

FLUID MECHANICS FOR MECHANICAL ENGINEERING (ME 208F)

Section B:
Fluid Dynamics - I

FLUID DYNAMICS

- **Control volume**
- A control volume is a finite region, chosen carefully by the analyst for a particular problem, with open boundaries through which mass, momentum, and energy are allowed to cross. The analyst makes a budget, or balance, between the incoming and outgoing fluid and the resultant changes within the control volume. Therefore one can calculate the gross properties (net force, total power output, total heat transfer, etc.) with this method.
- With this method, however, we do not care about the details inside the control volume (In other words we can treat the control volume as a "black box.")
- For the sake of the present analysis, let us consider a control volume that can be a tank, reservoir or a compartment inside a system, and consists of some definite *one-dimensional* inlets and outlets, like the one shown below:



- By steadiness, the total mass of fluid contained in the control volume must be invariant with time. Therefore there must be an exact balance between the total rate of flow into the control volume and that out of the control volume:

- Total Mass Outflow = Total Mass Inflow

$$\sum_{i=1}^M (\rho_i V_i A_i)_{\text{in}} = \sum_{i=1}^N (\rho_i V_i A_i)_{\text{out}}$$

- which translates into the following mathematical relation
- where M is the number of inlets, and N is the number of outlets. If the density of fluid is constant, conservation of mass also implies conservation of volume. Hence for a control volume with only one-dimensional inlets and outlets,

$$\sum_{i=1}^M (V_i A_i)_{\text{in}} = \sum_{i=1}^N (V_i A_i)_{\text{out}} \quad \text{or} \quad \sum_{i=1}^M (Q_i)_{\text{in}} = \sum_{i=1}^N (Q_i)_{\text{out}}$$

- For example, in a pipe of varying cross sectional area, the continuity equation requires that, if the density is constant, between any two sections 1 and 2 along the pipe

