FLUID MECHANICS FOR MECHANICAL ENGINEERING (ME 208F)

Section C:
Flow Through Pipes- III

Darcy-Weisbach Equation

τ_0	ρ	V	μ	D	e
$ML^{-1}T^{-2}$	ML^{-3}	LT^{-1}	$ML^{-1}T^{-1}$	L	L

$$\tau_0 = F(\rho, V, \mu, D, e)$$

$$\pi_4 = F(\pi_1, \pi_2)$$

Repeating variables : ρ, V, D

$$\pi_1 = \text{Re}; \quad \pi_2 = \frac{e}{D}; \quad \pi_3 = \frac{\tau_0}{\rho V^2}$$

$$\frac{\tau_0}{\rho V^2} = F(\text{Re}, \frac{e}{D})$$

$$\tau_0 = \rho V^2 F(\text{Re}, \frac{e}{D})$$

$$h_f = \frac{4L}{\gamma D}\tau_0$$

$$= \frac{4L}{\gamma D}\rho V^2 F(\text{Re}, \frac{e}{D})$$

$$= \frac{L}{D}\frac{V^2}{2g}\left[8F(\text{Re}, \frac{e}{D})\right]$$

$$h_f = f\frac{L}{D}\frac{V^2}{2g}$$

Darcy-Weisbach Eq.

$$f = 8F(\text{Re}, \frac{e}{D})$$

Friction factor

Laminar Flow in Pipes

- Laminar flow -- Newton's law of viscosity is valid:

$$\tau = \mu \frac{dV}{dy} = -\frac{r\gamma}{2}\frac{dh}{ds}$$

$$\frac{dV}{dy} = -\frac{dV}{dr}$$

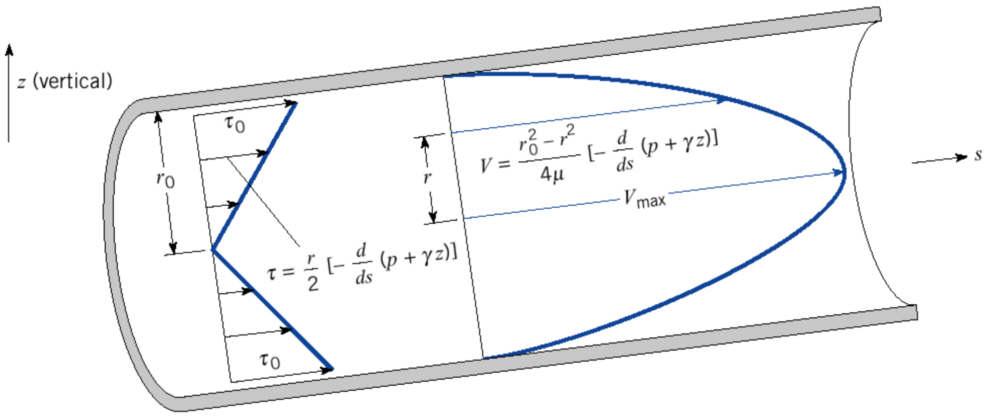
$$\frac{dV}{dr} = \frac{r\gamma}{2\mu}\frac{dh}{ds}$$

$$dV = \frac{r\gamma}{2\mu}\frac{dh}{ds}dr$$

$$V = \frac{r^2\gamma}{4\mu}\frac{dh}{ds} + C \qquad C = -\frac{r_0^2\gamma}{4\mu}\frac{dh}{ds}$$

$$V = -\frac{r_0^2\gamma}{4\mu}\frac{dh}{ds}\left[1 - \left(\frac{r}{r_0}\right)^2\right]$$

$$V = V_{max}\left[1 - \left(\frac{r}{r_0}\right)^2\right]$$



- Velocity distribution in a pipe (laminar flow) is parabolic with maximum at center.

Discharge in Laminar Flow

$$V = -\frac{\gamma}{4\mu}\frac{dh}{ds}(r_0^2 - r^2)$$

$$Q = \int V dA = \int_0^{r_0} -\frac{\gamma}{4\mu}\frac{dh}{ds}(r_0^2 - r^2)(2\pi r dr)$$

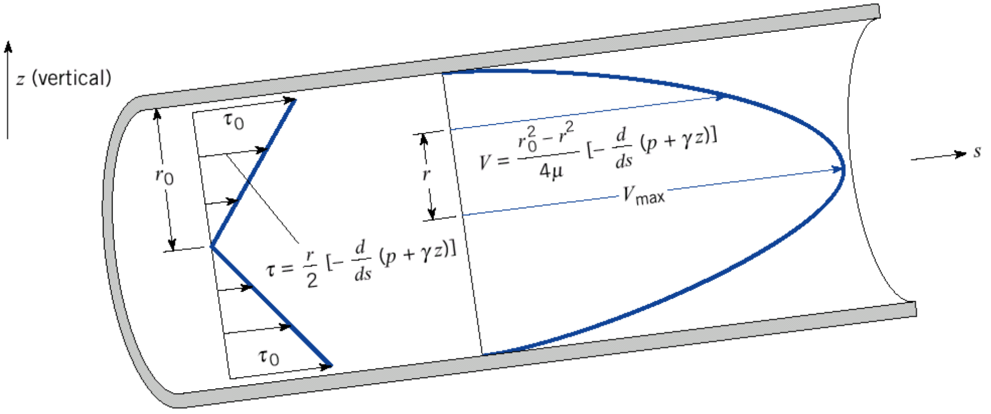
$$= \frac{\pi\gamma}{4\mu}\frac{dh}{ds}\frac{(r^2 - r_0^2)^2}{2}\Bigg|_0^{r_0}$$

$$Q = -\frac{\pi\gamma r_0^4}{8\mu}\frac{dh}{ds}$$

$$= -\frac{\pi\gamma D^4}{128\mu}\frac{dh}{ds}$$

$$\bar{V} = \frac{Q}{A}$$

$$\bar{V} = -\frac{\gamma D^2}{32\mu}\frac{dh}{ds}$$



Head Loss in Laminar Flow

$$\bar{V} = -\frac{\gamma D^2}{32\mu}\frac{dh}{ds}$$

$$\frac{dh}{ds} = -\bar{V}\frac{32\mu}{\gamma D^2}$$

$$dh = -\bar{V}\frac{32\mu}{\gamma D^2}ds$$

$$h_2 - h_1 = -\bar{V}\frac{32\mu}{\gamma D^2}(s_2 - s_1)$$

$$h_1 = h_2 + h_f$$

$$h_f = \frac{32\mu L\bar{V}}{\gamma D^2}$$

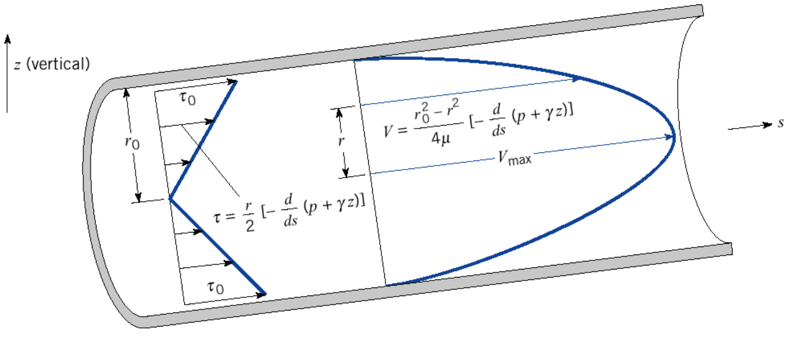
$$h_f = \frac{32\mu L\bar{V}}{\gamma D^2}$$

$$= \frac{32\mu L\bar{V}}{\gamma D^2}\frac{\rho\bar{V}^2/2}{\rho\bar{V}^2/2}$$

$$= 64\left(\frac{\mu}{\rho\bar{V}D}\right)\left(\frac{L}{D}\right)\rho\bar{V}^2/2$$

$$= \frac{64}{\mathrm{Re}}\left(\frac{L}{D}\right)\rho\bar{V}^2/2$$

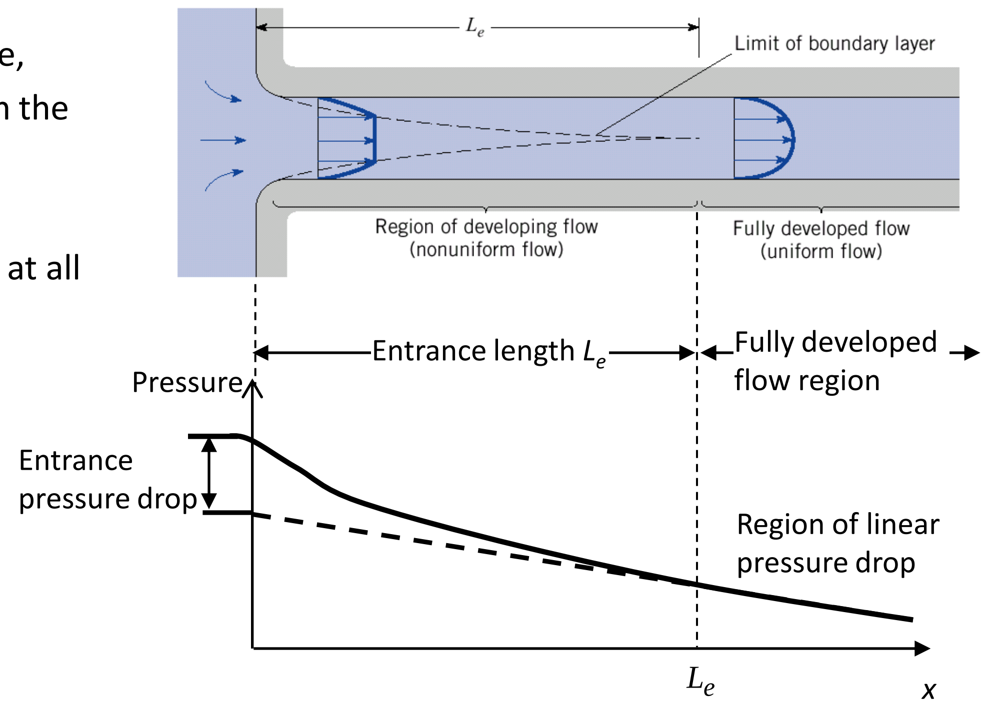
$$h_f = f\frac{L}{D}\frac{\rho\bar{V}^2}{2} \qquad f = \frac{64}{\mathrm{Re}}$$



Pipe Entrance

- Developing flow
  - Includes boundary layer and core,
  - viscous effects grow inward from the wall
- Fully developed flow
  - Shape of velocity profile is same at all points along pipe

$$\frac{L_e}{D} \approx \begin{cases} 0.06\,\mathrm{Re} & \text{Laminar flow} \\ 4.4\mathrm{Re}^{1/6} & \text{Turbulent flow} \end{cases}$$

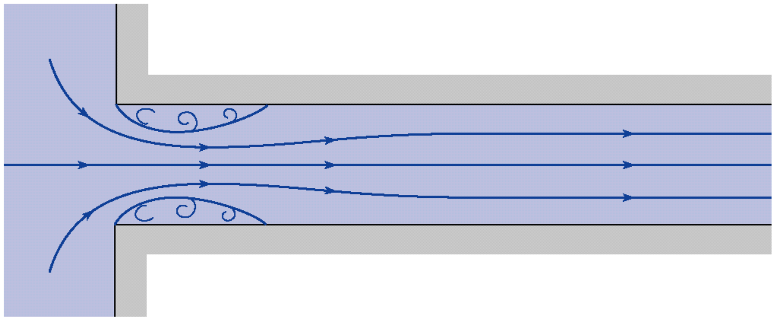


Entrance Loss in a Pipe

- In addition to frictional losses, there are minor losses due to
  - Entrances or exits
  - Expansions or contractions
  - Bends, elbows, tees, and other fittings
  - Valves
- Losses generally determined by experiment and then corellated with pipe flow characteristics
- Loss coefficients are generally given as the ratio of head loss to velocity head

$$K = \frac{h_L}{\frac{V^2}{2g}} \quad \text{or} \quad h_L = K\frac{V^2}{2g}$$

- *K* – loss coefficent
  - *K* ~ 0.1 for well-rounded inlet (high *Re*)
  - *K* ~ 1.0 abrupt pipe outlet
  - *K* ~ 0.5 abrupt pipe inlet



Abrupt inlet, *K* ~ 0.5

Elbow Loss in a Pipe

- A piping system may have many minor losses which are all correlated to $V^2/2g$
- Sum them up to a total system loss for pipes of the same diameter

$$h_L = h_f + \sum_m h_m = \frac{V^2}{2g}\left[f\frac{L}{D} + \sum_m K_m \right]$$

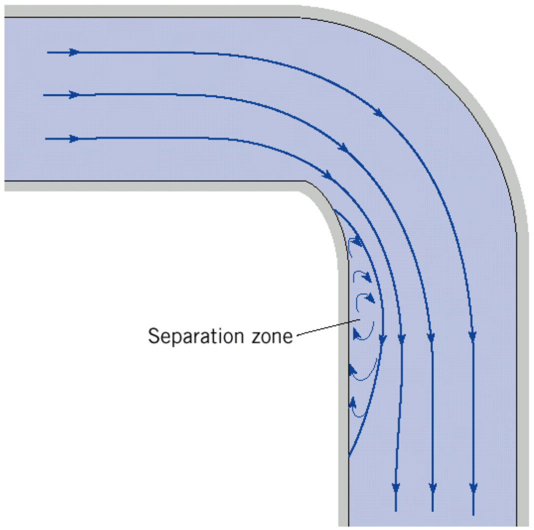
- Where,

h_L = Total head loss

h_f = Frictional head loss

h_m = Minor head loss for fitting m

K_m = Minor head loss coefficient for fitting m



EGL & HGL for Losses in a Pipe

- Entrances, bends, and other flow transitions cause the EGL to drop an amount equal to the head loss produced by the transition.
- EGL is steeper at entrance than it is downstream of there where the slope is equal the frictional head loss in the pipe.
- The HGL also drops sharply downstream of an entrance