Chapter 1 - Introduction

- Load-Stress and Load-Deflection Relationships
  - Mechanics of Materials Methods
  - Continuum Mechanics/Elasticity methods
  - Energy Methods
- Stress-Strain Relations
  - Elastic and Inelastic Response
  - Material Properties
  - Load-Carrying Members
- Failure Modes and Theories

Load-Stress Relationships

1. The equations of equilibrium or equations of motion.
2. Compatibility or continuity conditions (no overlaps or voids).
3. Constitutive equations.

Mechanics of Materials Methods

Based on simplifying assumptions related to the geometry of the deformation so that a strain distribution for a cross section of a member can be found.

Uses of Mechanics of Materials Methods

- Non-symmetric bending (Ch. 7)
- Shear Center (Ch. 8)
- Curved Beams (Ch. 9)
- Beams on Elastic Foundations (Ch. 10)

Continuum Mechanics (Elasticity) Approach

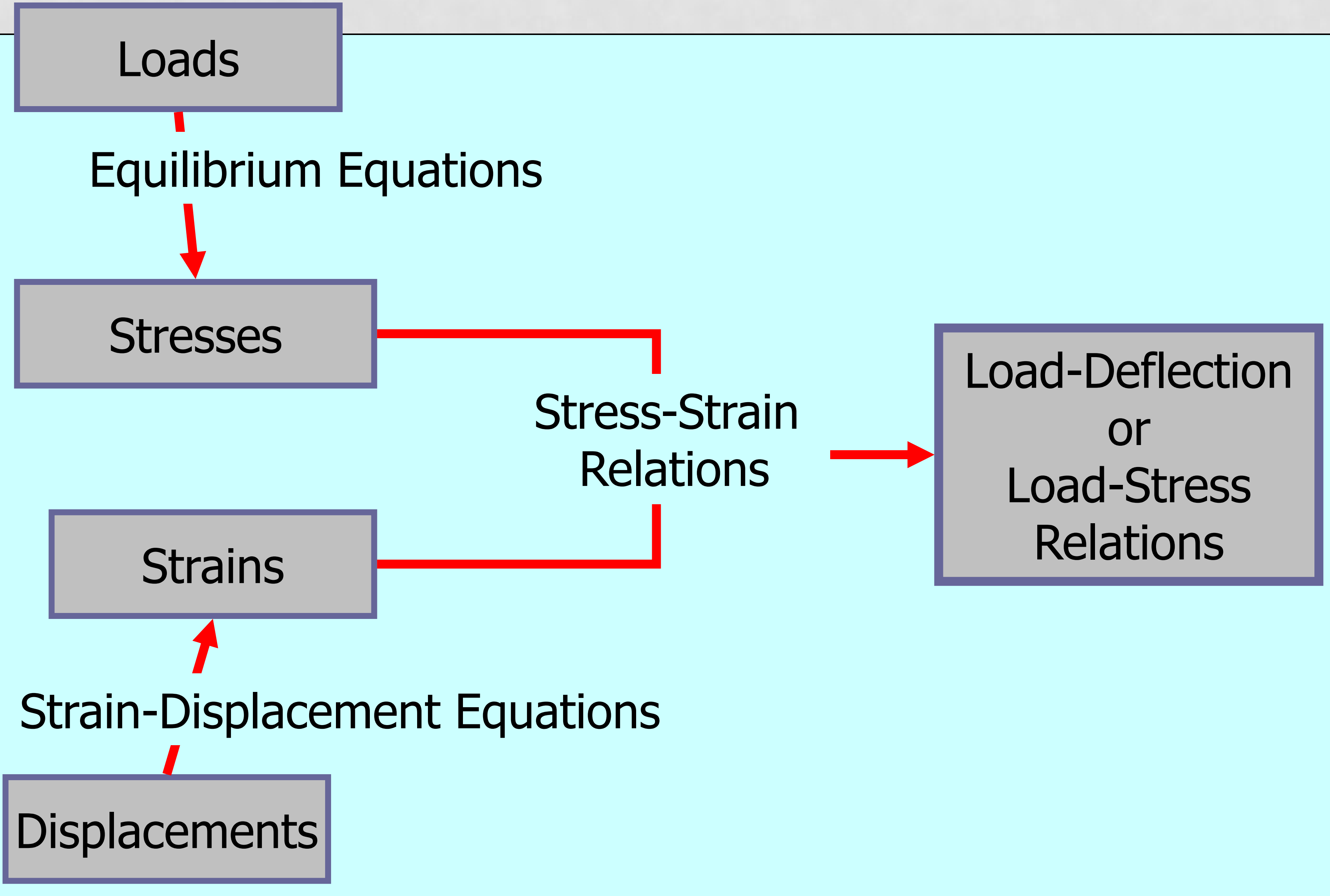
Based on infinitesimal volume element at a point in the body with faces normal to coordinate axes. Differential equations of equilibrium and compatibility are applied and solutions sought.

Uses of Continuum Mechanics Methods

- Noncircular torsion (Ch. 6)
- Thick-walled Cylinders (Ch. 11)
- Contact Stresses (Ch. 17)
- Stress Concentrations (Ch. 14)

Stress-Strain Relations

Stress components must be related to the strain components.


Loads
Equilibrium Equations
Stresses
Stress-Strain
Relations
Load-Deflection
or
Load-Stress
Relations
Strains
Strain-Displacement Equations
Displacements

In Elementary Mechanics of Material, we have studied.

Axial load

$\sigma = P/A$, $e = PL/AE$

$\varepsilon = P/AE = \sigma/E$

Bending moment

$\sigma = - My/I$, $\tau = VQ/bI$

$d^2v/dx^2 = M/EI$

Torsional load

$\tau = Tr/J$, $\psi = TL/GJ$

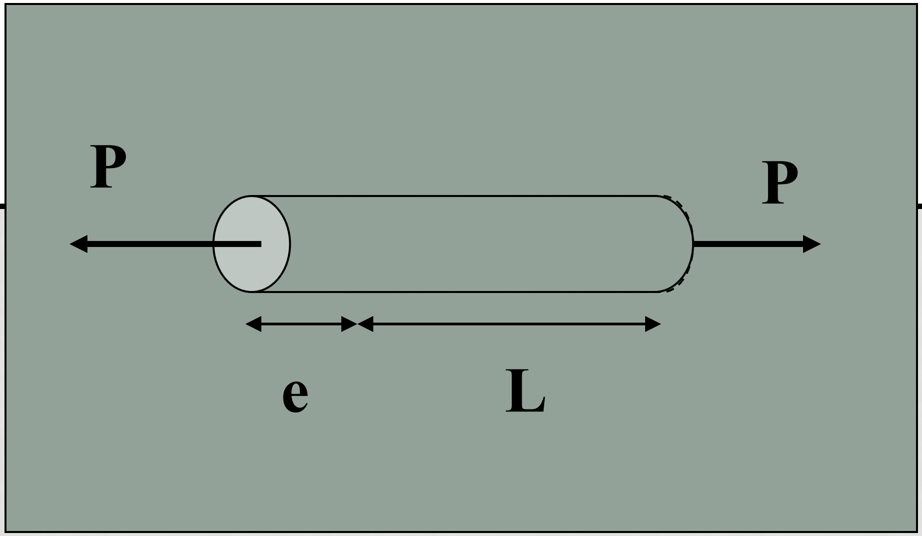
$\gamma = \tau/G = \psi r/L$

These formulas are based on certain simplifying assumptions and are applicable only under certain restrictions.

Axial load

Assumptions:

- Uniform and Prismatic (straight) bar, rod, tube etc.
- Homogenous material.
- Load ‘P’ directed axially along the centroidal axis of cross section.
- Elastic loading.

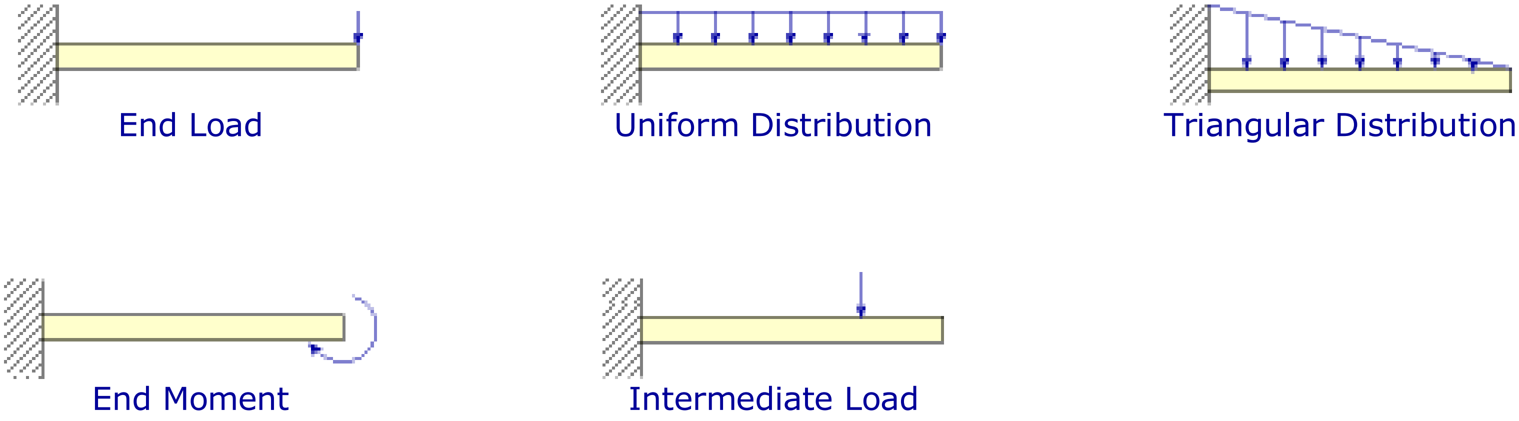


Bending moment

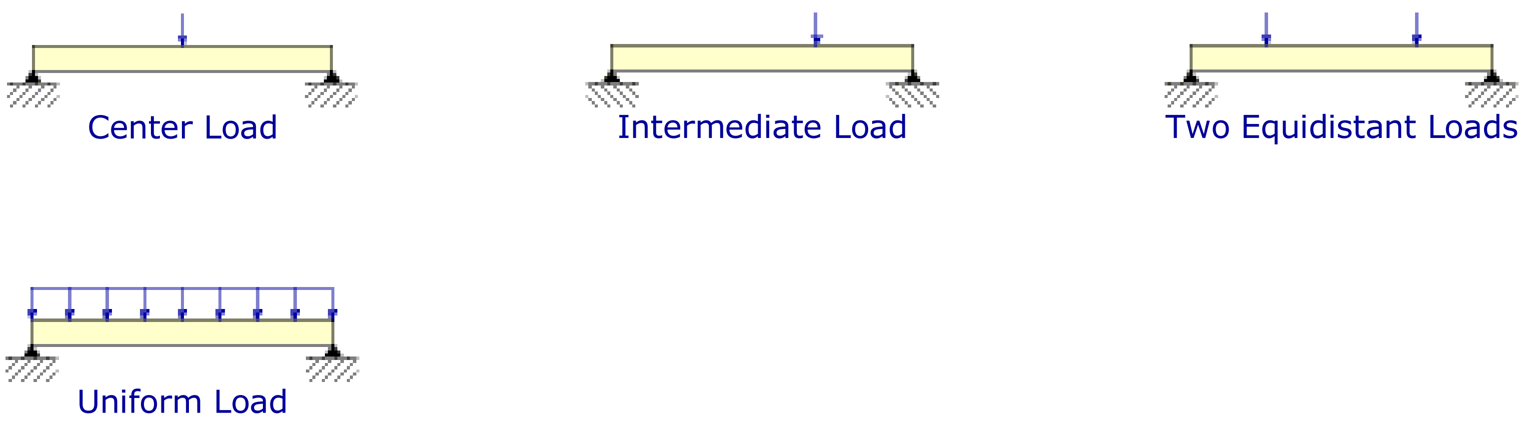
Assumptions:

- **Uniform prismatic beam ($L > 10 \times b$).**
- **Carries load that produce deflection perpendicular to its longitudal axis.**
- **Bending relative to principal axis only**
- **Linear elastic material.**
- **Applicable to small deflection and as long as deflection is in circular arc, ie. d^2v/dx^2 is a good approximation of the beam curvature.**

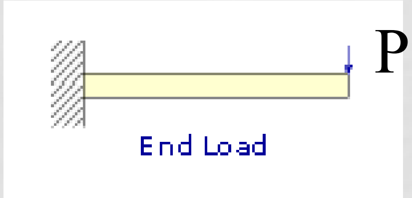
Cantilever beam problem:



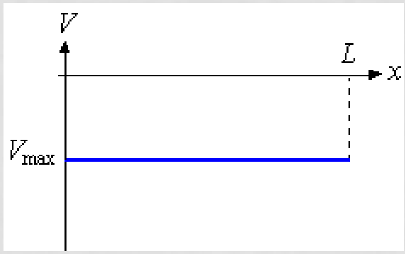
Simply supported beam problem:



Loading Diagram



SFD



$$V(x) = -P$$

$$V_{max} = V(x) = -P$$

BMD

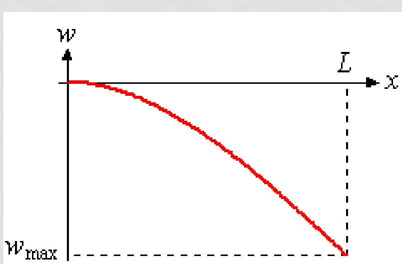
$$M(x) = P(L - x)$$

$$M_{max} = M(0) = PL$$

Slope

$$\theta(x) = -\frac{P(2L-x)x}{2EI}$$

Deflection



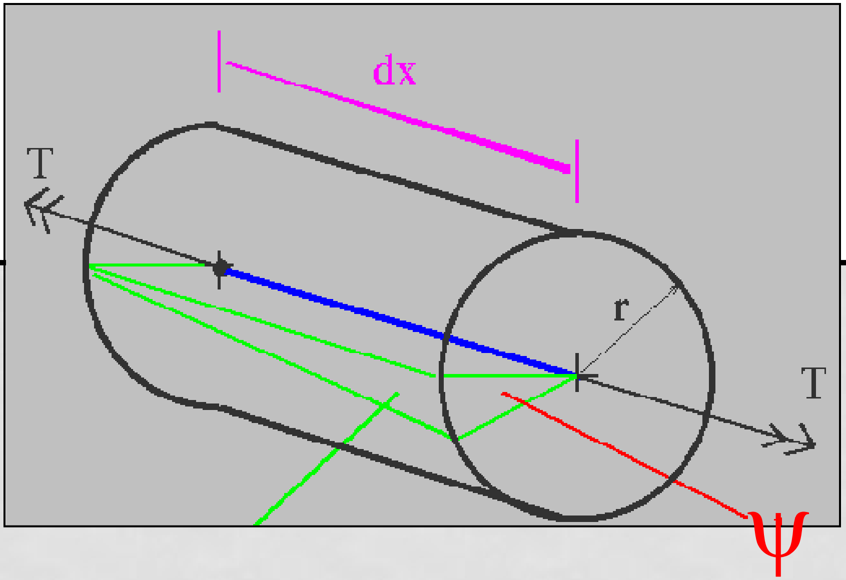
$$V(x) = -\frac{Px^2(3L-x)}{6EI}$$

$$V_{max} = w(L) = -\frac{PL^3}{3EI}$$

Torsional load

Assumptions:

- **Prismatic & circular (solid or hollow but thick) torsional member**
- **Homogenous material**
- **Sections at which torques are applied are remote from ends.**
- **Angle of twist is small.**



Methods of analysis

(Load-stress load-deflection relations)

Method of mechanics of material

- Equations of equilibrium
- Continuity condition
- σ/ ε = E

Method of continuum mechanics & Elasticity

- Equilibrium equation for elemental volume
- Differential compatibility equation
- Generalized Hooke’s law

Energy methods

Also called scalar method.

Plane cross section of member remain plane after deformation

Material Properties

- Modulus of Elasticity
- Poisson’s Ratio
- Shear Modulus
- Percent Elongation

- Yield Strength
- Ultimate tensile Strength
- Yield Point
- Modulus of Resilience
- Necking

Things We Can Learn from Stress-Strain Diagrams:

Almost all of the mechanical properties of a material can be obtained from its axial and shear stress-strain diagrams. Poisson's ratio is the only exception

- ✓Proportional-limit stress & strain in tension and compr.
- ✓Yield stress and strain in tension and compression
- ✓Ultimate stress and strain in tension and compression
- ✓Failure stress and strain in tension and compression
- ✓Resilience
- ✓Toughness
- ✓Elongation
- ✓Ductility and brittleness

Engineering Stress and Strain

$$\sigma = \frac{P}{A_0}$$

$$\varepsilon = \frac{\Delta L}{L} = \frac{e}{L}$$

True Stress and Strain

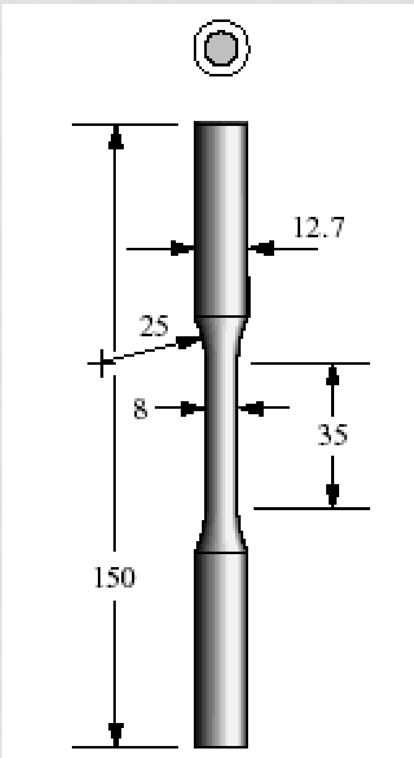
$$\sigma_t = \frac{P}{A_t}$$

$$L_t = L + e$$

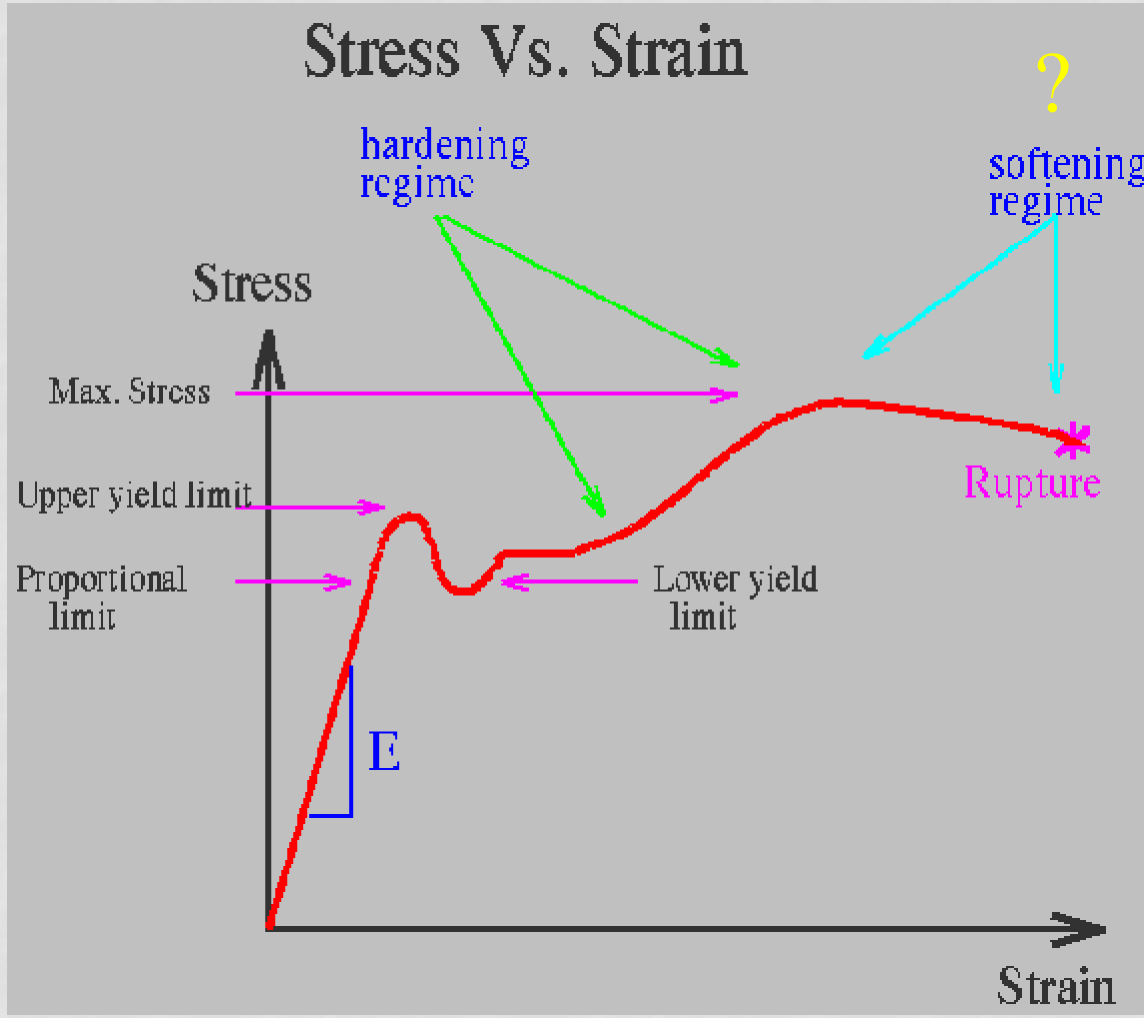
$$d\varepsilon_t = \frac{dL_t}{L_t}$$

$$\varepsilon_t = \int_{L}^{L_t} d\varepsilon_t = \ln\left(\frac{L + e}{L}\right) = \ln(1 + \varepsilon)$$

Structural steel

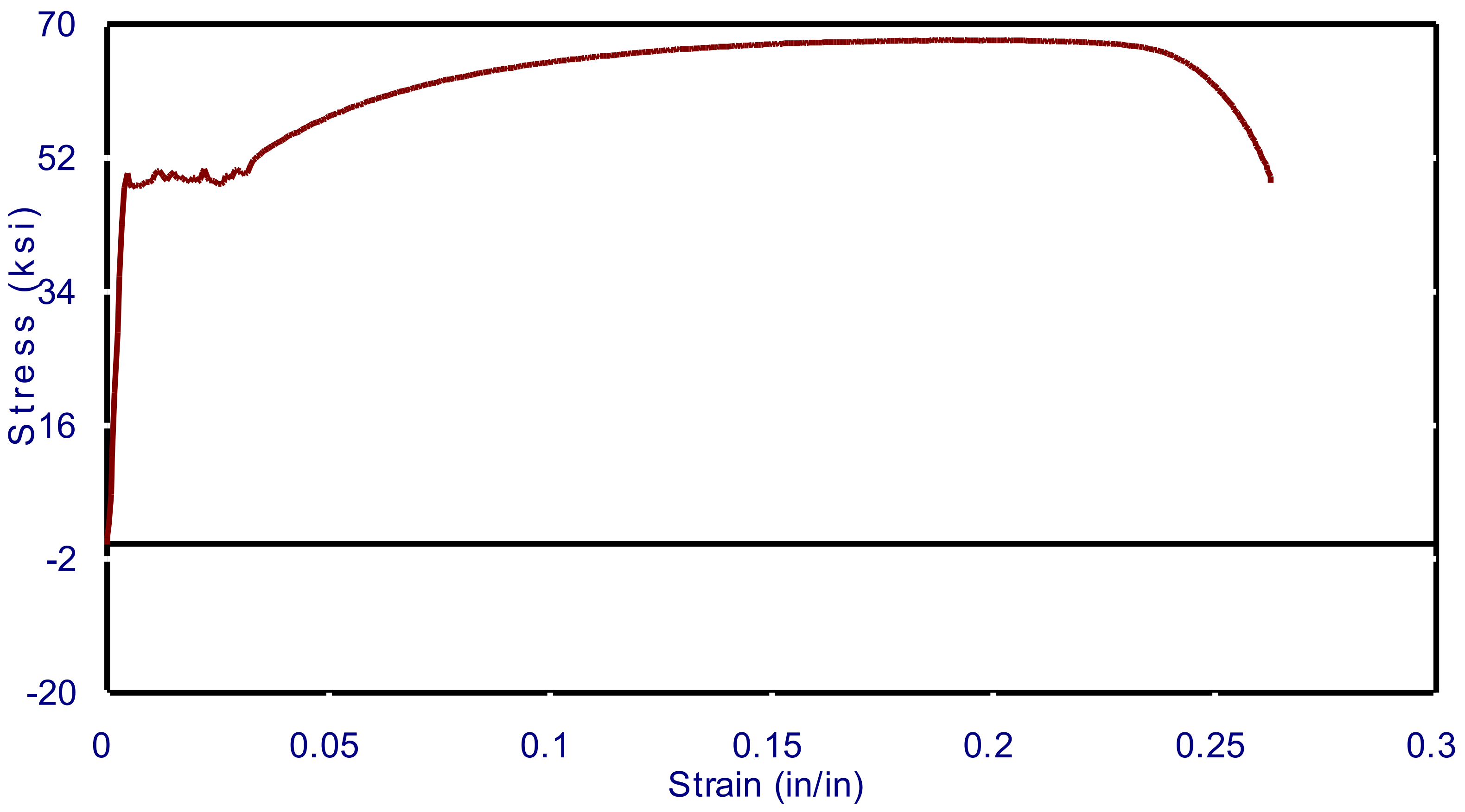


All dim. in mm

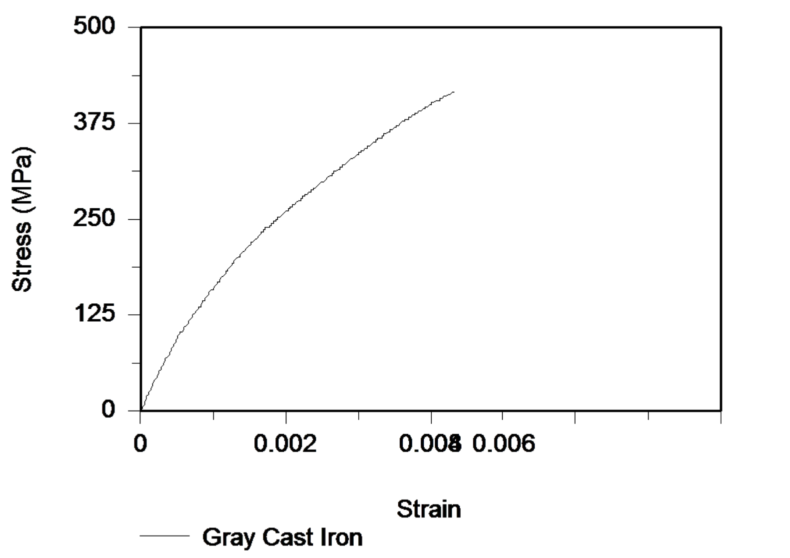


Structural steel

Experimental Stress-Strain plot for 1018 steel



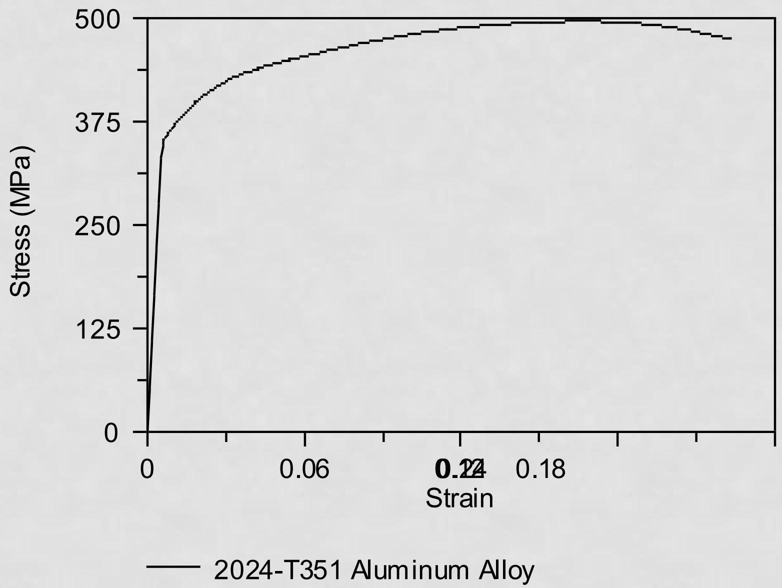
Characteristic stress-strain curve for brittle material



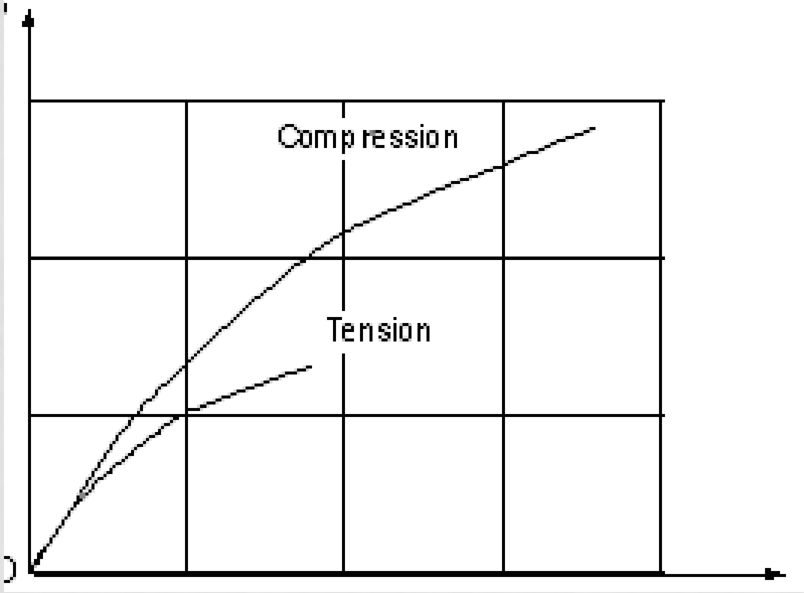
Cast Iron

Characteristic stress-strain curve for ductile material

Aluminum

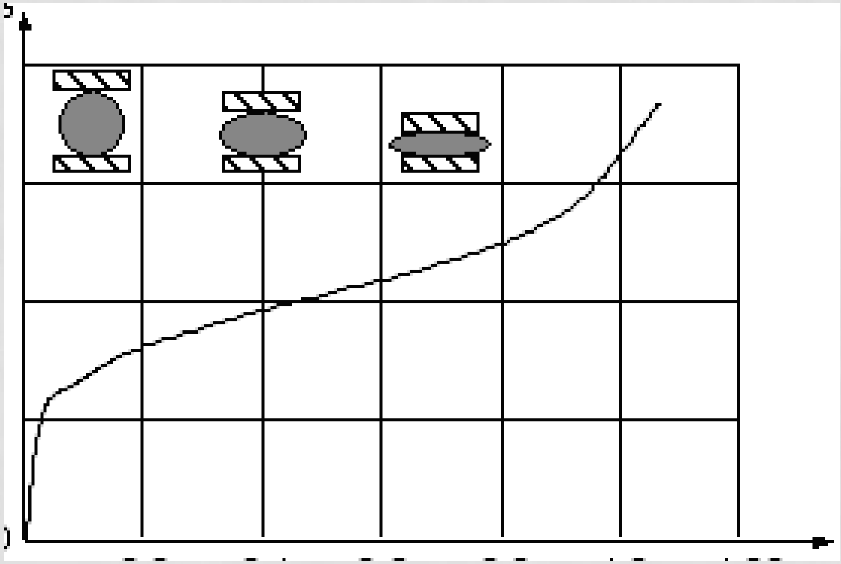


Stress-strain curve for compression

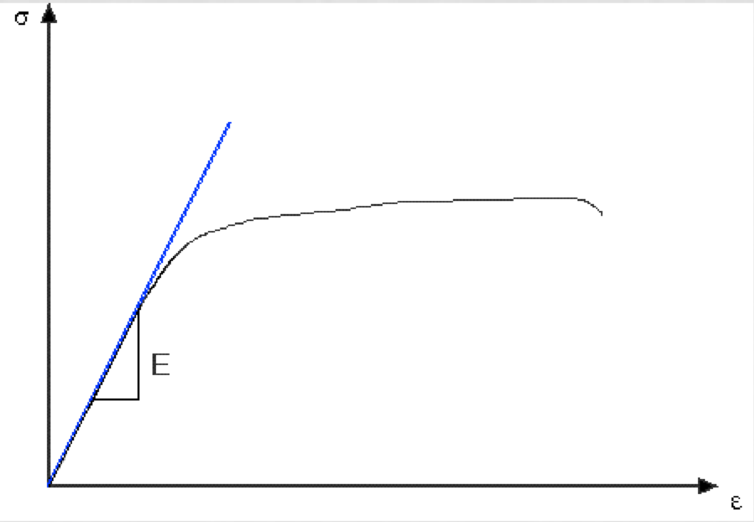


Cast Iron

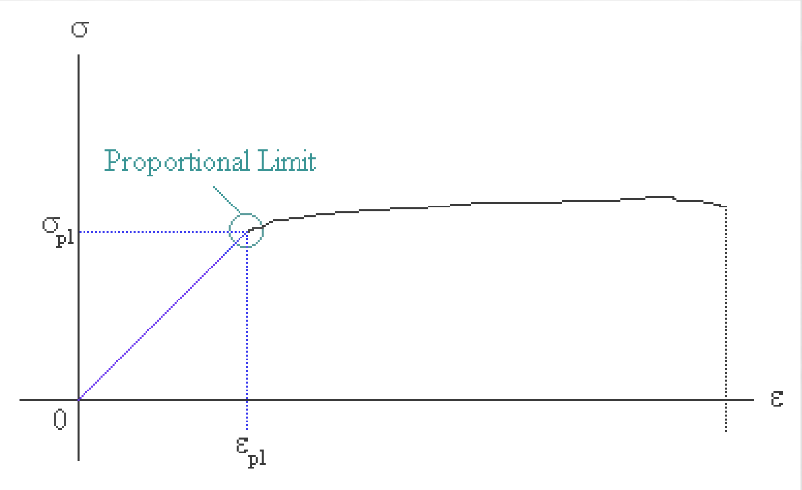
Copper



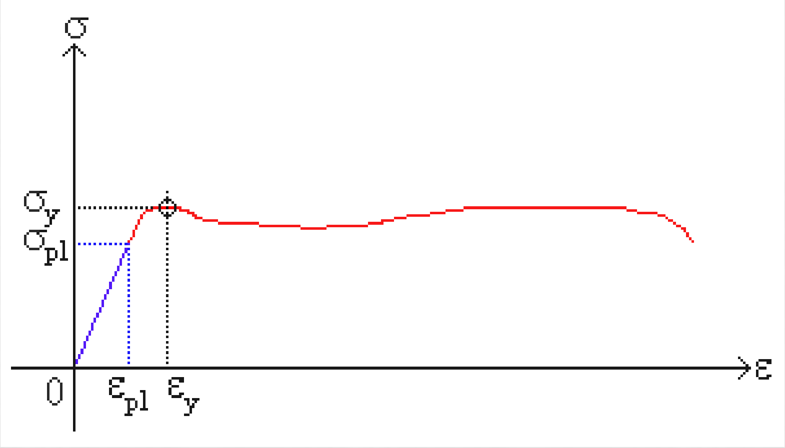
Young's Modulus

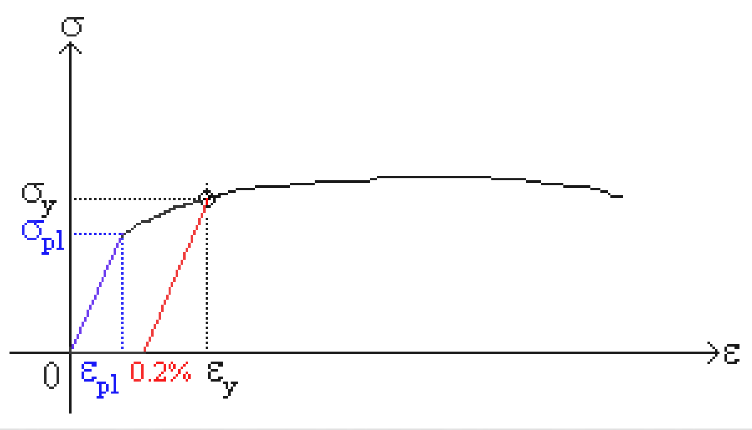


Proportional limit

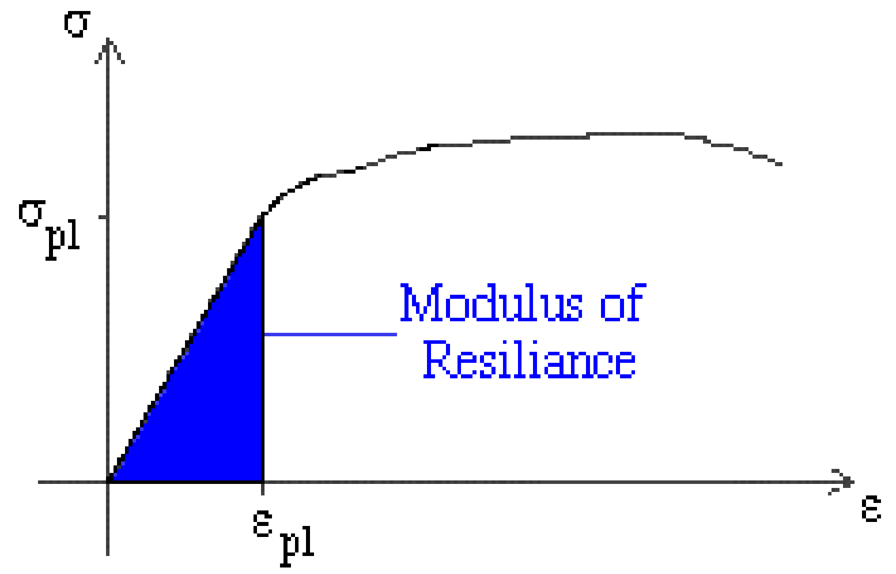


Yield Strength



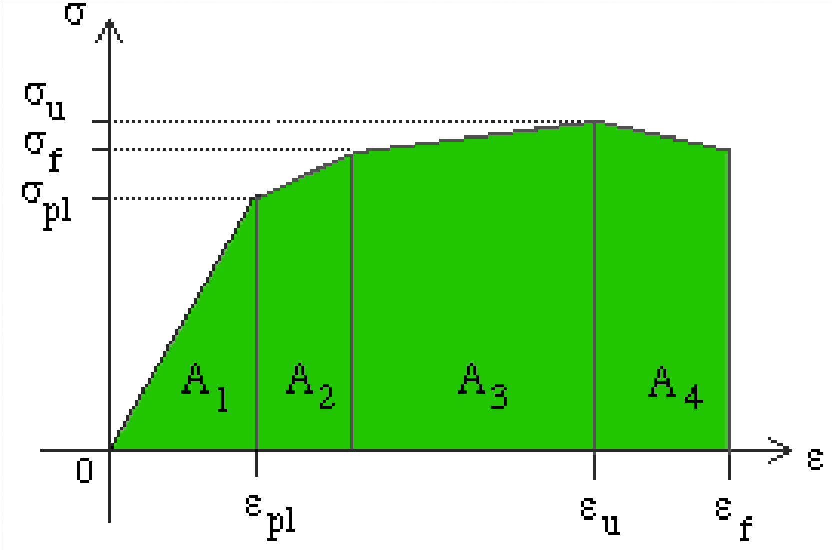


Resilience



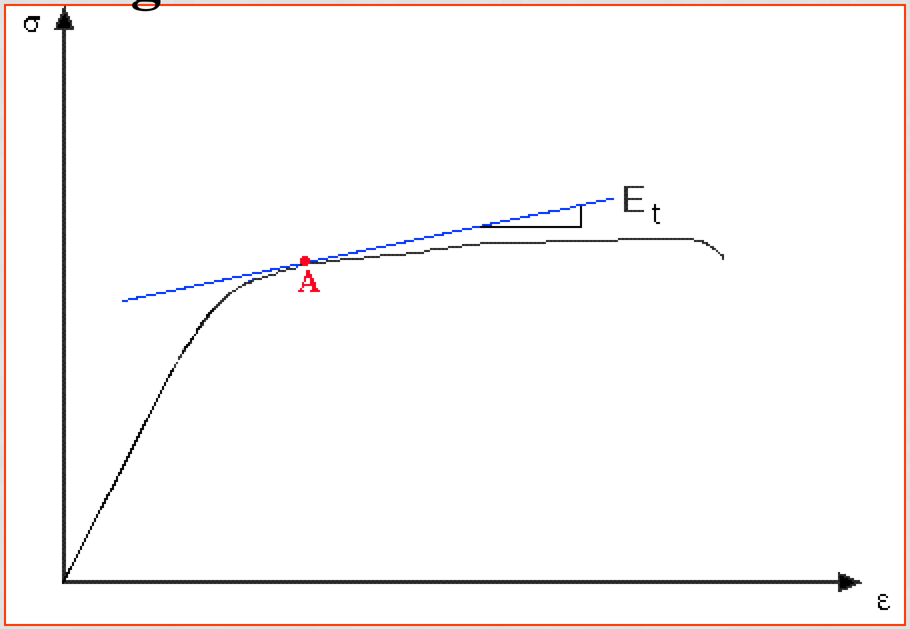
Amount of energy absorbed by a material in the elastic region

Toughness



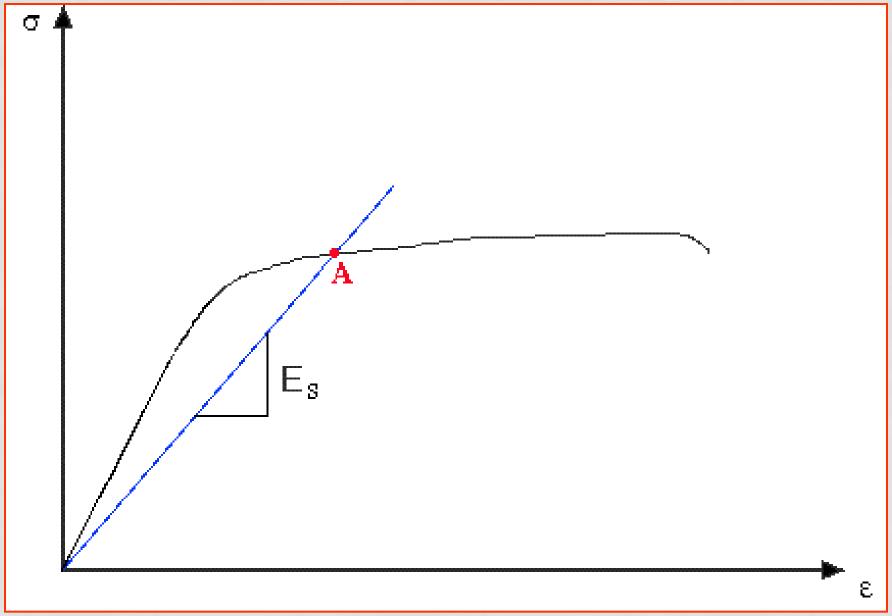
Amount of energy absorbed by a material up to the fracture

Tangent Modulus



Slope of a line tangent to the stress-strain curve at the point of interest. It is used to describe the stiffness of a material in the plastic range

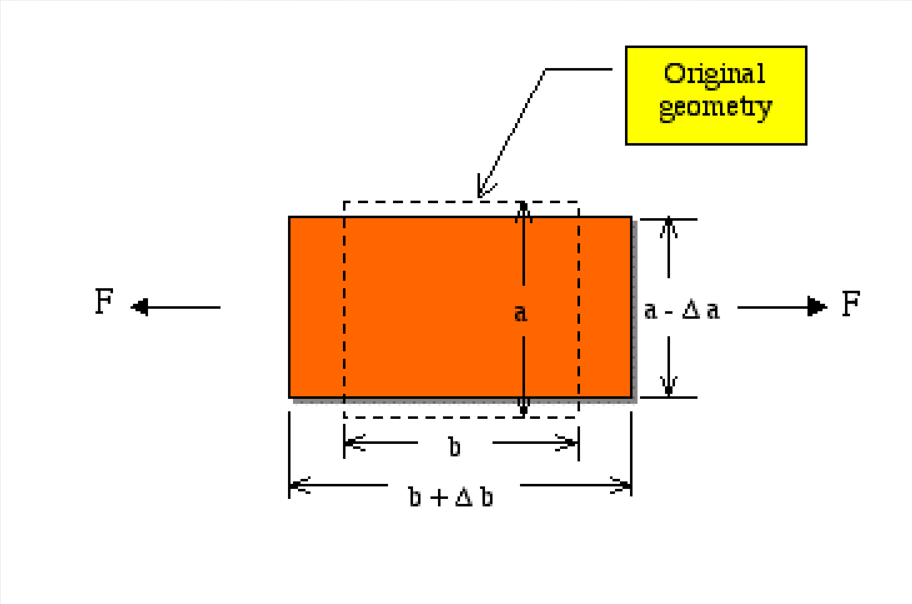
Secant Modulus



Slope of a line from the origin of the stress-strain diagram and intersecting the curve at the point of interest. It is also used to describe the stiffness of a material in the plastic range

Poisson's ratio

Poisson's ratio can be determined indirectly from stress-strain curve by knowing the change in the cross-sectional area of the specimen at a point along the elastic region of the stress-strain curve.



$$\nu = - \frac{\text{lateral strain}}{\text{axial strain}}$$

Some interesting points about Poisson's ratio

- The Poisson ratio for most metals falls between 0.25 to 0.35.
- Rubber has a Poisson ratio close to 0.5 and is therefore almost incompressible!
- Cork has a Poisson ratio close to zero. (This makes cork function well as a bottle stopper, since an axially-loaded cork will not swell laterally to resist bottle insertion.)
- The Poisson's ratio is bounded by two theoretical limits: it must be greater than -1, and less than or equal to 0.5,

(It is rare to encounter engineering materials with negative Poisson ratios.)

Failure and limits on design:

(isotropic material)

Stress based criteria

For isotropic materials, only two independent elastic constants are needed for describing the stress-strain relationship,
i.e., Hooke's Law $\sigma / \varepsilon = E$

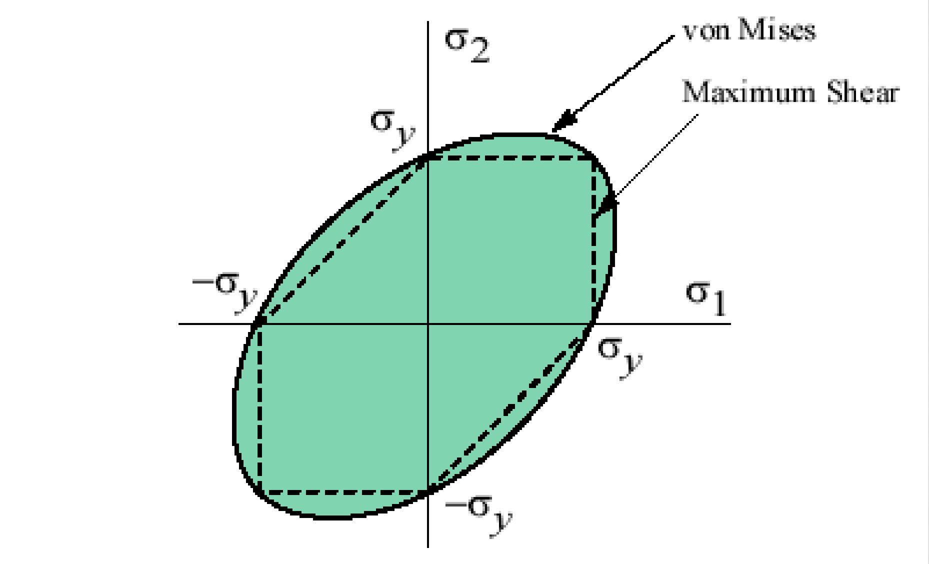
Material Type	Failure Theories
Ductile	Maximum shear stress criterion, von Mises criterion
Brittle	Maximum normal stress criterion, Mohr's theory

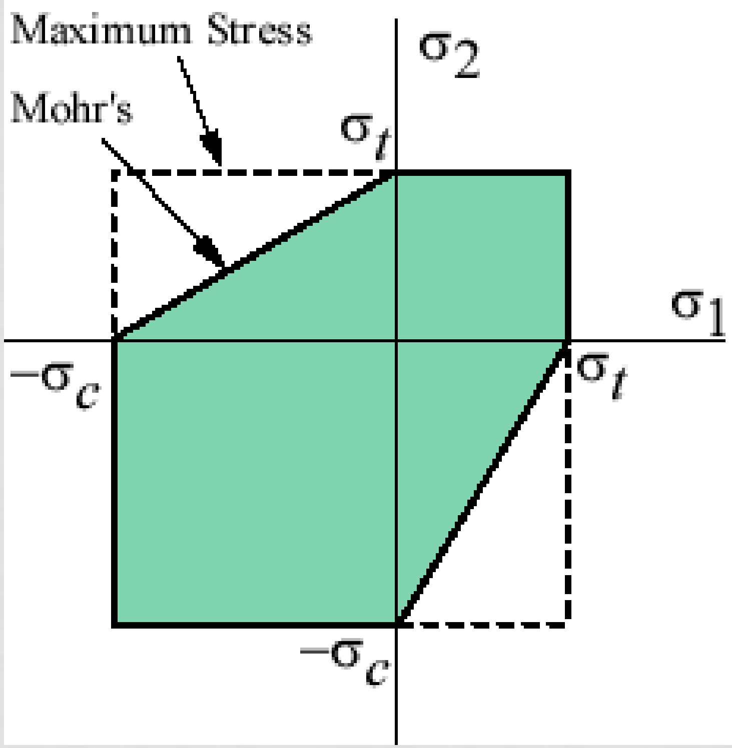
Coulomb-Mohr criteria

Non-Stress based criteria

Stiffness, vibrational characteristics, fatigue resistance, creep resistance etc.

For Ductile material





For Brittle material

Common failure modes of a structural member:

1. Failure by excessive deflection
   - **Elastic deflection**
   - **Deflection by creep**
2. Failure by general yielding
3. Failure by fracture
   - **Sudden (brittle) fracture**
   - **Fracture of cracked member**
   - **Progressive fracture, Fatigue**
4. Failure by instability (buckling)