FLUID MECHANICS FOR MECHANICAL ENGINEERING (ME 208F)

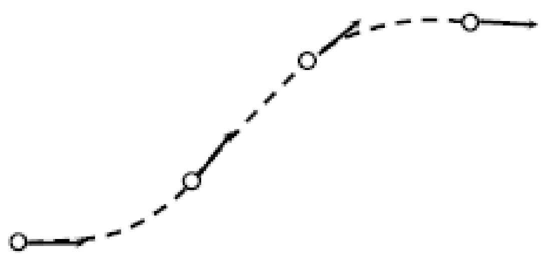
Section A:
Fluid Kinematics - II

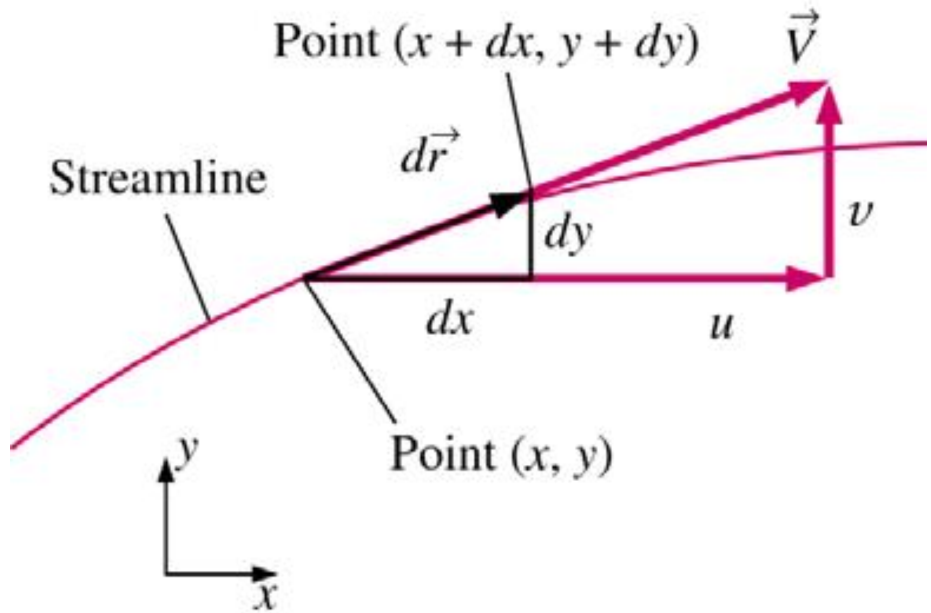
Flow Visualization

There are four different types of flow lines that may help to describe a flow field.

1) Streamline:

- A streamline is a line that is everywhere tangent to the velocity vector at a given instant of time. A streamline is hence an instantaneous pattern.

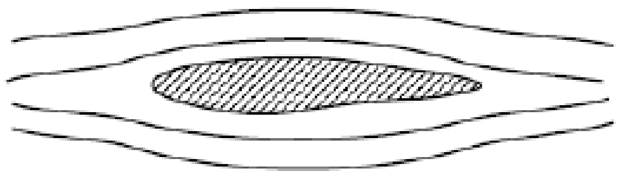




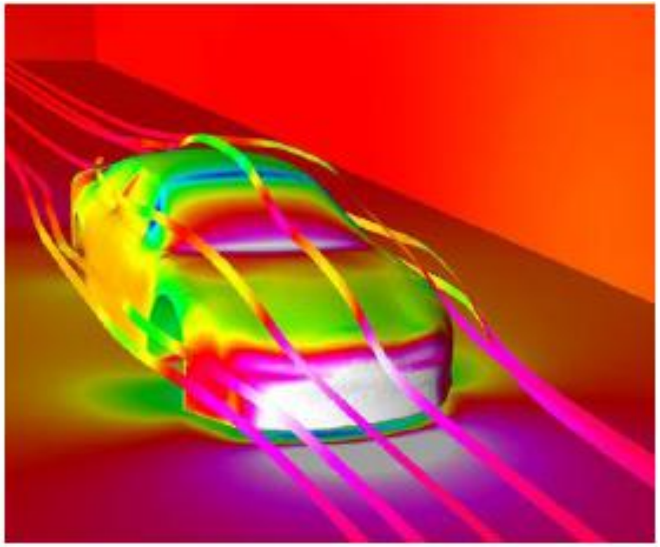
Flow Visualization

When fluid is flowing past a solid boundary, e.g. the surface of an aerofoil or the wall of a pipe, fluid obviously does not flow into or out of the surface. So very close to a boundary wall the flow direction must be parallel to the boundary.

- ***Close to a solid boundary streamlines are parallel to that boundary***



2-D streamlines around a cross-section of an aircraft wing shaped body



Surface pressure contours and streamlines

Flow Visualization

Some things to know about streamlines

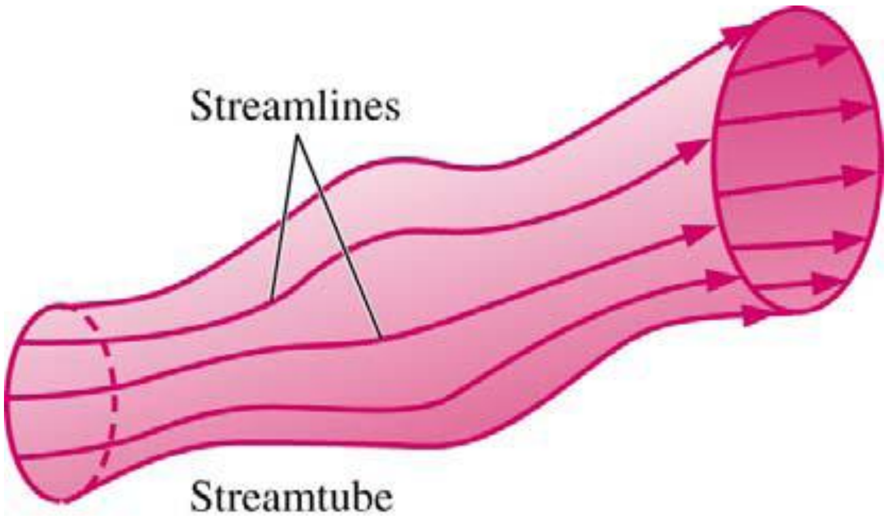
- *Because the fluid is moving in the same direction as the streamlines, fluid can not cross a streamline.*
- *Streamlines can not cross each other. If they were to cross this would indicate two different velocities at the same point. This is not physically possible.*
- *The above point implies that any particles of fluid starting on one streamline will stay on that same streamline throughout the fluid.*

Flow Visualization

A useful technique in fluid flow analysis is to consider only a part of the total fluid in isolation from the rest.

This can be done by imagining a tubular surface formed by streamlines along which the fluid flows.

This tubular surface is known as a streamtube, which is a tube whose walls are streamlines. Since the velocity is tangent to a streamline, no fluid can cross the walls of a streamtube.

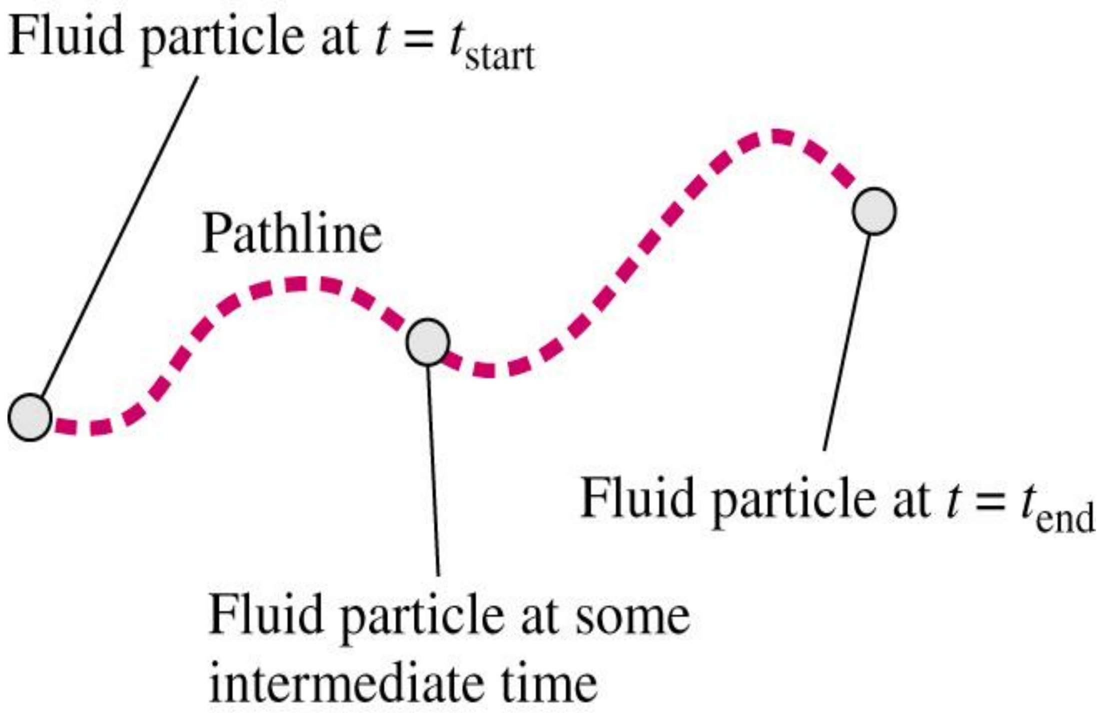


A ***stream tube*** is a *fluid mass bounded by a group of streamlines.*

Flow Visualization

3) Pathline

- A pathline is the actual path traversed by a given (marked) fluid particle.
- A pathline is hence also an integrated pattern.
- A pathline represents an integrated history of where a fluid particle has been.



Elementary Equations of Motion

- In analyzing fluid motion, we might take one of two approaches:
  1. seeking to describe the detailed flow pattern at every point (*x,y,z*) in the field,
  2. working with a finite region, making a balance of *flow in* versus *flow out*, and determining gross flow effects such as the force, or torque on a body, or the total energy exchange.

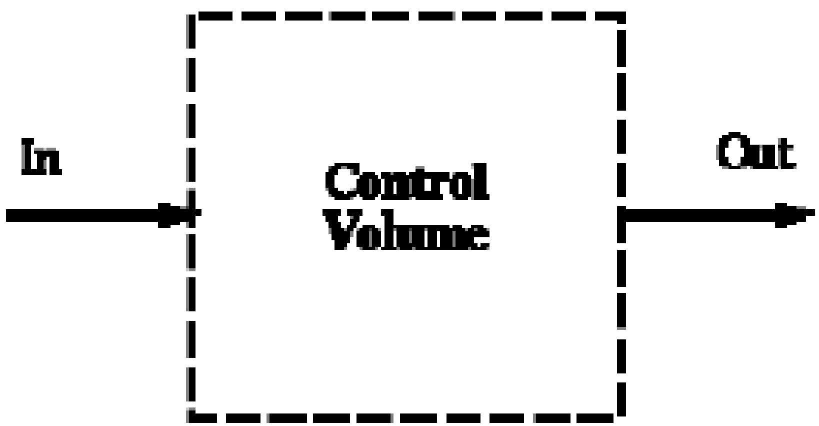
The second approach is the "**control-volume**" method and is the subject of this section. The first approach is the "**differential**" approach and will be covered in a higher level fluid mechanics course.

- We shall derive the three basic control-volume relations in fluid mechanics:
- the principle of conservation of mass, from which the continuity equation is developed;
- the principle of conservation of energy, from which the energy equation is derived;
- the principle of conservation of linear momentum, from which equations evaluating dynamic forces exerted by flowing fluids may be established.

Elementary Equations of Motion

1) Control volume

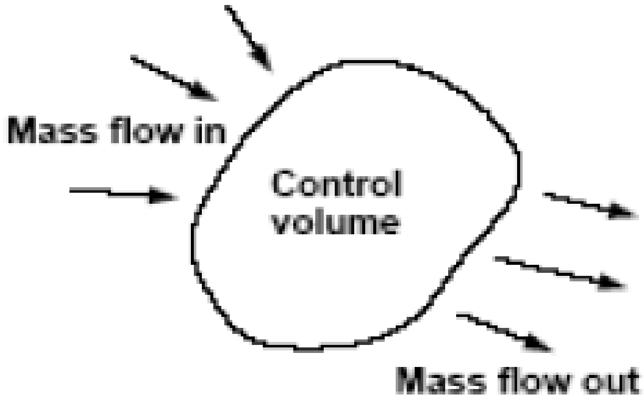
- A control volume is a finite region, chosen carefully by the analyst for a particular problem, with open boundaries through which mass, momentum, and energy are allowed to cross. The analyst makes a budget, or balance, between the incoming and outgoing fluid and the resultant changes within the control volume. Therefore one can calculate the gross properties (net force, total power output, total heat transfer, etc.) with this method.
- With this method, however, we do not care about the details inside the control volume (In other words we can treat the control volume as a "black box.")



Elementary Equations of Motion

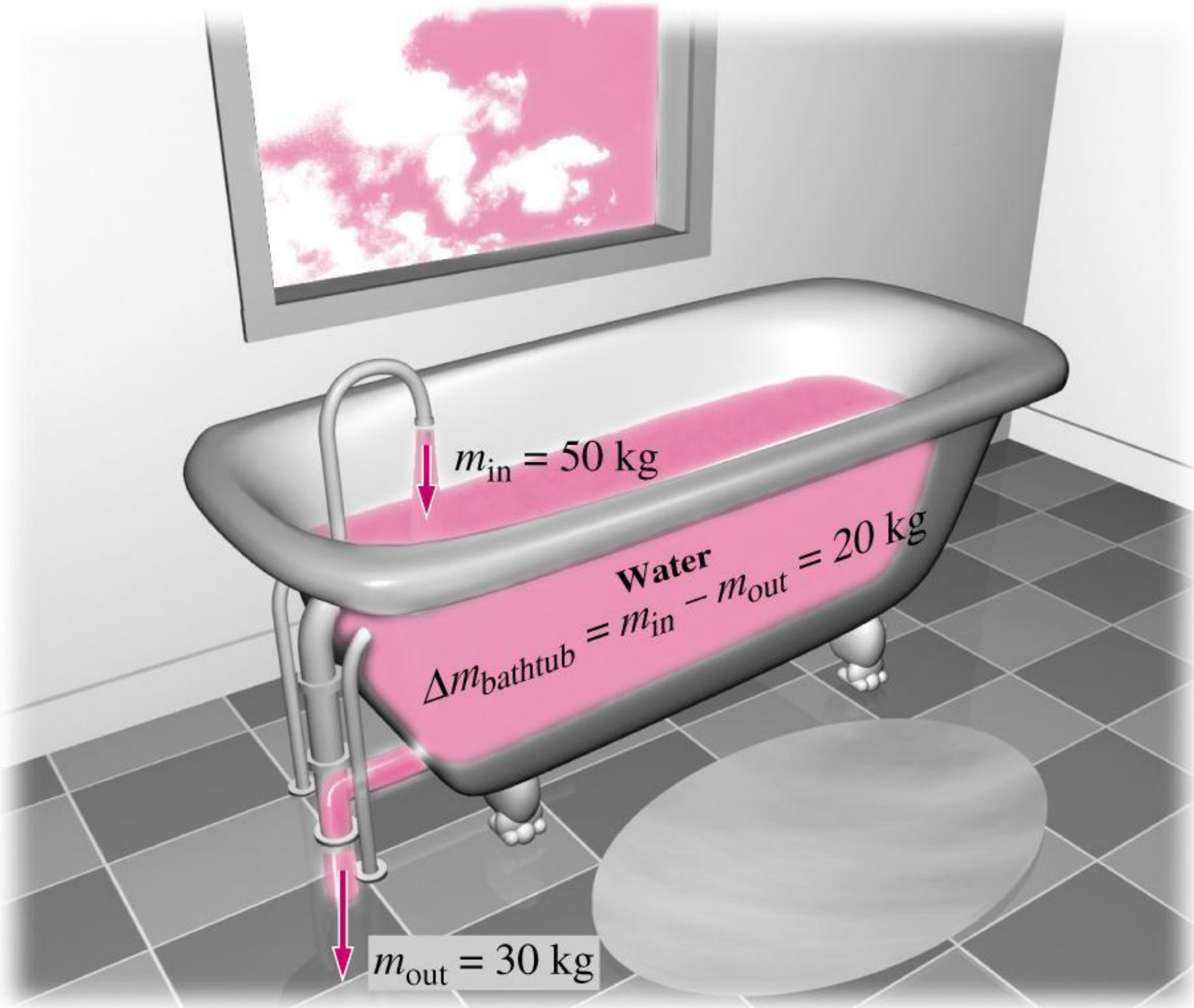
1) Control volume

- This principle of *conservation of mass* says ***Matter cannot be created or destroyed***
- We use it in the analysis of flowing fluids.
- The principle is applied to fixed region in the flow, known as ***control volumes*** (or surfaces), like that in the figure Shown:
- And for any control volume:

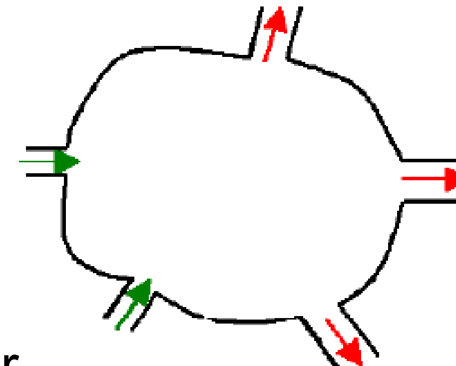


Mass entering per unit time	=	Mass leaving per unit time	+	Increase of mass in the control volume per unit time

Elementary Equations of Motion



Elementary Equations of Motion



Let us denote for each of the inlets and outlets:-

V = velocity of fluid in a stream

A = sectional area of a stream

p = pressure of the fluid in a stream

ρ = density of the fluid

Then, the **volume flow rate**,or **discharge** (volume of flow crossing a section per unit time) is given by

$$Q = VA$$

Similarly, the mass flow rate (n v crossing a section per unit time) is given by

$$\dot{m} = \rho V A = \rho Q$$

- Then, the **momentum fl** momentum of flow crossing a section per unit time, is given by $\dot{m} V$.
- For simplicity, we shall from here on c sider **steady** and **incompressible** flows only.

Elementary Equations of Motion

- DISCHARGE AND MEAN VELOCITY

Discharge:

¢The total quantity of fluid flowing in unit time past any cross section of a stream is called the *discharge* or *flow* at that section.

It can be measured either:

¢in terms of mass (mass rate of flow, *m*)

¢or in terms of volume (volume rate of flow, *Q*)

Elementary Equations of Motion

- 1. Mass flow rate:

¢ It is a method to measure the rate at which water is flowing along a pipe.

¢ It is the mass of fluid flowing per unit time.

$$\dot{m} = \frac{dm}{dt} = \frac{\text{mass of fluid}}{\text{time taken to collect the fluid}} \implies \text{Time} = \frac{\text{mass of fluid}}{\text{mass flow rate}}$$

Example:
An empty bucket weighs 2.0kg. After 7 seconds of collecting water the bucket weighs 8.0kg, then:

$$\dot{m} = \frac{dm}{dt} = \frac{\text{mass of fluid}}{\text{time taken to collect the fluid}} = \frac{8-2}{7} = 0.857 \text{ kg/s}$$

Elementary Equations of Motion

- 2.Volume flow rate - Discharge.

¢ It is another method to measure the rate at which water is flowing along a pipe. It is ***more commonly*** use The discharge is the volume of fluid flowing per unit time.

- The discharge is the volume of fluid flowing per unit time.

$$Q == \frac{\text{volume of fluid}}{\text{time}} \Longrightarrow \text{Time} = \frac{\text{volume of fluid}}{\text{discharge}}$$

- Also Note that: $\dot{m} = \rho Q$

Example:
If the density of the fluid in the above example is 850 kg/m^3, then:

$$Q = \frac{\dot{m}}{\rho} = \frac{0.857}{850} = 1.008 \times 10^{-3} \text{ m}^3/\text{s} = 1.008 \text{ l/s}$$

Elementary Equations of Motion

- 3. DISCHARGE AND MEAN VELOCITY

Discharge in pipe

If the area of cross section of the pipe at point **X** is A, and the mean velocity here is $\boldsymbol{u_m}$, during a time $\boldsymbol{t}$, a cylinder of fluid will pass point **X** with a volume $A \times \boldsymbol{u_m} \times \boldsymbol{t}$. The volume per unit time (the discharge) will thus be :

$$Q = \frac{volume}{time} = \frac{A \times u_m \times t}{t} \Rightarrow Q = A \times u_m$$

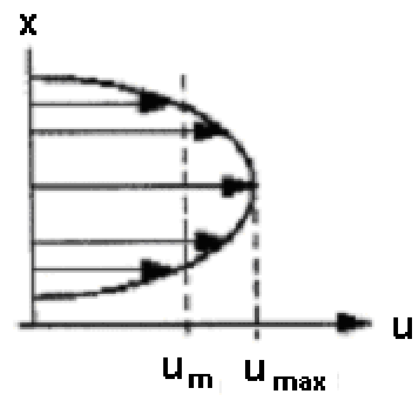
or: $u_m = \dfrac{Q}{A}$

Let: $u_m = V \Rightarrow V = \dfrac{Q}{A}$

This idea, that mean velocity multiplied by the area gives the discharge, applies to all situations - not just pipe flow.

Elementary Equations of Motion

- Note how carefully we have called this the mean velocity. This is because the velocity in the pipe is not constant across the cross section.
- Crossing the centre line of the pipe, the velocity is zero at the walls, increasing to a maximum at the centre then decreasing symmetrically to the other wall.



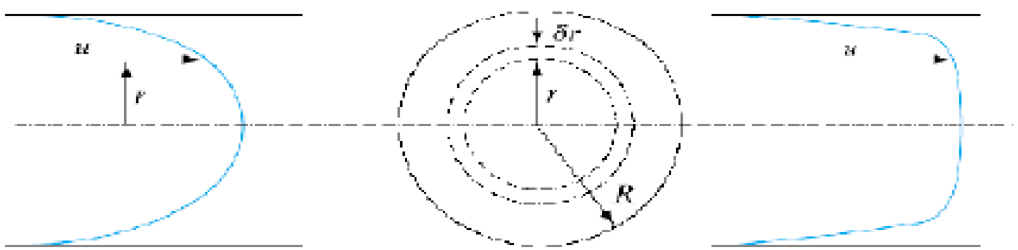
A typical velocity profile across a pipe for laminar flow

If u is the velocity at any radius r, the flow dQ through an annular element of radius r and thickness dr will be:

$$dQ = \text{Area of elememt} \times \text{Velocity}$$

$$= 2\pi r dr \times u$$

$$Q = 2\pi \int_0^R urdr$$



(a) Laminar flow (b) Turbulent flow

Elementary Equations of Motion

Continuity equation:

- By steadiness, the total mass of fluid contained in the control volume must be invariant with time. Therefore there must be an exact balance between the total rate of flow into the control volume and that out of the control volume:
- Total Mass Outflow = Total Mass Inflow

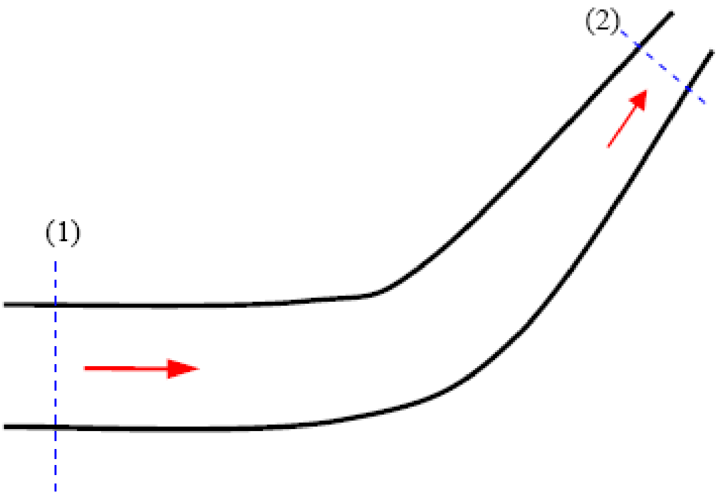
which translates into the following mathematical relation

$$\sum_{i=1}^{M}(\rho_i V_i A_i)_{\text{in}} = \sum_{i=1}^{N}(\rho_i V_i A_i)_{\text{out}}$$

$$\sum_{i=1}^{M}(V_i A_i)_{\text{in}} = \sum_{i=1}^{N}(V_i A_i)_{\text{out}}$$

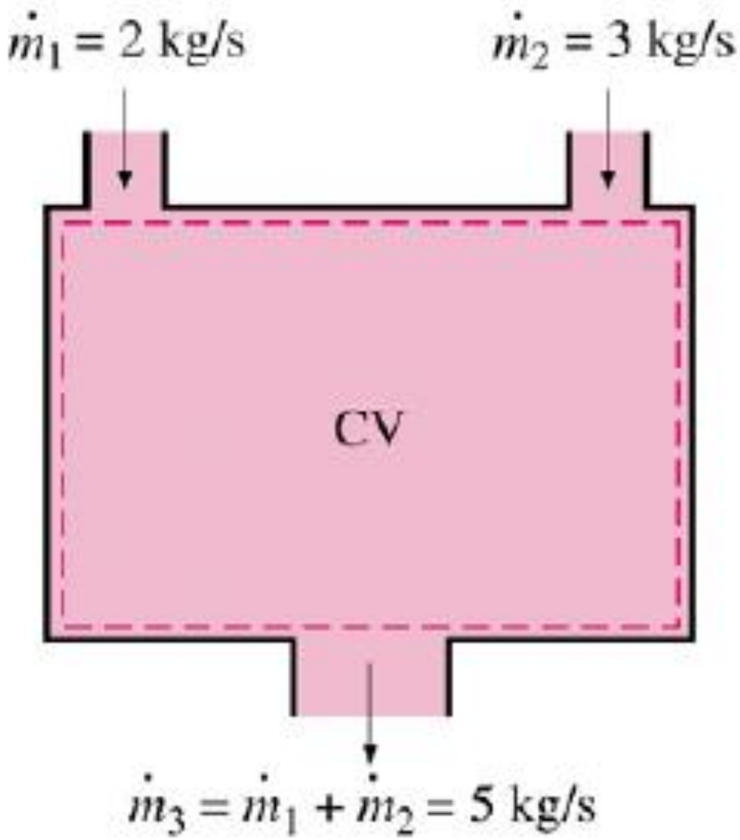
$$\sum_{i=1}^{M}(Q_i)_{\text{in}} = \sum_{i=1}^{N}(Q_i)_{\text{out}}$$

$$Q = V_1 A_1 = V_2 A_2 = \text{constant}$$



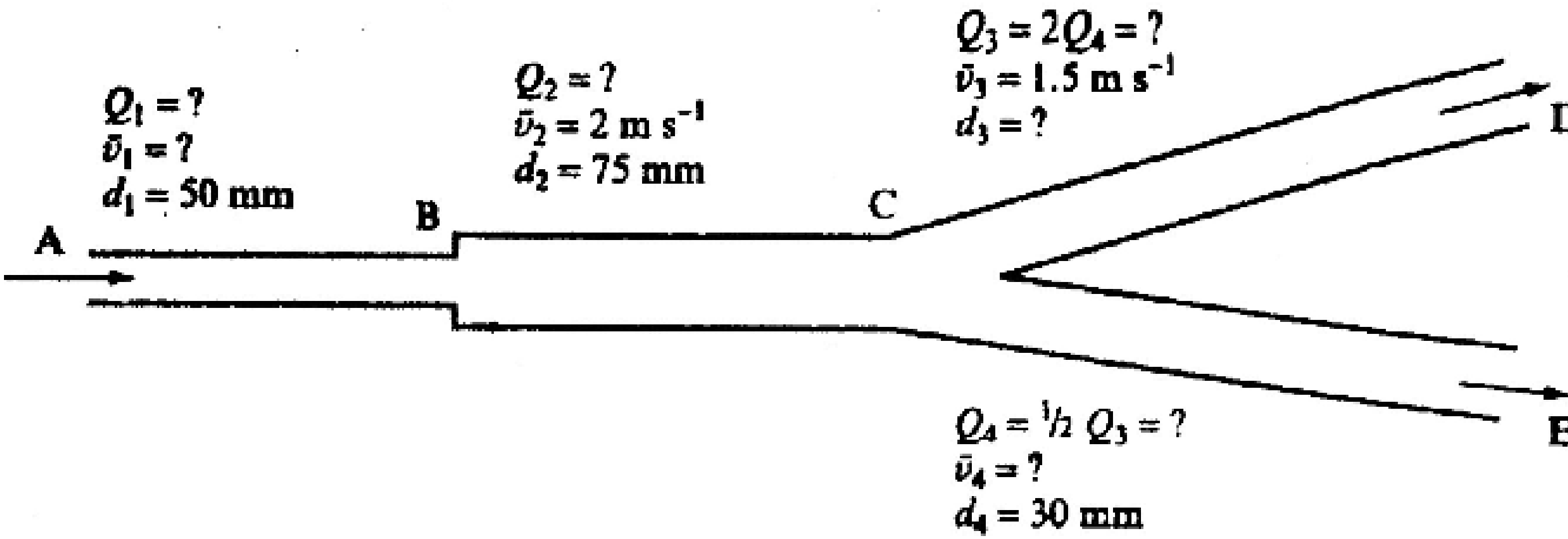
Elementary Equations of Motion

- Another example involving two inlets and one outlet is shown below.



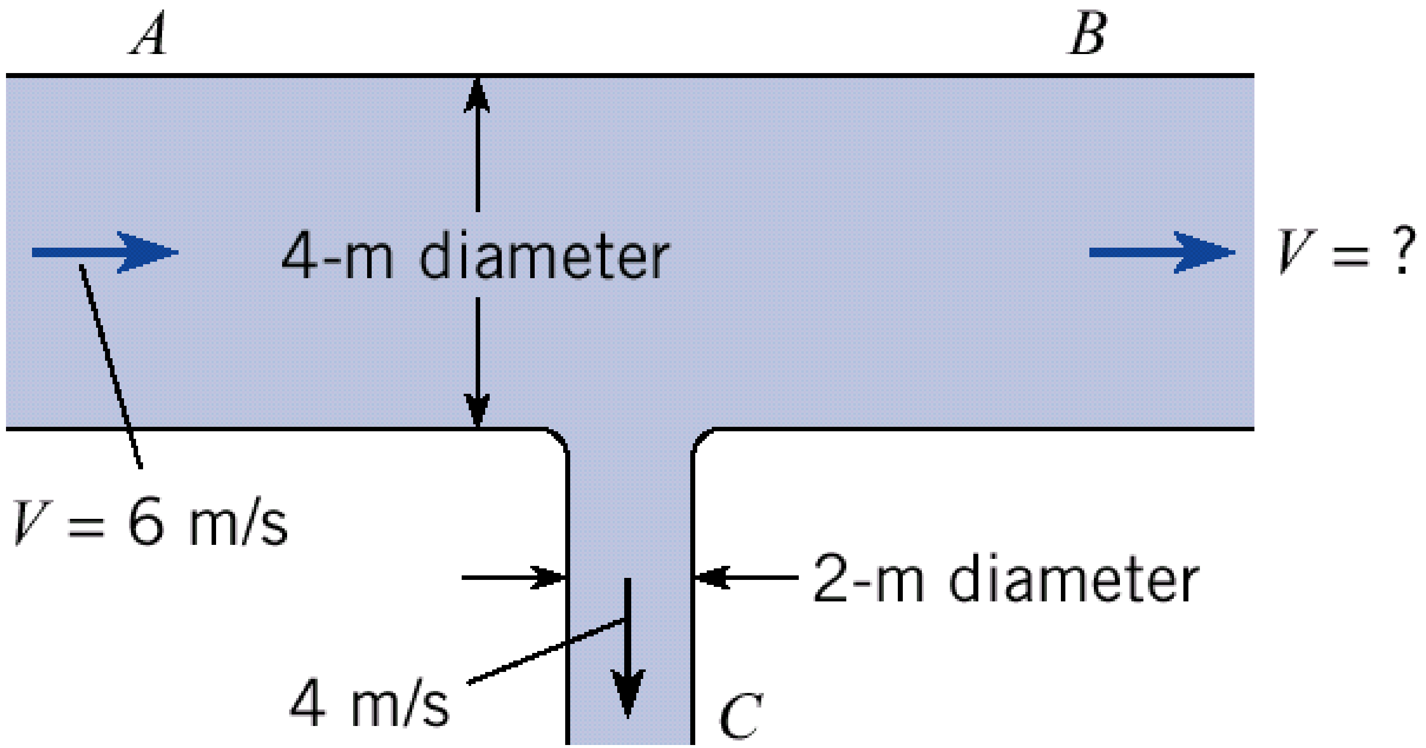
Elementary Equations of Motion

- Example 1:Find the missing values



Elementary Equations of Motion

- Example 2:Find the missing values



Elementary Equations of Motion

- Example 3:Find the missing values

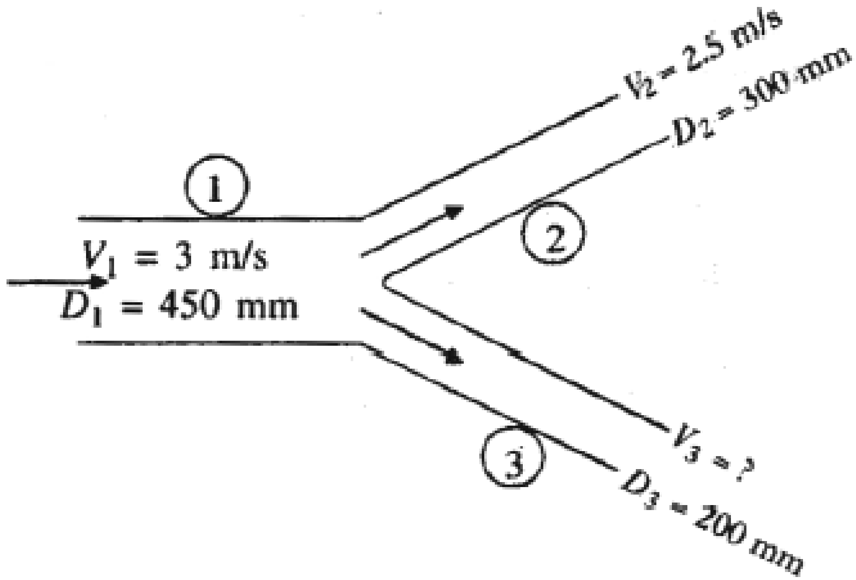
A pipe (1) 450 mm in diameter branches into two pipes (2 and 3) of diameters 300 mm and 200 mm respectively as shown in Fig. If the average velocity in 450 mm diameter pipe is 3 m/s find:

(i) Discharge through 450 mm diameter pipe;

(ii) Velocity in 200 mm diameter pipe if the average velocity in 300 mm pipe is 2·5 m/s.

$$A_1 = \frac{\pi}{4} \times 0{\cdot}45^2 = 0{\cdot}159 \text{ m}^2$$

$$V_1 = 3 \text{ m/s}$$



$$A_2 = \frac{\pi}{4} \times 0{\cdot}3^2 = 0{\cdot}0707 \text{ m}^2$$

$$V_2 = 2{\cdot}5 \text{ m/s}$$

$$A_3 = \frac{\pi}{4} \times 0{\cdot}2^2 = 0{\cdot}0314 \text{ m}^2$$

Elementary Equations of Motion

(i) Discharge through pipe (1), Q_1

$$Q_1 = A_1 V_1 = 0{\cdot}159 \times 3 = \mathbf{0{\cdot}477\ m^3/s}$$

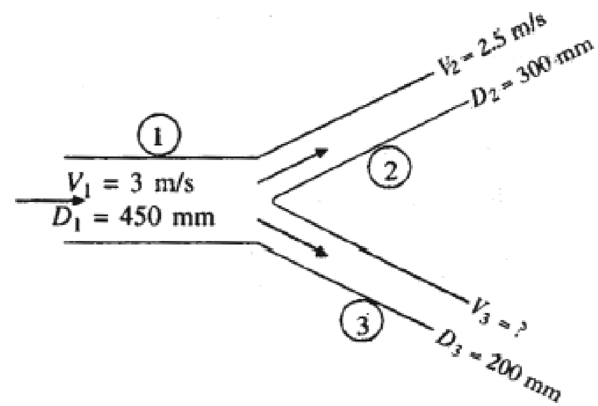
(ii) Velocity in pipe of diameter 200 mm i.e. V_3

Let Q_1, Q_2 and Q_3 be the discharge in pipes 1, 2 and 3 respectively.

$$Q_1 = Q_2 + Q_3$$

$$Q_1 = 0{\cdot}477\ m^3/s$$

$$Q_2 = A_2 V_2 = 0{\cdot}0707 \times 2{\cdot}5 = 0{\cdot}1767\ m^3/s$$



$$0{\cdot}477 = 0{\cdot}1767 + Q_2$$

$$Q_3 = 0{\cdot}477 - 0{\cdot}1767 \simeq 0{\cdot}3\ m^3/s \qquad Q_3 = A_3 V_3$$

$$V_3 = \frac{Q_3}{A_3} = \frac{0{\cdot}3}{0{\cdot}0314} = 9{\cdot}55\ m/s$$