The Laws of Thermodynamics

Principles of Thermodynamics

- Energy is conserved
  - FIRST LAW OF THERMODYNAMICS
  - Examples: Engines (Internal -> Mechanical)
    Friction (Mechanical -> Internal)
- All processes must increase ***entropy***
  - SECOND LAW OF THERMODYNAMICS
  - Entropy is measure of disorder
  - Engines can not be 100% efficient

Converting Internal Energy to Mechanical

$$\Delta U = Q - P\Delta V$$

Work done by expansion

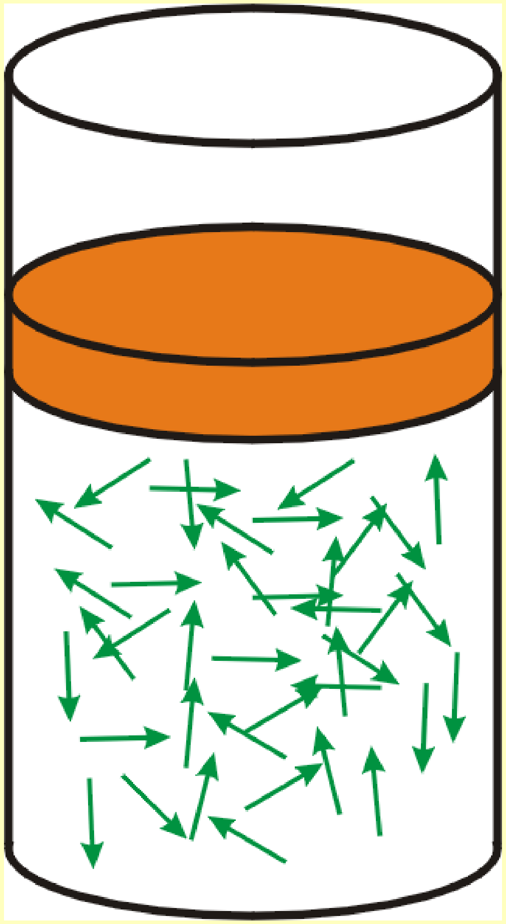
$$W = F\Delta x, \quad F = PA, \quad \Delta x = \Delta V / A$$

$$W = P\Delta V$$

Example

A cylinder of radius 5 cm is kept at pressure with a piston of mass 75 kg.

a) What is the pressure inside the cylinder?
b) If the gas expands such that the cylinder rises 12.0 cm, what work was done by the gas?
c) What amount of the work went into changing the gravitational PE of the piston?
d) Where did the rest of the work go?



Solution

Given: M =75, A = p·0.05^2, Dx=0.12, P_{atm} = 1.013x10^5 Pa

a) Find P_{gas}

$$P_{gas} = \frac{Mg}{A} + P_{atm}$$

= 1.950x10^5 Pa

b) Find W_{gas}

$$W = P\Delta V$$

$$W = P_{gas} A \Delta x$$

= 183.8 J

c) Find $W_{gravity}$

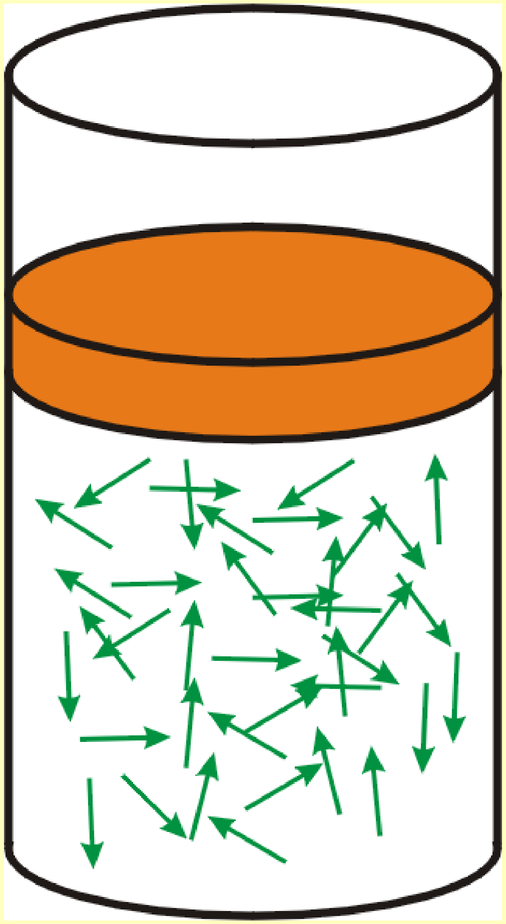
$$W = mgh$$

= 88.3 J

d) Where did the other work go?

Compressing the outside air

Quiz Review

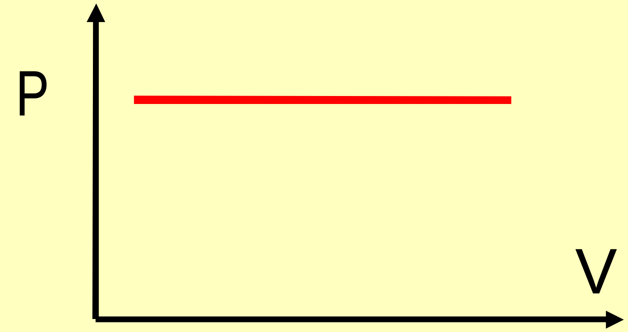


A massive copper piston traps an ideal gas as shown to the right. The piston is allowed to freely slide up and down and equilibrate with the outside air. Pick the most correct statement?

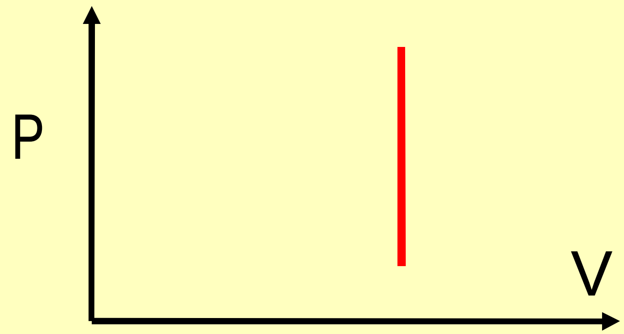
A. The pressure is higher inside the cylinder than outside.
B. The temperature inside the cylinder is higher than outside the cylinder.
C. If the gas is heated by applying a flame to cylinder, and the piston comes to a new equilibrium, the inside pressure will have increased.
D. All of the above.
E. A and C.

Some Vocabulary

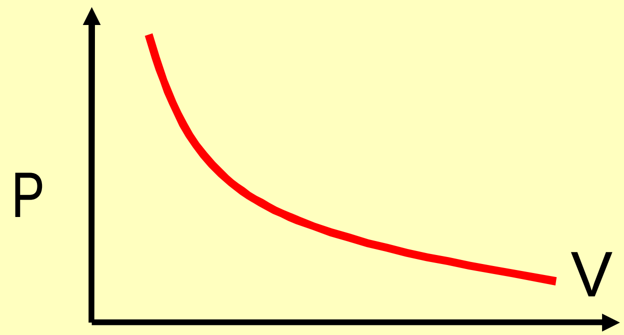
- Isobaric
  - o P = constant



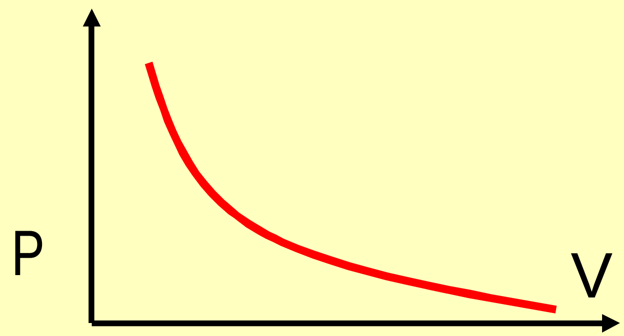
- Isovolumetric
  - o V = constant



- Isothermal
  - o T = constant

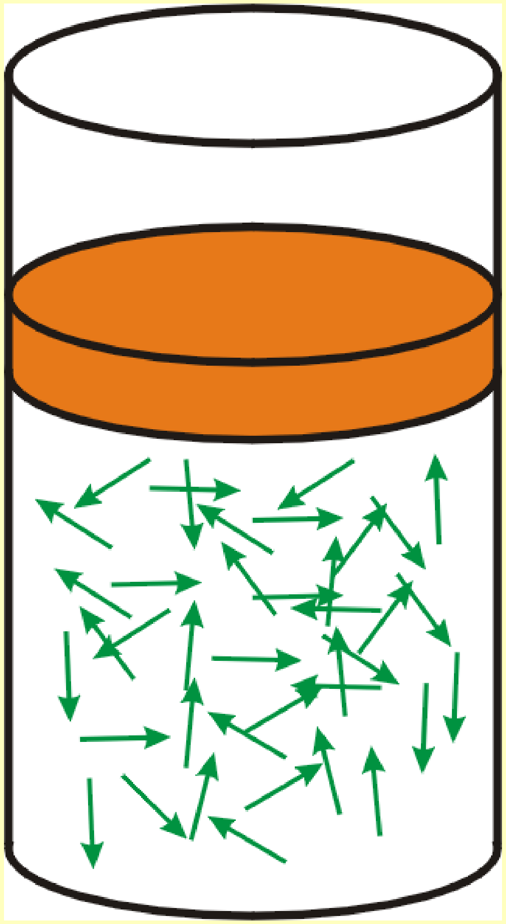


- Adiabatic
  - o Q = 0



Example

Outside Air: Room T, Atm. P



A massive piston traps an amount of Helium gas as shown. The piston freely slides up and down.

The system initially equilibrates at room temperature (a)

Weight is slowly added to the piston, ***isothermally*** compressing the gas to half its original volume (b)

A. P_b (> < =) P_a

B. T_b (> < =) T_a

C. W_{ab} (> < =) 0

D. U_b (> < =) U_a

E. Q_{ab} (> < =) 0

$$\Delta U = Q - P\Delta V$$

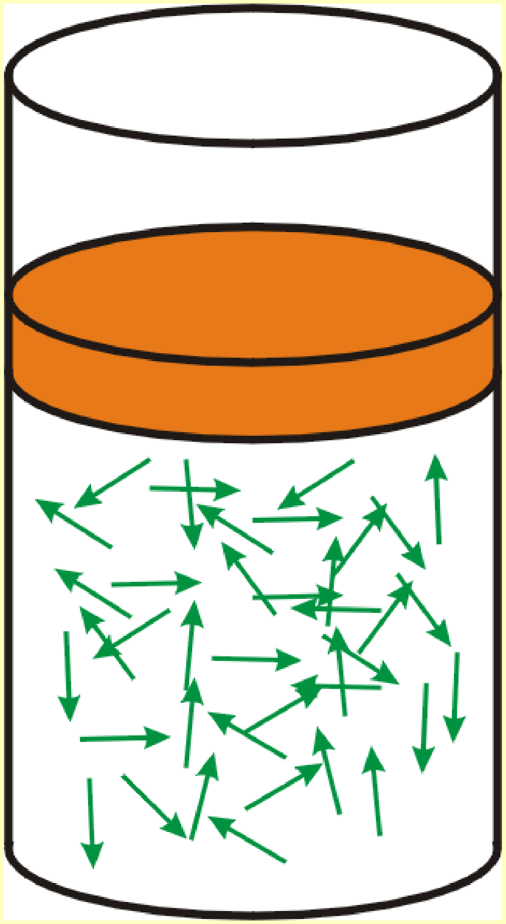
$$W = P\Delta V$$

Vocabulary: W_{ab} is work done by gas between a and b

Q_{ab} is heat added to gas between a and b

Example

Outside Air: Room T, Atm. P



A massive piston traps an amount of Helium gas as shown. The piston freely slides up and down.

The system initially equilibrates at room temperature (a)

Weight is slowly added to the piston, ***adiabatically*** compressing the gas to half its original volume (b)

A. P_b (> < =) P_a

B. W_{ab} (> < =) 0

C. Q_{ab} (> < =) 0

D. U_b (> < =) U_a

E. T_b (> < =) T_a

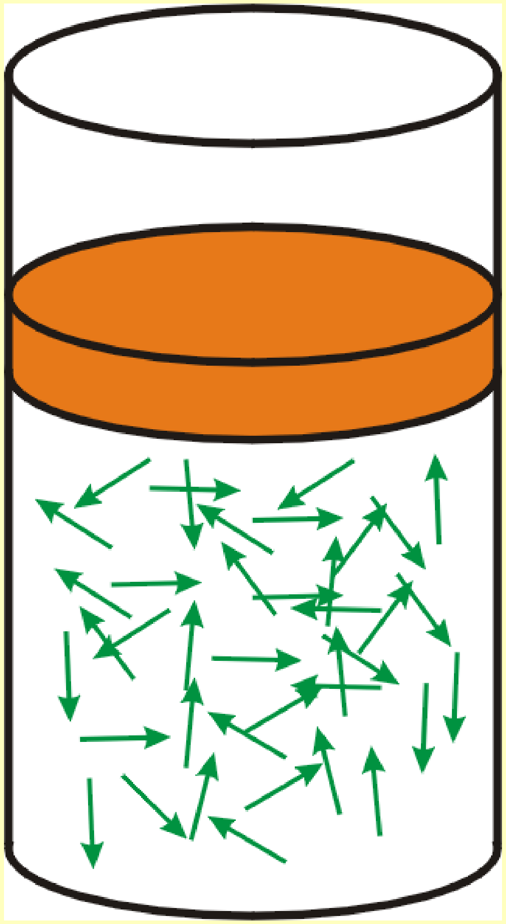
$$\Delta U = Q - P\Delta V$$
$$W = P\Delta V$$

Vocabulary: W_{ab} is work done by gas between a and b

Q_{ab} is heat added to gas between a and b

Example

Outside Air: Room T, Atm. P



A massive piston traps an amount of Helium gas as shown. The piston freely slides up and down.
The system initially equilibrates at room temperature (a)
The gas is cooled, ***isobarically*** compressing the gas to half its original volume (b)

A. P_b (> < =) P_a
B. W_{ab} (> < =) 0
C. T_b (> < =) T_a
D. U_b (> < =) U_a
E. Q_{ab} (> < =) 0

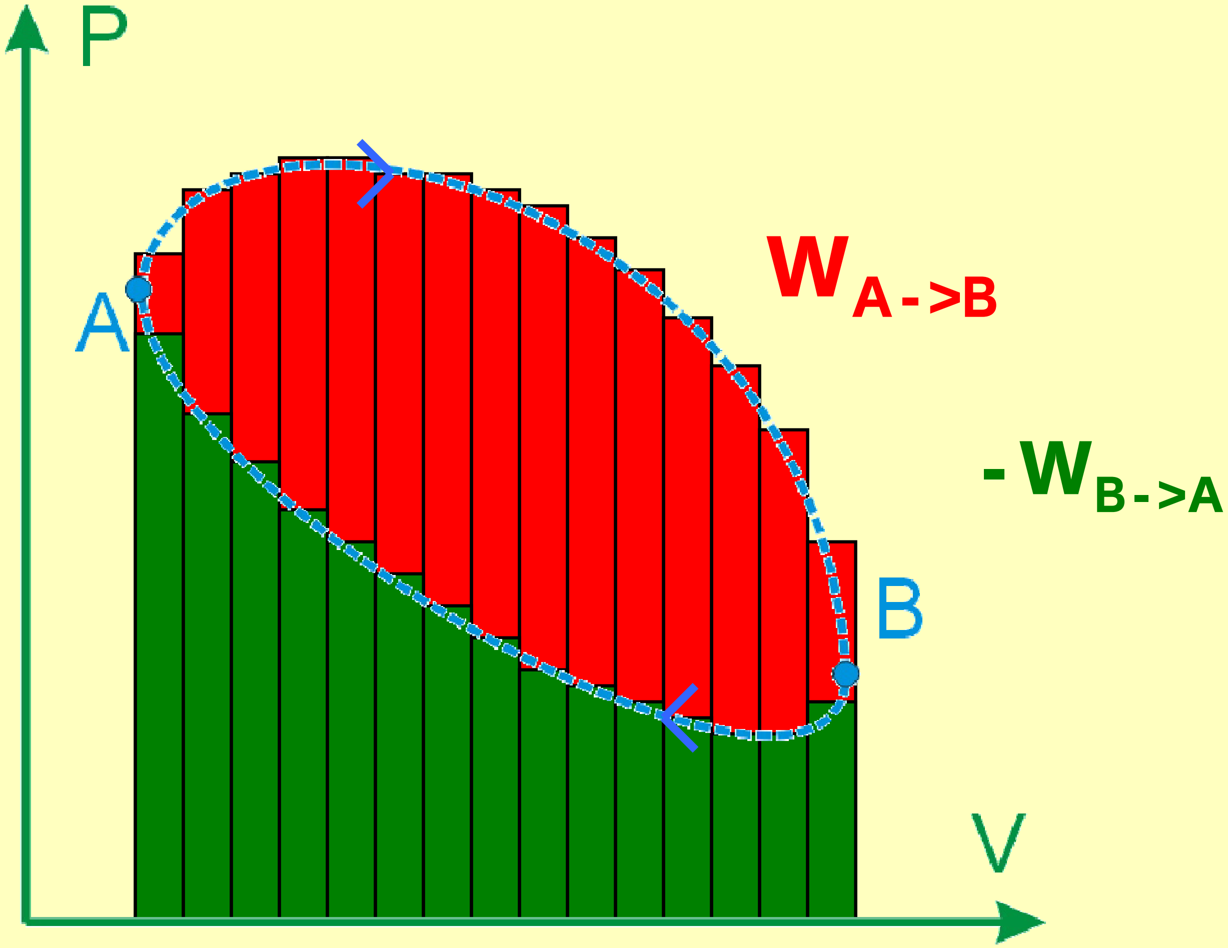
$$PV = nRT$$

$$\Delta U = Q - P\Delta V$$
$$W = P\Delta V$$

Vocabulary: W_{ab} is work done by gas between a and b
Q_{ab} is heat added to gas between a and b

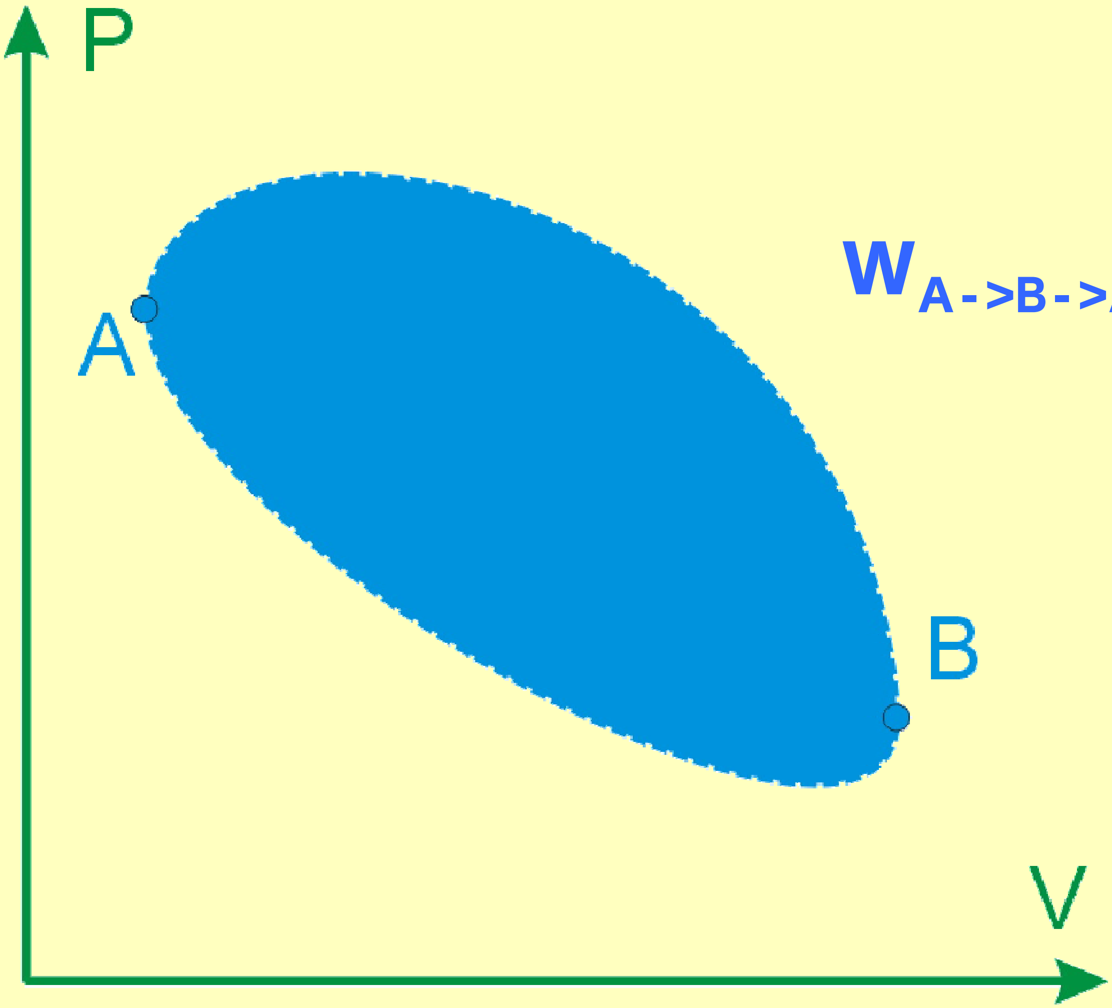
Work from closed cycles

Consider cycle A -> B -> A



Work from closed cycles

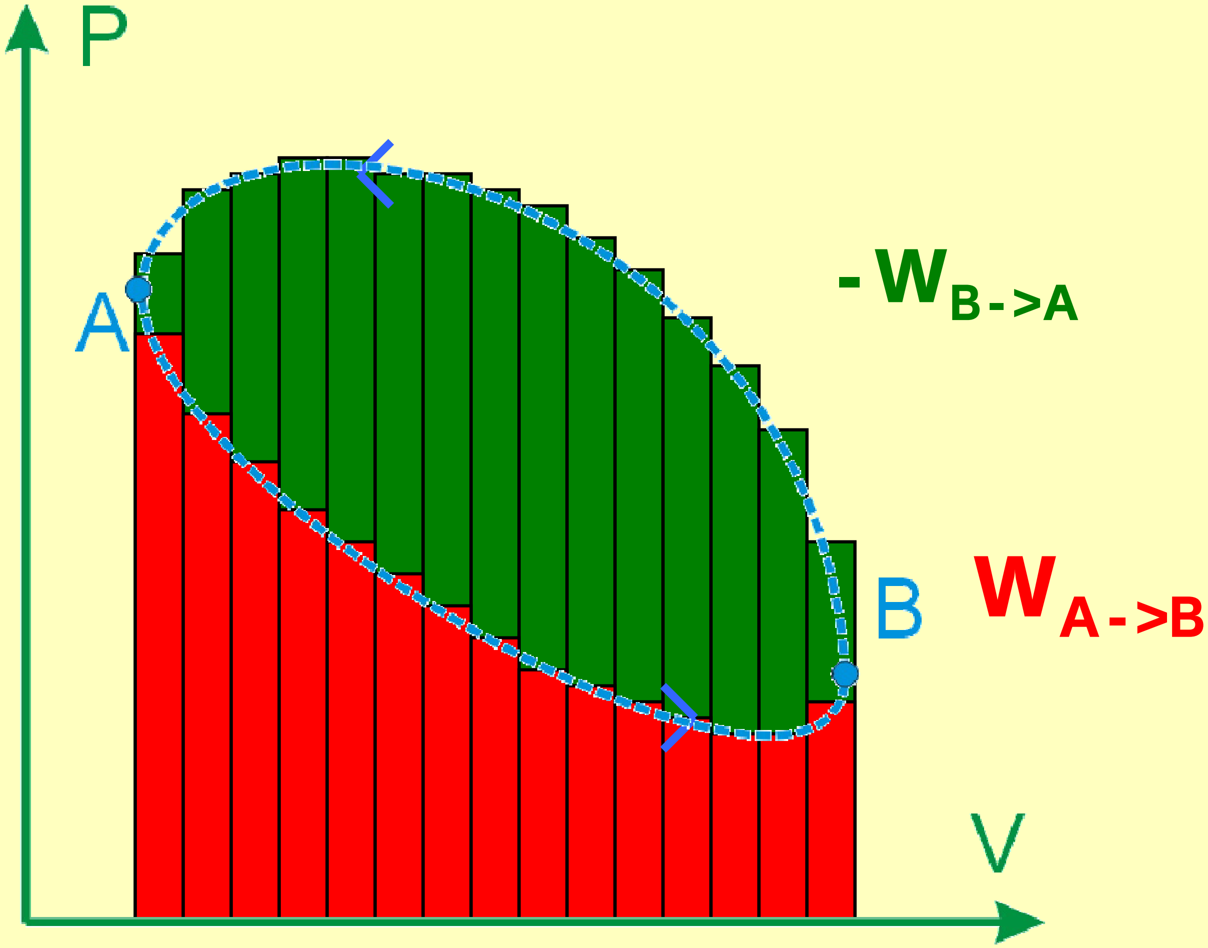
Consider cycle A -> B -> A



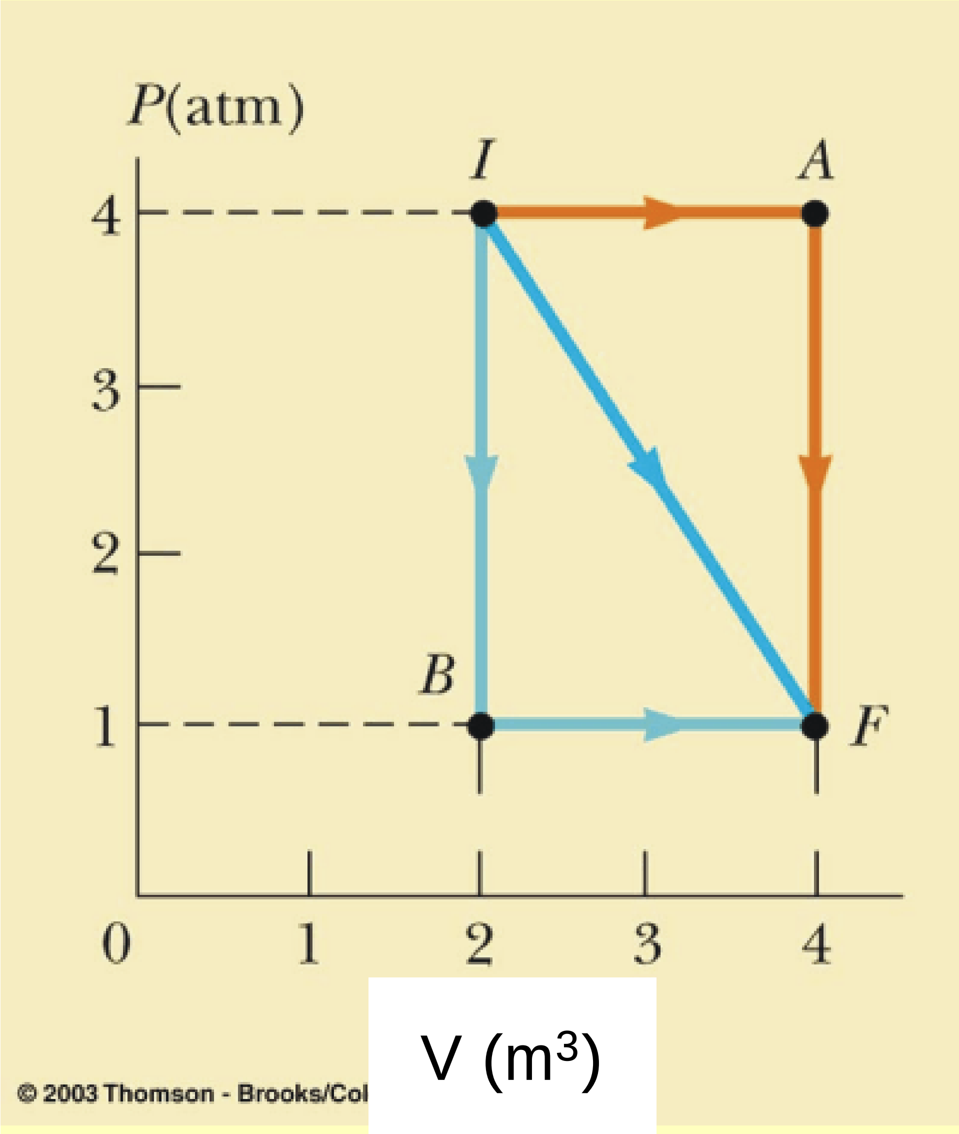
$W_{A\to B\to A}$ = **Area**

Work from closed cycles

Reverse the cycle, make it counter clockwise



Example



a) What amount of work is performed by the gas in the cycle IAFI?

$$W = \text{enclosed area}$$

$W_{IAFI} = 3P_{atm} = 3.04\text{x}10^5$ J

b) How much heat was inserted into the gas in the cycle IAFI?

$$\Delta U = Q - W$$

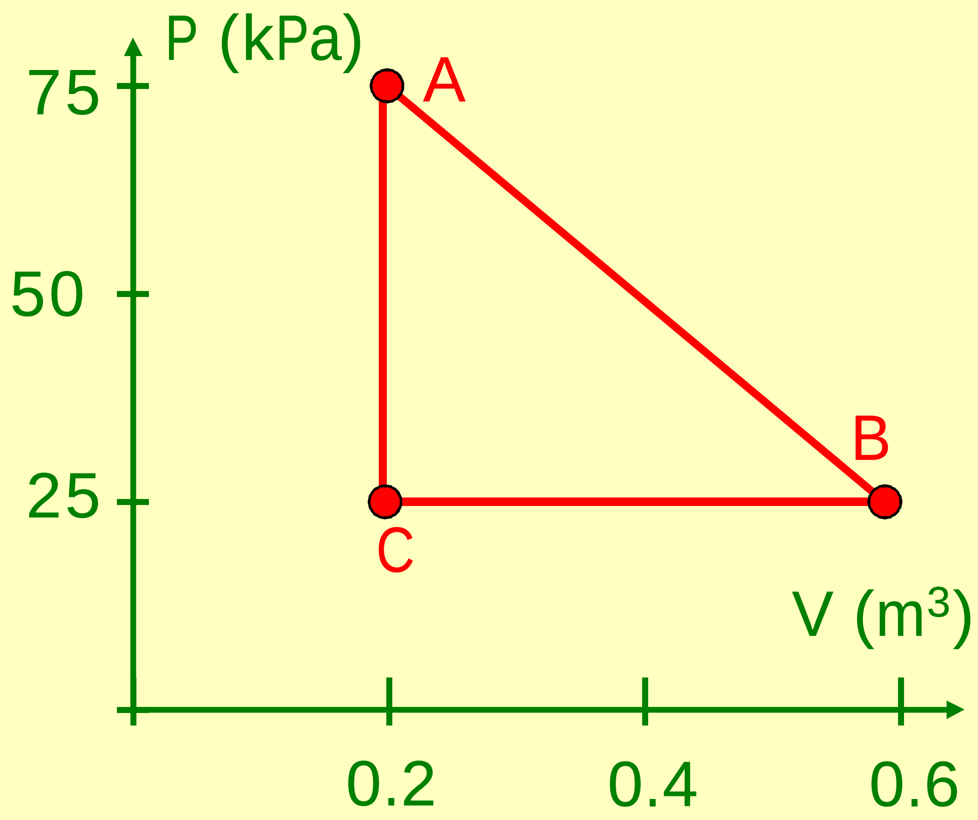
DU = 0   Q = $3.04\text{x}10^5$ J

c) What amount of work is performed by the gas in the cycle IBFI?

W = $-3.04\text{x}10^5$ J

One More Example

Consider a monotonic ideal gas which expands according to the PV diagram.



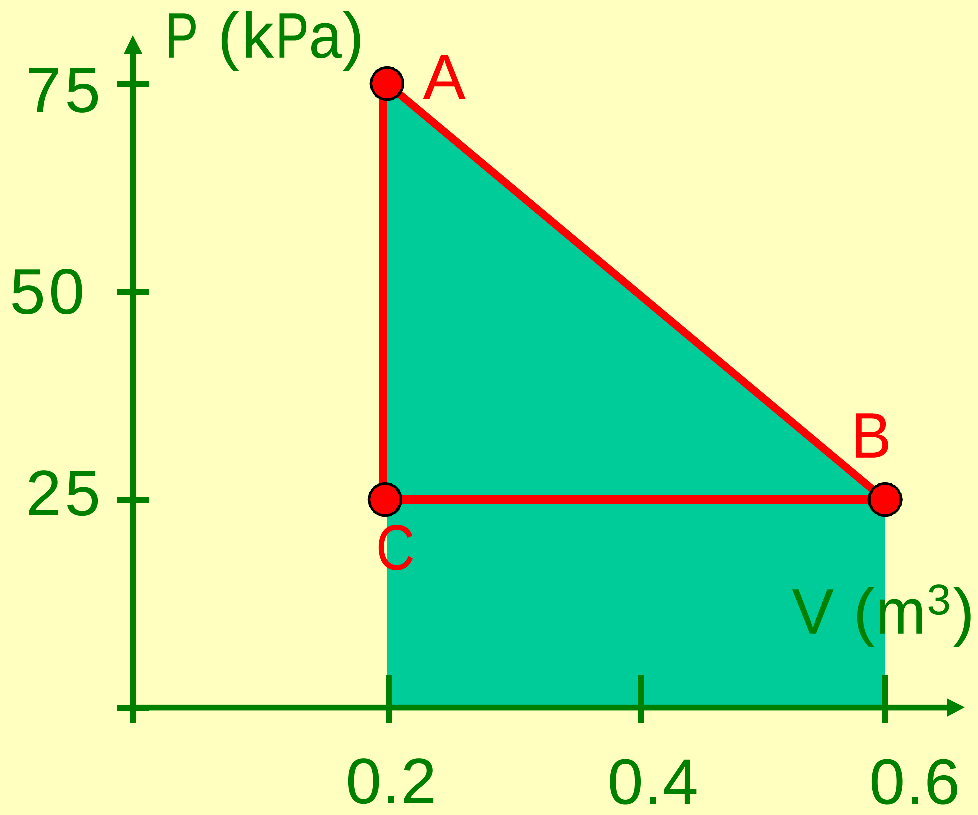
a) What work was done by the gas from A to B?
b) What heat was added to the gas between A and B?
c) What work was done by the gas from B to C?
d) What heat was added to the gas beween B and C?
e) What work was done by the gas from C to A?
f) What heat was added to the gas from C to A?

Solution

a) Find W_{AB}

W_{AB} = **Area** = 20,000 J

b) Find Q_{AB}



MonotonicGas:

$$U = \frac{3}{2}nRT$$

$$PV = nRT$$

$$U = \frac{3}{2}PV$$

- First find U_A and U_B

U_A = 22,500 J, U_B = 22,500 J, DU = 0

- Finally, solve for Q

$$\Delta U = Q - W$$

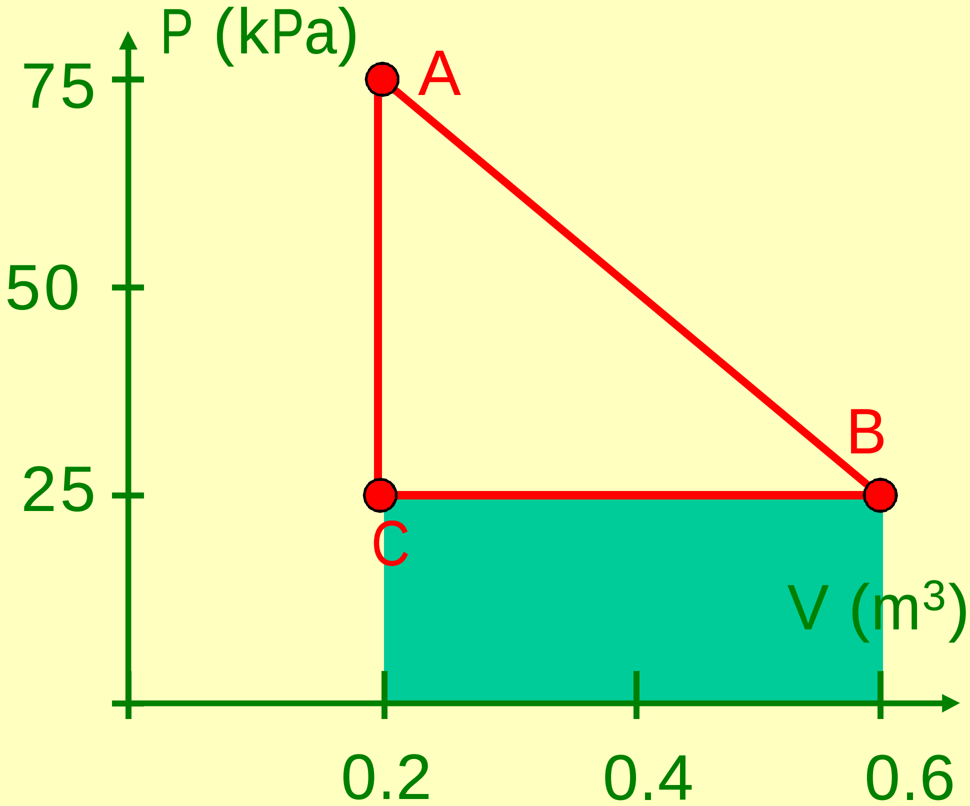
Q = 20,000 J

Solution

c) Find W_{BC}

W_{AB} = **-Area** = -10,000 J

d) Find Q_{BC}



$$U = \frac{3}{2}PV$$

$$\Delta U = Q - W$$

- First find U_B and U_C

U_B = 22,500 J, U_C = 7,500 J, DU = -15,000

- Finally, solve for Q

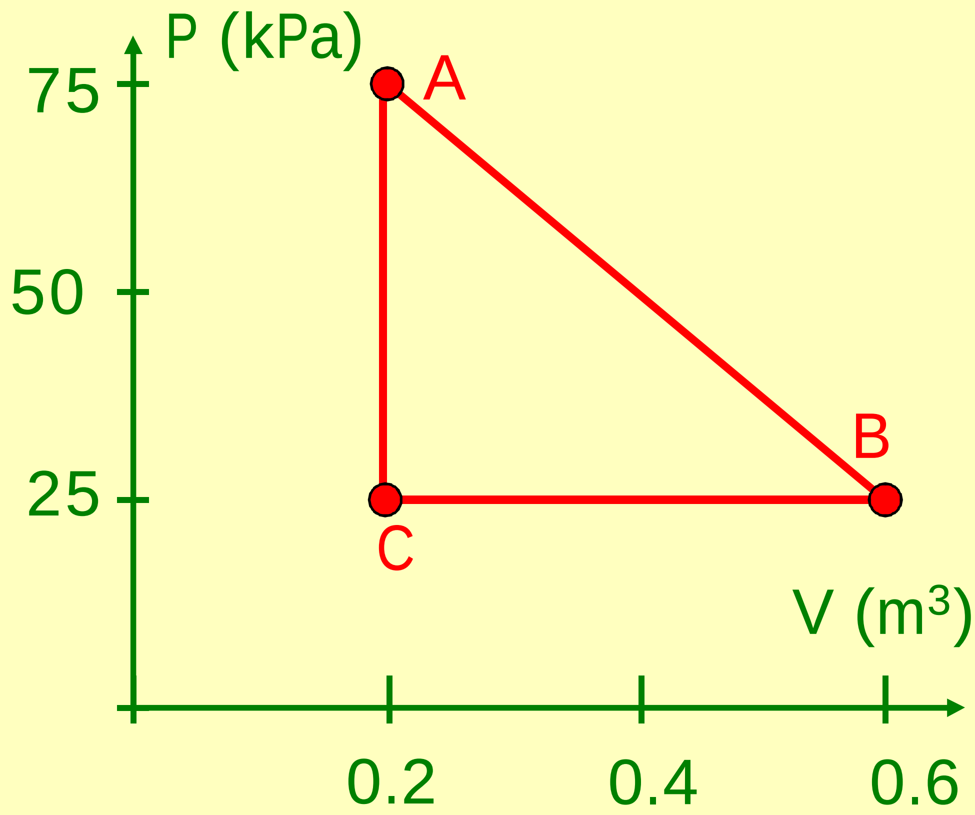
Q = -25,000 J

Solution

e) Find W_{CA}

W_{AB} = **Area** = 0 J

f) Find Q_{CA}



$$U = \frac{3}{2}PV$$

- First find U_C and U_A

U_C = 7,500 J, U_A = 22,500 J, DU = 15,000

$$\Delta U = Q - W$$

- Finally, solve for Q

Q = 15,000 J

Continued Example

Take solutions from last problem and find:

a) Net work done by gas in the cycle
b) Amount of heat added to gas

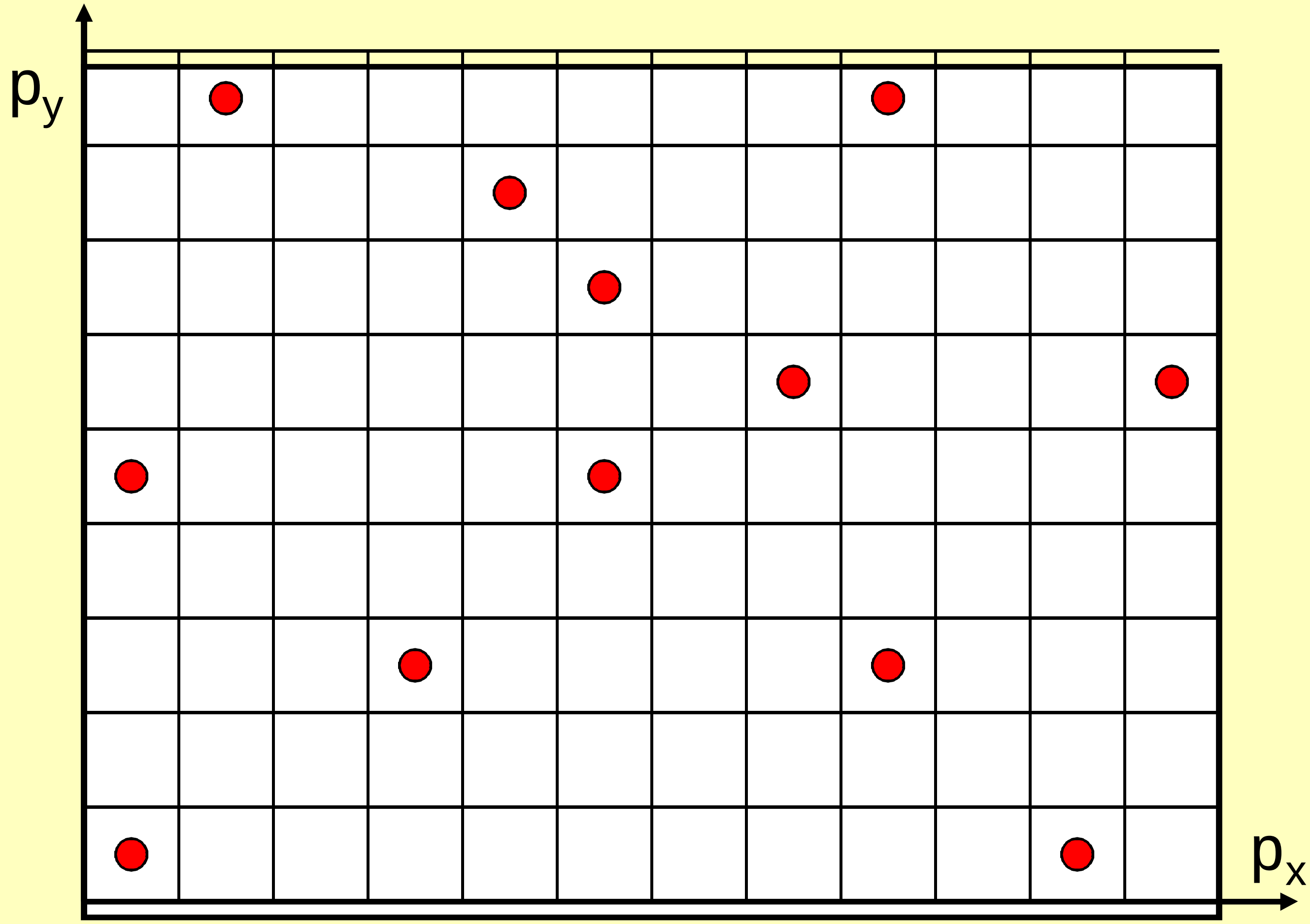
$W_{AB} + W_{BC} + W_{CA} = 10{,}000\ J$
$Q_{AB} + Q_{BC} + Q_{CA} = 10{,}000\ J$

This does NOT mean that the engine is 100% efficient!

Give Quiz

Entropy

- Measure of Disorder of the system (randomness, ignorance)
- $S = k_B \log(N)$
  N = # of possible arrangements for fixed E and Q



Entropy

- Total Entropy always rises!
  (2nd Law of Thermodynamics)
- Adding heat raises entropy

$$\Delta S = Q / T$$

Defines temperature in Kelvin!

Why does Q flow from hot to cold?

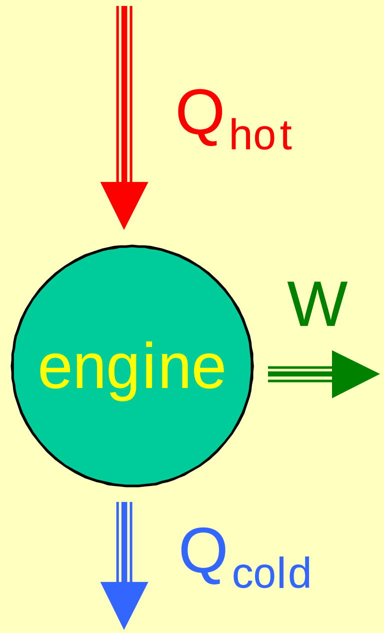
- Consider two systems, one with T_A and one with T_B
- Allow $Q > 0$ to flow from T_A to T_B
- Entropy changed by:

$$DS = Q/T_B - Q/T_A$$

- If $T_A > T_B$, then $DS > 0$
- System will achieve more randomness by exchanging heat until $T_B = T_A$

Efficiencies of Engines

- Consider a cycle described by:
  W, work done by engine
  Q_{hot}, heat that flows into engine from source at T_{hot}
  Q_{cold}, heat exhausted from engine at lower temperature, T_{cold}



- Efficiency is defined:

$$\text{engine}: e = \frac{W}{Q_{hot}} = \frac{Q_{hot} - Q_{cold}}{Q_{hot}} = 1 - \frac{Q_{cold}}{Q_{hot}}$$

Since $\Delta S = Q / T > 0$,

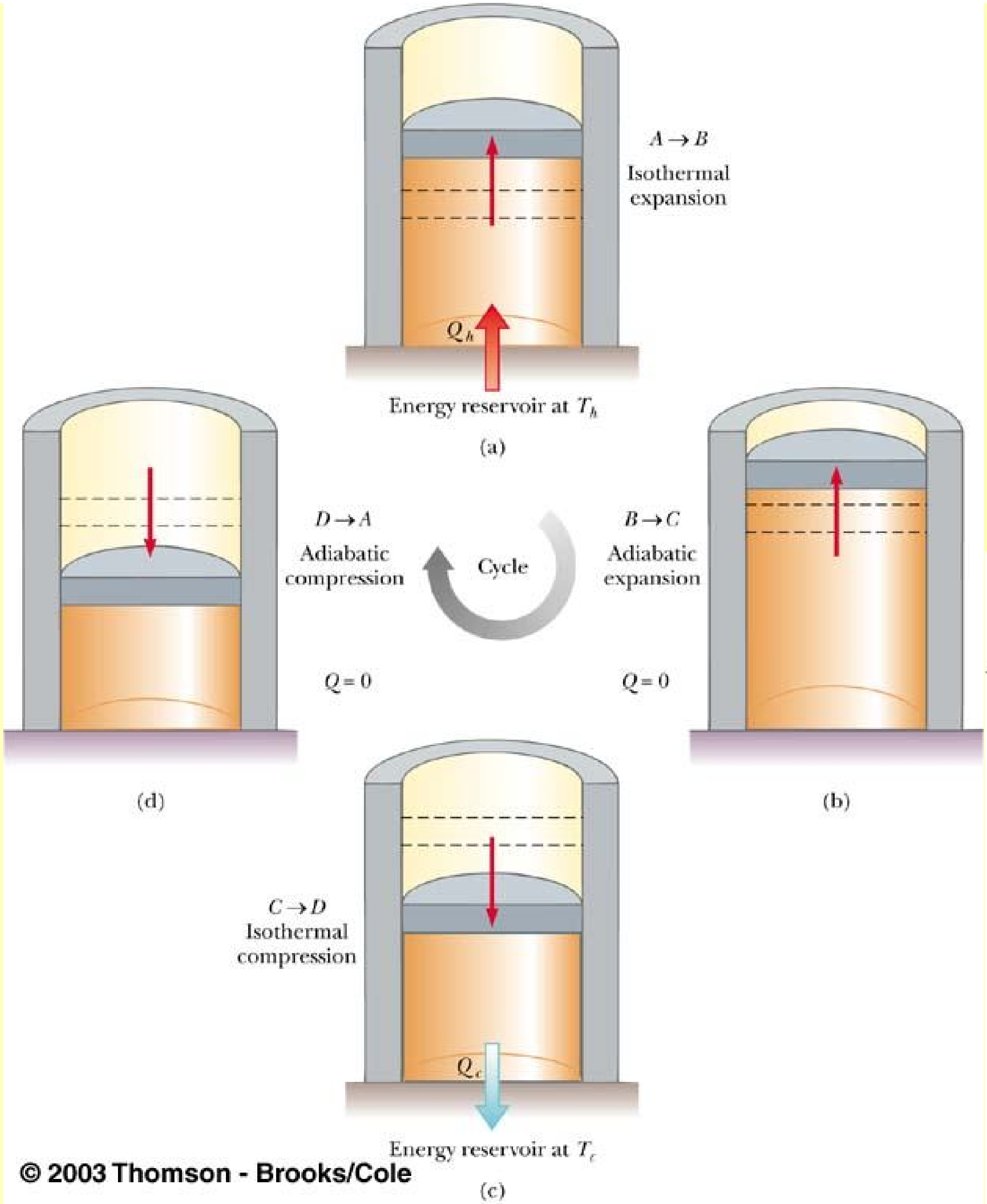
$$\frac{Q_{cold}}{T_{cold}} > \frac{Q_{hot}}{T_{hot}} \Rightarrow \frac{Q_{cold}}{Q_{hot}} > \frac{T_{cold}}{T_{hot}} \Rightarrow \text{engines}: e < 1 - \frac{T_{cold}}{T_{hot}}$$

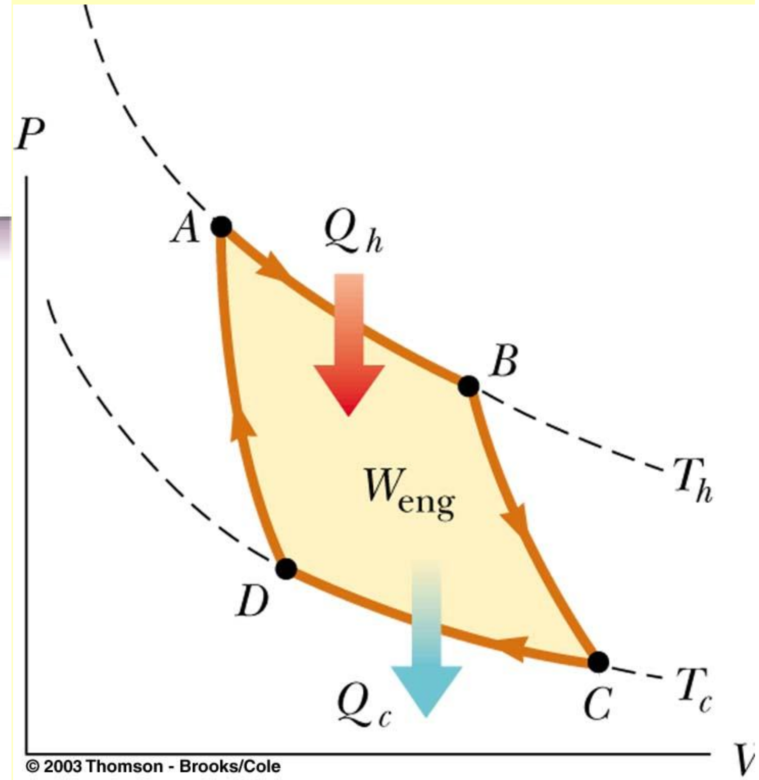
Carnot Engines

- Idealized engine
- Most efficient possible

$$e = \frac{W}{Q_{hot}} = 1 - \frac{T_{cold}}{T_{hot}}$$

Carnot Cycle





Example

An ideal engine (Carnot) is rated at 50% efficiency when it is able to exhaust heat at a temperature of 20 ºC. If the exhaust temperature is lowered to -30 ºC, what is the new efficiency.

Solution

Given, e=0.5 when T_{cold}=293 ºK, Find e when T_{cold}= 243 ºK

First, find T_{hot}

$$e = 1 - \frac{T_{cold}}{T_{hot}}$$

$$T_{hot} = T_{cold}\frac{1}{1-e}$$ = 586 ºK

Now, find e given T_{hot}=586 ºK and T_{cold}=243 ºK

$$e = 1 - \frac{T_{cold}}{T_{hot}}$$

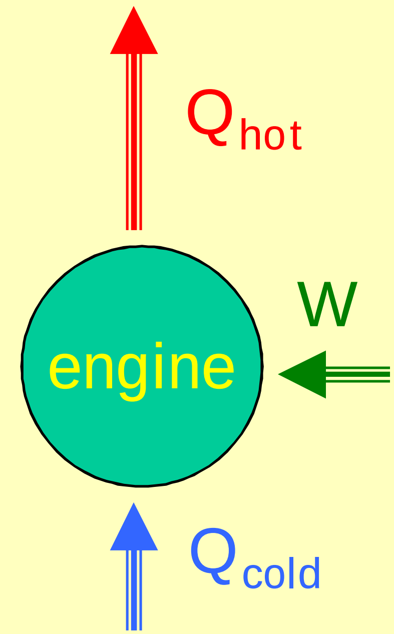
e = 0.585

Refrigerators

Given: Refrigerated region is at T_{cold}

Heat exhausted to region with T_{hot}

Find: Efficiency



$$\text{refrigerator: } e = \frac{Q_{cold}}{W} = \frac{Q_{cold}}{Q_{hot} - Q_{cold}} = \frac{1}{Q_{hot}/Q_{cold} - 1}$$

Since $\Delta S = Q/T > 0$,

$$\frac{Q_{hot}}{T_{hot}} > \frac{Q_{cold}}{T_{cold}} \Rightarrow \frac{Q_{hot}}{Q_{cold}} > \frac{T_{hot}}{T_{cold}} \Rightarrow \text{refrigerator: } e < \frac{1}{T_{hot}/T_{cold} - 1}$$

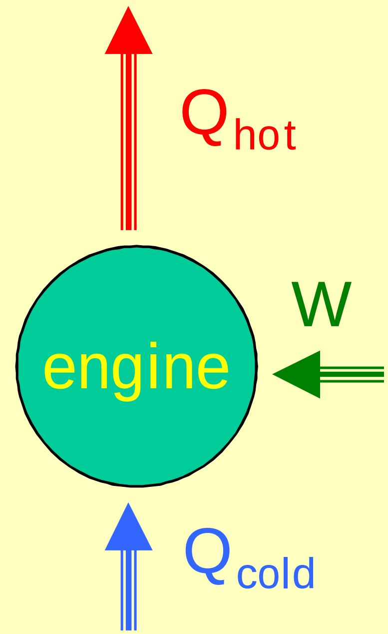
Note: Highest efficiency for small T differences

Heat Pumps

Given: Inside is at T_{hot}

Outside is at T_{cold}

Find: Efficiency



heat pump :

$$e = \frac{Q_{hot}}{W} = \frac{Q_{hot}}{Q_{hot} - Q_{cold}} = \frac{1}{1 - Q_{cold} / Q_{hot}}$$

Since $\Delta S = Q / T > 0$,

$$\frac{Q_{hot}}{T_{hot}} > \frac{Q_{cold}}{T_{cold}} \Rightarrow \frac{Q_{hot}}{Q_{cold}} > \frac{T_{hot}}{T_{cold}} \Rightarrow$$

heat pump :

$$e < \frac{1}{1 - T_{cold} / T_{hot}}$$

Like Refrigerator: Highest efficiency for small DT

Example

A modern gas furnace can work at practically 100% efficiency, i.e., 100% of the heat from burning the gas is converted into heat for the home. Assume that a heat pump works at 50% of the efficiency of an ideal heat pump.

If electricity costs 3 times as much per kw-hr as gas, for what range of outside temperatures is it advantageous to use a heat pump?
Assume $T_{inside} = 295\ ^{o}K$.

Solution

Find T for which e = 3 for heat pump.
Above this T: use a heat pump
Below this T: use gas

heat pump :

$$e < \frac{1}{1 - T_{cold} / T_{hot}}$$

$$e = 3 = 0.5 \cdot \frac{1}{1 - T / 295}$$

50% of ideal heat pump

$$T = 295 \frac{5}{6} = 245.8\,°\mathrm{K} = -27\,°\mathrm{C}$$