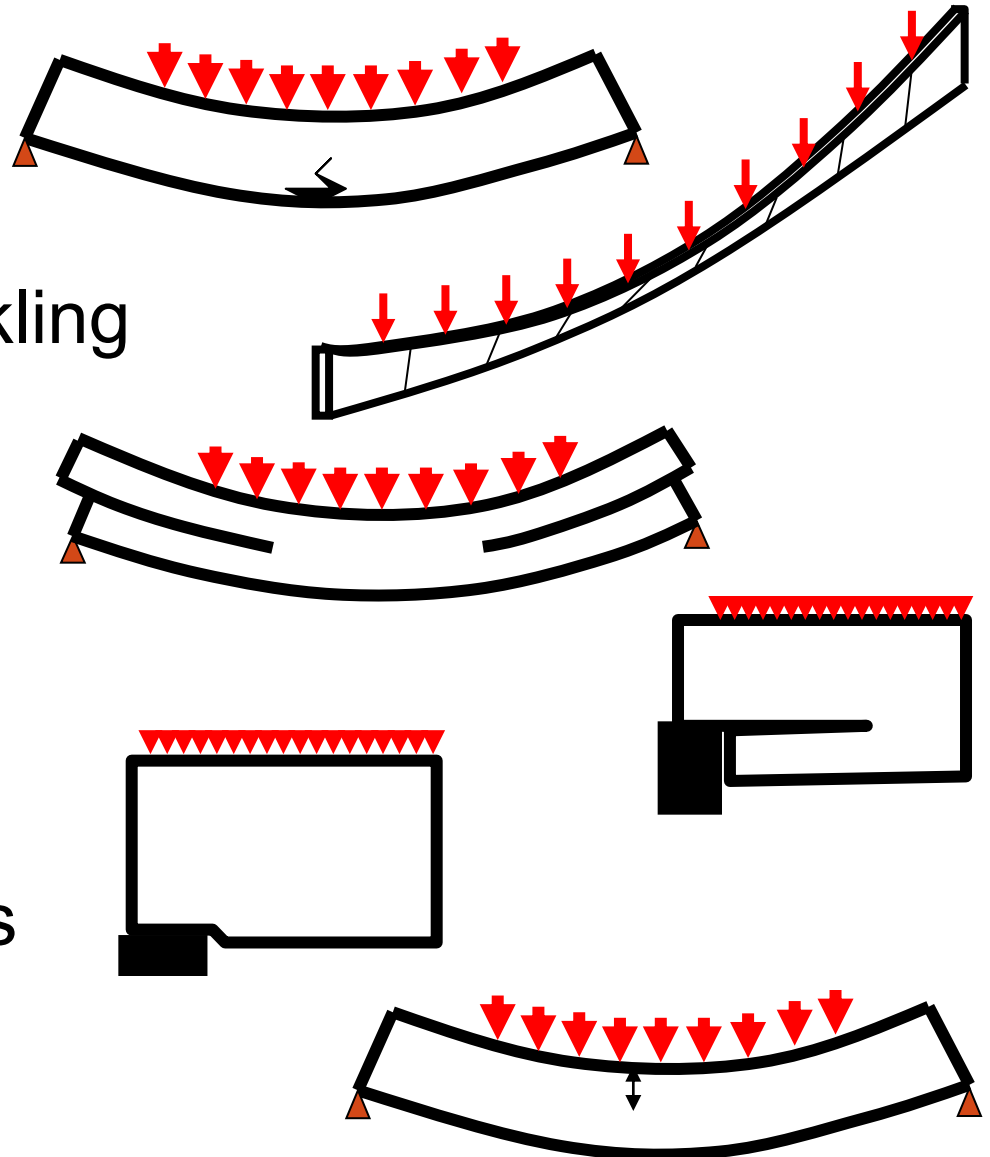


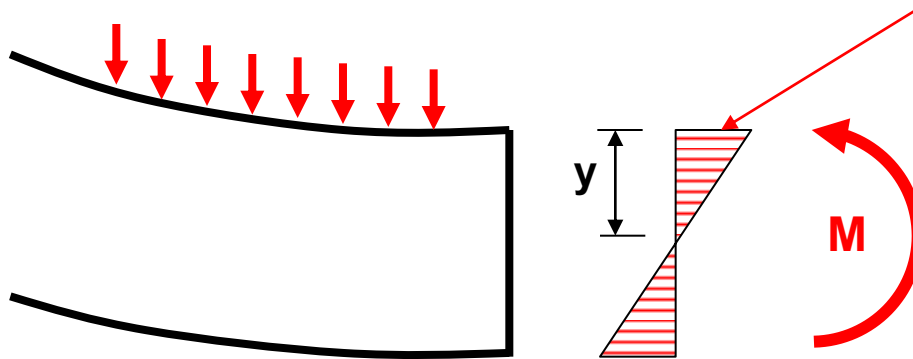
TIMBER BEAMS:

- Bending failure
- Lateral torsional buckling
- Shear failure
- Notch failure
- Bearing failure
- Excessive deflections



Bending Strength

Linear elastic stresses



$$\sigma_{\max} = \frac{My}{I} = \frac{M}{S}$$

$$M = \sigma_{\max} S$$

for rectangular sections

$$S = \frac{bh^2}{6}$$

Design Equation:

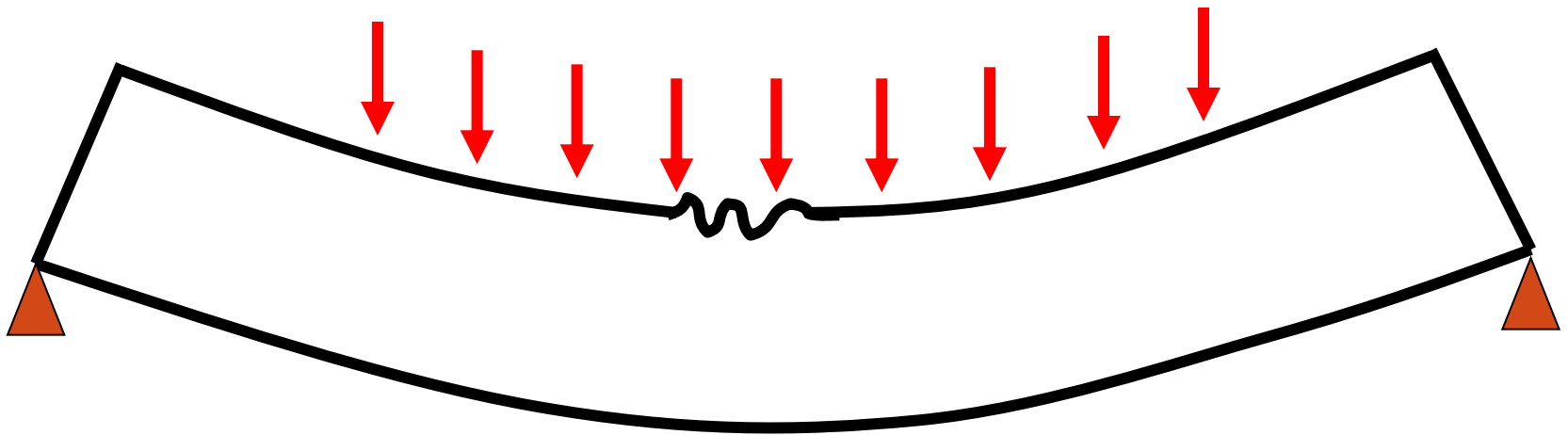
$$M_r = \varphi F_b S$$

Where F_b is the characteristic bending strength

For timber it is $F_b = f_b (K_D K_H K_{Sb} K_T)$

Bending failure in compression

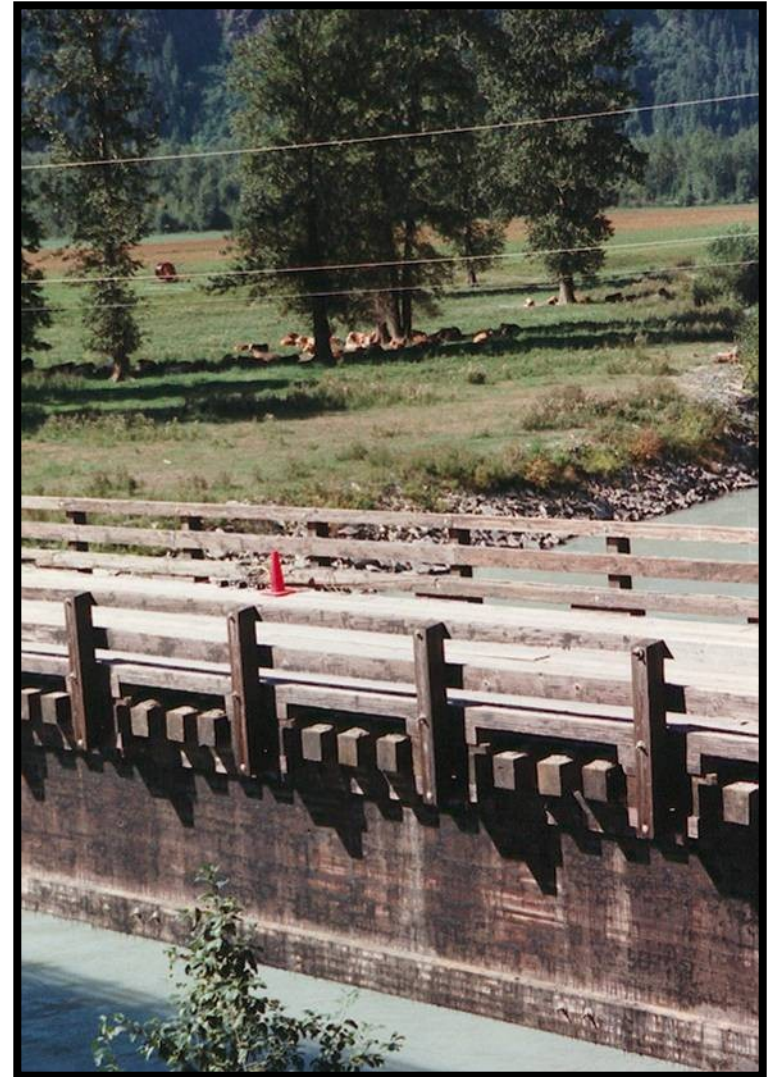
- Only likely for very high grade material
- Benign failure mode





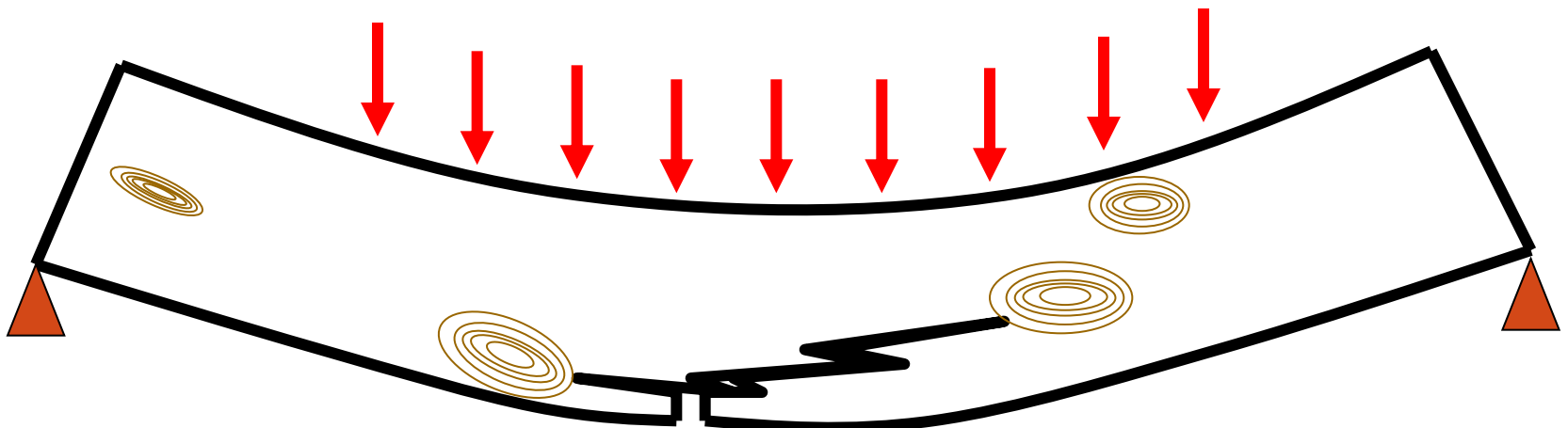
**Logging bridge
near Pemberton, BC**

Glulam I-beam

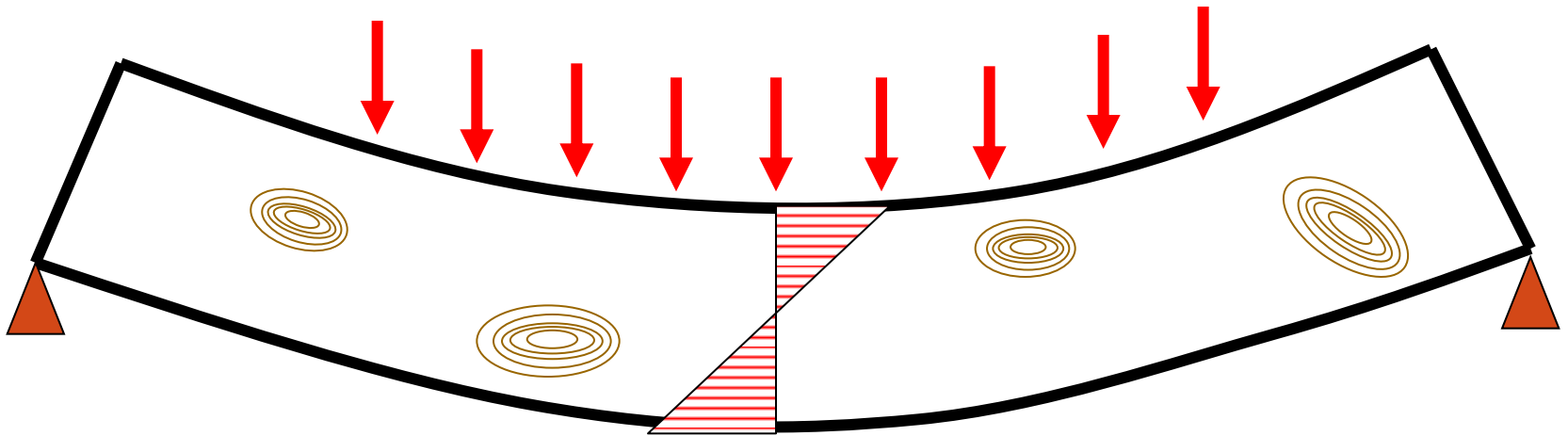


Bending failure in tension

- Most likely failure mode
- Brittle
- Combination of tension and shear, although tension fracture is the initiating mode



Bending capacity



$$M_r = \phi F_b S K_{zb} K_L$$

Lateral
torsional
buckling

where

$$\phi = 0.9$$

and

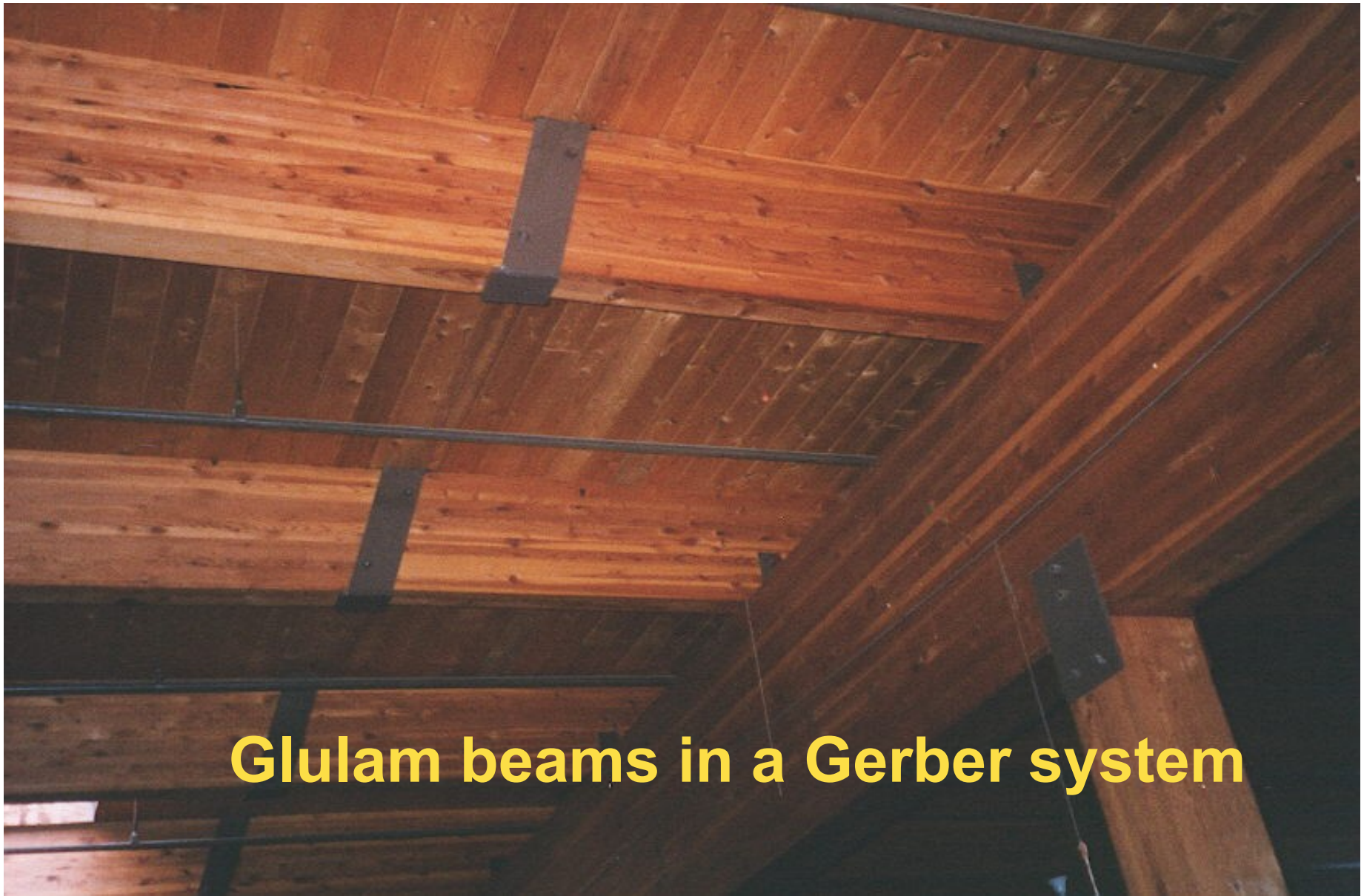
$$F_b = f_b (K_D K_H K_{Sb} K_T)$$

Table 5.3.1C
Specified Strengths and Modulus of Elasticity
for Beam and Stringer Grades (MPa)

Species identification	Grade	Bending at extreme fibre,* f_b	Longitudinal shear, f_v	Compression			Modulus of elasticity	
				Parallel to grain, f_c	Perpendicular to grain, f_{cp}	Tension parallel to grain, f_t	E	E_{05}
D Fir-L	SS	19.5		13.2		10.0	12 000	8 000
	No. 1	15.8	0.9	11.0	7.0	7.0	12 000	8 000
	No. 2	9.0		7.2		3.3	9 500	6 000
Hem-Fir	SS	14.5		10.8		7.4	10 000	7 000
	No. 1	11.7	0.7	9.0	4.6	5.2	10 000	7 000
	No. 2	6.7		5.9		2.4	8 000	5 500
S-P-F	SS	13.6		9.5		7.0	8 500	6 000
	No. 1	11.0	0.7	7.9	5.3	4.9	8 500	6 000
	No. 2	6.3		5.2		2.3	6 500	4 500
Northern	SS	12.8		7.2		6.5	8 000	5 500
	No. 1	10.8	0.6	6.0	3.5	4.6	8 000	5 500
	No. 2	5.9		3.9		2.2	6 000	4 000

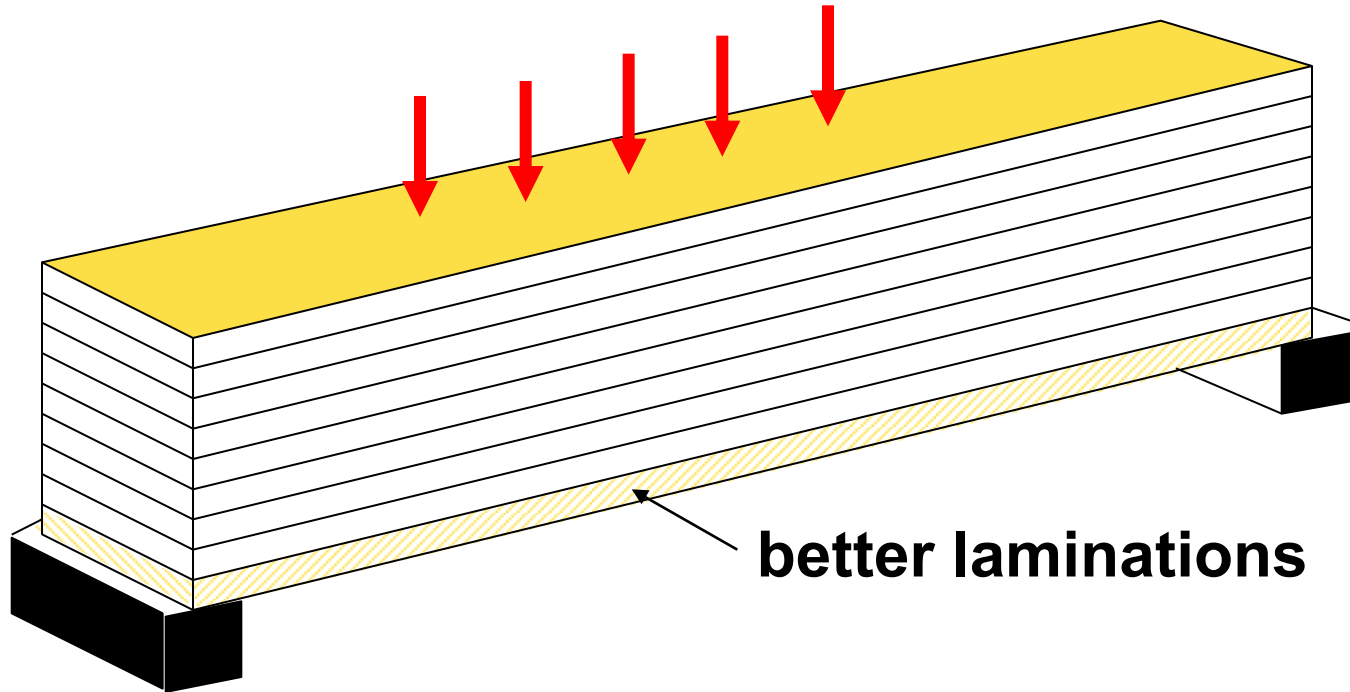
*Specified strengths for beams and stringers are based on loads applied to the narrow face. When beams and stringers are subject to loads applied to the wide face, the specified strength for bending at the extreme fibre and the specified modulus of elasticity shall be multiplied by the following factors:

	f_b	E or E_{05}
Select Structural	0.88	1.00
No. 1 or No. 2	0.77	0.90



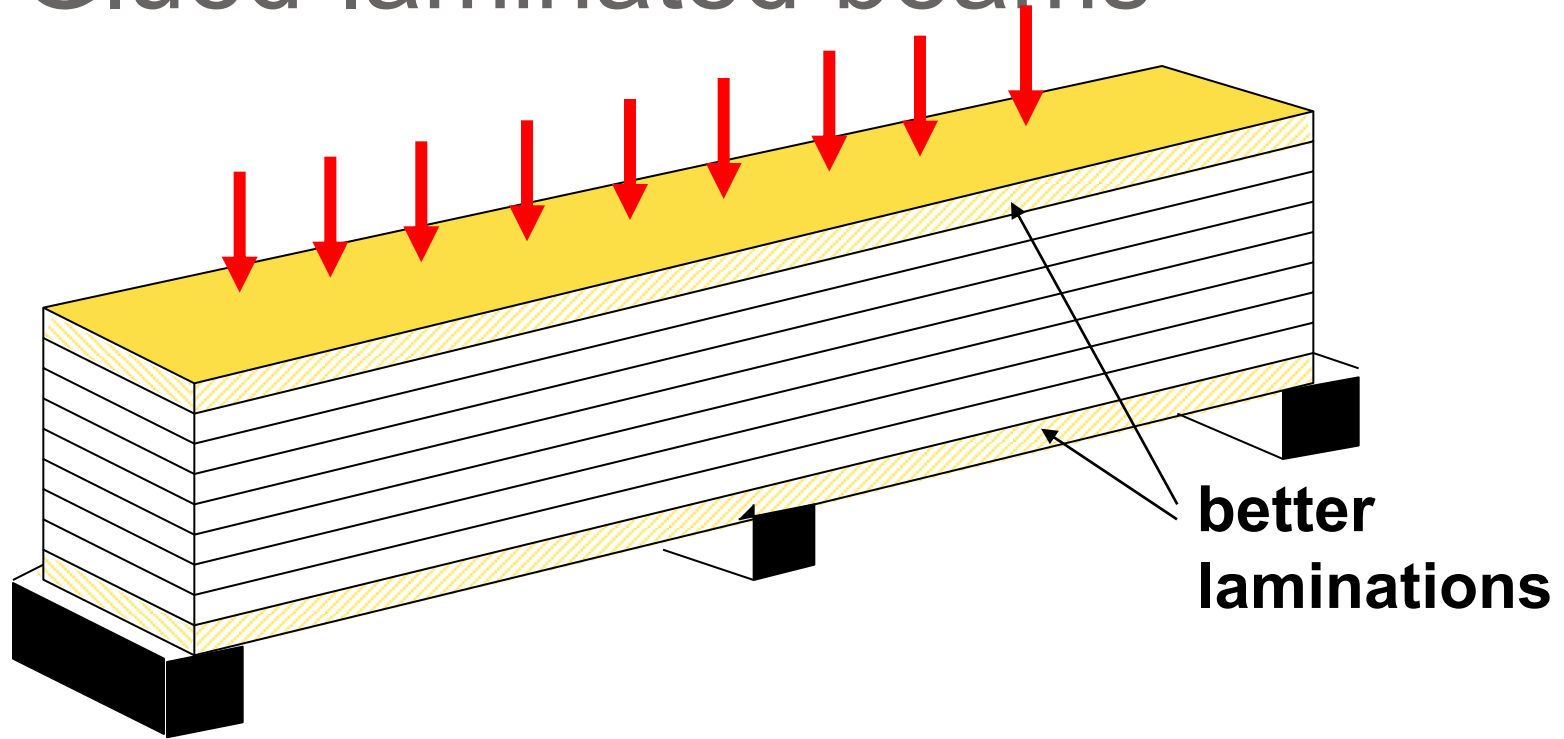
Glulam beams in a Gerber system

Glued-laminated beams



20f-E and 24f-E grades

Glued-laminated beams



20f-EX and 24f-EX grades

Table 6.3
Specified Strengths and Modulus of Elasticity
for Glued-Laminated Timber (MPa)

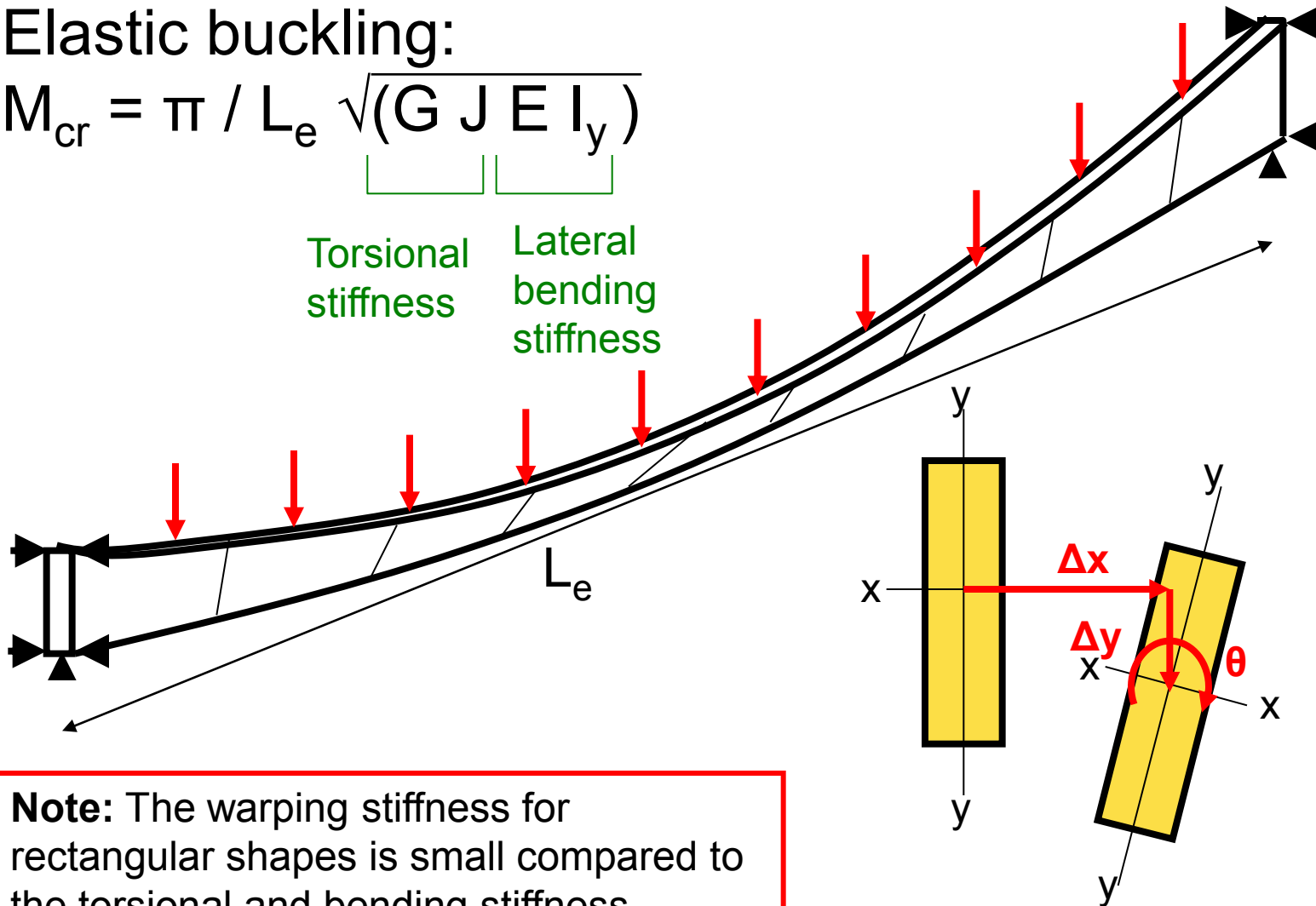
		Douglas Fir-Larch					
		24f-E	24f-EX	20f-E	20f-EX	18t-E	16c-E
Bending moment (pos.)	f_b	30.6	30.6	25.6	25.6	24.3	14.0
Bending moment (neg.)	f_b	14.0	30.6	14.0	25.6	24.3	14.0
Longitudinal shear	f_v	2.0	2.0	2.0	2.0	2.0	2.0
Compression parallel	f_c	30.2*	30.2*	30.2*	30.2*	30.2	30.2
Compression parallel combined with bending	f_{cb}	30.2*	30.2	30.2*	30.2	30.2	30.2
Compression perpendicular	f_{cp}						
— compression face bearing		7.0	7.0	7.0	7.0	7.0	7.0
— tension face bearing		7.0	7.0	7.0	7.0	7.0	7.0
Tension net section (See Clause 6.5.11)	f_m	20.4*	20.4	20.4*	20.4	23.0	20.4
Tension gross section	f_{tg}	15.3*	15.3	15.3*	15.3	17.9	15.3
Tension perpendicular to grain	f_{tp}	0.83	0.83	0.83	0.83	0.83	0.83
Modulus of elasticity	E	13 100	13 100	12 400	12 400	13 800	12 400

Lateral torsional buckling of timber beams

Elastic buckling:

$$M_{cr} = \pi / L_e \sqrt{\underbrace{G J}_{\text{Torsional stiffness}} \underbrace{E I_y}_{\text{Lateral bending stiffness}}}$$

Torsional stiffness
Lateral bending stiffness

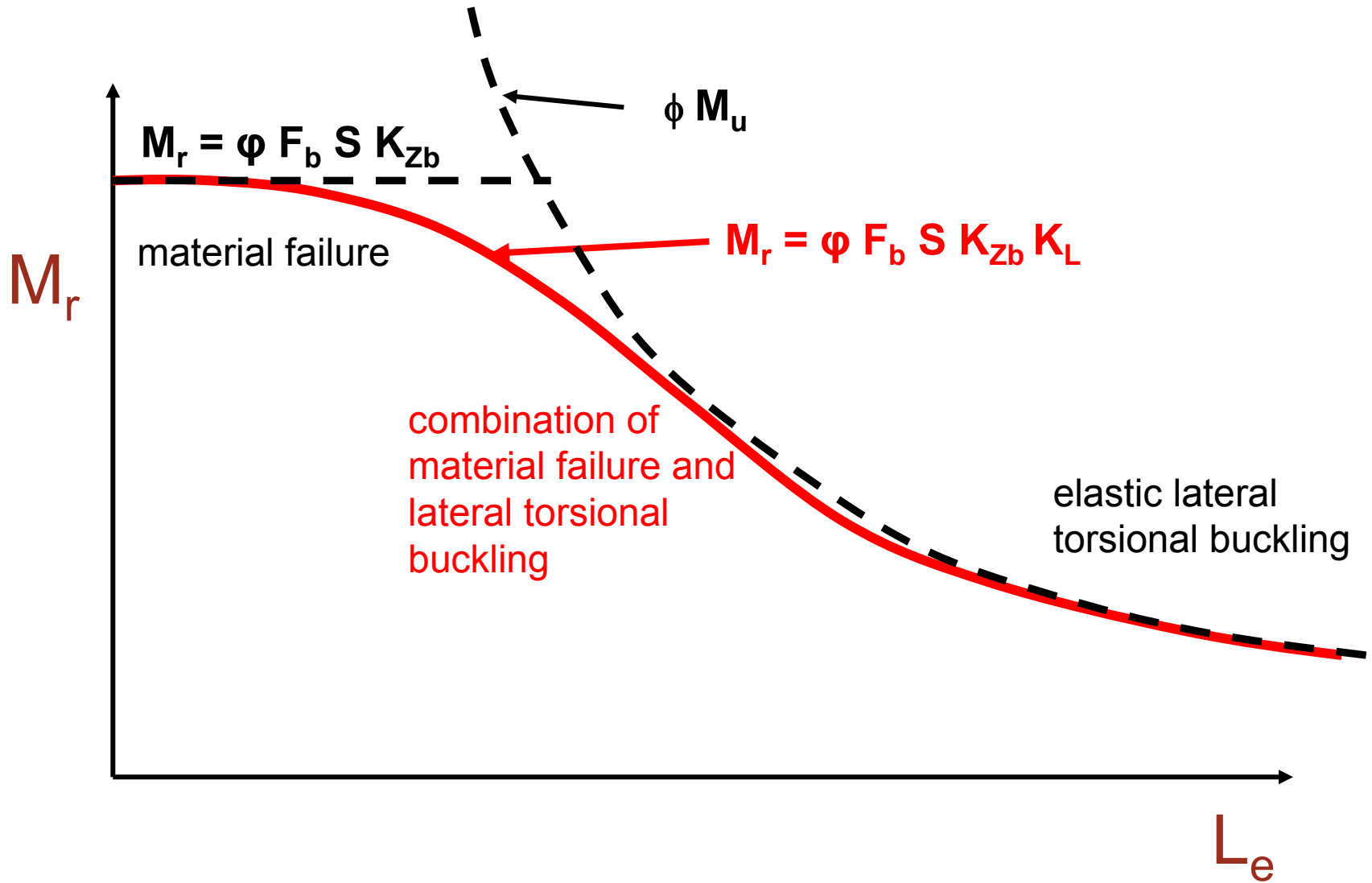


Note: The warping stiffness for rectangular shapes is small compared to the torsional and bending stiffness

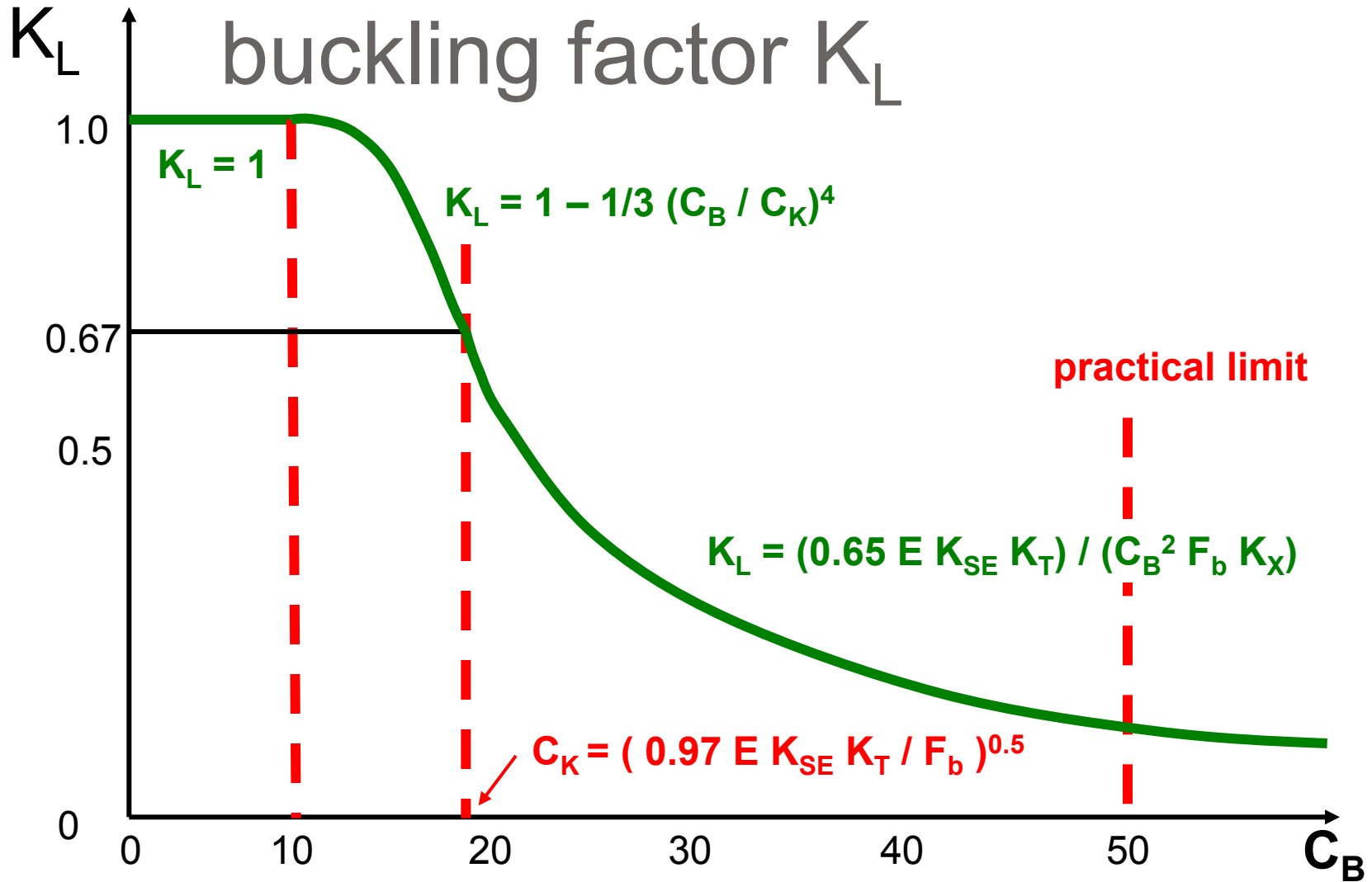


**Lateral torsional
buckling of
deep I-joists**

Capacity of a timber beam subject to lateral torsional buckling

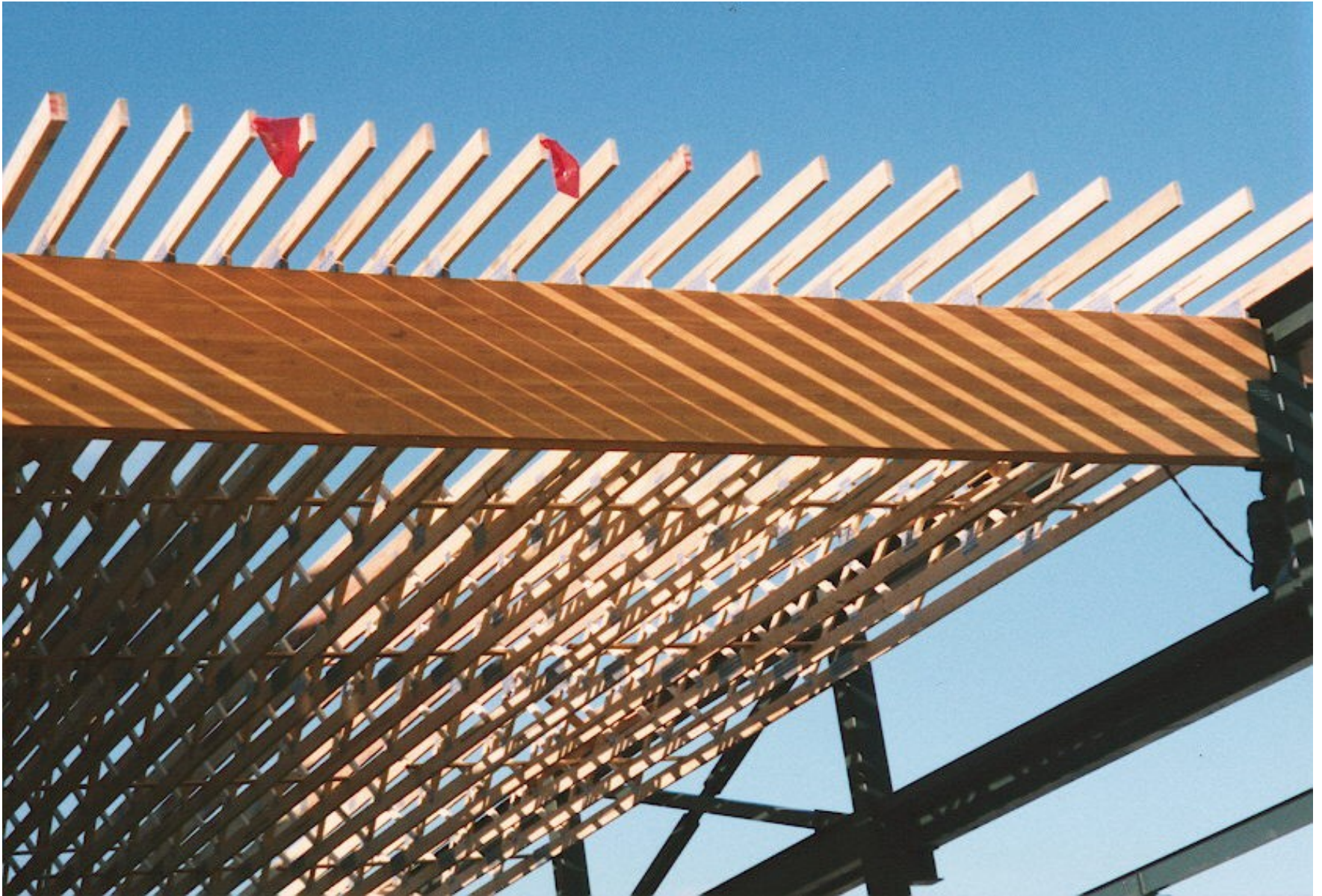


Lateral torsional buckling factor K_L





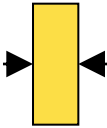

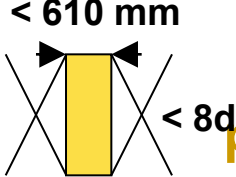

Slenderness ratio $C_B = (L_e d / b^2)^{0.5}$

Deep glulam beam



Prevention of lateral torsional buckling

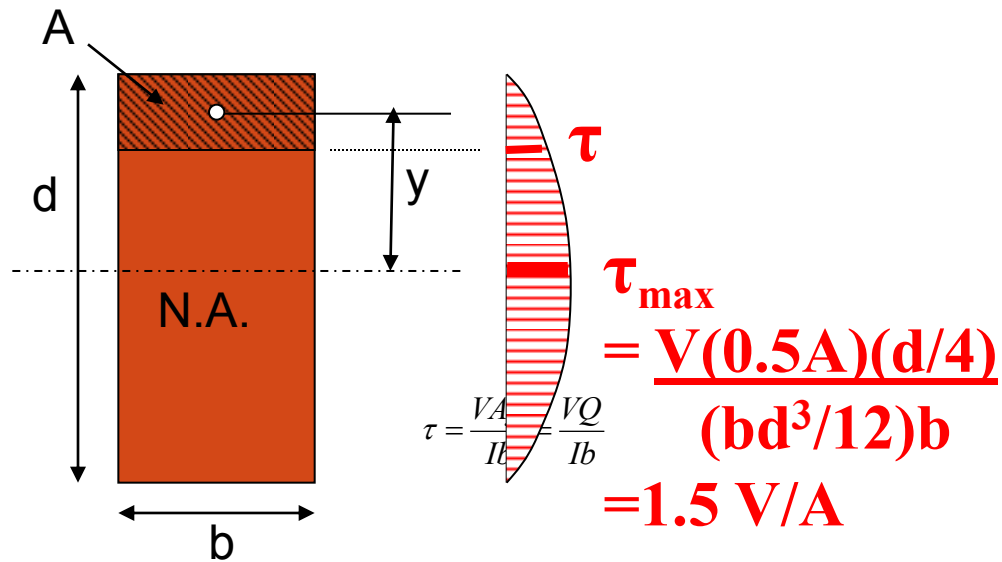
$K_L = 1.0$
when lateral support is provided as shown

d/b 	Lateral support at spacing:
< 4	 no support
< 5	 purlins or tie rods
< 6.5	$< 610 \text{ mm}$ compression edge held by decking or joists 
< 7.5	$< 610 \text{ mm}$ top edge plus bridging 
< 9	 both edges

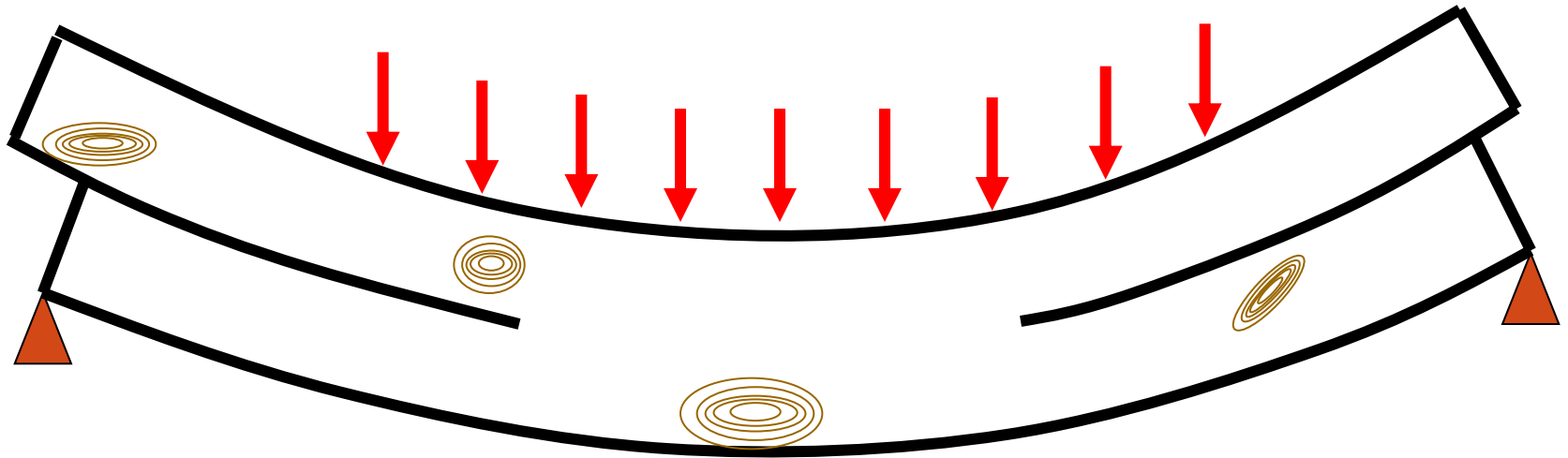
Bridging for floor joists



Shear stress in a beam



Shear in a timber beam



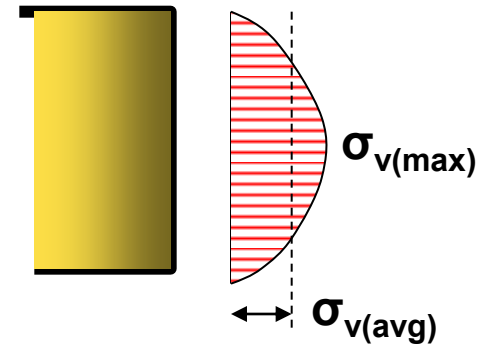
$$V_r = \varphi F_v \overbrace{2/3 A}^{A_s} K_{Zv}$$

where

$$\varphi = 0.9$$

and

$$F_v = f_v (K_D K_H K_{Sv} K_T)$$



$$\sigma_{v(max)} = 1.5 \sigma_{v(avg)}$$

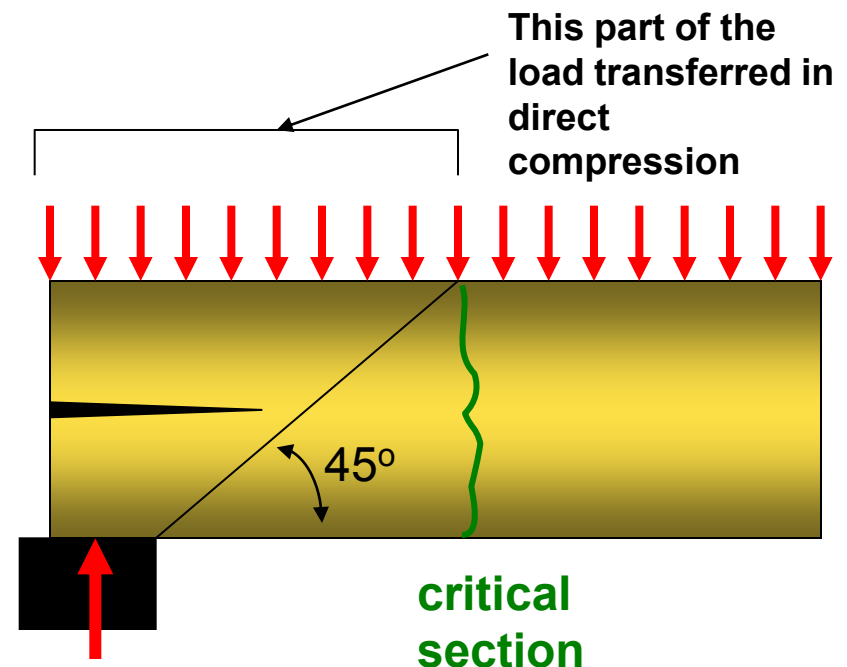
$$= 1.5 V / A$$



UNBC Prince George, BC

Shear failures

- One of the very weak properties of wood
- Shrinkage cracks often occur at the ends of beams in the zone of maximum shear stress
- **Direct compression transfer of loads in the end zones reduces the total shear force to be carried.**



Shear design of glulam beams

A simple approach for beams where the volume < 2.0 m³:

$$V_r = \phi F_v \frac{2}{3} A K_N$$

where

$$\phi = 0.9$$

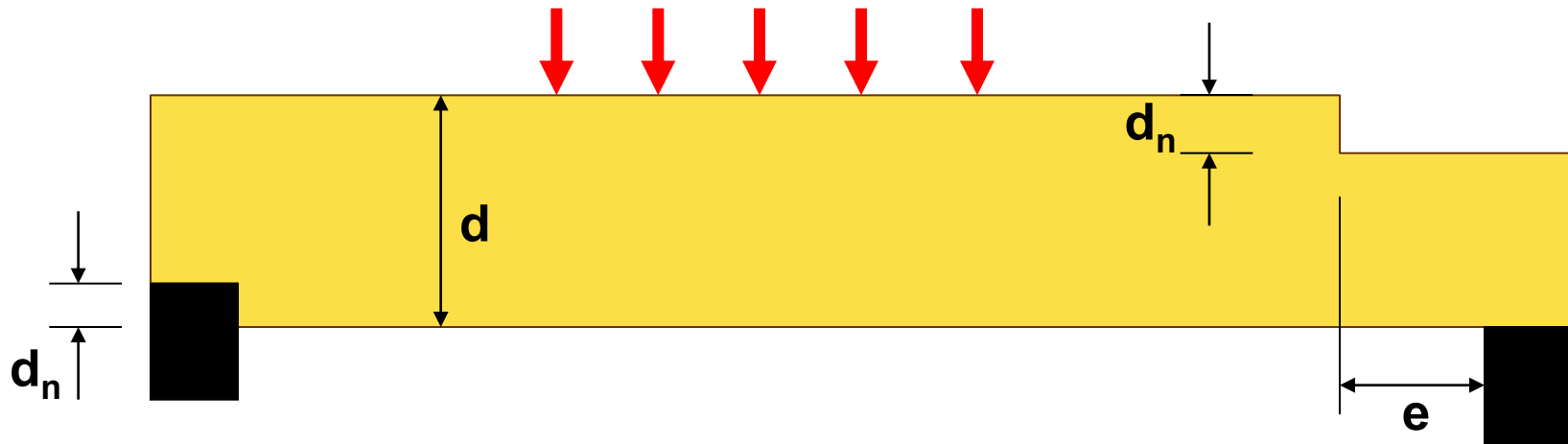
and

$$F_v = f_v (K_D K_H K_{Sv} K_T)$$

K_N = notch factor (see next section)

For larger beams this is usually quite conservative and a more sophisticated approach is used (see clause 6.5.7.3)

Notch factor for Glulam beams



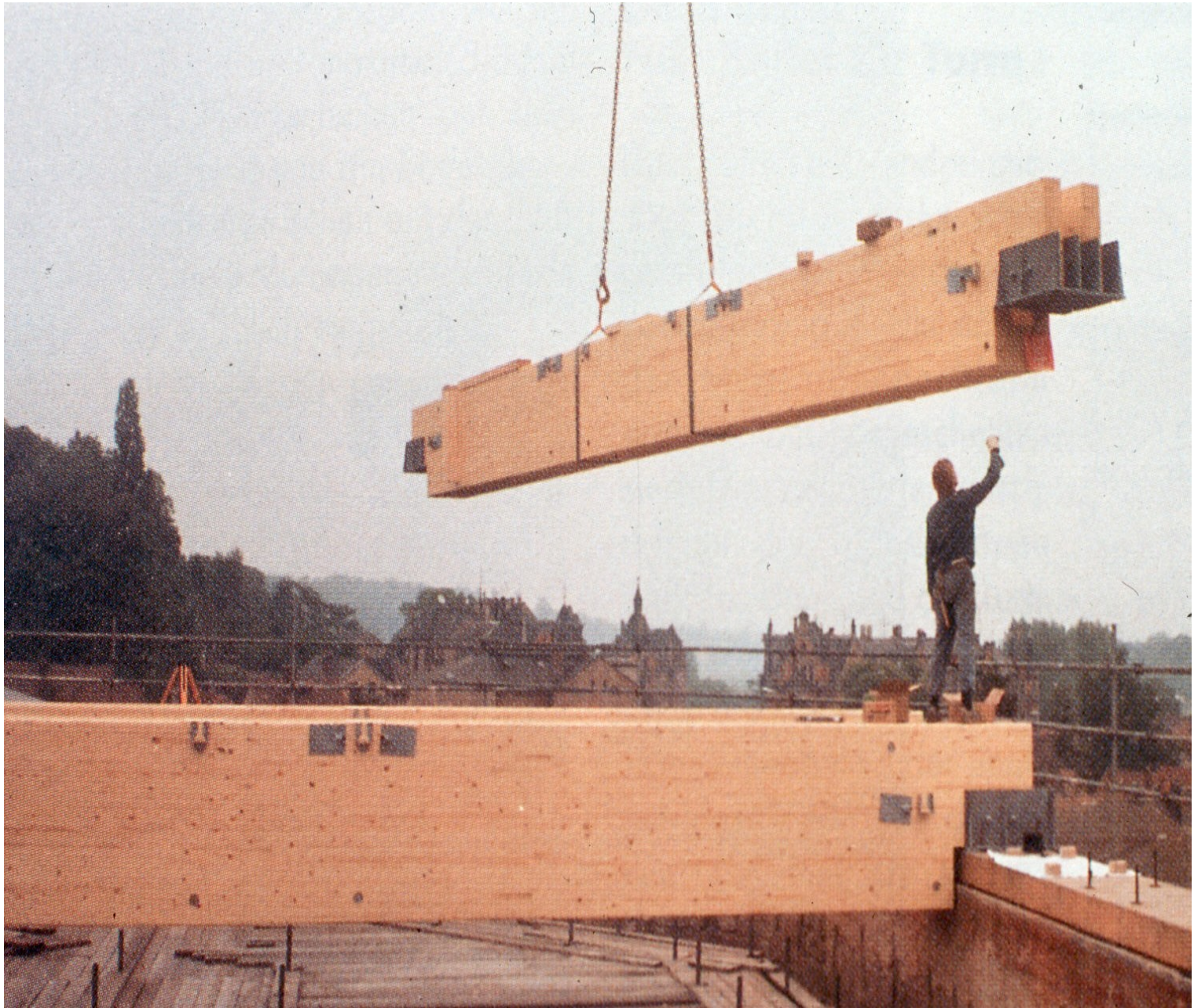
$$K_N = (1 - d_n/d)^2$$

For $e > d$:

$$K_N = (1 - d_n/d)$$

For $e < d$:

$$K_N = 1 - d_n e / [d(d - d_n)]$$



Notch effect in sawn lumber

- For notches on the tension side of supports (sawn lumber)
- In new code:
Reaction calculation

$$F_r = \phi F_t A K_N$$

$$\phi = 0.9$$

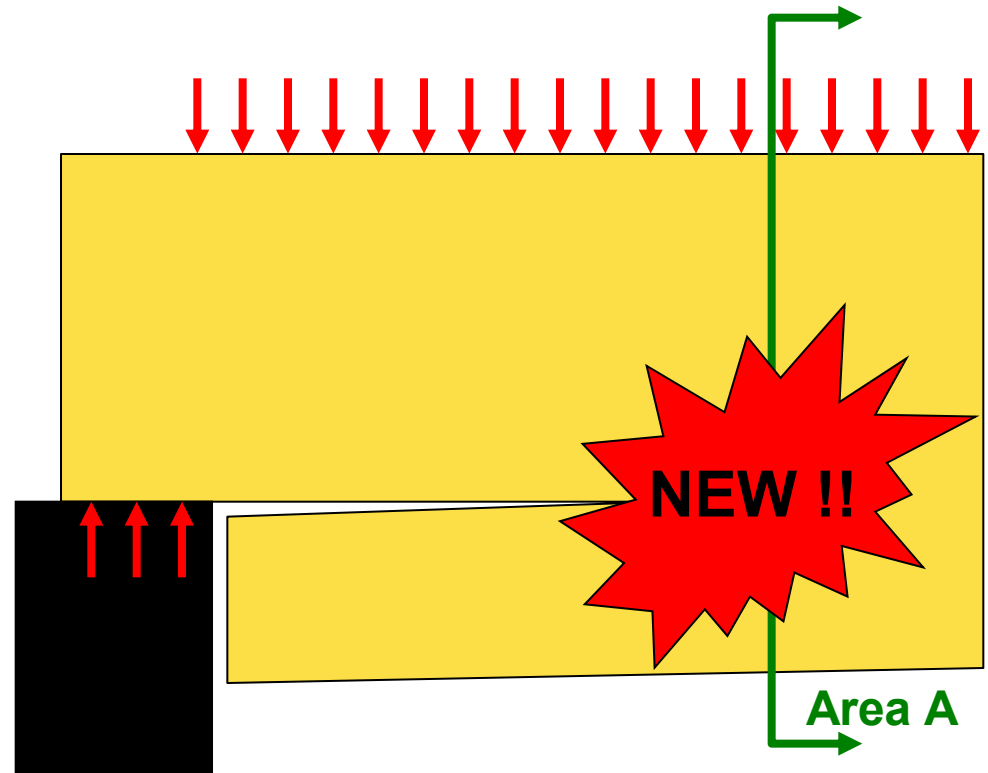
$$F_t = f_t (K_D K_H K_{St} K_T)$$

where f_t = specified reaction force strength = 0.5 MPa for sawn lumber

K_{St} = 1.0 for dry and 0.7 for wet service conditions

A = gross cross-section area

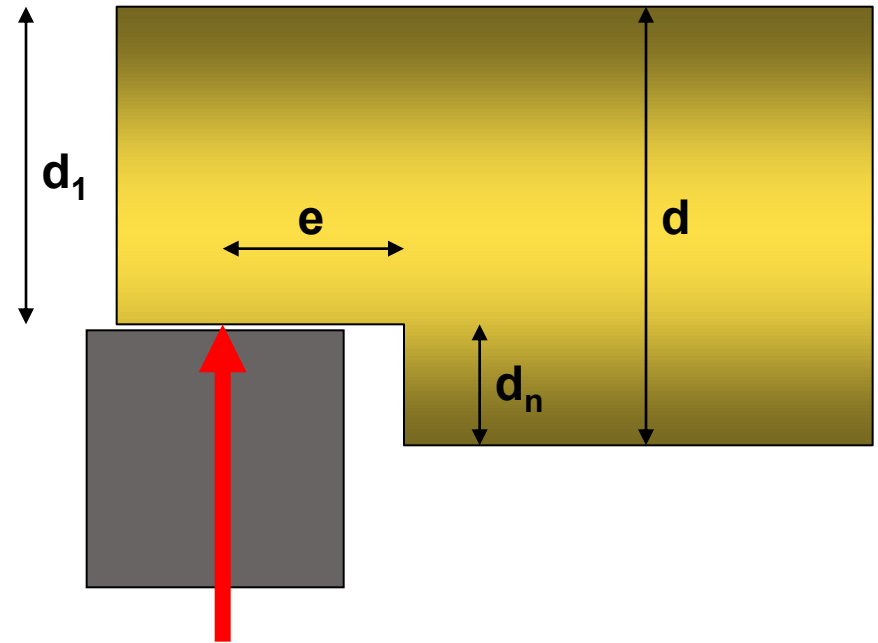
K_N = notch factor



Notch factor

 K_N

Based on Fracture
Mechanics theory

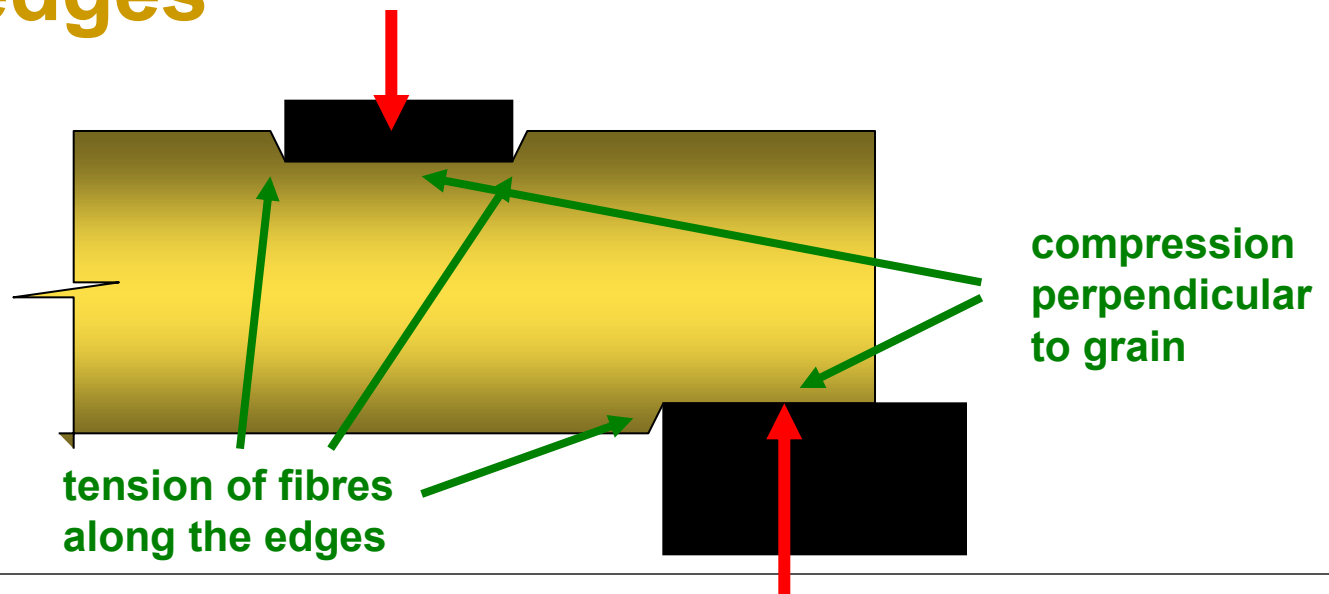


$$K_N = \left(0.006 \left(1.6 \left(\frac{1}{\alpha} - 1 \right) + \eta^2 \left(\frac{1}{\alpha^3} - 1 \right) \right) \right)^{-0.5}$$

$$\alpha = 1 - (d_n/d) \text{ and } \eta = e/d$$

Bearing failure in a timber beam

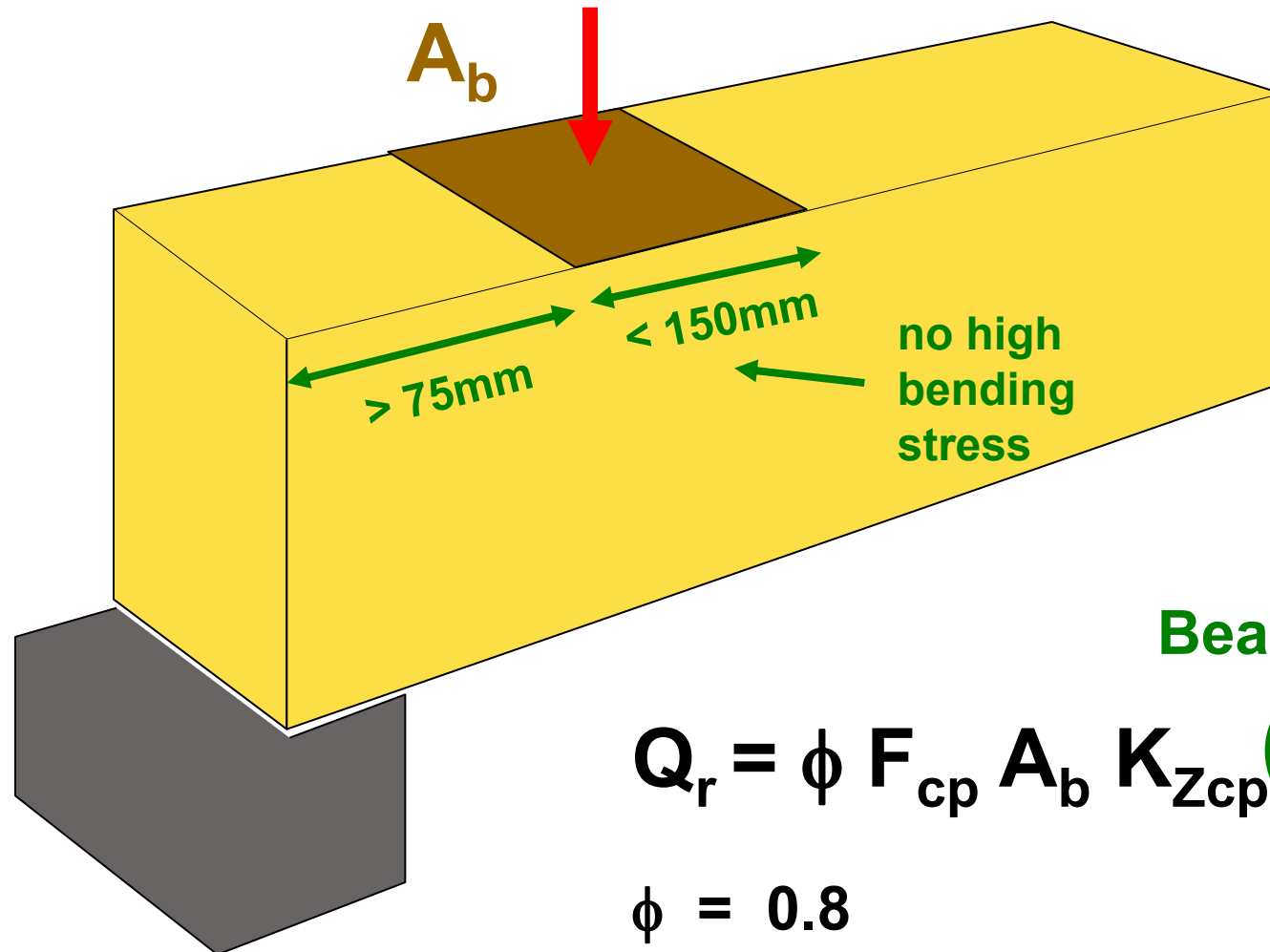
- The “soft” property of wood
- Often governs
- **Not only compression perpendicular to grain but also tension of the fibres along edges**







Bearing resistance



Bearing factor

$$Q_r = \phi F_{cp} A_b K_{Zcp} K_B$$

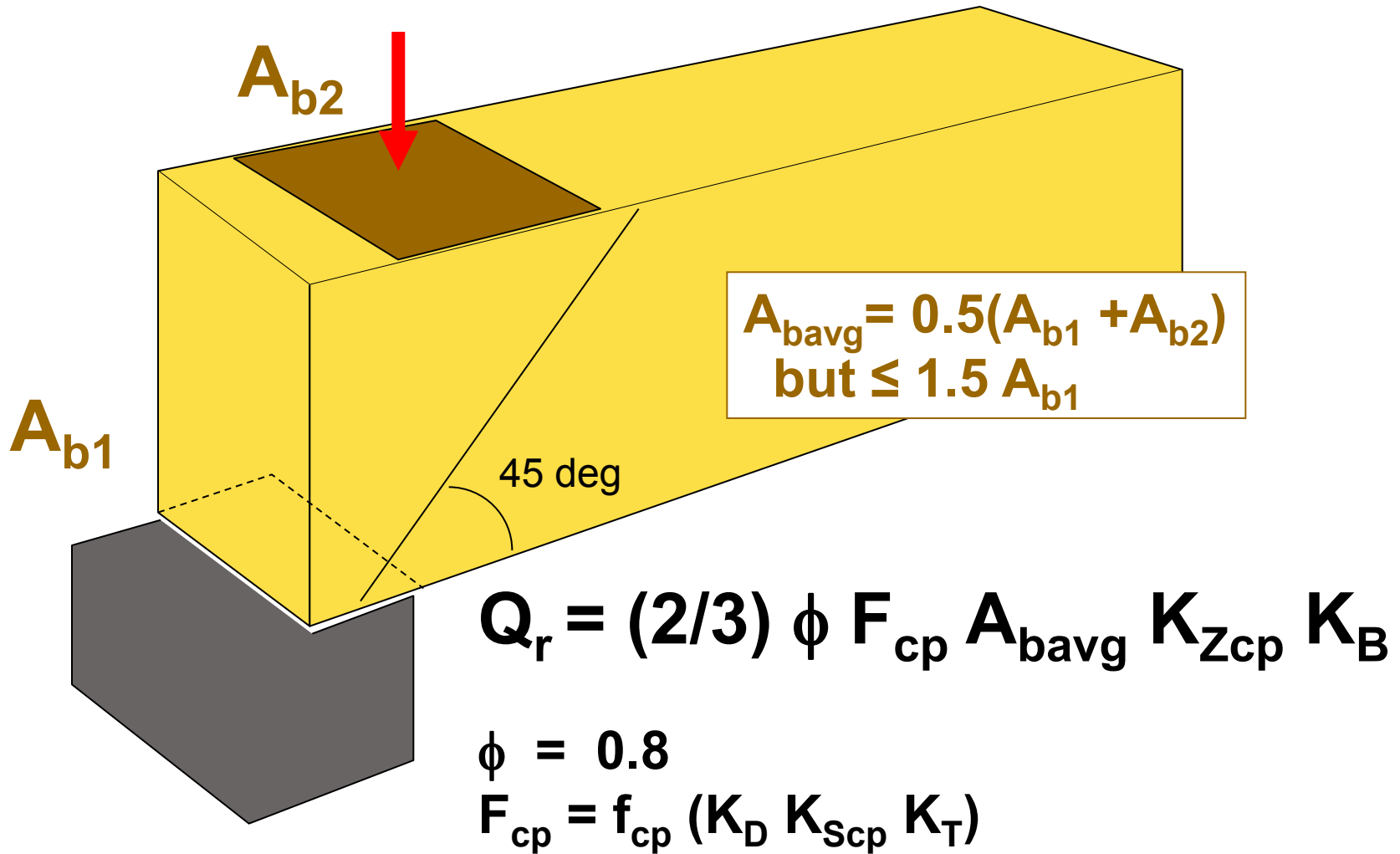
$$\phi = 0.8$$

$$F_{cp} = f_{cp} (K_{Scp} K_T)$$

Critical bearing areas in woodframe construction



Bearing resistance (double bearing)



Bearing factor K_B

Bearing length or diameter (mm)	Bearing factor K_B
< 12.5	1.75
25	1.38
38	1.25
50	1.19
75	1.13
100	1.10
> 150	1.0



Deflections

- A serviceability criterion
 - Avoid damage to cladding etc. ($\Delta \leq L/180$)
 - Avoid vibrations ($\Delta \leq L/360$)
 - Aesthetics ($\Delta \leq L/240$)
- Use unfactored loads
- Typically not part of the code

