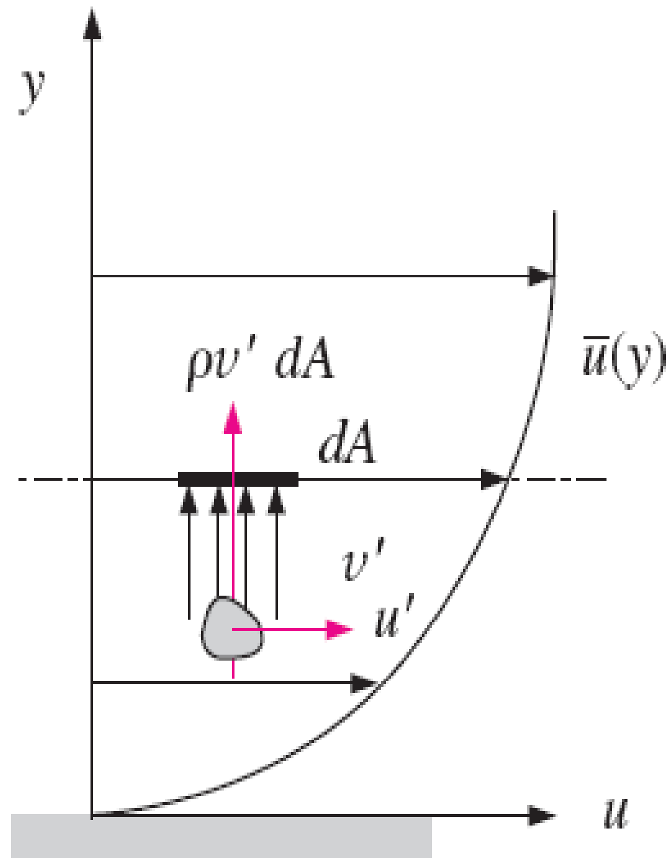
FLUID MECHANICS FOR MECHANICAL ENGINEERING (ME 208F)

Section D:
Turbulent Flow - II

Turbulent Shear Stress

- Consider turbulent flow in a horizontal pipe, and the upward eddy motion of fluid particles in a layer of lower velocity to an adjacent layer of higher velocity through a differential area *dA*
- Then the **turbulent shear stress** can be expressed as

$$\tau_{turb} = -\rho \overline{u'v'}$$



Note that $\overline{u'v'} \neq 0$ even though $\overline{u'} = 0$ and $\overline{v'} = 0$

Turbulent Shear Stress

- Experimental results show that $\overline{u'v'}$ usually a negative quantity.
- Terms such as or are called **Reynolds stresses** or **turbulent stress** $-\rho\overline{u'v'}$ $-\rho\overline{u'^2}$
- Many semi-empirical formulations have been developed that model the Reynolds stress in terms of average velocity gradients. Such models are called **turbulence models**.
- Momentum transport by eddies in turbulent flows is analogous to the molecular momentum diffusion.

Turbulent Shear Stress

- In many of the simpler turbulence models, turbulent shear stress is expressed as suggested by the French mathematician Joseph Boussinesq in 1877 as

$$\tau_{\text{turb}} = -\rho\overline{u'v'} = \mu_t \frac{\partial \overline{u}}{\partial y}$$

- where μ_t the **eddy viscosity** or **turbulent viscosity,** which accounts for momentum transport by turbulent eddies.
- The total shear stress can thus be expressed conveniently as

$$\tau_{\text{total}} = (\mu + \mu_t)\frac{\partial \overline{u}}{\partial y} = \rho(\nu + \nu_t)\frac{\partial \overline{u}}{\partial y}$$

- where $\nu_t = \mu$ [illegible] **ity** or **kinematic turbulent viscosity** (also called the *eddy diffusivity of momentum*).

Turbulent Shear Stress

- For practical purpose, eddy viscosity must be modeled as a function of the average flow variables; we call this *eddy viscosity closure*.
- For example, L. Prandtl introduced the concept of **mixing length** l_m, which is related to the average size of the eddies that are primarily responsible for mixing, and expressed the turbulent shear stress as

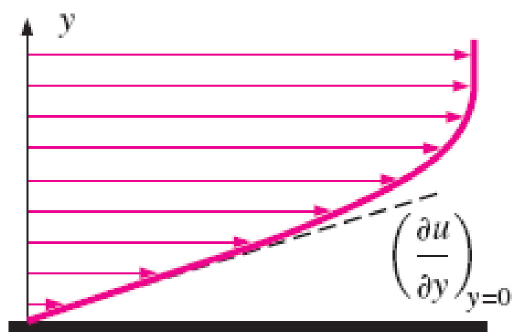
$$\tau_{\text{turb}} = \mu_t \frac{\partial \overline{u}}{\partial y} = \rho l_m^2 \left(\frac{\partial \overline{u}}{\partial y} \right)^2$$

- l_m is not a constant for a given flow and its determination is not easy.

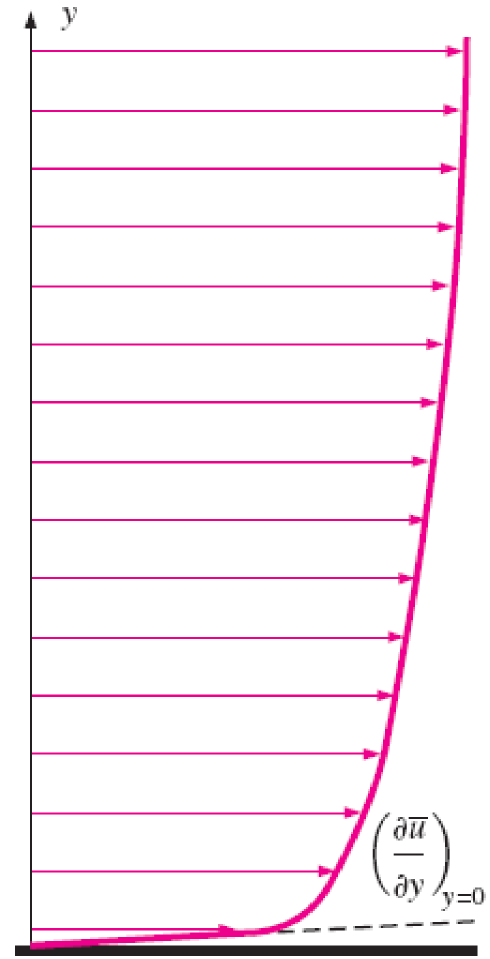
Turbulent Shear Stress

- Eddy motion and thus eddy diffusivities are much larger than their molecular counterparts in the core region of a turbulent boundary layer.
- The velocity profiles are shown in the figures. So it is no surprise that the wall shear stress is much larger in turbulent flow than it is in laminar flow.

- Molecular viscosity is a fluid property; however, eddy viscosity is a flow property.



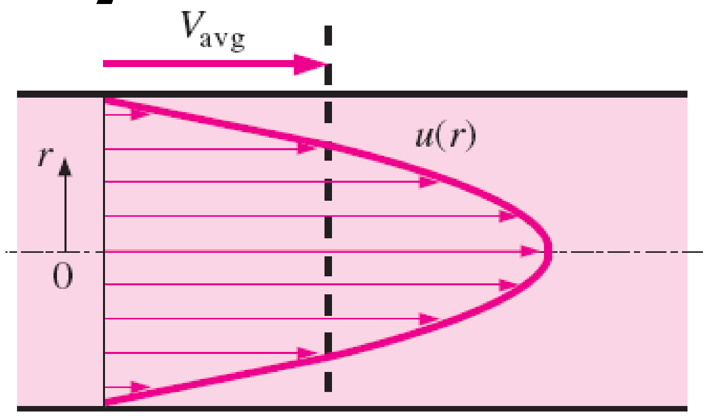
Laminar flow



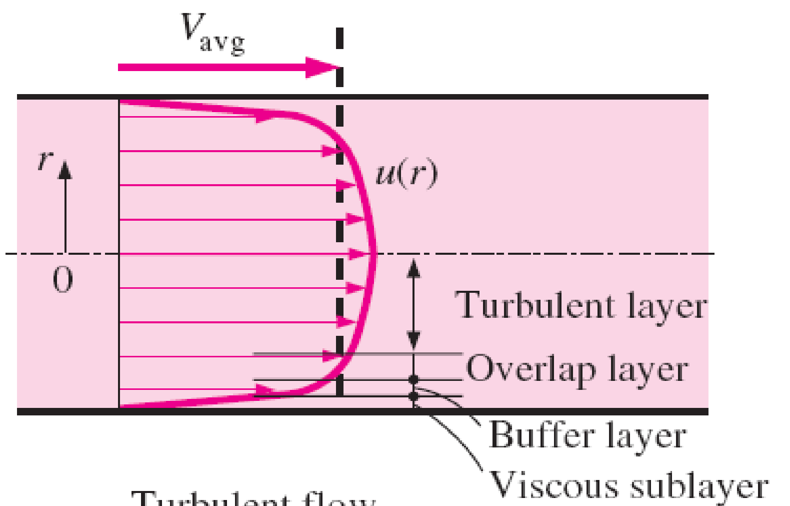
Turbulent flow

Turbulent Velocity Profile

- Typical velocity profiles for fully developed laminar and turbulent flows are given in Figures.
- Note that the velocity profile is parabolic in laminar flow but is much fuller in turbulent flow, with a sharp drop near the pipe wall.



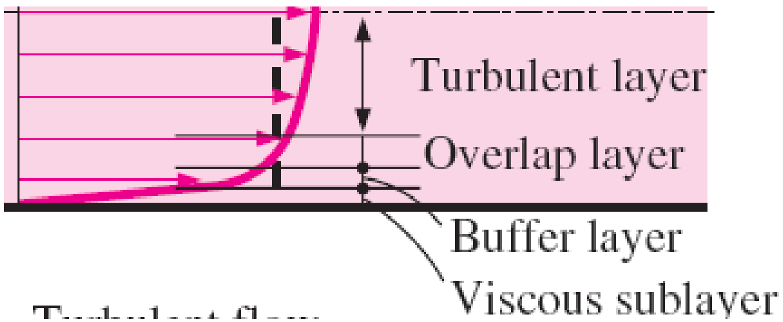
Laminar flow



Turbulent flow

Turbulent Velocity Profile

- Turbulent flow along a wall can be considered to consist of four regions, characterized by the distance from the wall.
  - **Viscous** (or **laminar** or **linear** or **wall**) sublayer: where viscous effects are dominant and the velocity profile in this layer is very nearly *linear*, and the flow is streamlined.
  - **Buffer layer:** viscous effects are still dominant: however, turbulent effects are becoming significant.
  - **Overlap** (or **transition**) **layer (or** the **inertial sublayer):** the turbulent effects are much more significant, but still not dominant.
  - **Outer** (or **turbulent**) **layer:** turbulent effects dominate over molecular diffusion (viscous) effects.



Turbulent Velocity Profile

- The Viscous sublayer (next to the wall):
  - The thickness of this sublayer is very small (typically, much less than 1 % of the pipe diameter), but this thin layer plays a dominant role on flow characteristics because of the large velocity gradients it involves.
  - The wall dampens any eddy motion, and thus the flow in this layer is essentially laminar and the shear stress consists of laminar shear stress which is proportional to the fluid viscosity.
  - The velocity profile in this layer to be very nearly linear, and experiments confirm that.

Turbulent Velocity Profile (Viscous sublayer)

- The velocity gradient in the viscous sublayer remains nearly constant at *du*/*dy* = *u*/*y*, and the wall shear stress can be expressed as

$$\tau_w = \mu \frac{u}{y} = \rho\nu \frac{u}{y} \qquad \text{or} \qquad \frac{\tau_w}{\rho} = \frac{\nu u}{y}$$

- where *y* is the distance from the wall. The square root of τ_w / ρ has the dimensions of velocity, and thus it is viewed as a fictitious velocity called the **friction velocity** expressed as $u_* = \sqrt{\tau_w/\rho}$.
- The velocity profile in the viscous sublayer can be expressed in dimensionless form as

$$\frac{u}{u_*} = \frac{yu_*}{\nu}$$

Turbulent Velocity Profile (Viscous sublayer)

- This equation is known as the **law of the wall,** and it is found to satisfactorily correlate with experimental data for smooth surfaces for $0 \leq yu_*/\nu \leq 5$.
- Therefore, the thickness of the viscous sublayer is roughly

$$y = \delta_{\text{sublayer}} = \frac{5\nu}{u_*} = \frac{25\nu}{u_\delta}$$

- where u_δ is the flow velocity at the edge of the viscous sublayer, which is closely related to the average velocity in a pipe. Thus we conclude the viscous sublayer is suppressed and it gets thinner as the velocity (and thus the Reynolds number) increases. Consequently, the velocity profile becomes nearly flat and thus the velocity distribution becomes more uniform at very high Reynolds numbers.

Turbulent Velocity Profile (Viscous sublayer)

- The quantity ν/u_* is called the **viscous length;** it is used to nondimensionalize the distance *y*; then we can get nondimensionalized velocity defined as

$$\textit{Nondimensionalized variables:} \qquad y^+ = \frac{yu_*}{\nu} \qquad \text{and} \qquad u^+ = \frac{u}{u_*}$$

- Then the normalized law of wall becomes simply

$$u^+ = y^+$$

- Note that y^+ resembles the Reynolds number expression.

Turbulent Velocity Profile

- The characteristics of the flow in viscous sublayer are very important since they set the stage for flow in the rest of the pipe. Any irregularity or roughness on the surface disturbs this layer and affects the flow. Therefore, unlike laminar flow, the friction factor in turbulent flow is a strong function of surface roughness.
- The roughness is a relative concept, and it has significance when its height ε is comparable to the thickness of the laminar sublayer (which is a function of the Reynolds number). All materials appear "rough" under a microscope with sufficient magnification. In fluid mechanics, a surface is characterized as being rough when $\varepsilon > \delta_{sublayer}$ and is said to be smooth when $\varepsilon < \delta_{sublayer}$. Glass and plastic surfaces are generally considered to be hydrodynamically smooth.

The Moody Chart

- The friction factor in fully developed turbulent pipe flow depends on the Reynolds number and the **relative roughness** ε/D, which is the ratio of the mean height of roughness of the pipe to the pipe diameter.
- It is no way to find a mathematical closed form for friction factor by theoretical analysis; therefore, all the available results are obtained from painstaking experiments.
- Most such experiments were conducted by Prandtl's student J. Nikuradse in 1933, followed by the works of others. The friction factor was calculated from the measurements of the flow rate and the pressure drop.
- Functional forms were obtained by curve-fitting experimental data.

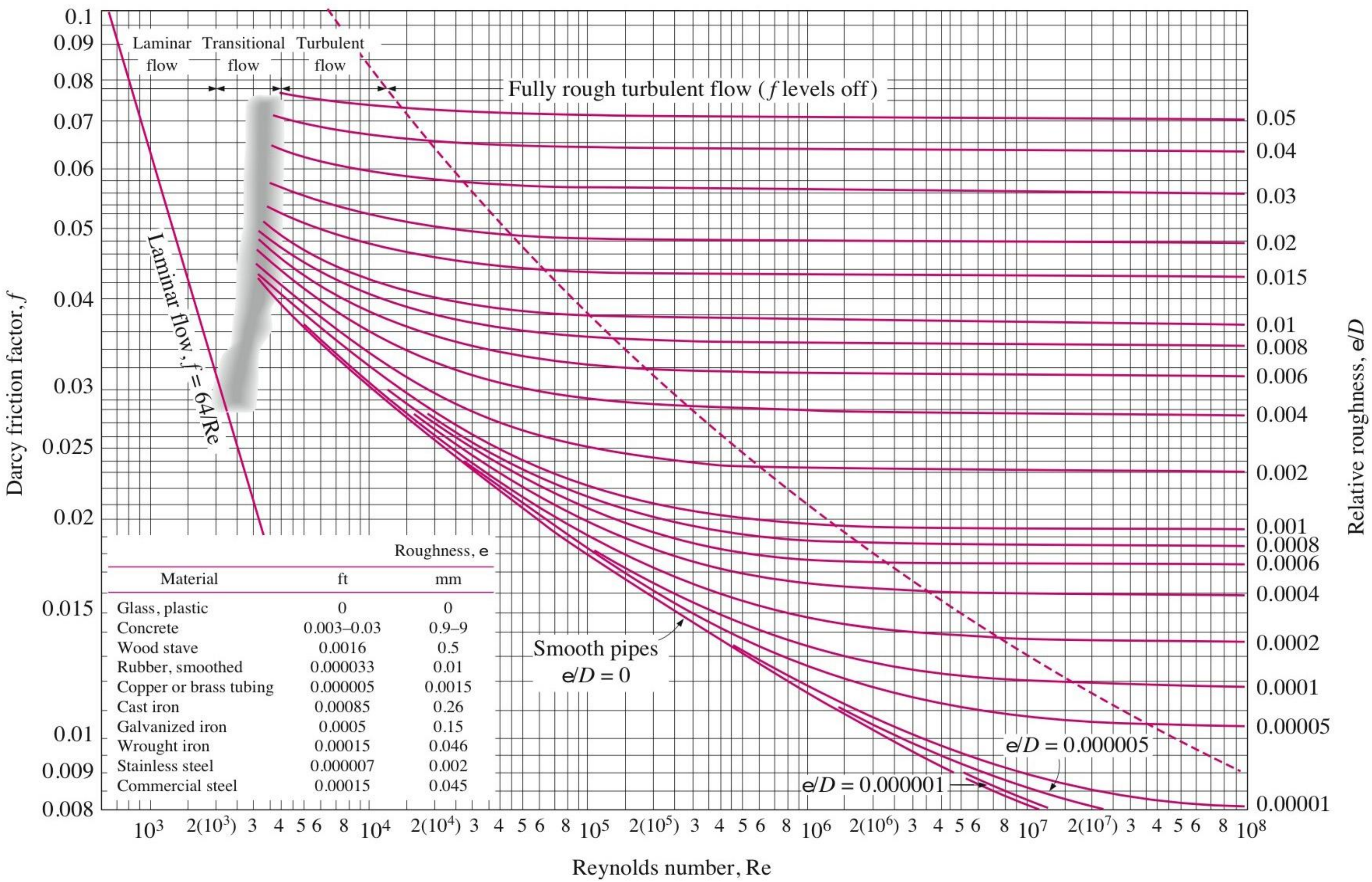
The Moody Chart

- In 1939, Cyril F. Colebrook combined the available data for transition and turbulent flow in smooth as well as rough pipes into the **Colebrook equation:**

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \qquad \text{(turbulent flow)}$$

- In 1942, the American engineer Hunter Rouse verified Colebrook's equation and produced a graphical plot of *f.*
- In 1944, Lewis F. Moody redrew Rouse's diagram into the form commonly used today, called Moody chart given in the appendix as Fig. A–12.

The Moody Chart



	Roughness, e	
Material	ft	mm
Glass, plastic	0	0
Concrete	0.003–0.03	0.9–9
Wood stave	0.0016	0.5
Rubber, smoothed	0.000033	0.01
Copper or brass tubing	0.000005	0.0015
Cast iron	0.00085	0.26
Galvanized iron	0.0005	0.15
Wrought iron	0.00015	0.046
Stainless steel	0.000007	0.002
Commercial steel	0.00015	0.045