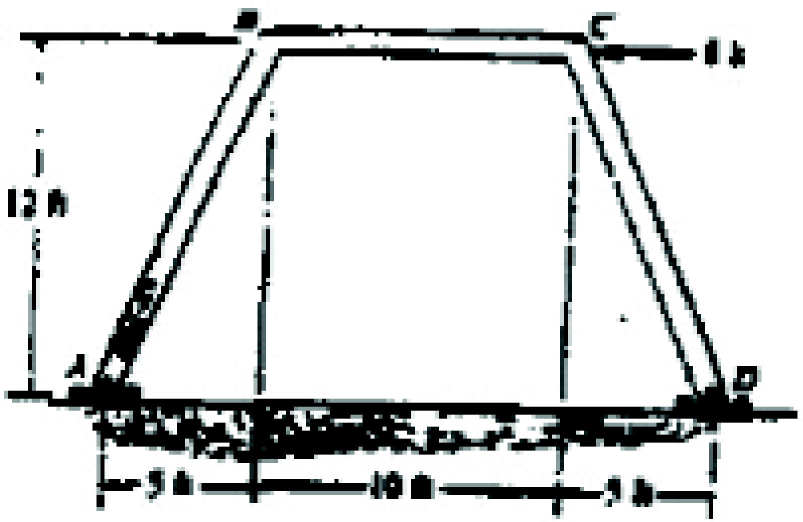
$M_{BC} = 110$ k·ft Ans

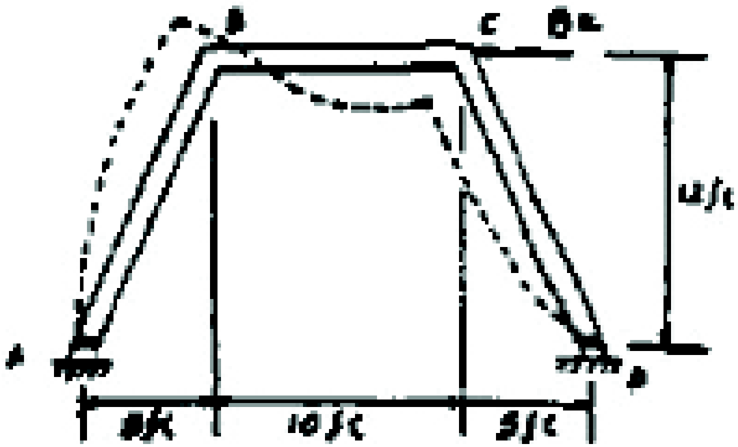
$M_{CB} = 135$ k·ft Ans

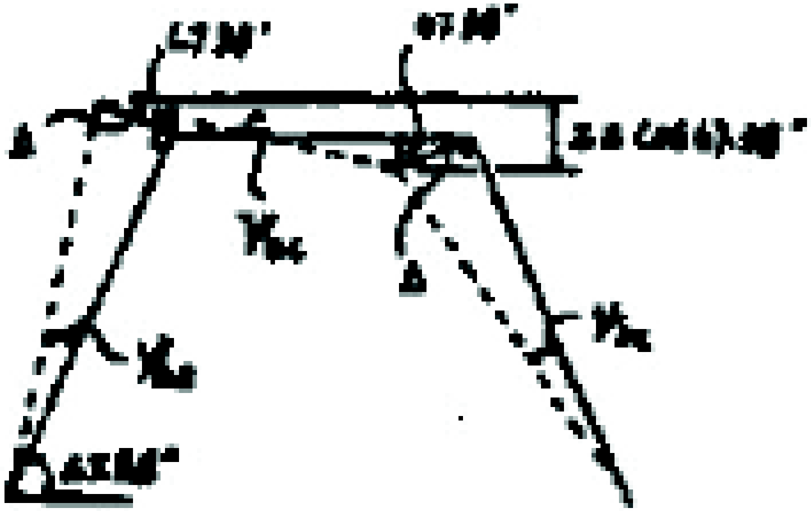
$M_{CD} = -135$ k·ft Ans

$M_{DC} = -243$ k·ft Ans

10–26. Determine the moment at each joint of the battered-column frame. The supports at A and D are pins. EI is constant.







$$(FEM)_{BA} = (FEM)_{BC} = (FEM)_{CB} = (FEM)_{CD} = 0$$

$$\psi_{AB} = \psi_{DC} = \frac{\Delta}{13} \qquad \psi_{BC} = \frac{2\Delta \cos 67.38^\circ}{10}$$

$$\psi_{AB} = \psi_{DC} = \psi_{BC}$$

$$M_N = 3E\left(\frac{I}{L}\right)(\theta_N - \psi) + (FEM)_N$$

$$M_{BA} = 3E\left(\frac{I}{13}\right)(\theta_B + \psi_{AB}) + 0$$

$$M_{BA} = 0.2308EI(\theta_B + \psi_{AB}) \qquad (1)$$

$$M_N = 2E\left(\frac{I}{L}\right)(2\theta_N + \theta_F - 3\psi) + (FEM)_N$$

$$M_{BC} = 2E\left(\frac{I}{10}\right)(2\theta_B + \theta_C - 3\psi_{AB}) + 0$$

$$M_{BC} = 0.2EI(2\theta_B + \theta_C - 3\psi_{AB}) + 0 \qquad (2)$$

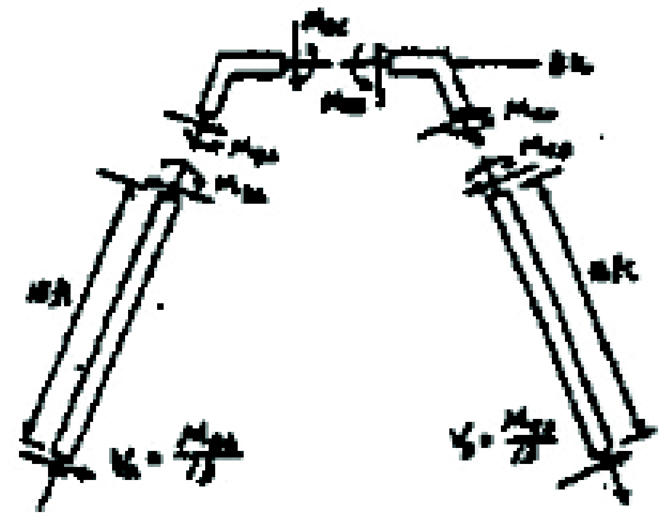
$$M_{CB} = 2E\left(\frac{I}{10}\right)(2\theta_C + \theta_B - 3\psi_{AB}) + 0$$

$$M_{CB} = 0.2EI(2\theta_C + \theta_B - 3\psi_{AB}) + 0 \qquad (3)$$

$$M_N = 3E\left(\frac{I}{L}\right)(\theta_N - \psi) + (FEM)_N$$

$$M_{CD} = 3E\left(\frac{I}{13}\right)(\theta_C + \psi_{AB}) + 0$$

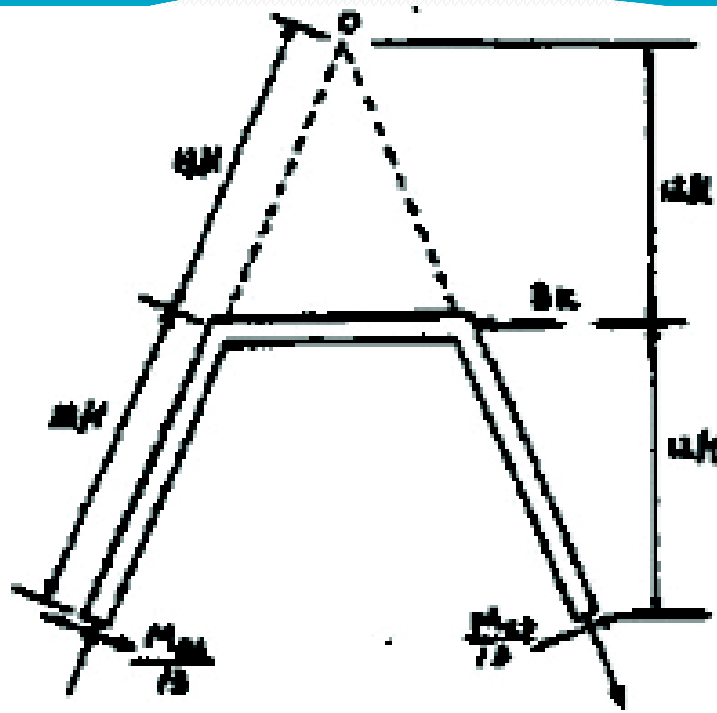
$$M_{CD} = 0.2308EI(\theta_C + \psi_{AB}) \qquad (4)$$



Equilibrium

$$M_{BA} + M_{BC} = 0 \qquad (5)$$

$$M_{CD} + M_{CB} = 0 \qquad (6)$$



$$\curvearrowleft + \Sigma M_O = 0: \quad \frac{M_{BA}}{13}(26) + \frac{M_{CD}}{13}(26) - 8(12) = 0$$

$$2M_{BA} + 2M_{CD} - 96 = 0 \qquad (7)$$

Solving these equations :

$$\theta_B = \theta_C = \frac{32}{EI}$$

$$\psi_{AB} = \frac{72}{EI}$$

$M_{BA} = 24 \text{ k}\cdot\text{ft}$ Ans

$M_{BC} = -24 \text{ k}\cdot\text{ft}$ Ans

$M_{CB} = -24 \text{ k}\cdot\text{ft}$ Ans

$M_{CD} = 24 \text{ k}\cdot\text{ft}$ Ans