FLUID MECHANICS FOR MECHANICAL ENGINEERING (ME 208F)

Section B:
Fluid Dynamics - II

- **Bernoulli and energy equations**
- Let us first derive the *Bernoulli* equation, which is one of the most well-known equations of motion in fluid mechanics, and yet is often misused. It is thus important to understand its limitations, and the assumptions made in the derivation.

- The assumptions can be summarized as follows:
  - Inviscid flow (ideal fluid, frictionless)
  - Steady flow (unsteady Bernoulli equation will not be discussed in this course)
  - Along a streamline
  - Constant density (incompressible flow)
  - No shaft work or heat transfer

- The Bernoulli equation is based on the application of Newton's law of motion to a fluid element on a streamline.

- Let us consider the motion of a fluid element of length *ds* and cross-sectional area *dA* moving at a local speed *V*, and *x* is a horizontal axis and *z* is pointing vertically upward. The forces acting on the element are the pressure forces *pdA* and *(p+dp)dA*, and the weight *w* as shown. Summing forces in the direction of motion, the *s*-direction, there results

$$pdA-(p+dp)dA-\rho g\ ds\ dA\cos\theta=\rho\ ds\ dA\ a_s$$

- where a_s is the acceleration of the element in the *s*-direction. Since the flow is steady

$$a_s=V\frac{dV}{ds}$$

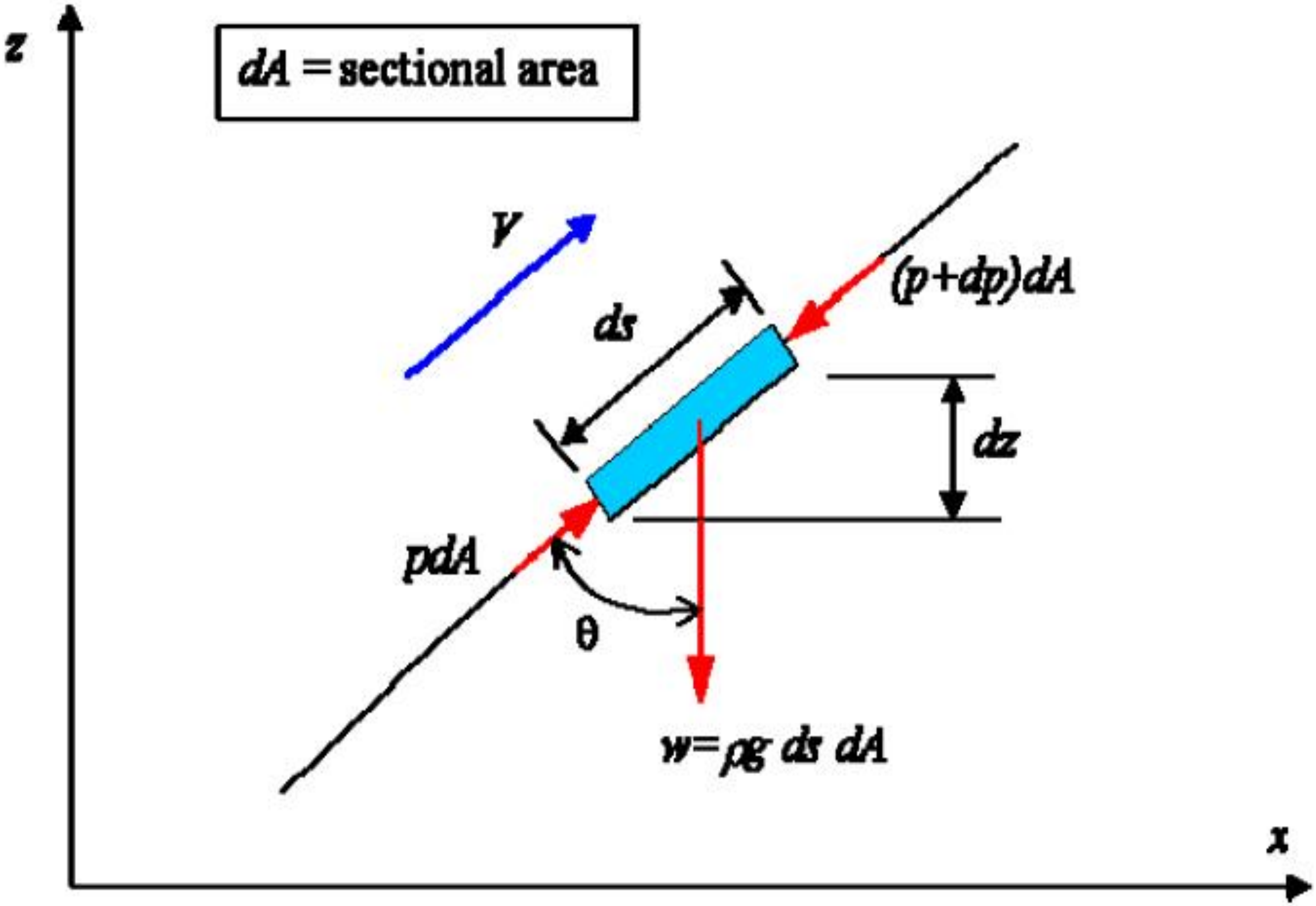
$$\frac{dp}{\rho g}+dz+\frac{V}{g}dV=0$$

- where a_s is the acceleration of the element in the *s*-direction. Since the flow is steady, only convective acceleration exists

Also, it is easy to see that $\cos\theta = dz/ds$. On substituting and dividing the equation by $\rho g dA$, we can obtain Euler's equation:

Note that Euler's equation is valid also for compressible flow.

$$\frac{p}{\rho g} + z + \frac{V^2}{2g} = \text{constant}$$


z
dA = sectional area
V
(p+dp)dA
ds
dz
pdA
θ
w=ρg ds dA
x