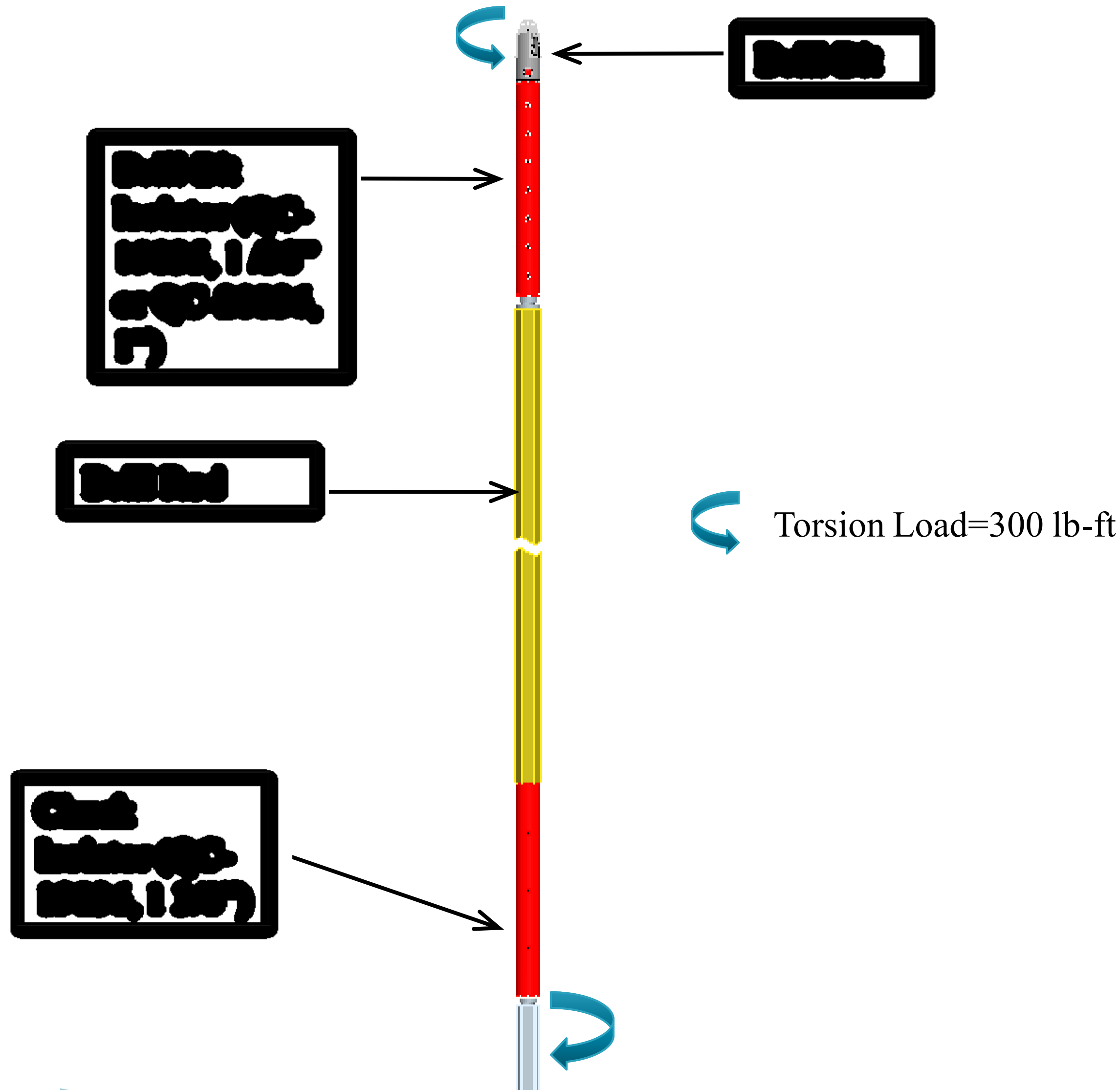
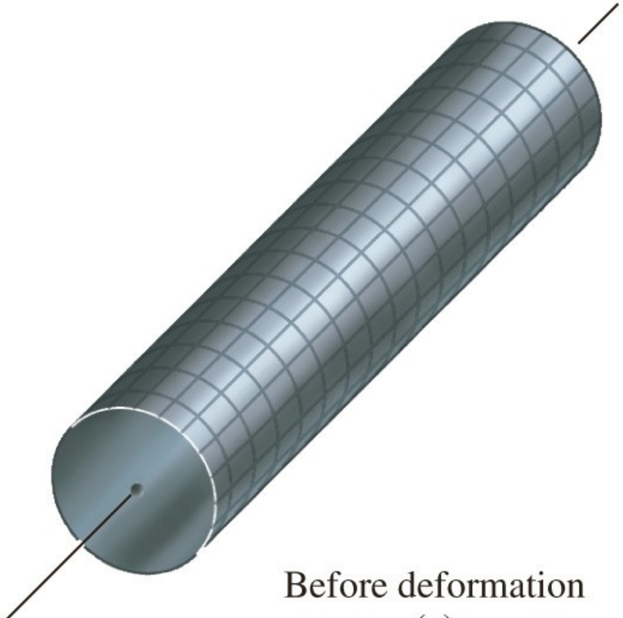
1 3/8” Isolator (Assembly)

Torsional induced shear stresses throughout – where is it maximum?

Other loadings to consider? (welcome to my nightmare)

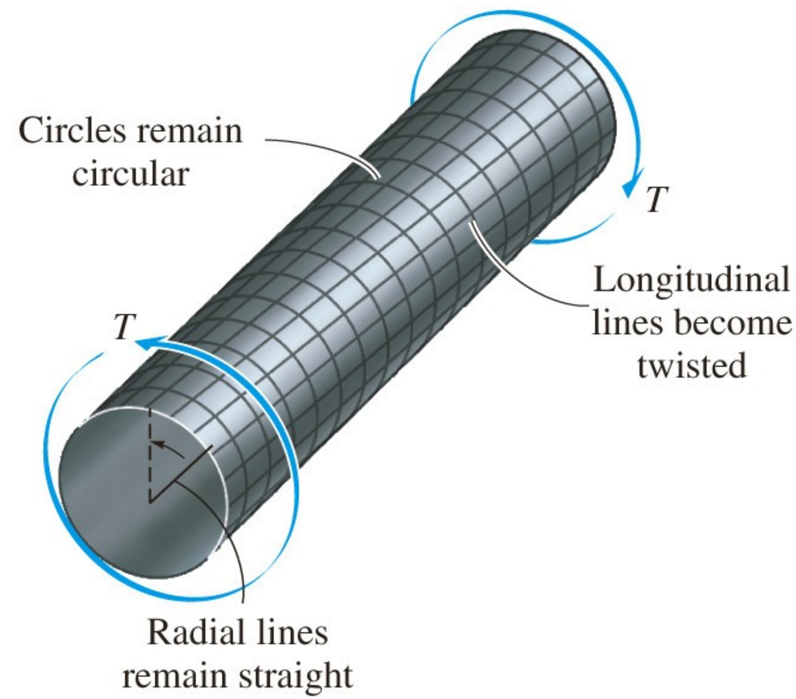


Ch 5 - Torsion

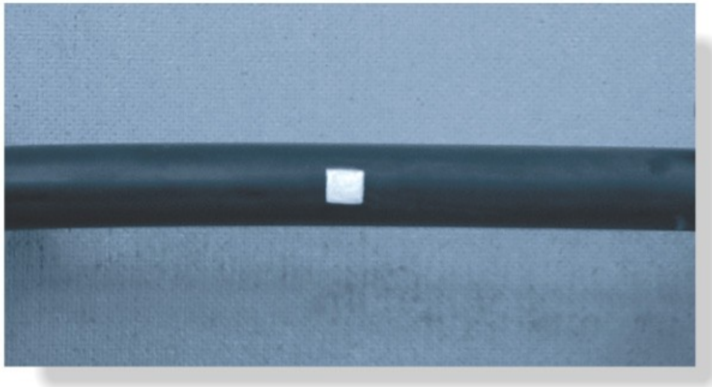


Before deformation
(a)

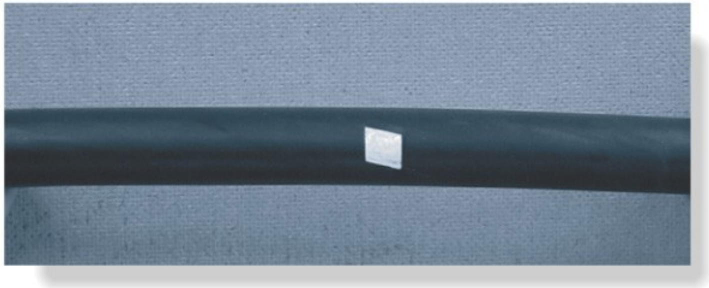
Recall: External loads (T) produce internal loads which produce deformation, strain and stress.



After deformation
(b)

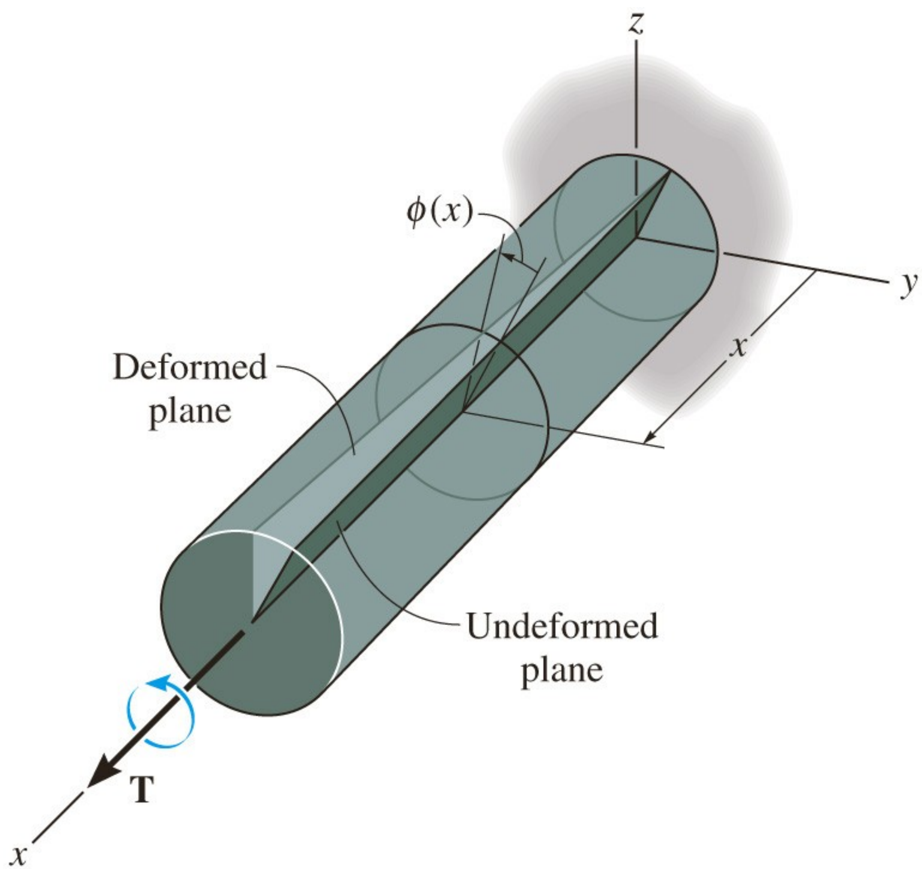


Before Torque



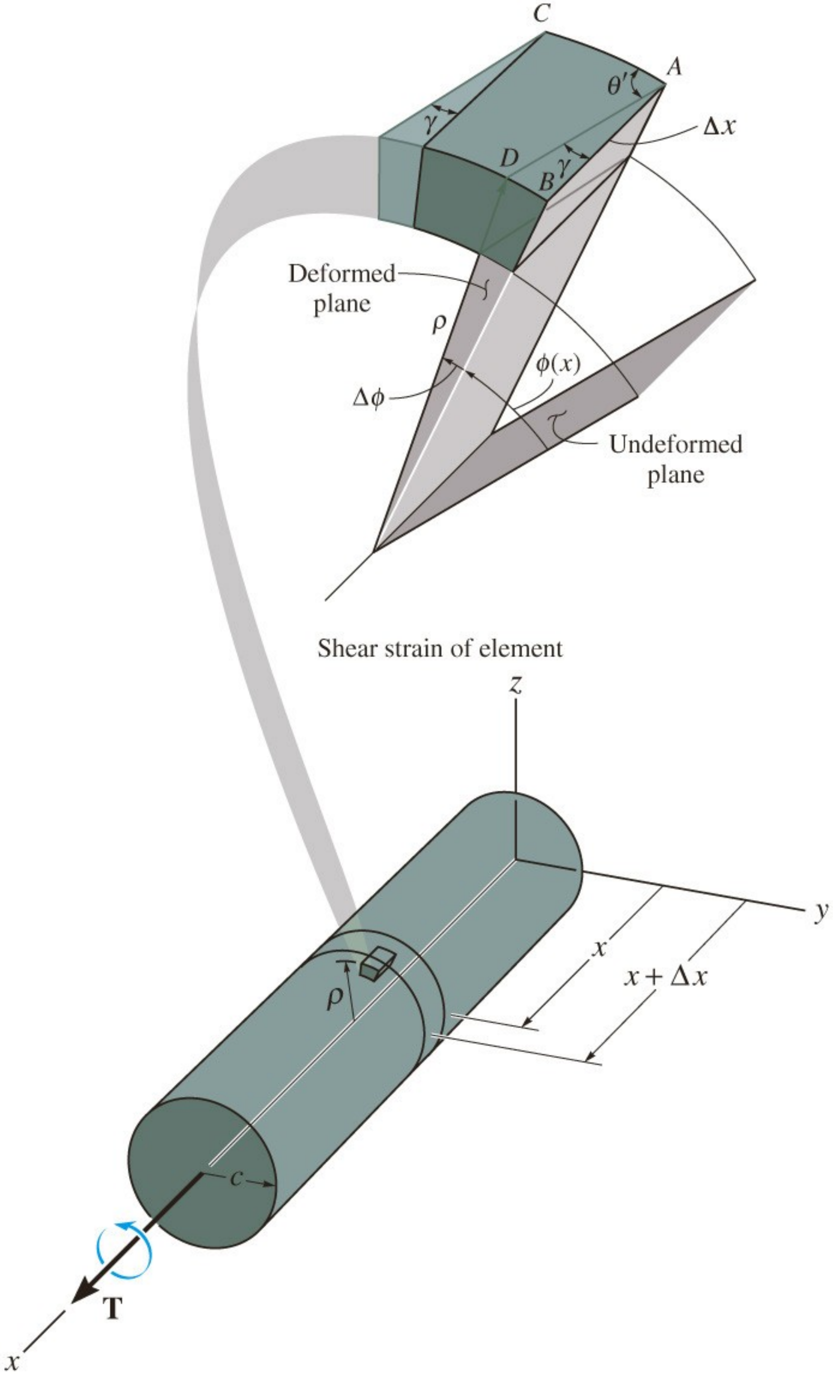
After Torque

5.1 Torsional Deformation of a Circular Shaft



The angle of twist $\phi(x)$ increases as x increases.

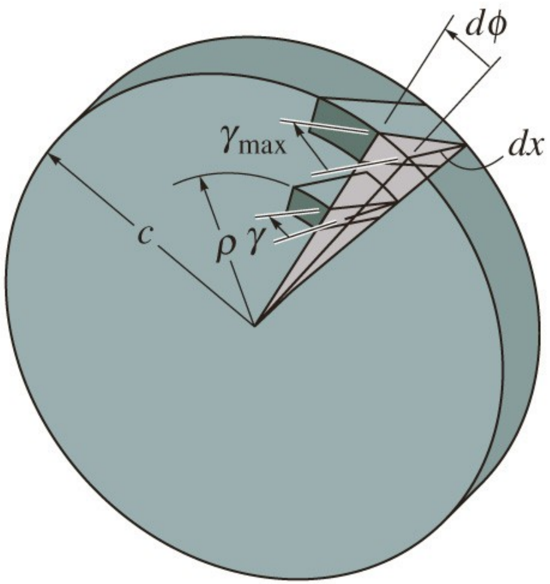
$\phi(x)$ = angle of twist (varies linearly along the length, 0 at x = 0, max at x = L)



Recall γ = shear strain (rad)

$$\gamma = \frac{\rho \Delta \phi}{\Delta x} = \rho \frac{d\phi}{dx}$$

Notice, shear strain, γ varies linearly with radial distance, ρ, and is max on the outer surface!!



The shear strain for the material increases linearly with ρ, i.e., $\gamma = (\rho/c)\gamma_{max}$

Notice, shear strain, γ varies linearly with radial distance, ρ, and is max on the outer surface!!

Distance from center to point of interest

$$\gamma = \left(\frac{\rho}{c}\right)\gamma_{max}$$

Distance from center to outer fiver (i.e. outer radius)

What have we learned so far?

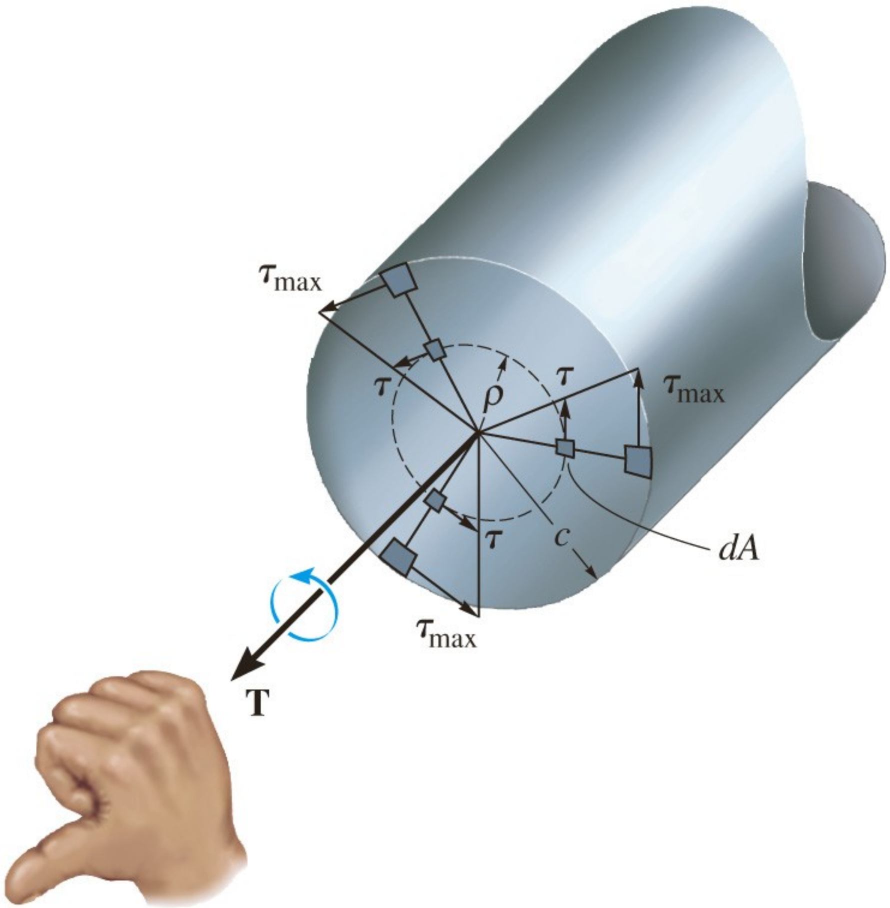
- ϕ = angle of twist varies from zero at fixed support to max at end.
- γ = shear strain varies from zero at center to max at outer fiber.

What about stress???

- Deformation = strain
- Strain = stress

- If you can visualize deformation, you can visualize stress
- The stress is a shear stress!!

5.2 The Torsion Formula

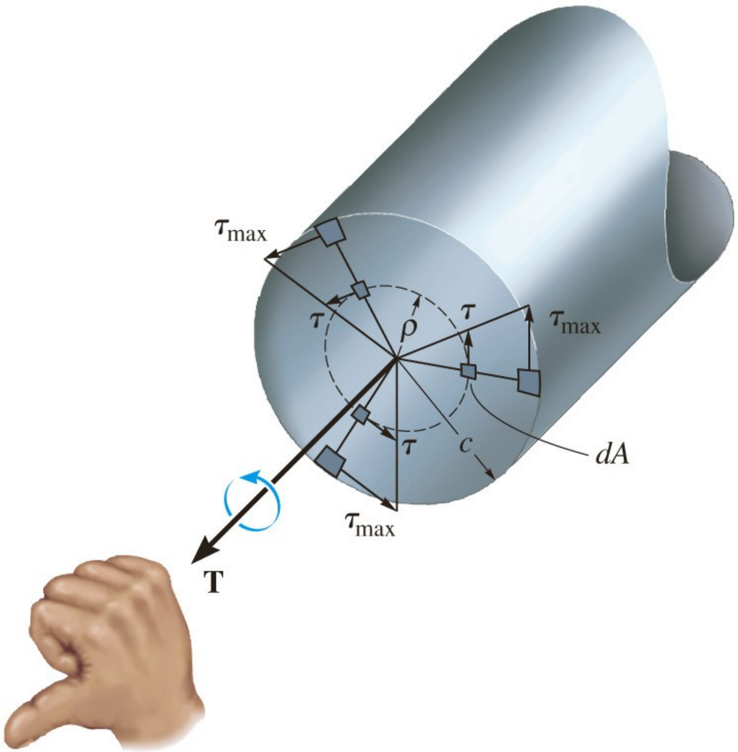


Shear stress varies linearly along each radial line of the cross section.

If linear elastic, Hooke's law applies, t = Gy

Therefore, stress follows same profile as strain!!

$$\tau = \left(\frac{\rho}{c}\right)\tau_{\max}$$



Shear stress varies linearly along each radial line of the cross section.

Derivation – simple Torque balance. The torque produced by the stress distribution over the entire cross section must be equal to the resultant internal torque, or:

$$T = \int_A \rho(\tau dA) = \int_A \rho\left(\frac{\rho}{c}\right)\tau_{max}\, dA$$

$$T = \frac{\tau_{max}}{c}\int_A \rho^2 dA$$

This is simply polar moment of inertia, J (an area property)

The torsion formula (see derivation):

Torque (N-m, N-mm or lb-in, lb-ft, etc)

Outer radius of shaft (m or in)

$$\tau_{\max} = \frac{Tc}{J}$$

Max shear stress in shaft (MPa, psi/ksi, etc.)

Polar moment of inertia (m^4 or in^4)

or

$$\tau = \frac{T\rho}{J}$$

J = polar moment of inertia

- Solid shaft:

$$J = \frac{\pi}{2} c^4$$

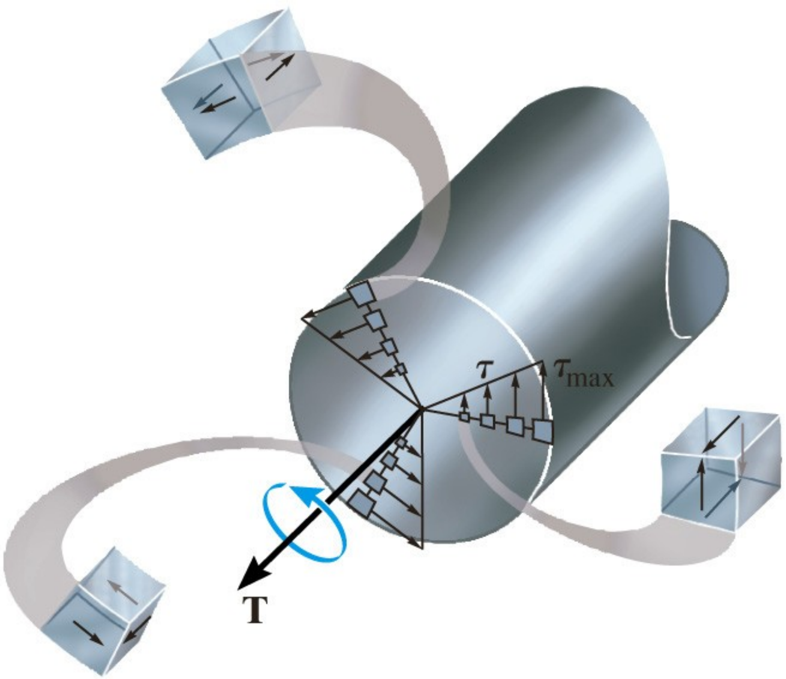
- Hollow shaft:

$$J = \frac{\pi}{2}\left(c_o^4 - c_i^4\right)$$

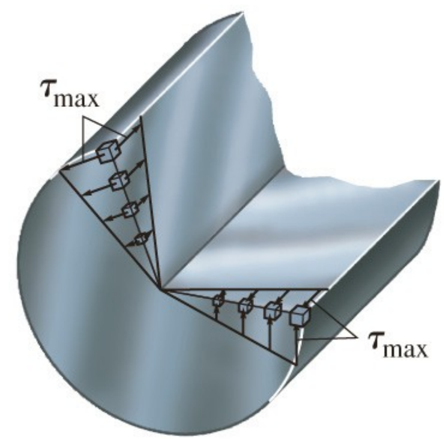
Stress Profiles:

Shear stress profile – YOU MUST UNDERSTAND THIS!!!!

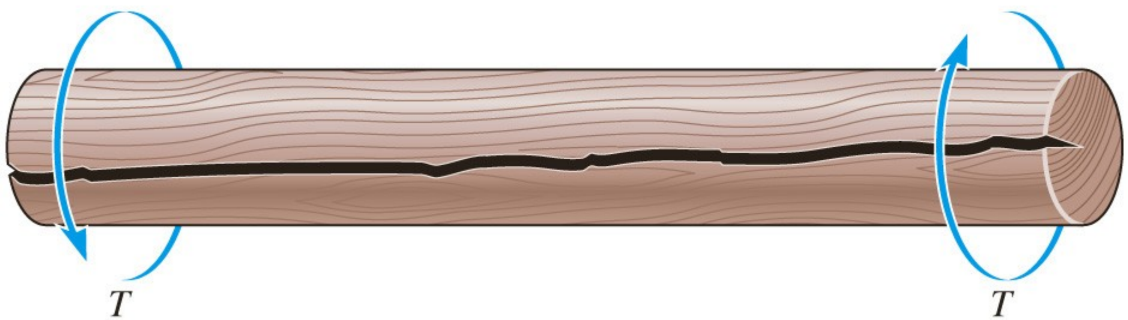
Where is shear stress max? zero? How does it vary along the length and circumference?



(a)



Shear stress varies linearly along each radial line of the cross section.
(b)



Failure of a wooden shaft due to torsion.

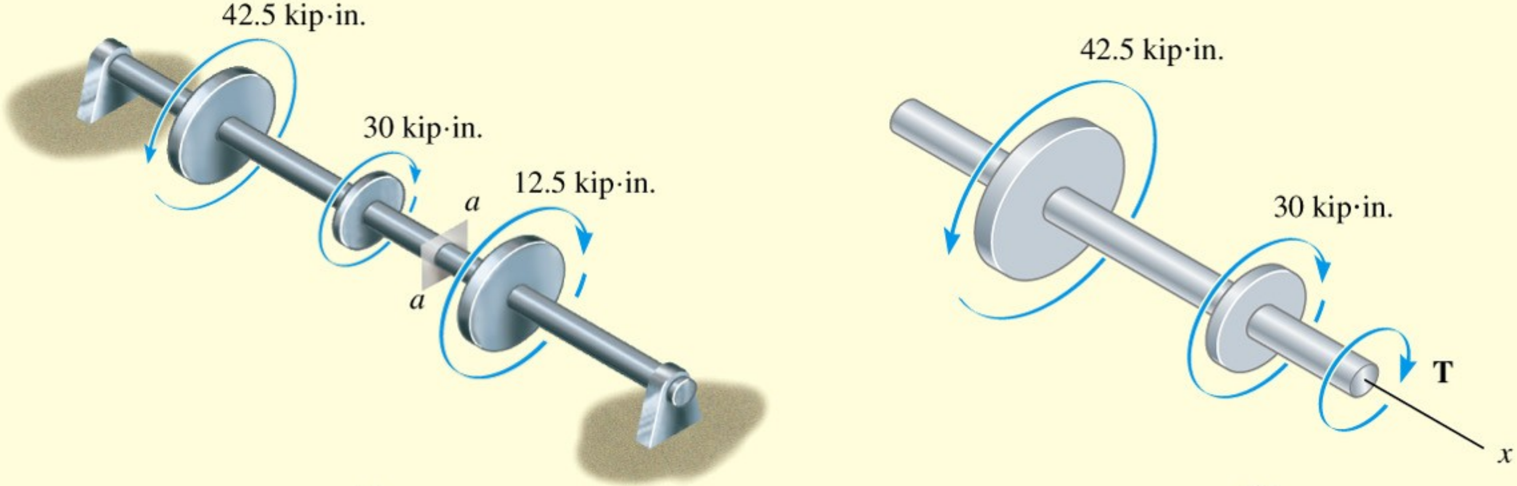
Examples:

1. Analysis: $\tau_{\max} = \frac{Tc}{J}$ Want << τ_{allow}

2. Design: $\frac{J}{c} = \frac{T}{\tau_{allow}}$ Find geometry

E X A M P L E 5.3

The shaft shown in Fig. 5–12*a* is supported by two bearings and is subjected to three torques. Determine the shear stress developed at points *A* and *B*, located at section *a*–*a* of the shaft, Fig. 5–12*b*.



(a) (b)

Fig. 5–12

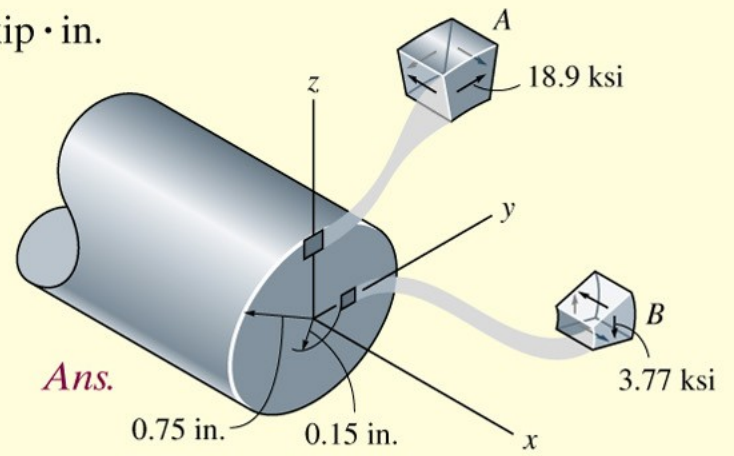
Solution

Internal Torque. The bearing reactions on the shaft are zero, provided the shaft's weight is neglected. Furthermore, the applied torques satisfy moment equilibrium about the shaft's axis.

The internal torque at section a–a will be determined from the free-body diagram of the left segment, Fig. 5–12b. We have

$$\Sigma M_x = 0; \quad 42.5 \text{ kip}\cdot\text{in.} - 30 \text{ kip}\cdot\text{in.} - T = 0 \qquad T = 12.5 \text{ kip}\cdot\text{in.}$$

Fig. 5–12



(c)

Section Property. The polar moment of inertia for the shaft is

$$J = \frac{\pi}{2}(0.75 \text{ in.})^4 = 0.497 \text{ in}^4$$

Shear Stress. Since point A is at $\rho = c = 0.75$ in.,

$$\tau_A = \frac{Tc}{J} = \frac{(12.5 \text{ kip}\cdot\text{in.})(0.75 \text{ in.})}{(0.497 \text{ in}^4)} = 18.9 \text{ ksi} \qquad \textit{Ans.}$$

Likewise for point B, at $\rho = 0.15$ in., we have

$$\tau_B = \frac{T\rho}{J} = \frac{(12.5 \text{ kip}\cdot\text{in.})(0.15 \text{ in.})}{(0.497 \text{ in}^4)} = 3.77 \text{ ksi} \qquad \textit{Ans.}$$

The directions of these stresses on each element at A and B, Fig. 5–12c, are established from the direction of the resultant internal torque **T**, shown in Fig. 5–12b. Note carefully how the shear stress acts on the planes of each of these elements.

EXAMPLE 5.4

The pipe shown in Fig. 5–13*a* has an inner diameter of 80 mm and an outer diameter of 100 mm. If its end is tightened against the support at *A* using a torque wrench at *B*, determine the shear stress developed in the material at the inner and outer walls along the central portion of the pipe when the 80-N forces are applied to the wrench.

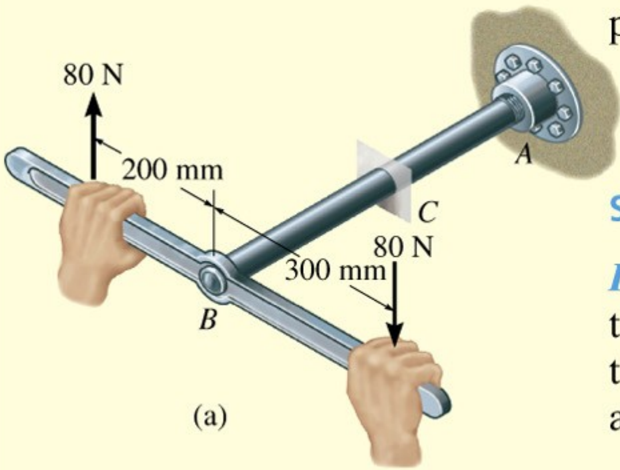


Fig. 5–13

Solution

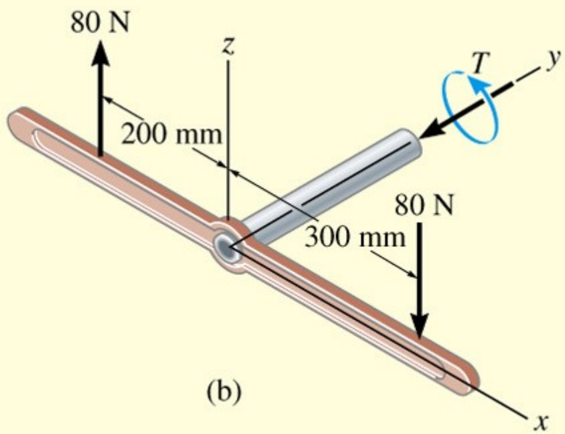
Internal Torque. A section is taken at an intermediate location *C* along the pipe's axis, Fig. 5–13*b*. The only unknown at the section is the internal torque **T**. Force equilibrium and moment equilibrium about the *x* and *z* axes are satisfied. We require

$$\Sigma M_y = 0; \qquad 80\text{ N}(0.3\text{ m}) + 80\text{ N}(0.2\text{ m}) - T = 0$$

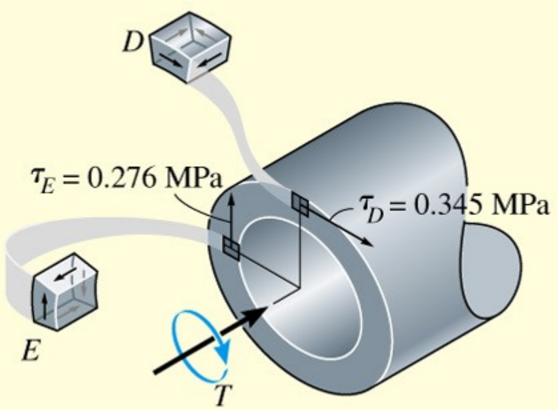
$$T = 40\text{ N}\cdot\text{m}$$

Section Property. The polar moment of inertia for the pipe's cross-sectional area is

$$J = \frac{\pi}{2}[(0.05\text{ m})^4 - (0.04\text{ m})^4] = 5.80(10^{-6})\text{ m}^4$$



(b)



(c)

Fig. 5–13

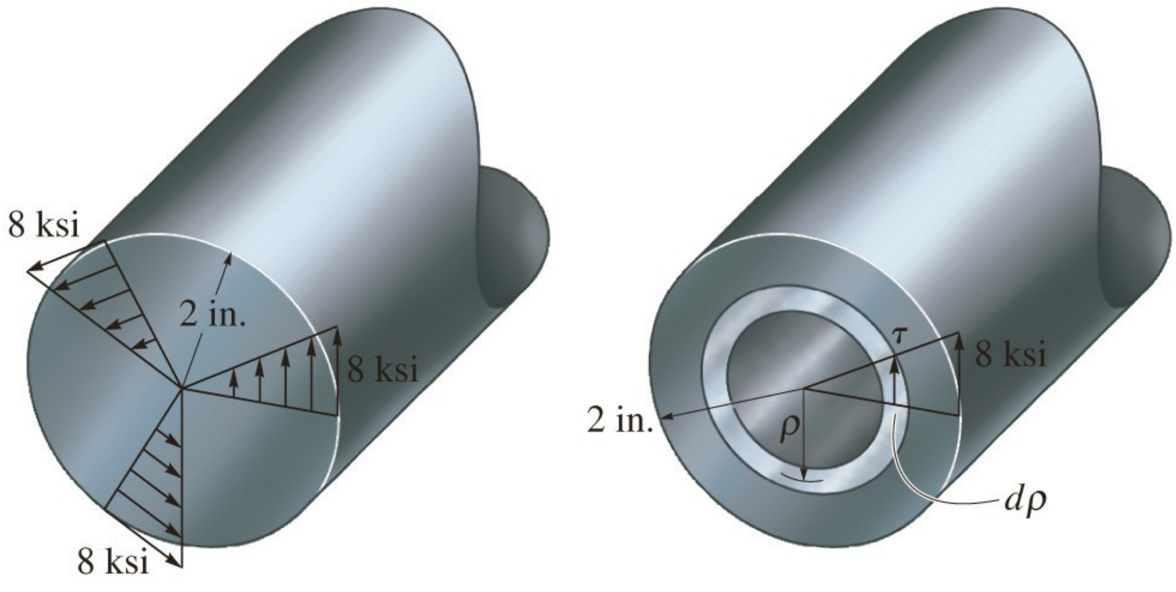
Shear Stress. For any point lying on the outside surface of the pipe, $\rho = c_o = 0.05$ m, we have

$$\tau_o = \frac{Tc_o}{J} = \frac{40\text{ N}\cdot\text{m}(0.05\text{ m})}{5.80(10^{-6})\text{ m}^4} = 0.345\text{ MPa} \qquad \textit{Ans.}$$

And for any point located on the inside surface, $\rho = c_i = 0.04$ m, so that

$$\tau_i = \frac{Tc_i}{J} = \frac{40\text{ N}\cdot\text{m}(0.04\text{ m})}{5.80(10^{-6})\text{ m}^4} = 0.276\text{ MPa} \qquad \textit{Ans.}$$

To show how these stresses act at representative points D and E on the cross-sectional area, we will first view the cross section from the front of segment CA of the pipe, Fig. 5–13a. On this section, Fig. 5–13c, the resultant internal torque is equal but opposite to that shown in Fig. 5–13b. The shear stresses at D and E contribute to this torque and therefore act on the shaded faces of the elements in the directions shown. As a consequence, notice how the shear-stress components act on the other three faces. Furthermore, since the top face of D and the inner face of E are in stress-free regions taken from the pipe's outer and inner walls, no shear stress can exist on these faces or on the other corresponding faces of the elements.



(a) (b)

5.3 Power Transmission

- Nothing new, just calculate Torque, T, from power equation:

Power (watts, ft-lb/s or hp)

$$P = T\omega$$

Torque (N-m, lb-ft)

Angular velocity (rad/s or Hz)

Careful with units!

Note: 1 hp = 550 ft-lb/s

$$\omega = 2\pi f$$

f = Hz or rev/s

Examples (English):

- Shaft powered by 5 hp electric motor spins at 10 Hz, find Torque in shaft.
- $P = T\omega$

5 hp (550 ft-lb/s/hp)
= 2,750 ft-lb/s

10 Hz (2π rad/rev) = 62.83 rad/s

$$T = \frac{2750 \text{ ft-lb/s}}{62.83 \text{ rad/s}} = 43.76 \text{ lb-ft}$$

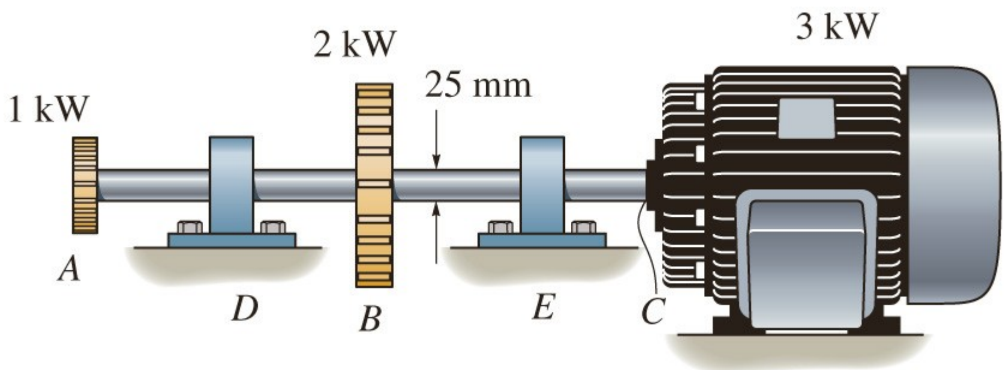
Examples (SI):

- Shaft powered by 500 W electric motor spins at 10 Hz, find Torque in shaft.
- $P = T\omega$

10 Hz (2π rad/rev) = 62.83 rad/s

$$T = \frac{500 \text{ N-m/s}}{62.83 \text{ rad/s}} = 7.96 \text{ N-m}$$

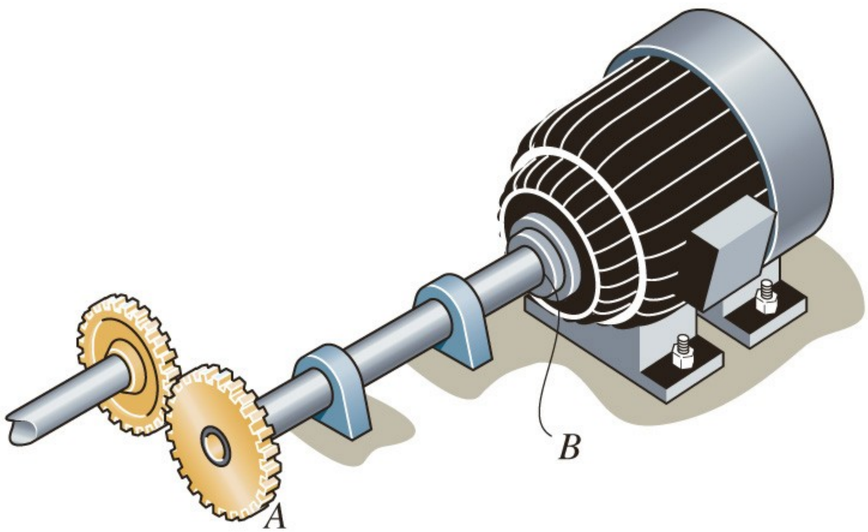
Your HW prob 5.39:
Find stress throughout shaft:



Steps:

1. Find Torque throughout shaft
2. Solve for stress throughout shaft

Prob 5.42: The motor delivers 500 hp to the steel shaft *AB* which is tubular and has an inside dia of 1.84 in and outside of 2 in. Find: *smallest* angular velocity at which the shaft can rotate if τ_{allow} = 25 ksi



Design or Analysis:

Steps:

1. Find allowable torque
2. Back solve for speed using P=Tn