FLUID MECHANICS FOR MECHANICAL ENGINEERING (ME 208F)

Section B:
Compressible Fluid Flows - II

Stagnation and Sonic Properties

- The stagnation properties at a point are defined as those which are to be obtained if the local flow were imagined to cease to zero velocity isentropically. As we will see in the later part of the text, stagnation values are useful reference conditions in a compressible flow.
- Let us denote stagnation properties by subscript zero. Suppose the properties of a flow (such as T, p , ρ etc.) are known at a point, the stangation enthalpy is, thus, defined as

$$h_0 = h + \frac{1}{2}V^2$$

where h is flow enthalpy and V is flow velocity.

- For a perfect gas , this yields,

$$c_p T_0 = c_p T + \frac{1}{2} V^2$$

which defines the **Stagnation Temperature**

Now, $\frac{T_0}{T}$ can be expressed as

$$\frac{T_0}{T} = 1 + \frac{V^2}{2 c_p T} = 1 + \frac{\gamma - 1}{2} \cdot \frac{V^2}{\gamma RT}$$

$$c_p - \frac{c_p}{\gamma} = R$$

$$c_p = \frac{\gamma R}{\gamma - 1}$$

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} Ma^2 \qquad \left(Ma = \frac{V}{a} = \frac{V}{\sqrt{\gamma RT}} \right)$$

If we know the local temperature (**T**) and Mach number (**Ma**) , we can find out the stagnation temperature $\mathbf{T_0}$.

- Consequently, **isentropic(adiabatic) relations** can be used to obtain **stagnation pressure** and **stagnation density** as

$$\frac{p_0}{p} = \left(\frac{T_0}{T}\right)^{\frac{\gamma}{\gamma-1}} = \left[1 + \frac{\gamma-1}{2} Ma^2\right]^{\frac{\gamma}{\gamma-1}}$$

$$\frac{\rho_0}{\rho} = \left(\frac{T_0}{T}\right)^{\frac{1}{\gamma-1}} = \left[1 + \frac{\gamma-1}{2} Ma^2\right]^{\frac{1}{\gamma-1}}$$

- Values of T_0/T , p_0/p and ρ_0/ρ as a function of Mach number can be generated using the above relationships and the tabulated results are known as **Isentropic Table**