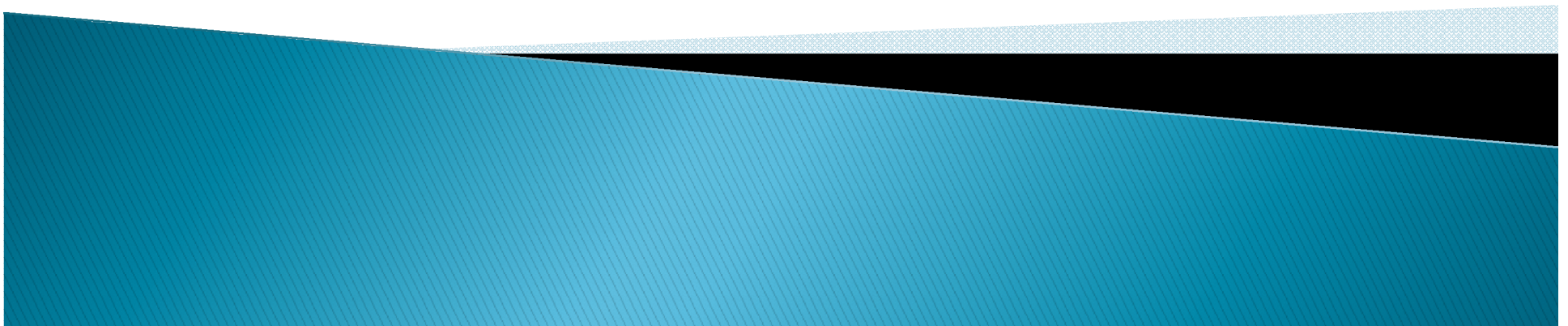


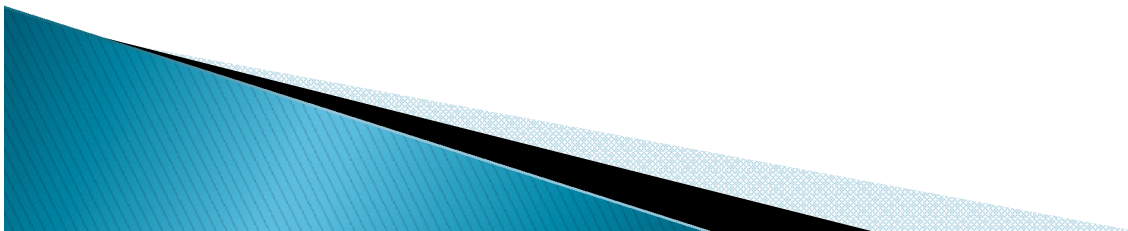
ME16A: CHAPTER THREE

BENDING MOMENTS AND SHEARING FORCES IN BEAMS



3.1 BEAM

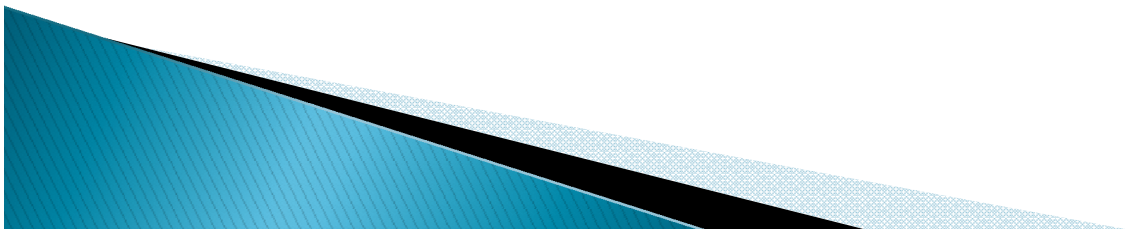
- ▶ A structural member which is long when compared with its lateral dimensions, subjected to transverse forces so applied as to induce bending of the member in an axial plane, is called a beam.



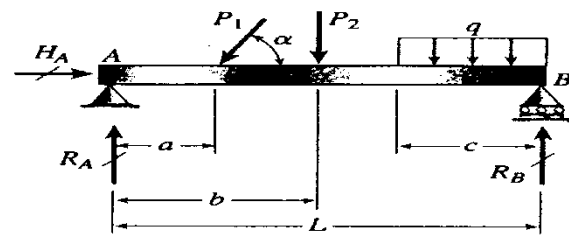
3.1.1

TYPES OF BEAMS

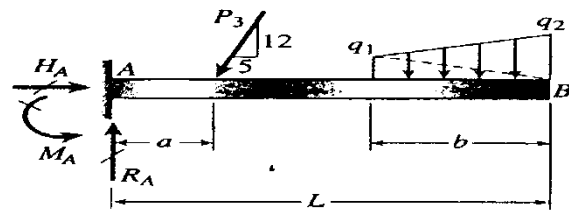
- ▶ Beams are usually described by the manner in which they are supported.
- ▶ For instance, a beam with a pin support at one end and a roller support at the other is called a **simply supported beam or a simple beam** (Figure 3.1a).
- ▶ A **cantilever beam** (Figure 3.1b) is one that is fixed at one end and is free at the other. The free end is free to translate and rotate unlike the fixed end that can do neither.



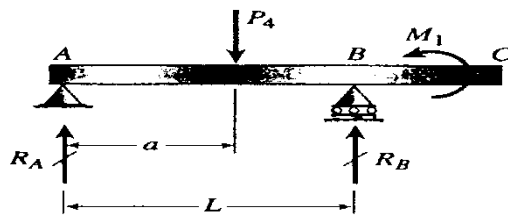
Types of Beams



(a)

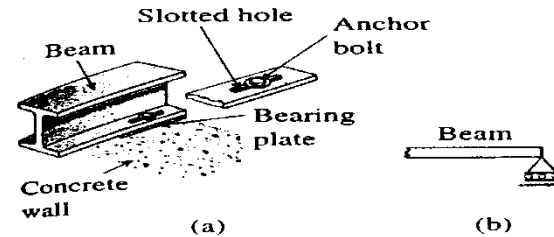


(b)



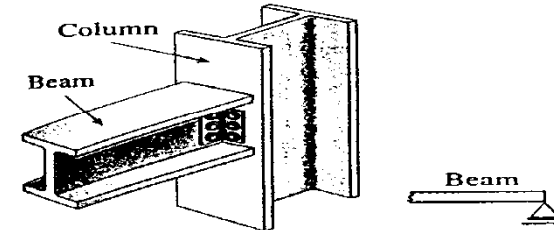
(c)

Fig. 3.1 Types of beams: (a) simple beam, (b) cantilever beam, and (c) beam with an overhang



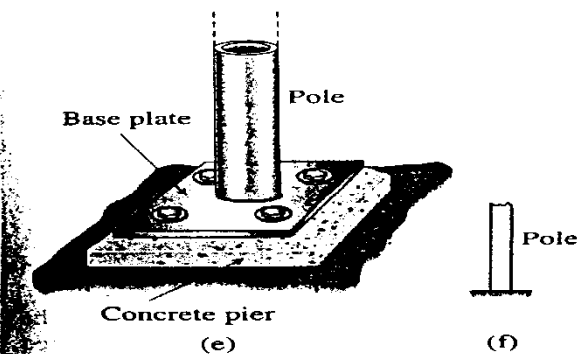
(a)

(b)



(c)

(d)



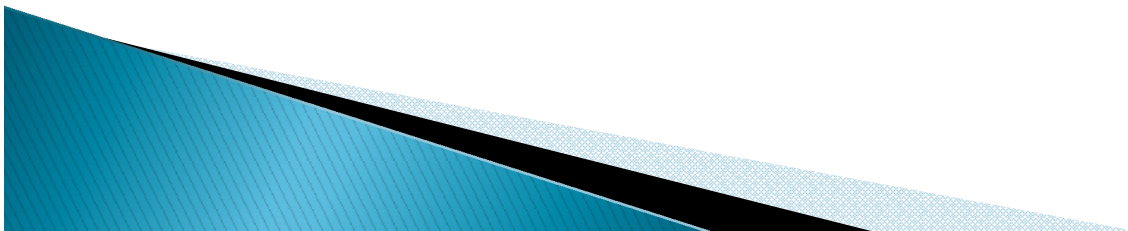
(e)

(f)

Beam supported on a wall: (a) actual construction, and (b) representation as a roller support. Beam-to-column connection: (c) actual construction, and (d) representation as a pin support. Pole anchored to a pier: (e) actual construction, and (f) representation as a fixed support.

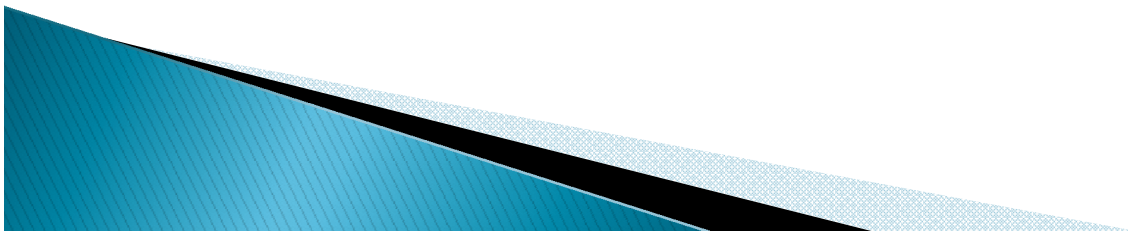
TYPES OF BEAMS

- ▶ The third example is a **beam with an overhang** (Figure 3.1c).
- ▶ The beam is simply supported at points A and B but it also projects beyond the support at B.
- ▶ The overhanging segment is similar to a cantilever beam except that the beam axis may rotate at point B.



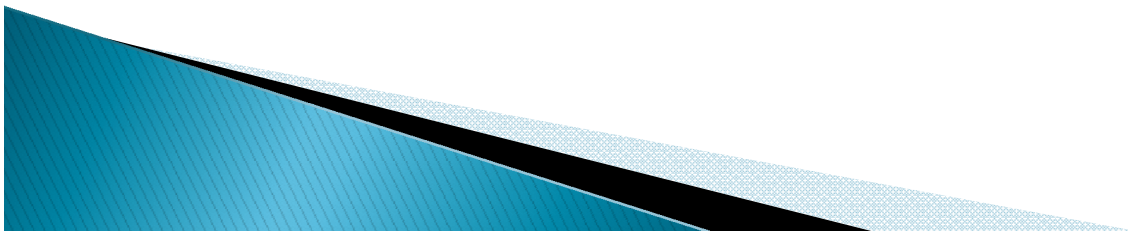
3.1 TYPES OF LOADS:

- ▶ A load can be classified as:
- ▶ **(i) Concentrated:** which is regarded as acting wholly at one. Examples are loads P , P_2 , P_3 and P_4 in Figure 3.1.
- ▶ **(ii) Distributed Load:** A load that is spread along the axis of the beam, such as q in Figure 3.1 a. Distributed loads are measured by their intensity, which is expressed in force per unit distance e.g. kN/m.



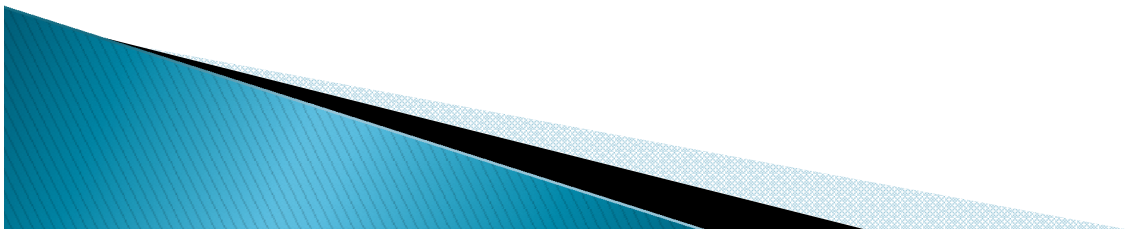
TYPES OF LOADS CONTD.

- ▶ A uniformly distributed load, or uniform load has constant intensity, q per unit distance (Figure 3.1. a).
- ▶ A linearly varying load (Figure 3.1 b) has an intensity which changes with distance.
- ▶ **(iii) Couple:** This is illustrated by the couple of moment M acting on the overhanging beam in Figure 3.1 c).



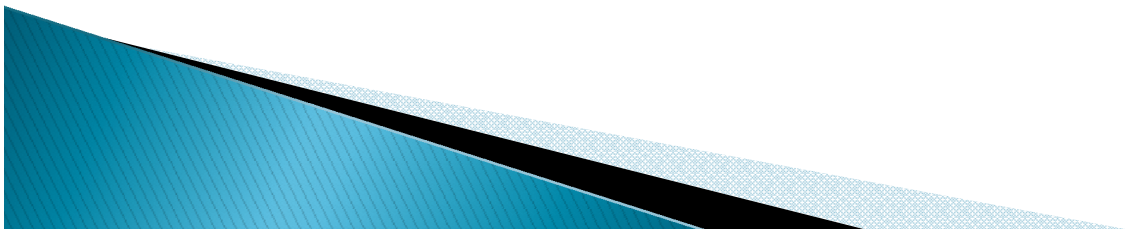
3.1 SHEAR FORCES AND BENDING MOMENTS

- ▶ When a beam is loaded by forces or couples, stresses and strains are created throughout the interior of the beam.
- ▶ To determine these stresses and strains, the internal forces and internal couples that act on the cross sections of the beam must be found.



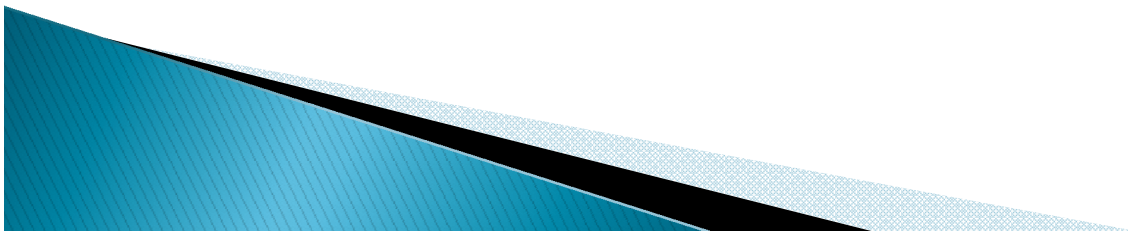
SHEAR FORCES AND BENDING MOMENTS CONTD.

- ▶ To find the internal quantities, consider a cantilever beam in Figure 3.2 .
- ▶ Cut the beam at a cross-section mn located at a distance x from the free end and isolate the left hand part of the beam as a free body (Figure 3.2 b).
- ▶ The free body is held in equilibrium by the force P and by the stresses that act over the cut cross section.



SHEAR FORCES AND BENDING MOMENTS CONTD.

- ▶ The resultant of the stresses must be such as to maintain the equilibrium of the free body.
- ▶ The resultant of the stresses acting on the cross section can be reduced to a shear force V and a bending moment M .
- ▶ The stress resultants in statically determinate beams can be calculated from equations of equilibrium.

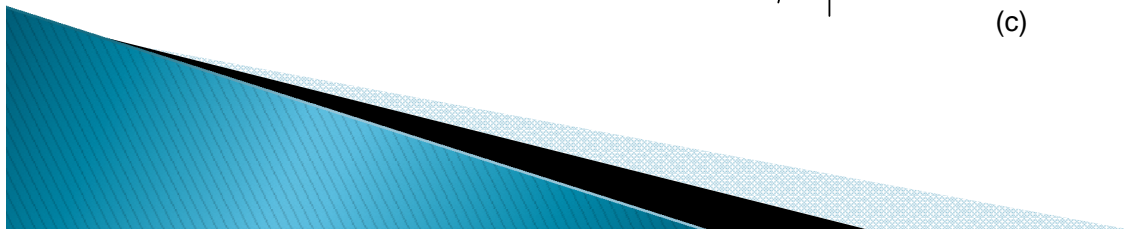
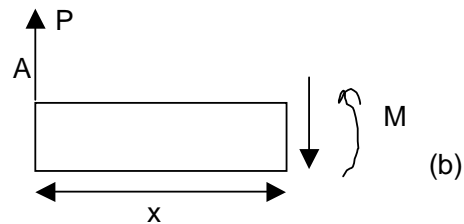
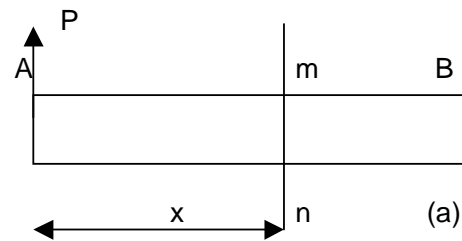


Shear Force V and Bending Moment, M in a Beam

Summing forces in the vertical direction and also taking moments about the cut section:

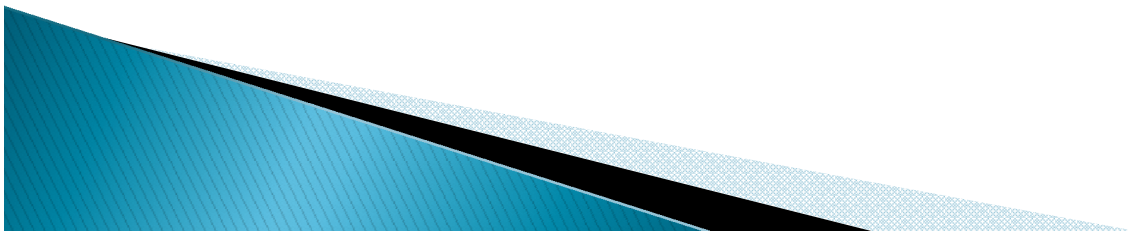
$$\sum F_x = 0 \text{ i.e. } P - V = 0 \text{ or } V = P$$

$$\sum M = 0 \text{ i.e. } M - Px \text{ or } M = Px$$

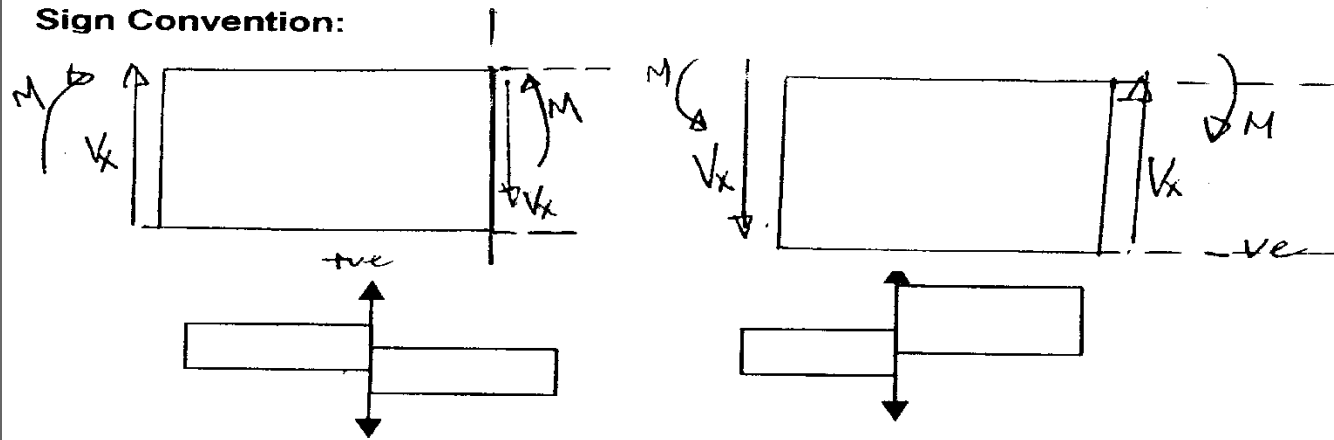


Shear Force and Bending Moment

- ▶ **Shear Force:** is the algebraic sum of the vertical forces acting to the left or right of the cut section
- ▶ **Bending Moment:** is the algebraic sum of the moment of the forces to the left or to the right of the section taken about the section



Sign Convention:



Positive shear

Negative shear

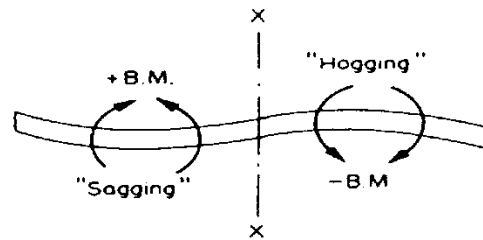


Fig. Beam with point of contraflexure at $X-X$.

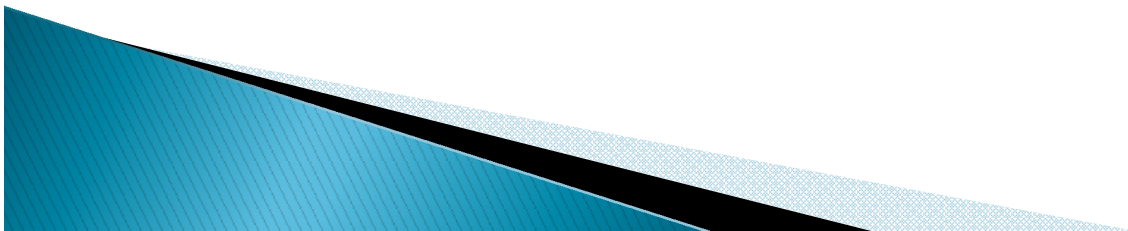
Sagging (positive)

Hogging (negative)

Figure 3.3

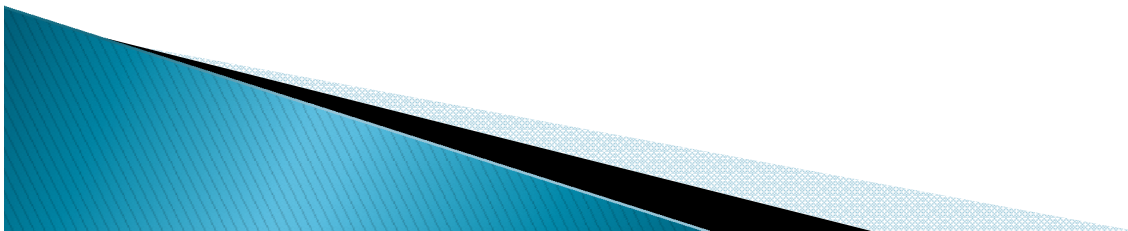
SIGN CONVENTION CONTD.

- ▶ Positive directions are denoted by an internal shear force that causes clockwise rotation of the member on which it acts, and an internal moment that causes compression, or pushing on the upper arm of the member.
- ▶ Loads that are opposite to these are considered negative.



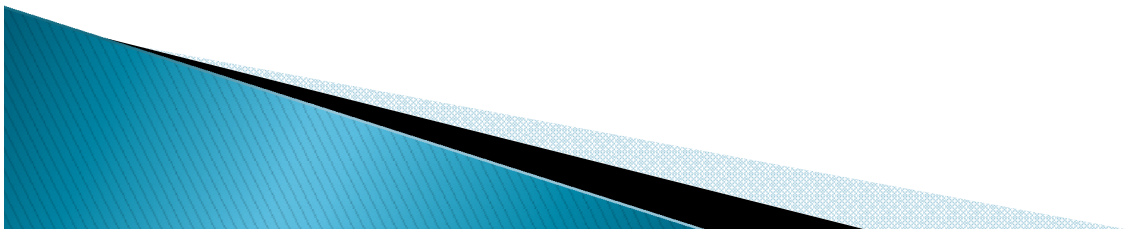
3.1 RELATIONSHIPS BETWEEN LOADS, SHEAR FORCES, AND BENDING MOMENTS

These relationships are quite useful when investigating the shear forces and bending moments throughout the entire length of a beam and they are especially helpful when constructing shear-force and bending moment diagrams in the Section 3.5.



RELATIONSHIPS CONTD.

- ▶ Consider an element of a beam cut between two cross sections that are dx apart (Figure 3.4a).
- ▶ The shear forces and bending moments acting on the sides of the element are shown in their positive directions.
- ▶ The shear forces and bending moments vary along the axis of the beam.



ELEMENTS OF A BEAM

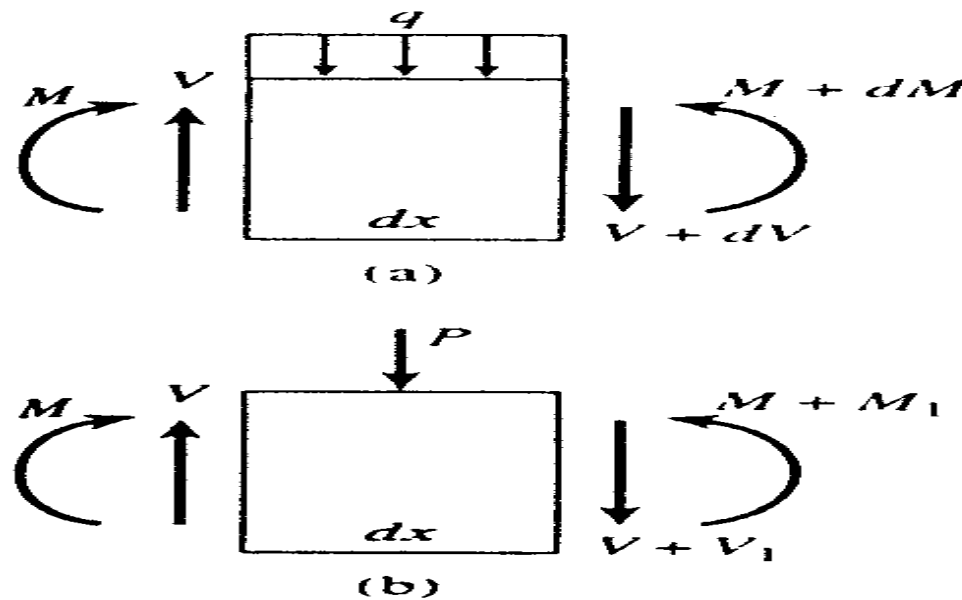
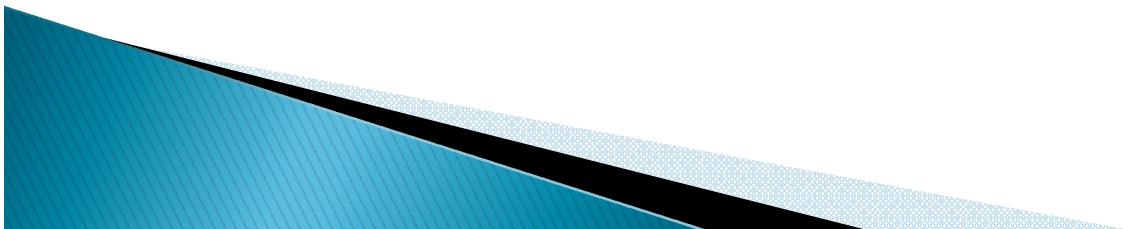


Fig. 34. Element of a beam used in deriving the relationships between loads, shear forces, and bending moments. (All loads and stress resultants are shown in their positive directions.)

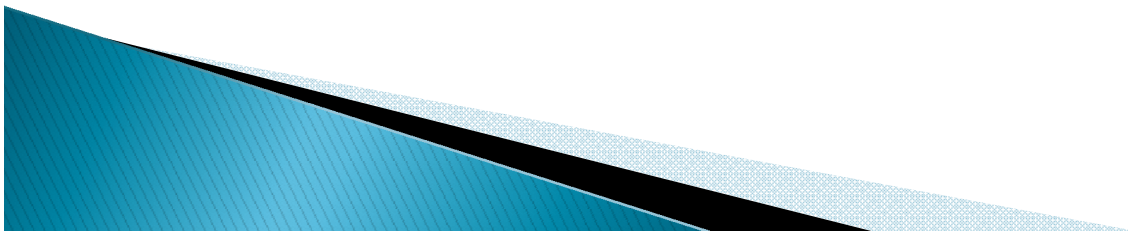
RELATIONSHIPS CONTD.

- ▶ The values on the right hand face of the element will therefore be different from those on the left hand face. In the case of distributed load, as shown in the figure, the increments in V and M are infinitesimal and so can be denoted as dV and dM respectively.



RELATIONSHIPS CONTD.

- ▶ The corresponding stress resultants on the right hand face are $V + dV$ and $M + dM$.
- ▶ In the case of concentrated load (Figure 3.4b), or a couple (Figure 3.4c), the increments may be finite, and so they are denoted V_1 and M_1 .
- ▶ The corresponding stress resultants on the RHS face are $V + V_1$ and $M + M_1$.



(a) Distributed Loads

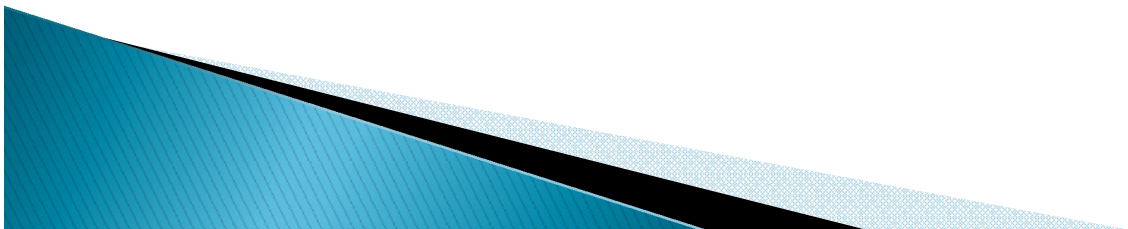
From Figure 34a

$$\sum F_y = 0 \text{ i.e. } V - qdx - (V+dV) = 0$$

$$-qdx - dV = 0 \text{ and}$$

$$\frac{dV}{dx} = -q \quad \dots\dots\dots (3.1)$$

This means that the rate of change of shear force at any point on the axis of the beam is equal to the negative of the intensity of the distributed load at that same point.



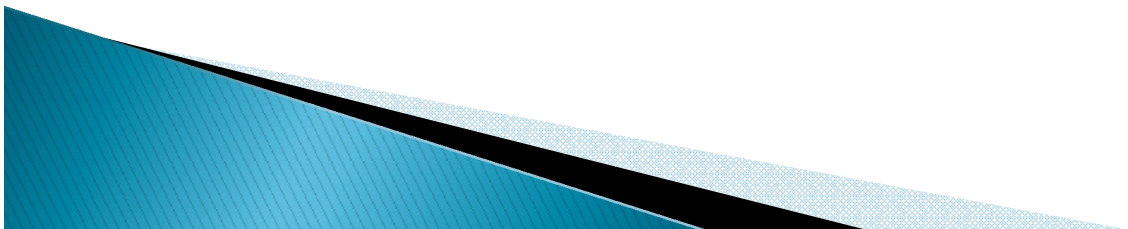
(a) Distributed Loads Contd.

If there is no distributed load on a segment of a beam (i.e. $q = 0$),

then $\frac{dV}{dx} = 0$ and the shear force is constant in that part of the beam.

Also, if the distributed load is uniform along part of the beam (q is constant),

then $\frac{dV}{dx}$ is also a constant and the shear force changes linearly in that part of the beam.



Distributed Loads Contd.

Taking Moments about the LHS of the element in Figure 3.4.a:

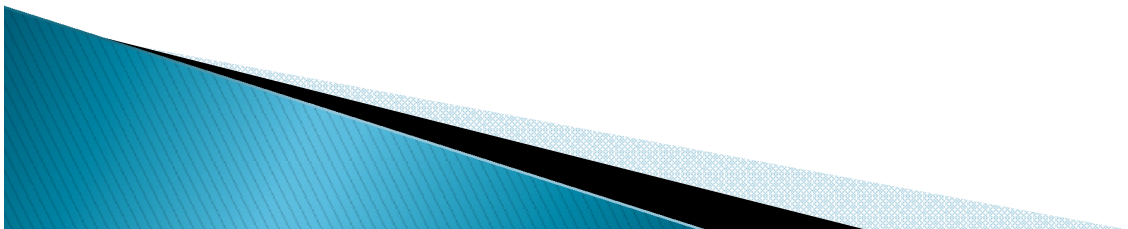
$$\sum M = 0 \text{ i.e. } -M - q dx (dx/2) - (V + dV) dx + M + dM = 0$$

Neglecting products of differentials since they are small compared to other terms:

$$-V dx + dM = 0 \text{ and:}$$

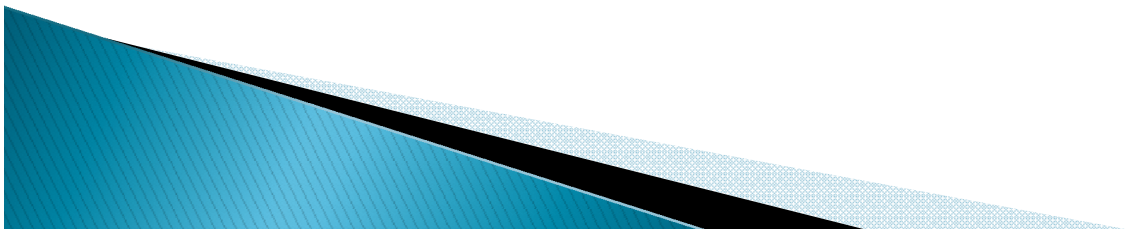
$$\frac{dM}{dx} = V \text{ (3.2)}$$

This equation means that the rate of change of the bending moment at any point on the axis of a beam is equal to the shear force at that same point. For instance, if the shear force is zero in a region of the beam, then the bending moment is constant in that region.



Distributed Loads Contd.

- ▶ Note that equation 3.2 applies only in regions where distributed loads or no loads act on the beam.
- ▶ At a point where a concentrated load acts, a sudden change (or discontinuity) in the shear force occurs and the derivative dM/dx is undefined at that point.



(b) Concentrated Loads (Figure 3.4 b)

$$\sum F_y = 0 \text{ i.e. } V - P - (V + V_1) = 0 \quad \text{or } V_1 = -P$$

This means that an abrupt change in the shear force occurs at any point where a concentrated load acts.

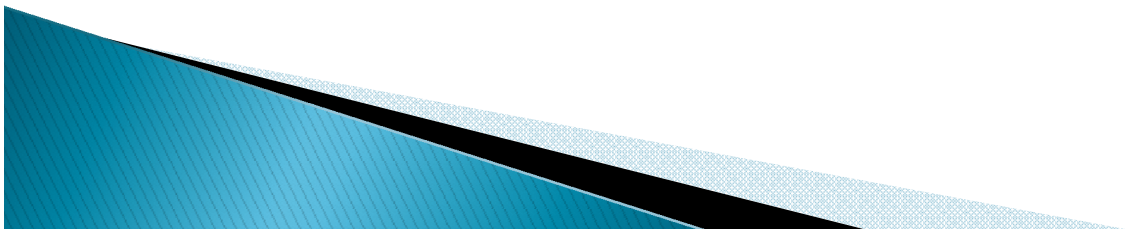
As one passes from left to right through the point of load application, the shear force decreases by an amount P.

Taking Moments about the LHS face of the element:

$$-M - P(dx/2) - (V + V_1)dx + M + M_1 = 0$$

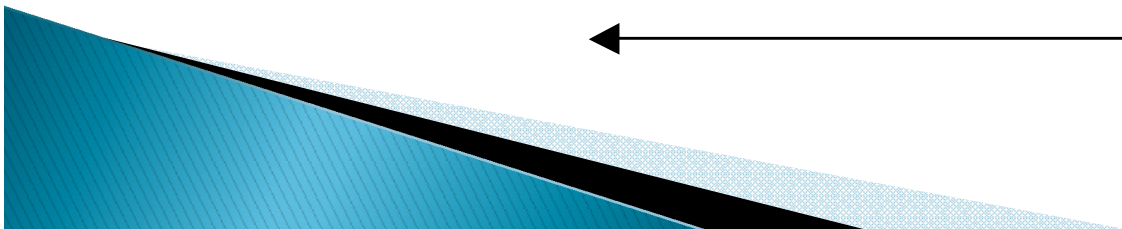
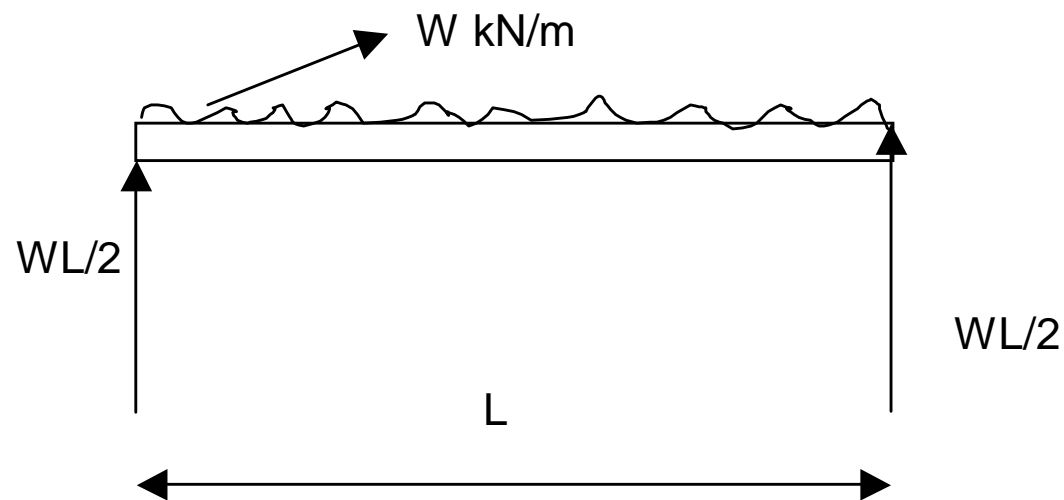
$$M_1 = P(dx/2) + Vdx + V_1dx$$

Since dx is small, M_1 is also small and this means that the **bending moment does not change as we pass through the point of application of a concentrated load.**



Example

- ▶ Determine the equation for Bending Moment and Shear force for the beam below:



Solution

$$\frac{dV_x}{dx} = -w$$

$$V_x = -wx + C_1$$

$$\frac{dM_x}{dx} = V_x$$

$$M_x = -w/2 x^2 + C_1 x + C_2$$

Boundary Condition: At $x = 0$, $M_x = 0$

- Simply supported beam

i.e. $C_2 = 0$

i.e. $M_x = -w/2 x^2 + C_1 x$

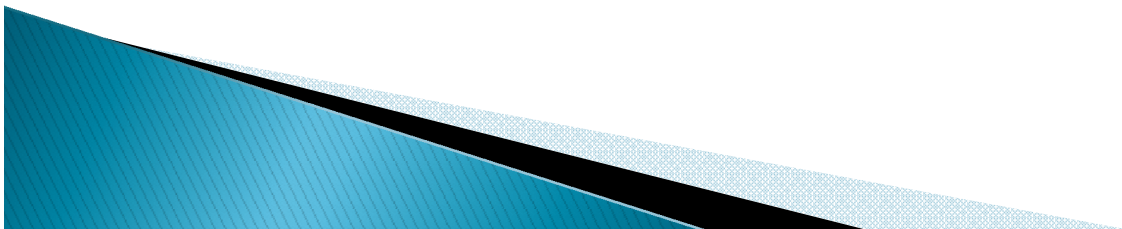
Boundary Condition: At $x = L$, $M_x = 0$

- Simply supported

i.e. $0 = -w/2 L^2 + C_1 L$ and $C_1 = w L/2$

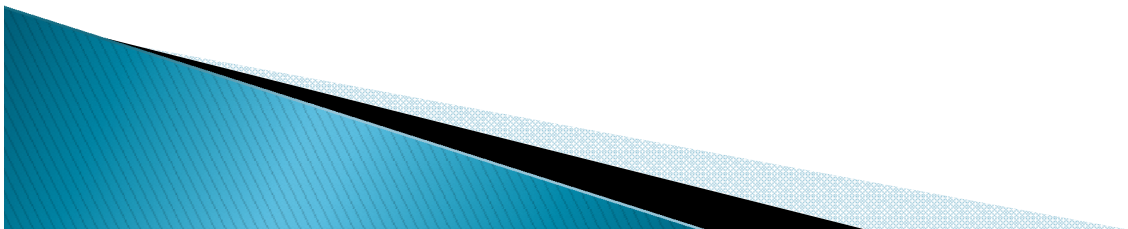
i.e. $M_x = w L/2 x - w x^2/2$

$$V_x = -wx + w L/2$$



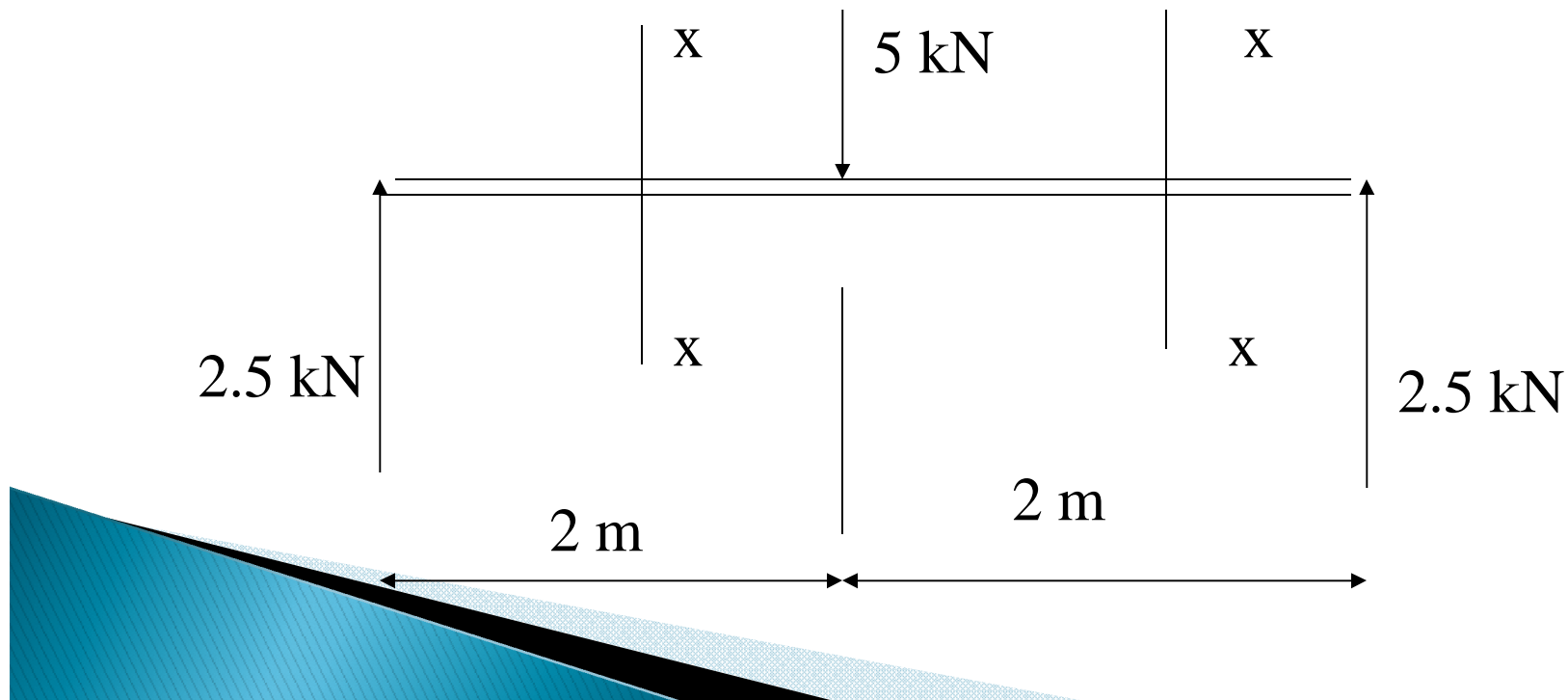
3.5 SHEAR FORCE AND BENDING MOMENT DIAGRAMS

- ▶ When designing a beam, there is the need to know how the bending moments vary throughout the length of the beam, particularly the maximum and minimum values of these quantities.



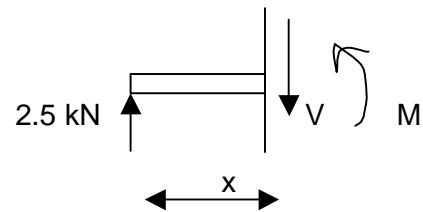
Example

- ▶ Draw the shear and bending moment diagrams for the beam shown in the Figure.



Solution

- (i) First determine the reactions at A and B. These are equal to 2.5 kN each.
- (ii) Cut the beam at an arbitrary section x after A but before B



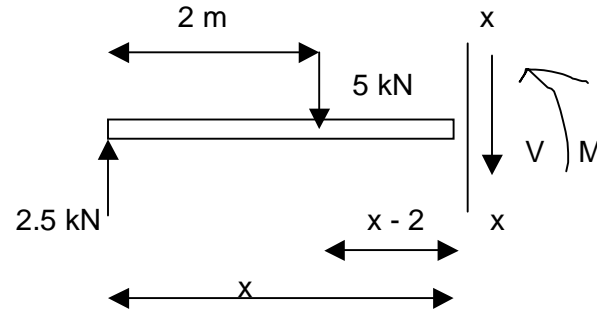
The unknown forces V and M are assumed to act in the positive sense on the right hand face of the segment according to the sign convention:

$$V = 2.5 \text{ kN} \quad (1)$$

$$\text{i.e. } M = 2.5 x \text{ kN.m} \quad (2)$$

Solution Contd.

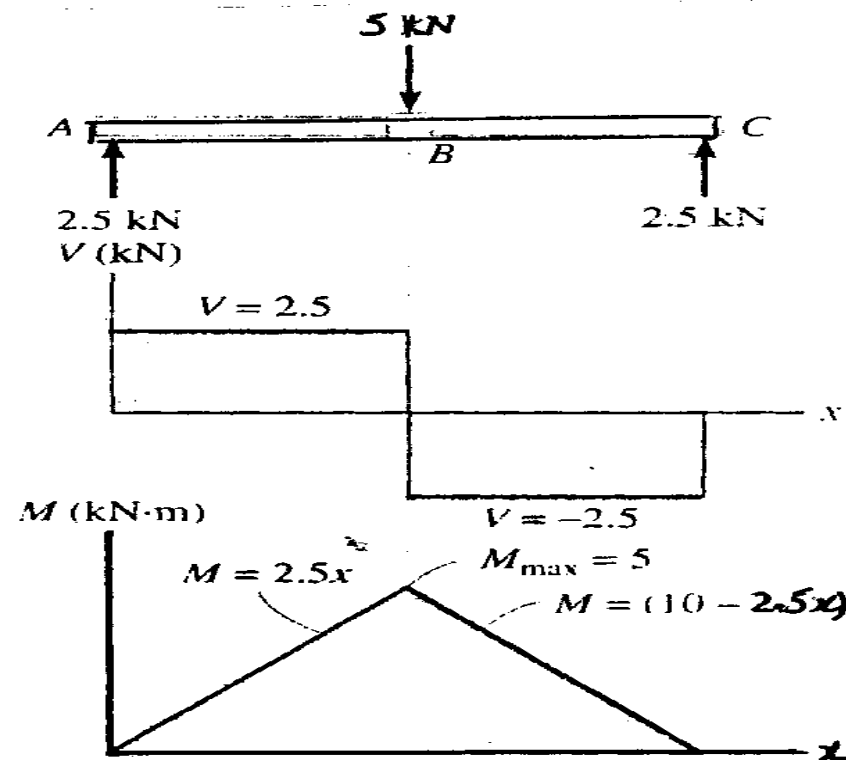
(i) Now choose another section along BC after the 5 kN load ($2 \text{ m} < x < 4 \text{ m}$)



$$V = 2.5 \text{ kN} - 5 \text{ kN} = -2.5 \text{ kN} \quad (3)$$

$$M = 2.5x - 5(x-2) = (10 - 2.5x) \text{ kN.m} \quad (4)$$

Shear Force and Bending Moment Diagrams



Simpler Method

Simpler Method For Drawing Shear Force and Bending Moment Diagrams

- (i) The Shear forces (V) can be determined by mental arithmetic using the convention that the upward force at the LHS section is positive and downward force is negative. Also downward force at the RHS of the beam is positive while the upward force is negative: Starting from the LHS of beam:

At A: $V = 2.5 \text{ kN}$

At B: , $V = 2.5 - 5 = -2.5 \text{ kN}$

At point C: $V = -2.5 \text{ kN}$ (upward force at the right of beam)

- (ii) For bending moment (BM), remember that at the LHS of a beam, clockwise moment is positive and anti-clockwise is negative. Starting from the LHS:

At A: B.M. is zero ... Simply supported beam

At B: $B.M = 2.5 \times 2 = 5 \text{ kN m}$

At C: B.M. is zero Simply supported beam

Example

- ▶ Draw the shear and bending moment diagrams for the beam AB

$M_{\max} = qL^2/8$ as shown on the diagram.

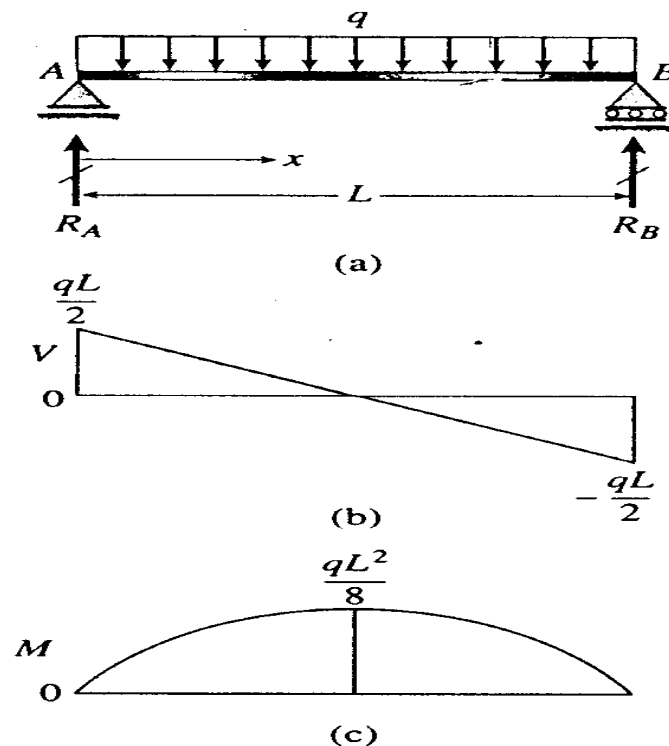
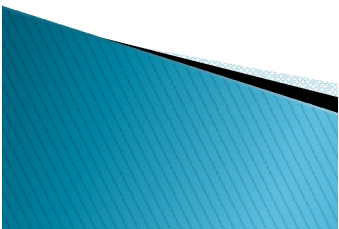
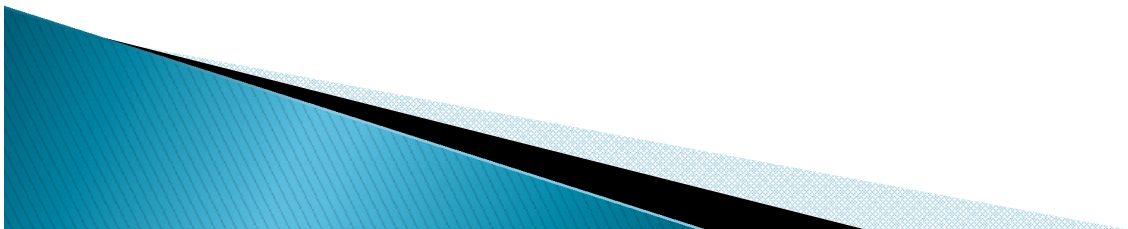


Fig. Shear-force and bending-moment diagrams for a simple beam with a uniform load



Solution

- ▶ Because the beam and its loading are symmetric, R_A and R_B are $q L/2$ each. The shear force and bending moment at distance x from the LHS are:
- ▶ $V = R_A - q x = q L/2 - q x$
- ▶ $M = R_A x - q x (x/2) = q L x /2 - q x^2/2$
- ▶ These equations, are valid throughout the length of the beam and are plotted as shear force and bending moment diagrams.



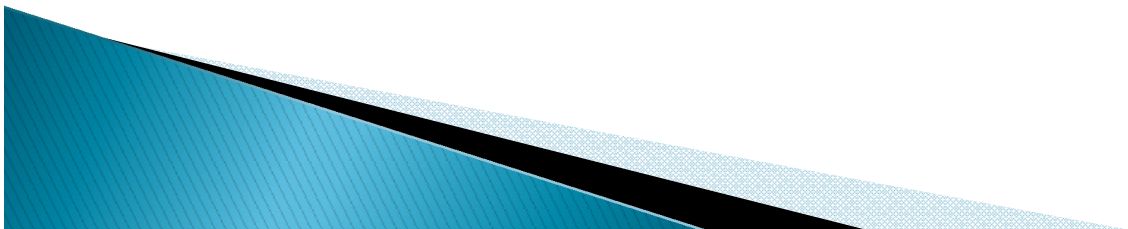
Solution Contd.

Note: The slope of the inclined straight line representing the shear force is $-q$ which agrees with equation 3.1. Also at each cross section, the slope of the bending moment diagram is equal to the shear force as shown in equation 3.2, thus:

$$\frac{dM}{dx} = \frac{d}{dx} \left(\frac{qLx}{2} - \frac{qx^2}{2} \right) = \frac{qL}{2} - qx = V$$

The maximum bending moment occurs at the midpoint of the beam; therefore, we substitute $x = L/2$ into the expression for M to obtain:

$$M_{\max} = qL^2/8 \quad \text{as shown on the diagram.}$$



Diagrams

$M_{\max} = qL^2/8$ as shown on the diagram.

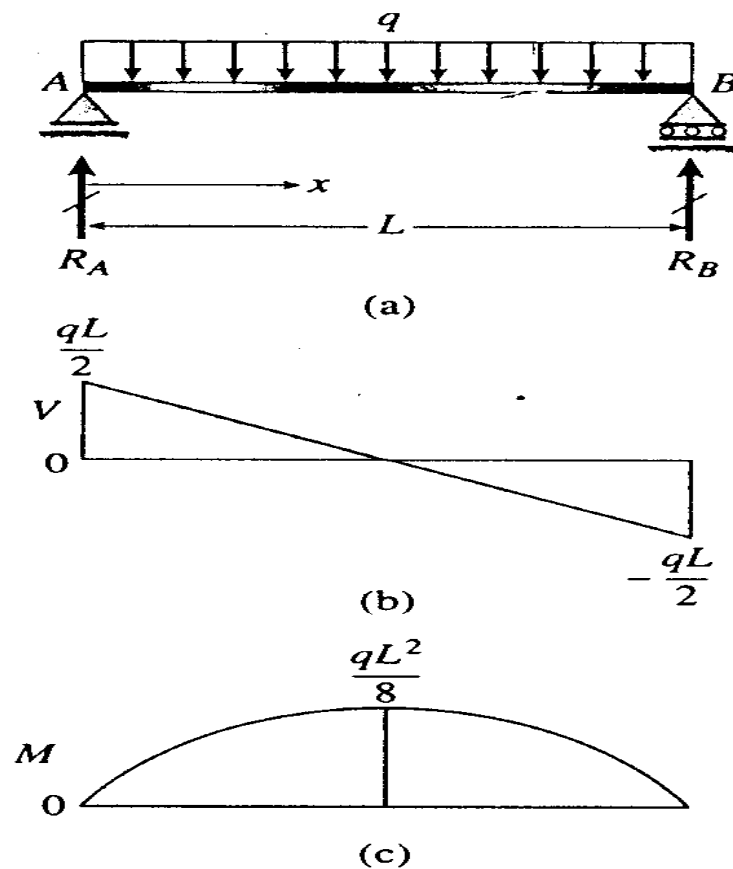
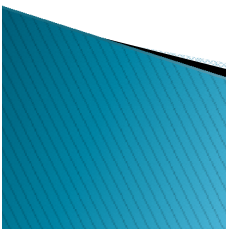
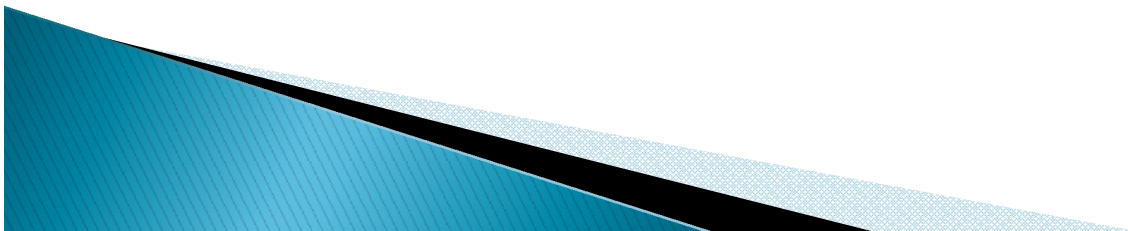
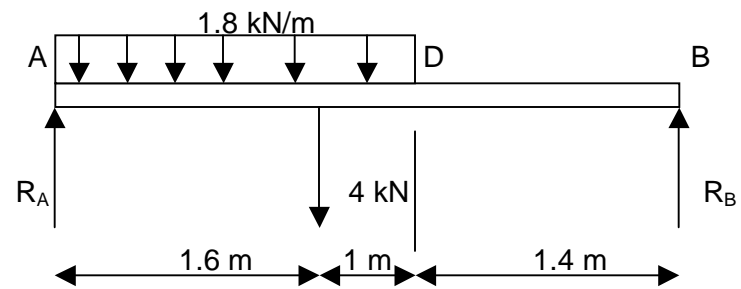


Fig. Shear-force and bending-moment diagrams for a simple beam with a uniform load



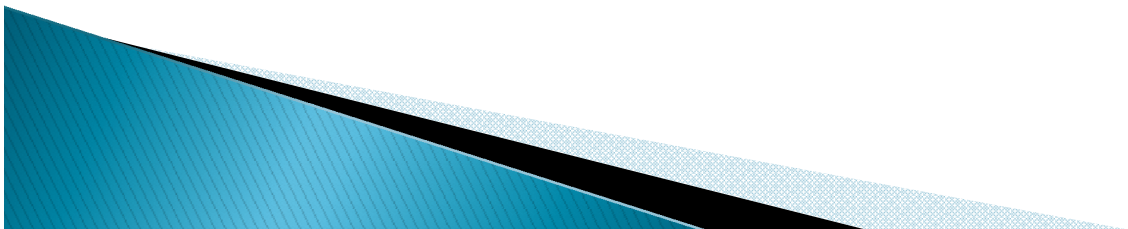
Example

Draw the shear and bending moment diagrams for the beam AB



Solution

- ▶ Find R_A and R_B
- ▶ $F_y = 0$ i.e. $R_A + R_B = (1.8 \times 2.6) + 4 \text{ kN} = 8.68 \text{ kN}$
- ▶ $M_B = 0$ i.e. $-4 R_A + 2.4 \times 4 + (1.8 \times 2.6) \times (4 - 1.3) = 0$
- ▶ $4 R_A = 9.6 + 12.63 = 22.23;$
- ▶ $R_A = \mathbf{5.56 \text{ kN}}$
- ▶ $R_B = 8.68 - 5.56 = \mathbf{3.12 \text{ kN}}$



Solution Contd.

(i) Using the usual convention: At point A: $V = 5.56 \text{ kN}$

$$\text{At point C, } V = 5.56 - (1.8 \times 1.6) = 5.56 - 2.88 = 2.68 \text{ kN}$$

$$\begin{aligned} \text{At point C also, because of the 4 kN load, the } V \text{ is also equal to } & 2.68 - 4 = \\ & = -1.32 \text{ kN} \end{aligned}$$

$$\text{At point D: } V = 5.56 - (1.8 \times 2.6) - 4 = 5.56 - 4.68 - 4 = -3.12 \text{ kN}$$

At point B: $V = -3.12 \text{ kN}$ - upward force on right of section.

(ii) For the Bending moment:

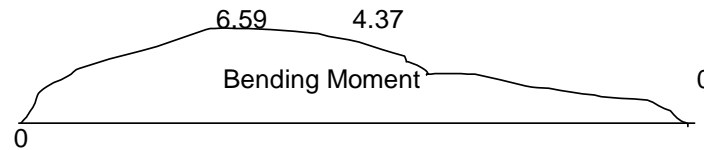
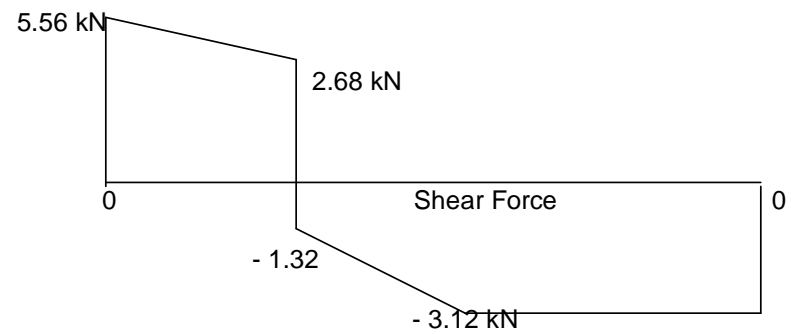
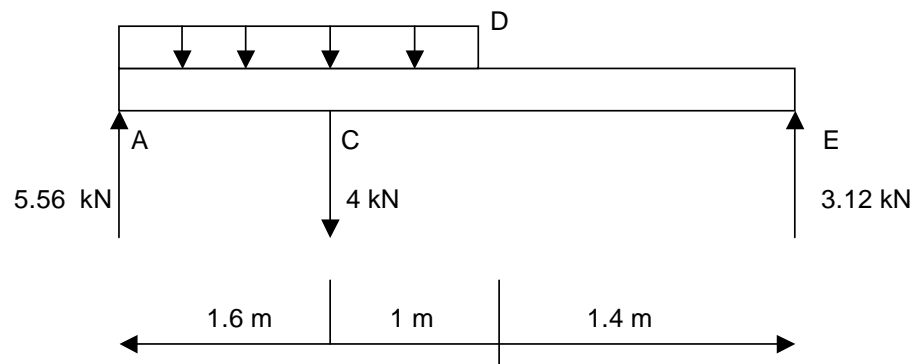
At point A: B.M. is zero Simply supported beam

$$\begin{aligned} \text{At point C: B.M.} &= (5.56 \times 1.6) - (1.8 \times 1.6) \times 0.8 = 8.896 - 2.304 \\ &= 6.59 \text{ kN m} \end{aligned}$$

$$\begin{aligned} \text{At point D: B.M} &= (5.56 \times 2.6) - (4 \times 1) - (1.8 \times 2.6) \times 1.3 \\ &= 14.456 - 4 - 6.084 = 4.37 \text{ kN m} \end{aligned}$$

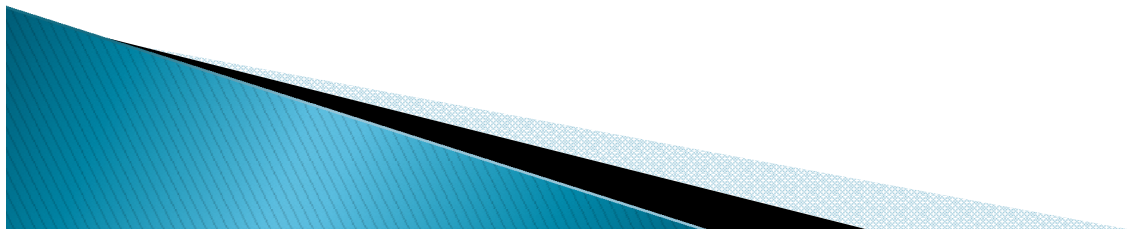
At point B, B.M is zero. Simply supported beam.

Shear Force & Bending Moment Diagrams



3.1 BENDING STRESSES IN BEAMS (FOR PURE BENDING)

- ▶ It is important to distinguish between pure bending and non-uniform bending.
- ▶ Pure bending is the flexure of the beam under a constant bending moment. Therefore, pure bending occurs only in regions of a beam where the shear force is zero because $V = dM/dx$.
- ▶ Non-uniform bending is flexure in the presence of shear forces, and bending moment changes along the axis of the beam.



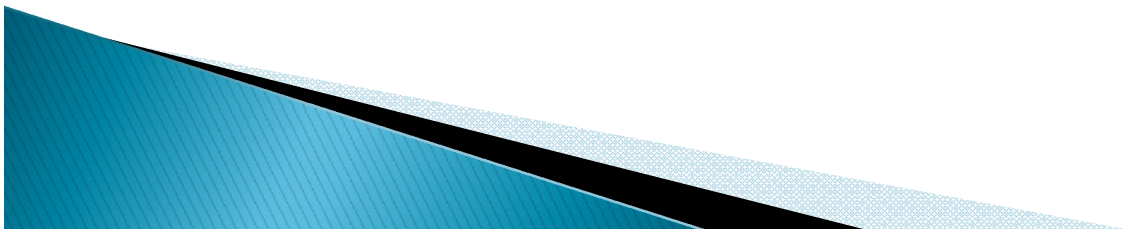
Assumptions in Simple Bending Theory

- ▶ (i) Beams are initially straight
- ▶ (ii) The material is homogenous and isotropic i.e. it has a uniform composition and its mechanical properties are the same in all directions
- ▶ (iii) The stress-strain relationship is linear and elastic
- ▶ (iv) Young's Modulus is the same in tension as in compression
- ▶ (v) Sections are symmetrical about the plane of bending



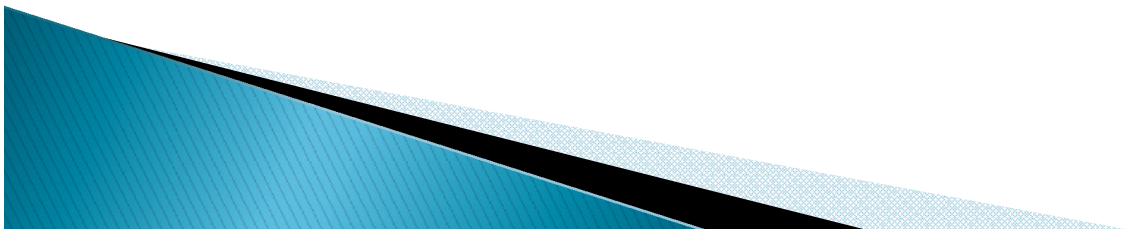
Assumptions in Simple Bending Theory Contd.

- ▶ Sections which are plane before bending remain plane after bending.
- ▶ The last assumption implies that each section rotates during bending about a neutral axis, so that the distribution of strain across the section is linear, with zero strain at the neutral axis.



Assumptions in Simple Bending Contd.

- ▶ The beam is thus divided into tensile and compressive zones separated by a neutral surface.
- ▶ The theory gives very accurate results for stresses and deformations for most practical beams provided that deformations are small.



Theory of Simple Bending

- ▶ Consider an initially straight beam, AB under pure bending.
- ▶ The beam may be assumed to be composed of an infinite number of longitudinal fibers.
- ▶ Due to the bending, fibres in the lower part of the beam extend and those in the upper parts are shortened.
- ▶ Somewhere in-between, there would be a layer of fibre that has undergone no extension or change in length.
- ▶ This layer is called neutral surface.



Theory of Bending

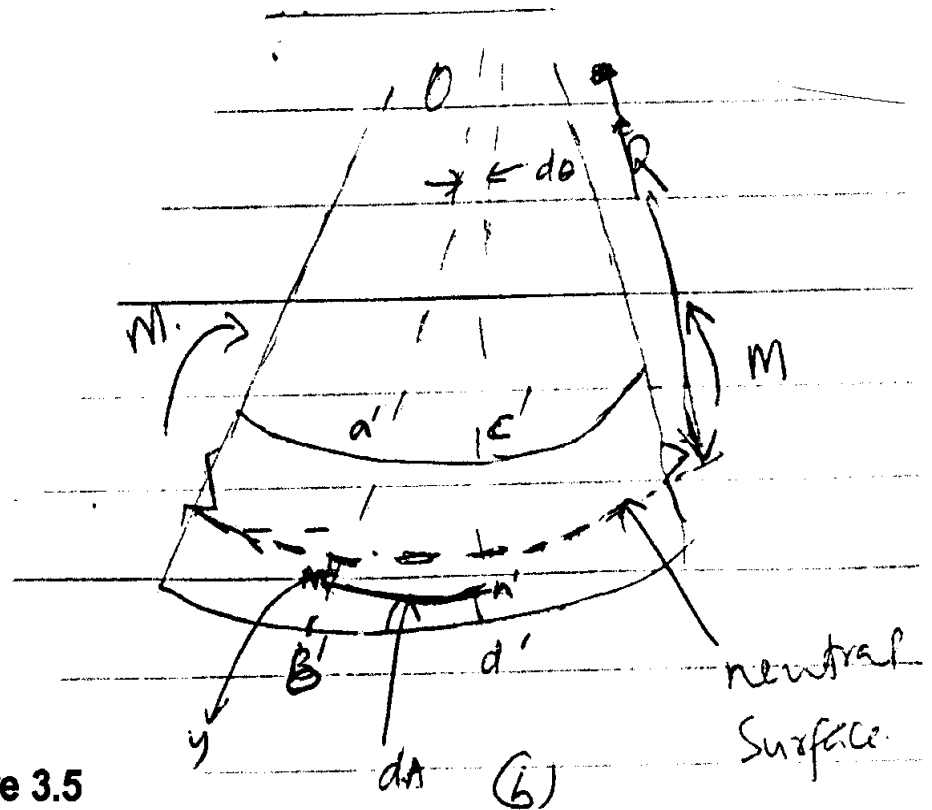
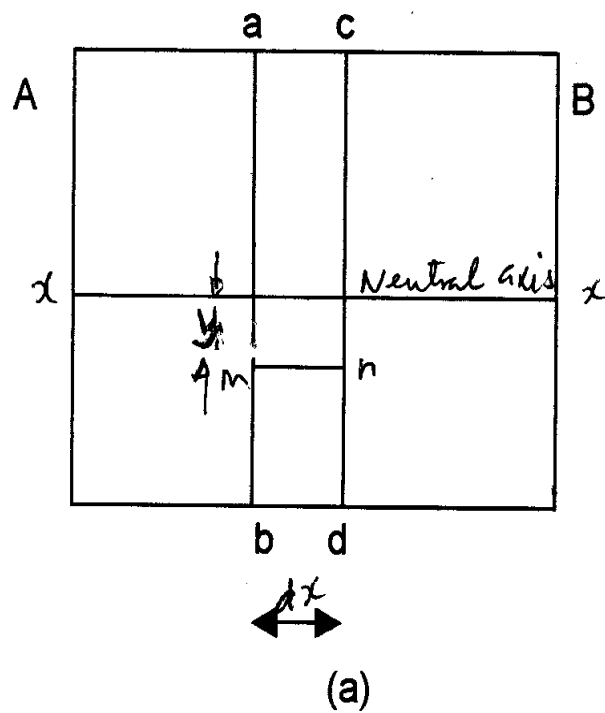
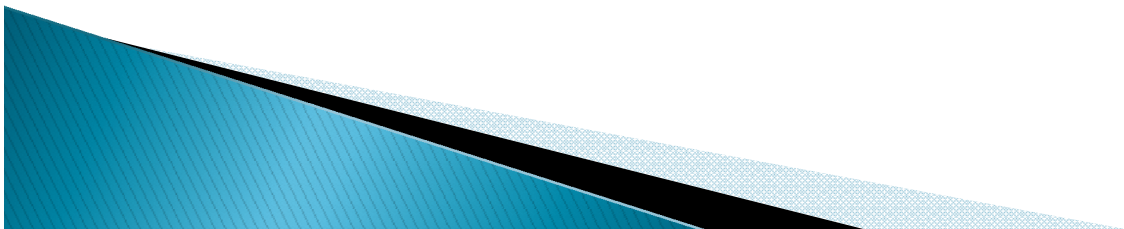


Figure 3.5

Theory of Simple Bending

- ▶ The line of intersection of the neutral surface with the cross-section is called Neutral Axis of the cross section.



Theory of Simple Bending

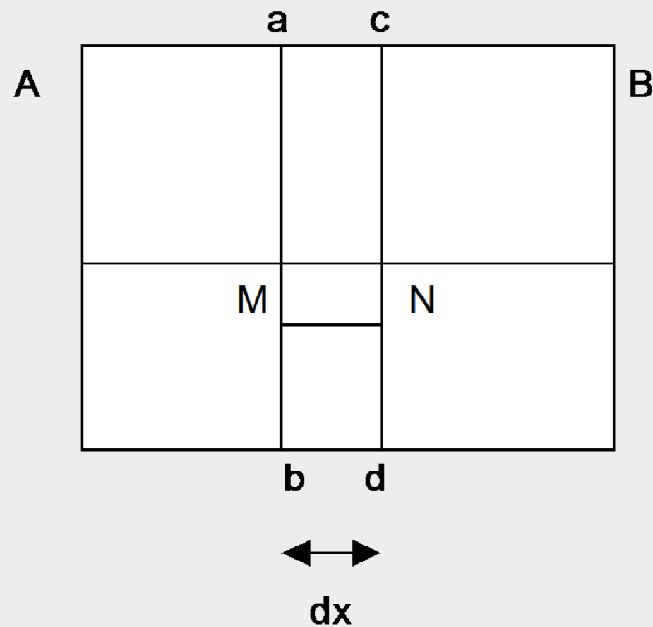


Figure 3.5 a

Let ab and cd be two cross sections at a distance dx apart (Figure 3.5a).

Let O be the centre of curvature and let R be the radius of the neutral surface.

Consider a fibre 'mn' at a distance 'y' from the neutral surface.

$$\text{Change in length of } mn = (R + y) d\theta - R d\theta = y d\theta$$

$$\text{Strain, } \epsilon = \frac{y d\theta}{R d\theta} = \frac{y}{R}$$

$$\text{Stress, } \sigma = E \epsilon = E \frac{y}{R} \dots\dots\dots(A)$$

Theory of Simple Bending Contd.

Equilibrium Equations

$$dF \text{ acting on } dA = \sigma dA = \frac{E}{R} y dA$$

$$F = \text{Normal force} = \frac{E}{R} \int_A y dA = 0 \quad \dots(1)$$

(as there is no normal force acting on the cross section)

$$dF = \sigma \cdot dA$$

$$dM = \sigma \cdot y \cdot dA$$

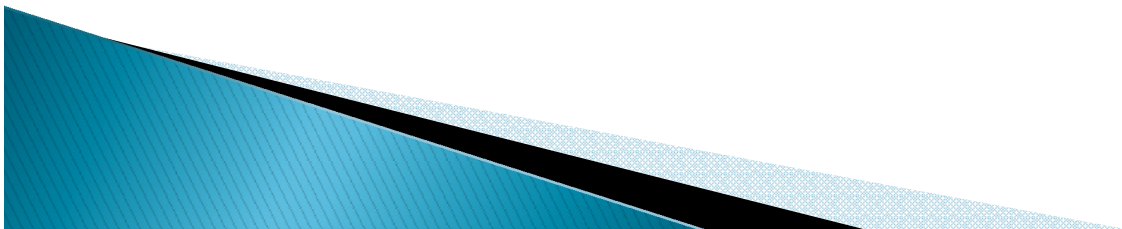
$$M = \int_A \sigma \cdot y \cdot dA \quad \dots(2)$$

$$\int_A \sigma \cdot dA = 0 \quad \int_A \sigma \cdot y \cdot dA = M$$

$$\int_A \sigma \cdot dA = 0, \text{ meaning that } \frac{E}{R} \int_A y dA = 0$$

$$\text{Since } \frac{E}{R} \text{ cannot be zero: } \int_A y dA = 0$$

This integral is called first moment of area of the cross section. Hence, neutral axis coincides with the centroidal axis.



Theory of Simple Bending Contd.

Second Condition

$$\int_A \sigma y dA = \frac{E}{R} \int_A y^2 dA = M$$

$\int_A y^2 dA$ is called the second moment of area or the moment of inertia of the cross

section about the neutral axis i.e. I_{NA}

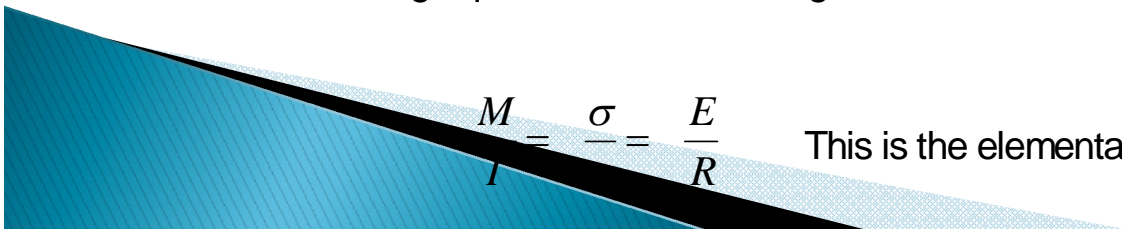
$$\int_A \sigma y dA = \frac{E}{R} \int_A y^2 dA = M$$

i.e. $\frac{E}{R} I = M \dots\dots(B)$

Combining equations A and B, we get:

$$\frac{M}{I} = \frac{\sigma}{R} = \frac{E}{R}$$

This is the elementary bending formula



Simple Bending: Calculation of Stress

To calculate the stress, use the first two equations. That is:

$$\sigma = \frac{M}{I} y \dots\dots\dots(C)$$

To calculate Radius of Curvature:

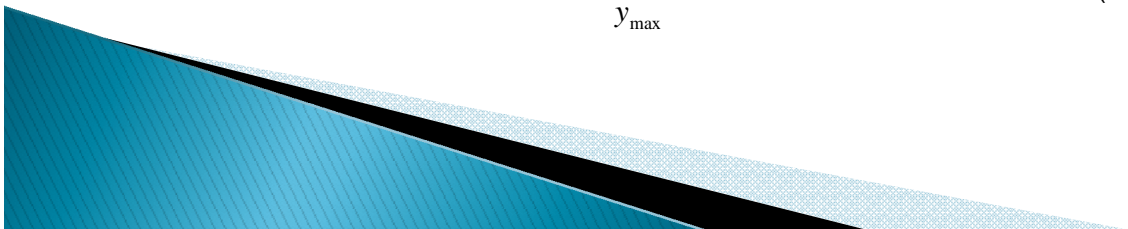
$$\phi = \frac{1}{R} = \frac{M}{E I} \dots\dots\dots(D)$$

This means that radius of curvature (ϕ) is directly proportional to M and inversely proportional to EI. EI is called the flexural rigidity.

$\sigma = \frac{M}{I} y$: Stresses are normally calculated on the extremes i.e. compression and tensile maximum stresses.

$$\text{i.e. } \sigma_{\max} = \frac{M}{I} y_{\max} = \frac{M}{Z}$$

Where: $Z = \frac{I}{y_{\max}}$ called 'Modulus of the Section' (Section Modulus)

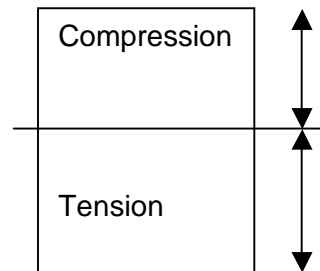


Simple Bending: Compression and Tension Zones

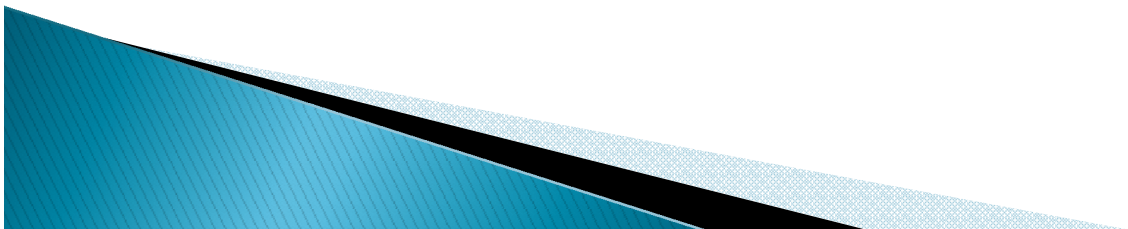
Moment of resistance of the cross section, $M = \sigma \cdot Z$

$\sigma \cdot Z$ will be equal or less than applied moment for the section to be safe.

The magnitudes of the maximum compressive stress and the minimum tensile stress are the same.

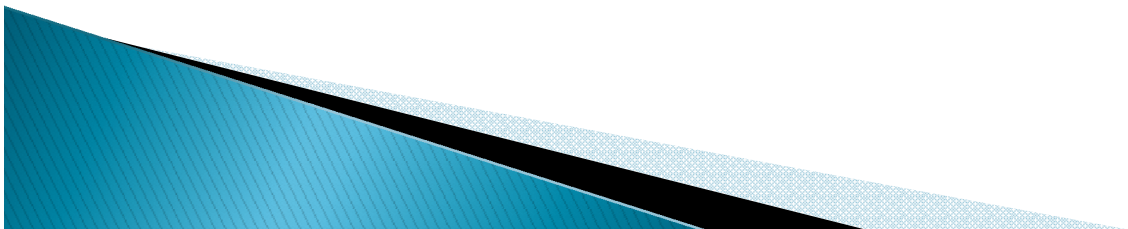


$$\sigma_{\max} = \frac{M}{I} \cdot h_1 = \frac{M}{Z_1}$$
$$\sigma_{\min} = \frac{M}{I} \cdot h_2 = \frac{M}{Z_2}$$



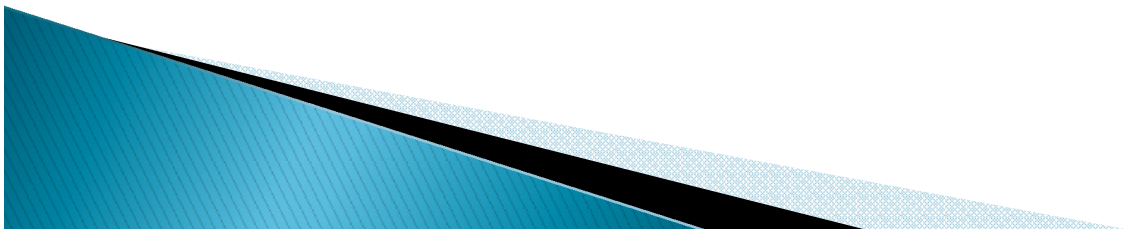
Non-Uniform Bending

- ▶ **Non-Uniform Bending:** In the case of non-uniform bending of a beam, where bending moment varies from section to section, there will be shear force at each cross section which will induce shearing stresses.
- ▶ Also these shearing stresses cause warping (or out-of-plane distortion) of the cross section so that plane cross sections do not remain plane even after bending.



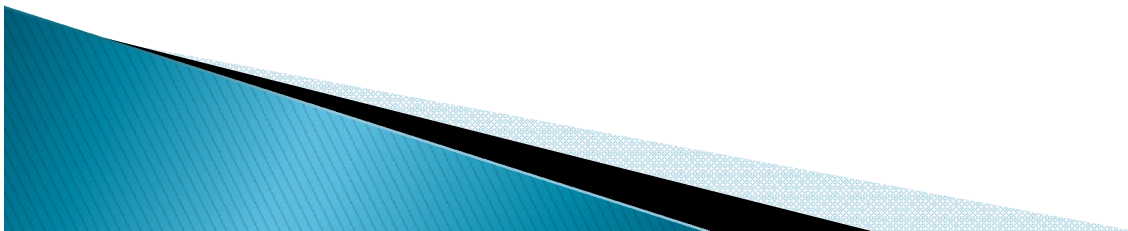
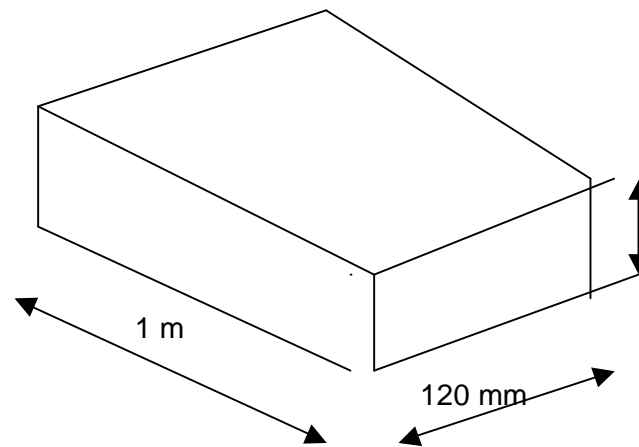
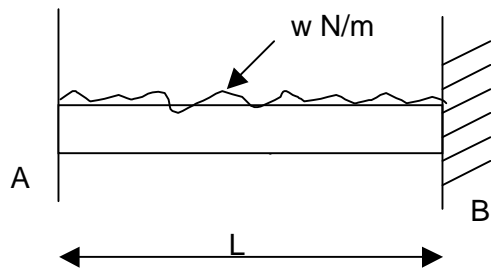
Non-Uniform Bending Contd.

- ▶ This complicates the problem but it has been found by more detailed analysis that normal stresses calculated from simple bending formula are not greatly altered by the presence of shear stresses.
- ▶ Thus we may justifiably use the theory of pure bending for calculating normal stresses in beams subjected to non-uniform bending.



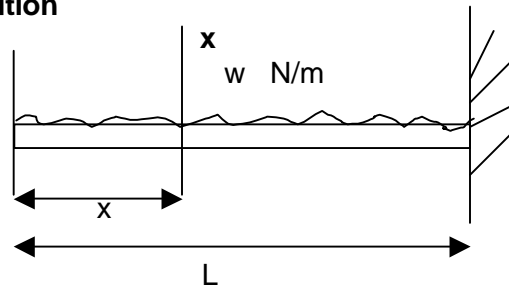
Example

Example: A cantilever AB of length, 1 m carries a uniformly distributed load, w N/m from the fixed end A to the free end B. If the Cantilever consists of a horizontal plate of thickness 10 mm and width 120 mm as shown in the figure below, calculate the total permissible distributed load, w if the maximum bending stress is not to exceed 100 MN/m^2 . Ignore the weight of the plate.



Solution

Solution

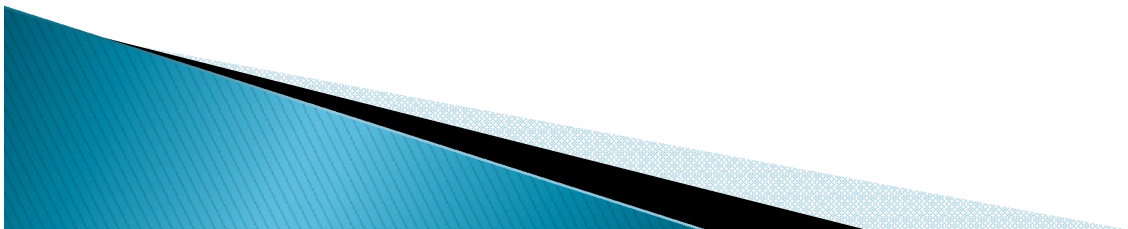


$$M = -wx^2/2 \quad (\text{for } 0 < x < L) \quad (\text{x is from right to left})$$

$$\text{At } x = 0, \quad Mx = 0$$

$$\text{At } x = L/2, \quad Mx = -w/2 (L/2)^2 = -wL^2/8$$

At $x = L$, $Mx = -wL^2/2$. The negative sign shows that the bending involves hogging rather than sagging.



Solution Concluded

So the maximum bending moment (M_{\max}) is $w L^2/2$ (Numerical maximum).

For $L = 1$ m, $M_{\max} = w/2$ N.m

Moment of inertia of a rectangular cross section (I_x) = $b d^3/12$

$$= \frac{0.12 \text{ m} \times 0.01^3}{12} = 1 \times 10^{-8} \text{ m}^4$$

Maximum stress, $\sigma_{\max} = M_{\max} y_{\max} / I_x$

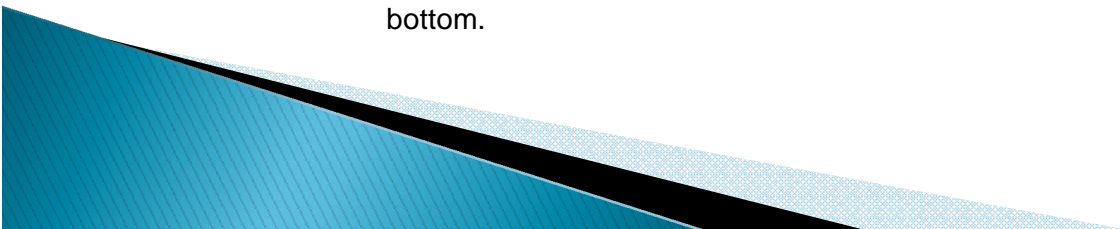
For a rectangle $y_{\max} = d/2 = 10/2 = 5$ mm (d is thickness)

σ_{\max} is given as 100×10^6 N/m²

$$M_{\max} (\text{N/m}^2) = \frac{\sigma_{\max} I}{y_{\max}} = \frac{100 \times 10^6 \text{ N/m}^2 \times 1 \times 10^{-8} \text{ m}^4}{0.005 \text{ m}}$$
$$= 200 \text{ N/m}^2 = w/2$$

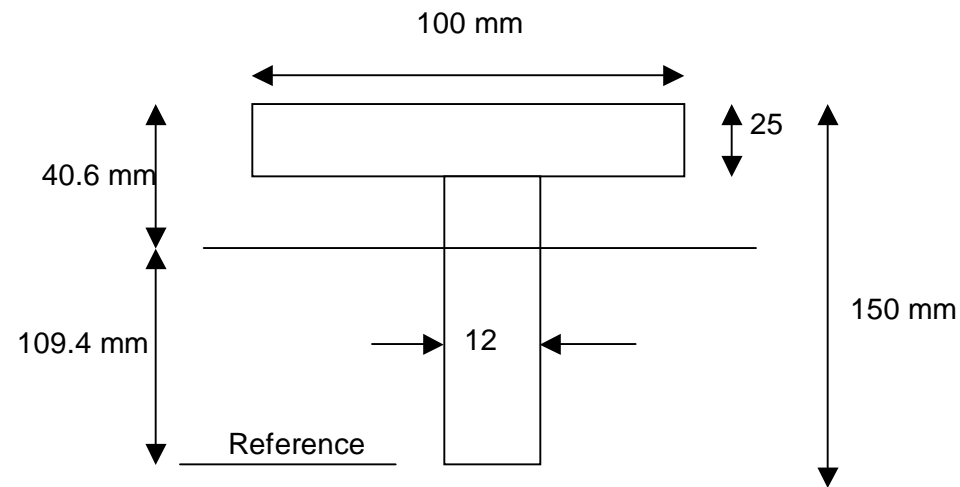
Therefore the permissible distributed load, **w = 400 N/m²**

Note: Since the maximum bending moment is negative, the maximum tensile stress occurs at the top of the beam while the maximum compressive stress occurs at the bottom.

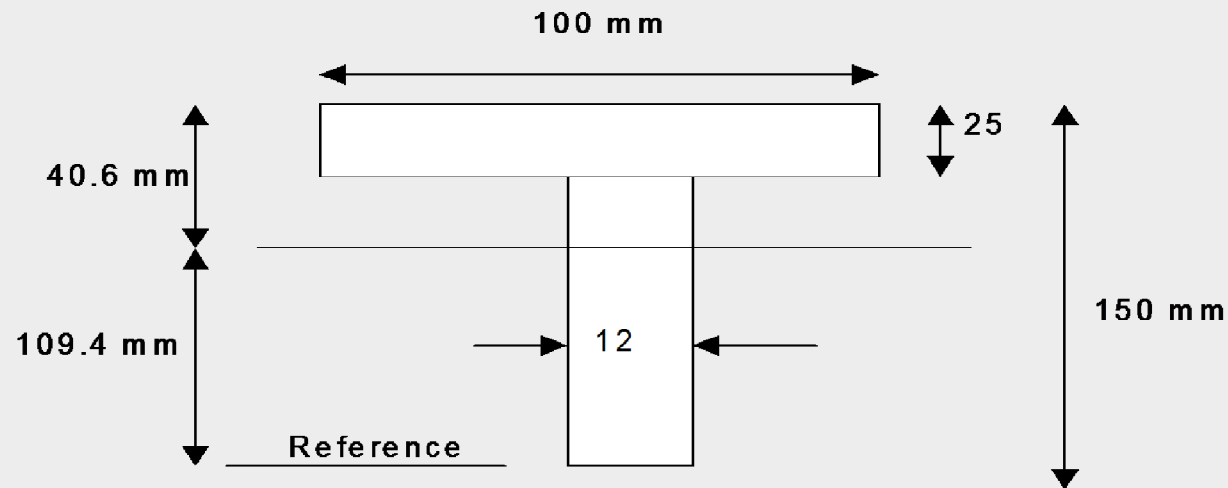


Example

Example: A uniform T-section beam is 100 mm wide and 150 mm deep with a flange thickness of 25 mm and a web thickness of 12 mm. If the limiting bending stresses for the material of the beam are 80 MN/m^2 in compression and 160 MN/m^2 , find the maximum uniformly distributed load (u.d.l) that the beam can carry over a simply supported span of 5 m.



Solution



Solution: The second moment of area, I used in the simple bending theory is about the neutral axis, thus in order to determine the I value of the T-section shown, it is necessary first to determine the position of the centroid and hence the neutral axis.

Find the Neutral Axis:

$$A y = A_1 y_1 + A_2 y_2$$

$$(100 \times 25) + (125 \times 12) y = (100 \times 25) \times 137.5 + (12 \times 125 \times 62.5)$$

$$(2500 + 1500) y = 343750 + 93750 = 437500$$

$$y = 109.4 \text{ mm} \quad (\text{see in figure})$$

Thus the N.A. is positioned, as shown, a distance of 109.4 mm above the base.

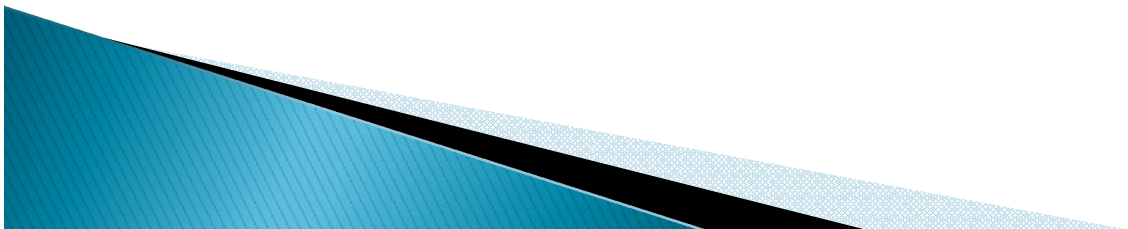
Solution Contd.

The second moment of area, I can now be found by dividing the section into convenient rectangles with their edges in the neutral axis.

$$\begin{aligned} I_{xx} &= \frac{12 \times 125^3}{12} + (12 \times 125) \times 46.9^2 + \frac{100 \times 25^3}{12} + (100 \times 25) \times 28.1^2 \\ &= 1953125 + 3299415 + 130208.3 + 1974025 \\ &= 7356773.3 \text{ mm}^4 = 7.36 \times 10^{-6} \text{ m}^4 \end{aligned}$$

Maximum compressive stress will occur at the upper surface, where $y = 40.6 \text{ mm}$ and using the limiting compressive stress value quoted:

$$M = \frac{\sigma I}{y} = \frac{80 \times 10^6 \times 7.36 \times 10^{-6}}{40.6 \times 10^{-3}} = 14.5 \text{ kNm}$$



Solution Concluded

This suggests a maximum allowable bending moment of 14.5 kN m. It is now necessary, however, to check the tensile stress criterion which must apply on the lower surface,

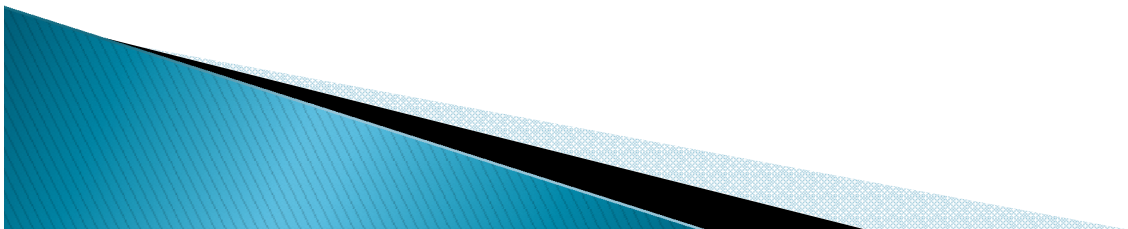
$$M = \frac{\sigma I}{y} = \frac{160 \times 10^6 \times 7.36 \times 10^{-6}}{109.4 \times 10^{-3}} = 10.76 \text{ kN m}$$

The greatest moment that can therefore be applied to retain stresses within both conditions quoted is therefore $M = 10.76 \text{ kN m}$

But for a simply supported beam with u.d.l, $M_{\max} = w L^2/8$

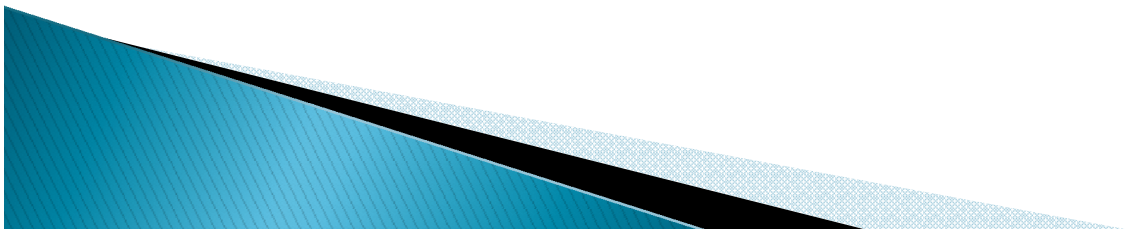
$$W = 8 M/L^2 = (8 \times 10.76 \times 10^3) / 5^2 = \mathbf{3.4 \text{ kN/m}}$$

The u.d.l must be limited to 3.4 kN/m.

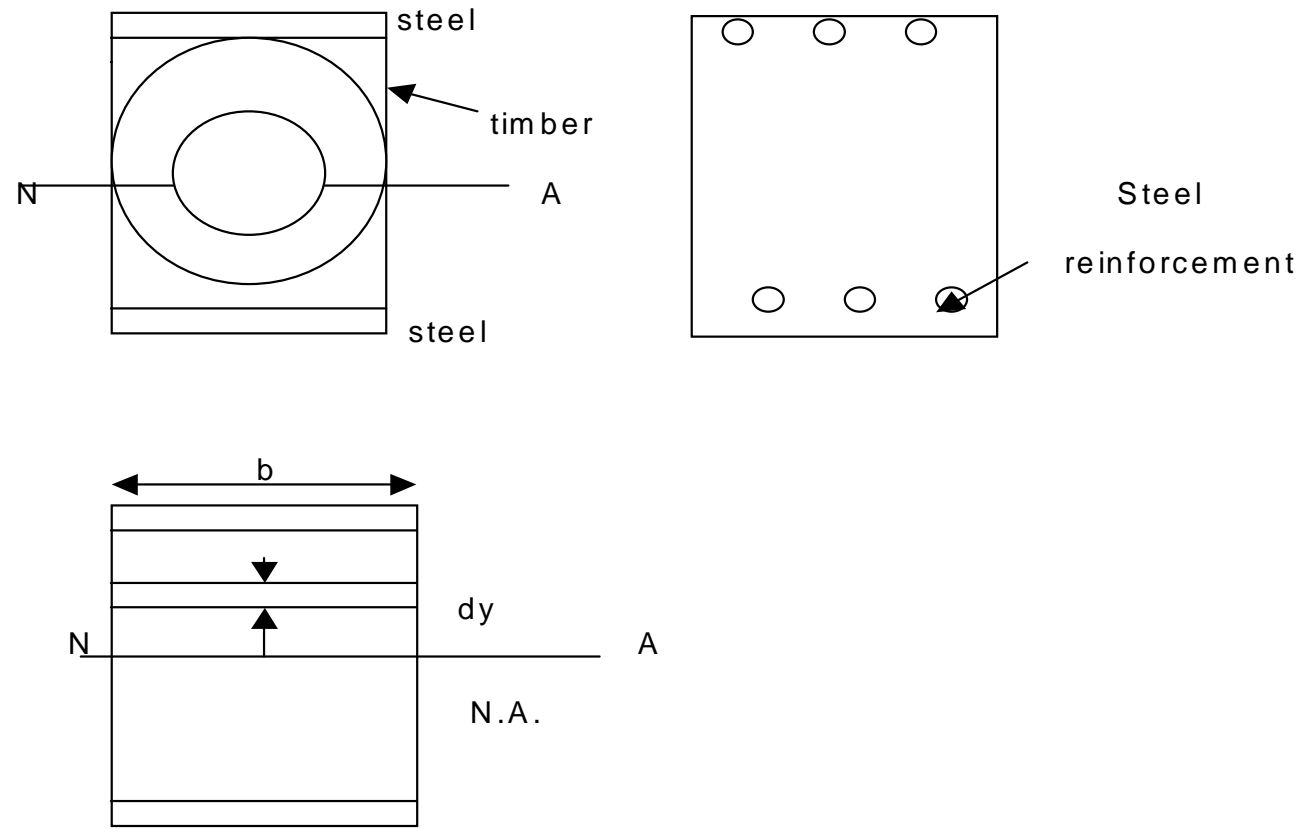


4.8. BENDING OF BEAMS OF TWO MATERIALS

- ▶ A composite beam is one which is constructed from a combination of materials. Since the bending theory only holds good when a constant value of Young's modulus applies across a section, it cannot be used directly to solve composite-beam problems when two different materials, and therefore different values of E , are present. The method of solution in such as case is to replace one of the materials by an equivalent section of the other.



Bending of Beams of Two Materials



At the interface, strain in timber = strain in steel

$$\frac{\sigma_t}{E_t} = \frac{\sigma_s}{E_s} \quad \text{Where } \sigma_t = \text{stress in timber, } \sigma_s = \text{stress in steel}$$

E_s = modulus of elasticity of steel, E_t = modulus of elasticity of timber

Beams of Two Materials Contd.

Equilibrium Conditions

$$\int_A \sigma \, dA = 0$$

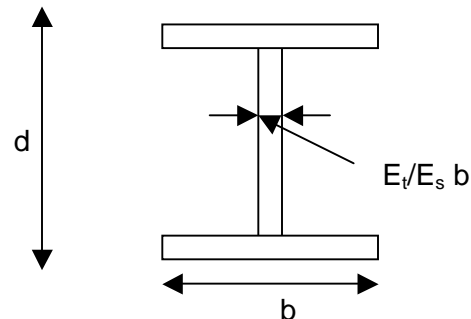
$$\int_A \sigma \, y \, dA = M \quad \sigma = \frac{E \, y}{R}$$

Force on an elemental area of timber, $dF_t = \sigma_t \, b \, dy = E_t \, y/R \, b \, dy$ ($E \times \text{strain} \times \text{area}$)

Force on an elemental area ($b \, dy$) of steel, $dF_s = \sigma_s \, b \, dy = b \, E_s \, y/R \, dy$

$$dF_t = E_t/E_s (E_s \, y/R \, b \, dy) = E_t/E_s \, b (E_s \, y/R \, dy)$$

This is equivalent to a steel section of width ($b \, E_t/E_s$)

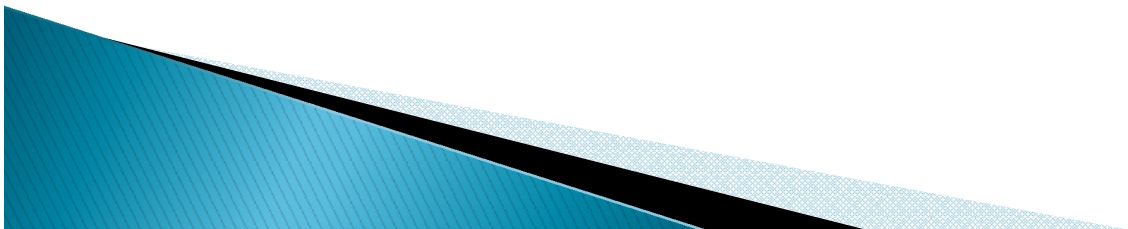


Equivalent steel section of the given composite beam

Beams of Two Materials Concluded

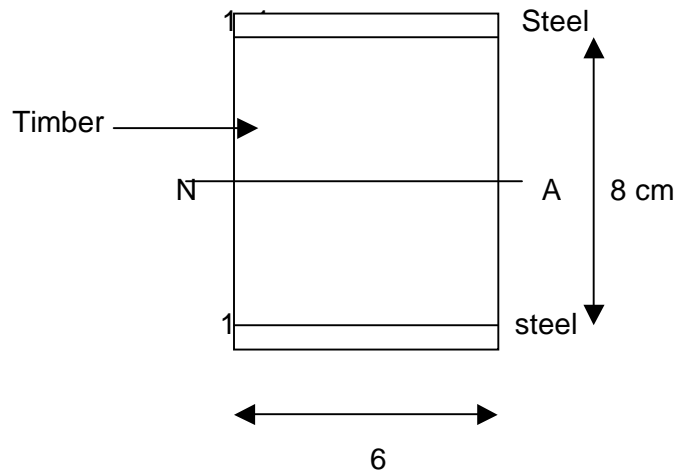
$$\frac{\sigma_t}{E_t} = \frac{\sigma_s}{E_s} \quad \text{and} \quad \sigma_t = \sigma_s \frac{E_t}{E_s}$$

E_s/E_t is called modular ratio.



Example

Example: Calculate the moment of resistance of the cross section and maximum stress in timber and steel. The allowable stress in steel is 150 N/mm^2

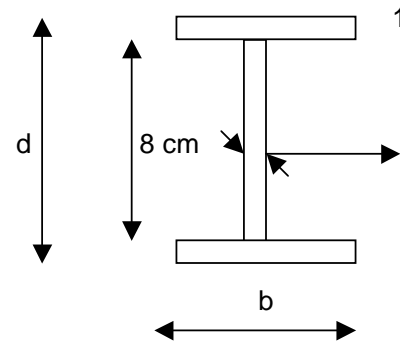


$$E_s = 210,000 \text{ N/mm}^2$$

$$E_t = 14,000 \text{ N/mm}^2$$

Solution

Equivalent Steel Section



$$b = (4,000 \times 6) / 210000 \\ = 0.4 \text{ cm}$$

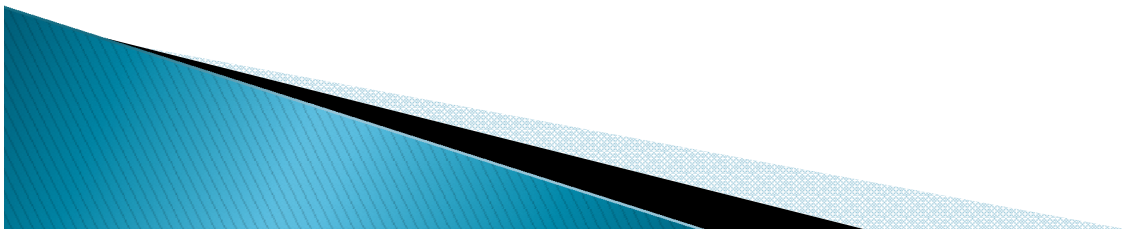
$$I_{NA} = 2 \left(\frac{6 \times 1^3}{12} + 6 \times 4.5^2 \right) + \frac{0.4 \times 8^3}{12} = 261.06 \text{ cm}^4$$

Allowable stress in steel = 150 N/mm²

$$M = \frac{\sigma I}{y}$$

$$M_R = \frac{150 \text{ N/mm}^2 \times 261.06 \times 10^4 \text{ mm}^3}{5 \times 10 \text{ mm}}$$

$$= 7.83 \times 10^6 \text{ N mm}$$



Solution Concluded

Stress in steel = 150 N/mm²

Stress in timber:

$$\frac{\sigma_t}{E_t} = \frac{\sigma_s}{E_s} \quad \text{and} \quad \sigma_t = \sigma_s \frac{E_t}{E_s}$$
$$= 150 \times 14,000/210,000 = 10 \text{ N/mm}^2$$

Maximum stress in timber in the given composite beam = $10 \times 4/5 = \mathbf{8 \text{ N/mm}^2}$

