FLUID MECHANICS FOR MECHANICAL ENGINEERING (ME 208F)

Section B:
Compressible Fluid Flows - II

Isentropic Flow of an Ideal Gas – Area Variation

- Basic Equations for Isentropic Flow

$$\rho_1 V_1 A_1 = \rho_2 V_2 A_2 = \rho V A = \dot{m} = \text{constant}$$

$$R_x + p_1 A_1 - p_2 A_2 = \dot{m} V_2 - \dot{m} V_1$$

$$h_{0_1} = h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2} = h_{0_2} = h_0$$

$$s_2 = s_1 = s$$

$$p = \rho R T$$

$$\Delta h = h_2 - h_1 = c_p \Delta T = c_p (T_2 - T_1)$$

$$\frac{p_1}{\rho_1^k} = \frac{p_2}{\rho_2^k} = \frac{p}{\rho^k} = \text{constant}$$

Isentropic Flow of an Ideal Gas – Area Variation

- Isentropic Flow

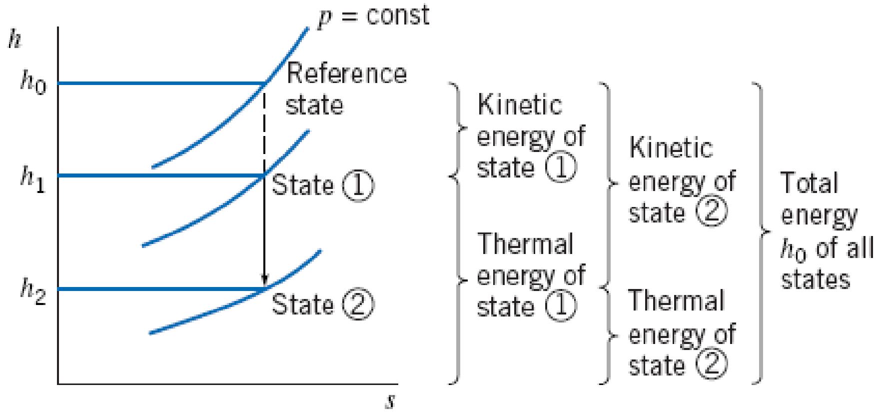

Fig. 13.2 Isentropic flow in the *hs* plane.

Isentropic Flow of an Ideal Gas – Area Variation

- Subsonic, Supersonic, and Sonic Flows

$$\frac{dV}{V} = -\frac{dA}{A}\frac{1}{[1 - M^2]}$$

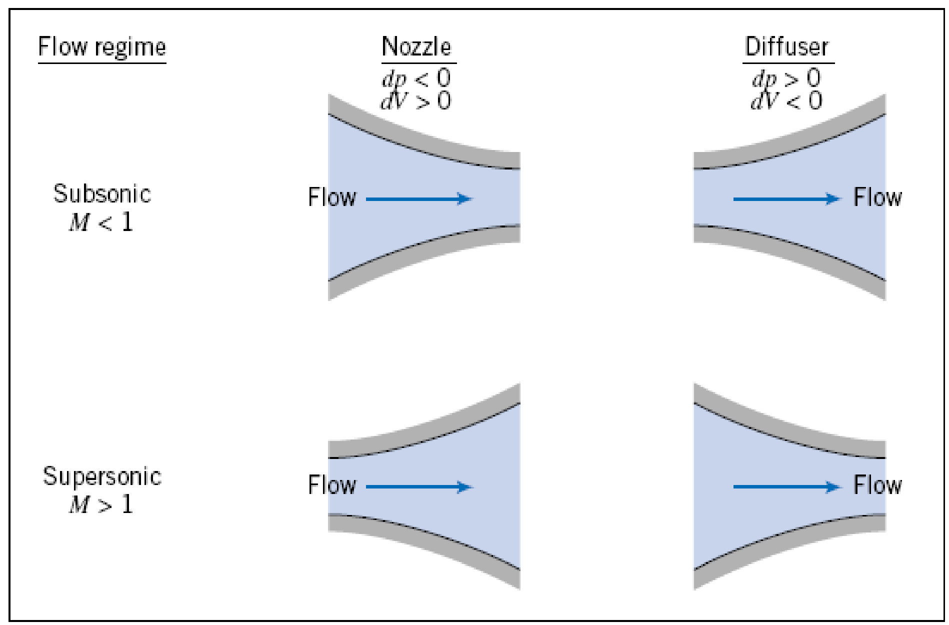

Fig. 13.3 Nozzle and diffuser shapes as a function of initial Mach number.

Isentropic Flow of an Ideal Gas – Area Variation

- Reference Stagnation and Critical Conditions for Isentropic Flow

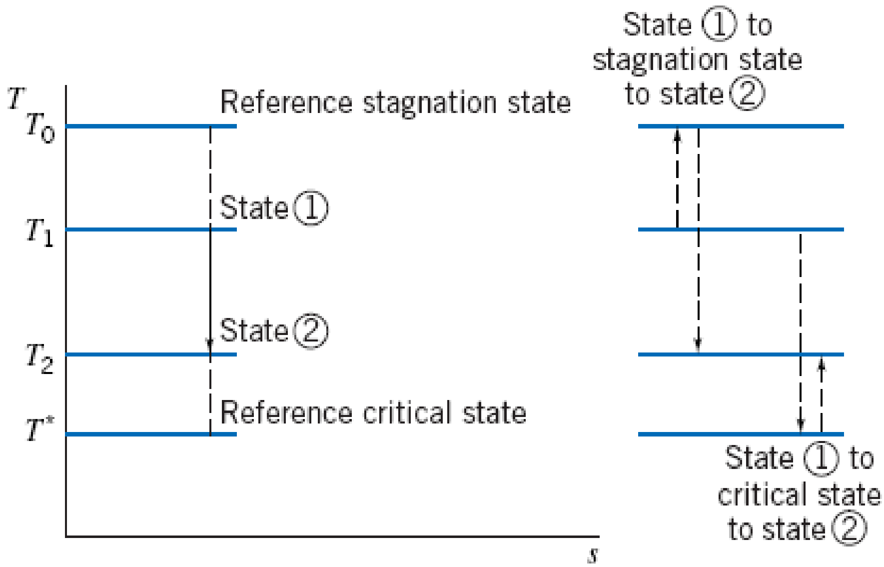

Fig. 13.4 Example of stagnation and critical reference states in the *Ts* plane.

Isentropic Flow of an Ideal Gas – Area Variation

- Property Relations

$$\frac{p_0}{p} = \left[1 + \frac{k-1}{2} M^2\right]^{k/(k-1)}$$

$$\frac{T_0}{T} = 1 + \frac{k-1}{2} M^2$$

$$\frac{\rho_0}{\rho} = \left[1 + \frac{k-1}{2} M^2\right]^{1/(k-1)}$$

$$\frac{A}{A^*} = \frac{1}{M}\left[\frac{1 + \frac{k-1}{2} M^2}{\frac{k+1}{2}}\right]^{(k+1)/2(k-1)}$$

Isentropic Flow of an Ideal Gas – Area Variation

- Isentropic Flow in a Converging Nozzle

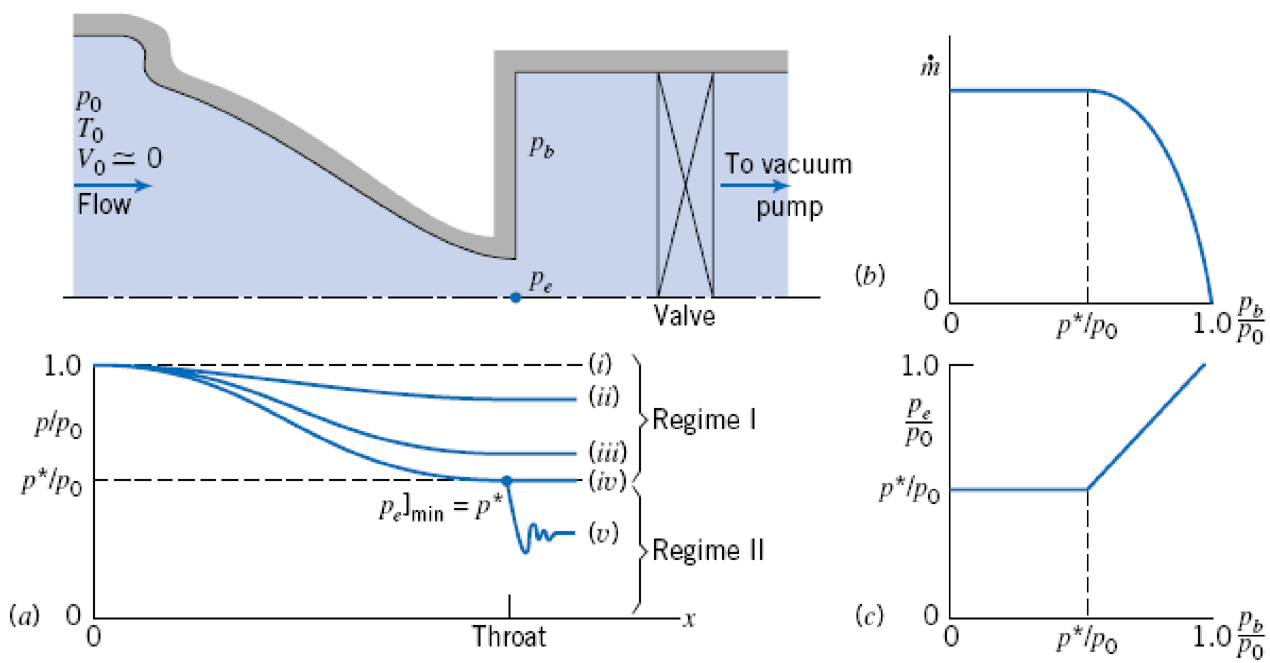

Fig. 13.6 Converging nozzle operating at various back pressures.

Isentropic Flow of an Ideal Gas – Area Variation

- Isentropic Flow in a Converging Nozzle

$$\left.\frac{p_e}{p_0}\right|_{\text{choked}} = \frac{p^*}{p_0} = \left(\frac{2}{k+1}\right)^{k/(k-1)}$$

$$\dot{m}_{\text{choked}} = A_e p_0 \sqrt{\frac{k}{RT_0}}\left(\frac{2}{k+1}\right)^{(k+1)/2(k-1)}$$

Isentropic Flow of an Ideal Gas – Area Variation

- Isentropic Flow in a Converging-Diverging Nozzle

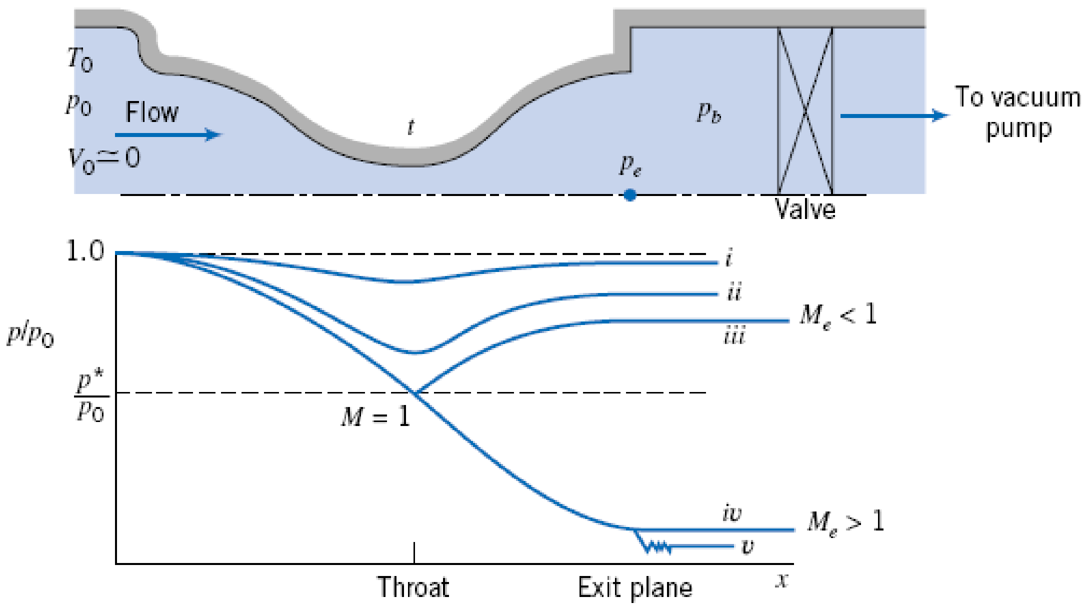

Fig. 13.8 Pressure distributions for isentropic flow in a converging-diverging nozzle.

Isentropic Flow of an Ideal Gas – Area Variation

- Isentropic Flow in a Converging-Diverging Nozzle

$$\dot{m}_{\text{choked}} = A_t p_0 \sqrt{\frac{k}{RT_0}} \left(\frac{2}{k+1} \right)^{(k+1)/2(k-1)}$$

Flow in a Constant-Area Duct with Friction

- Control Volume

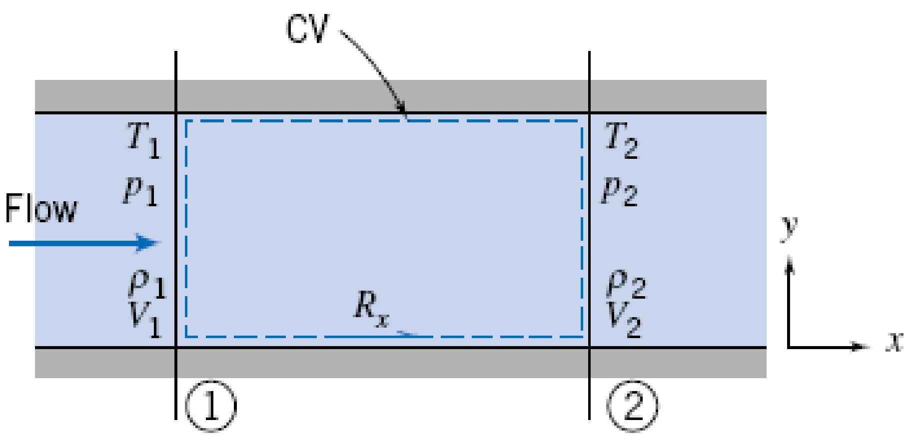

Fig. 13.9 Control volume used for integral analysis of frictional adiabatic flow.

Flow in a Constant-Area Duct with Friction

- Basic Equations for Adiabatic Flow

$$\rho_1 V_1 = \rho_2 V_2 = \rho V \equiv G = \frac{\dot{m}}{A} = \text{constant}$$

$$R_x + p_1 A - p_2 A = \dot{m} V_2 - \dot{m} V_1$$

$$h_{0_1} = h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2} = h_{0_2} = h_0$$

$$s_2 > s_1$$

$$p = \rho R T$$

$$\Delta h = h_2 - h_1 = c_p \Delta T = c_p (T_2 - T_1)$$

$$\Delta s = s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1}$$