Section-B

CURVES

RoleofCurvesinGeometricModelling

- Curvesareusedtodrawawire-framemodel
- Curvesareutilizedtogeneratesurfaces byperformingparametrictransformationsonthem.

ParametricandNon-parametricEquations ofaCurve

Non-parametricformofequations

- Explicitnon-parametricequation

$$y =c^1 +c^2x+ c^3x^2+c^4x^3$$

$$z= d^1+ d^2x+d^3x^2+ d^4x^3$$

Thusyandzarecalculated explicitlyintermsofx.

Thereisauniquesinglevalueofthedependentvariable foreachvalueoftheindependentvariable.

- Implicitnon-parametricequation

$$(x–x^c)^2+ (y–y^c)^2 =r^2$$

Nodistinctionismadebetweenthedependentandtheindependentvariables.

Equationsofcurveareintheformf(x,y,z)=0andp(x,y,z)=0yandzcan besolvedintermsofx.

Parametricformofequations

- Parametricequationsdescribethedependentand independentvariablesintermsofaparameter.
- Parametricequationsallowgreatversatilityin constructing spacecurvesthataremulti-valuedandeasilymanipulated.
- Parametriccurvescanbedefinedinaconstrainedperiod $(0 \leq t \leq 1)$;sincecurvesareusuallyboundedin computer graphics
- parametricformisthemostcommonform ofcurve representationingeometricmodelling.
- Examplesofparametricandnon-parametricequationsare

Non-Parametric	Parametric
Circle:$x^2 + y^2 = r^2$	$x=r\cos\theta, y = r\sin\theta$
	Where,θis theparameter.

TypesofCurves

➢ AnalyticalCurves:

- canberepresentedbyasimplemathematicalequation, suchas,acircle oranellipse.
- Theyhaveafixedformandcannotbemodifiedtoachievea shapethatviolatesthemathematicalequations.

➢ SyntheticCurves

• Interpolatedcurves:

- Aninterpolatedcurveisdrawnbyinterpolatingthegivendatapoin tsandhasafixedform,dictatedbythegivendatapoints.
- Thesecurveshavesomelimitedflexibilityinshape creation,dictatedbythedatapoints.

• ApproximatedCurves:

- Thesecurvesprovidethemostflexibilityindrawingcurves ofverycomplexshapes.
- Themodelofacurvedautomobile fendercanbeeasily createdwiththehelpofapproximatedcurvesandsurfaces.

ParametricEquationsofAnalyticalCurves

1. Equationof aStraightLine:

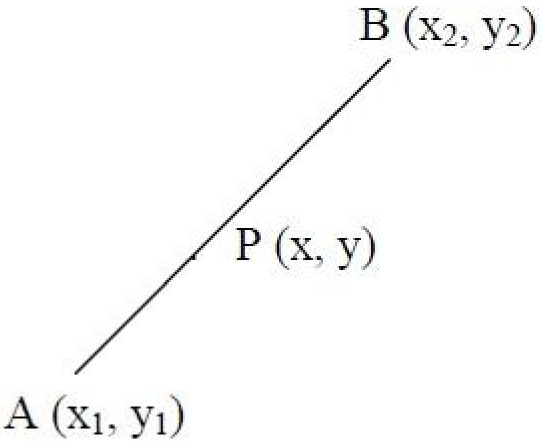
$$P(t) = A + (B-A)\, t$$

The parametric equation of line AB can be derived as,

$$x = x_1 + (x_2 - x_1)\, t$$

$$y = y_1 + (y_2 - y_1)\, t$$

where, $0 \leq t \leq 1$



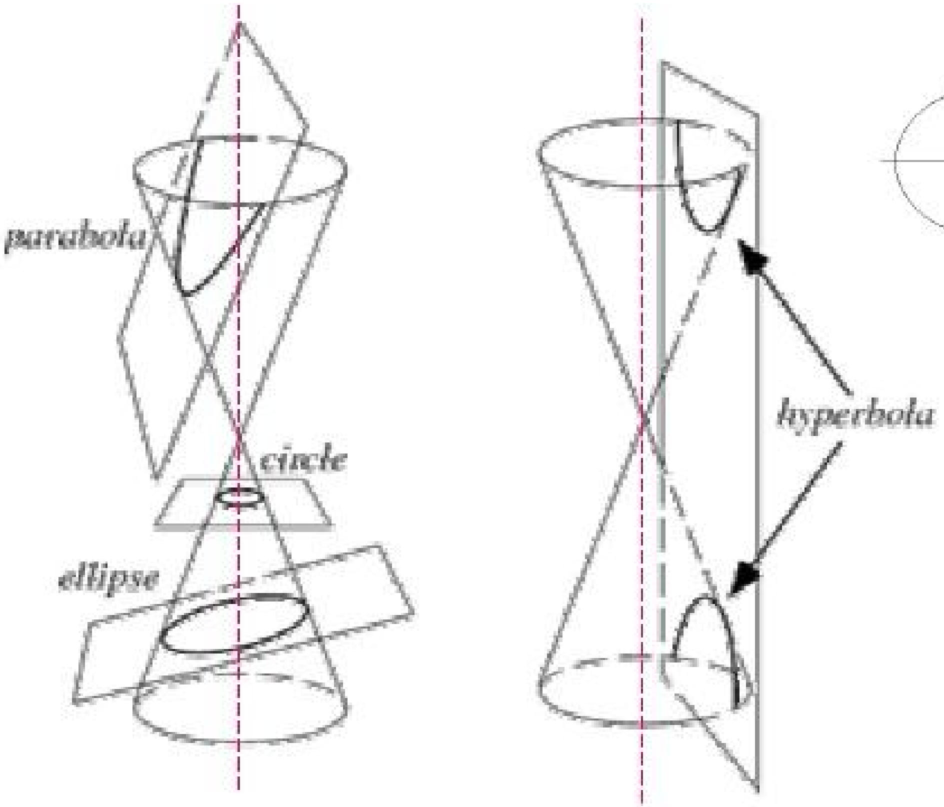
The point P on the line is sweeped from A to B,
as the value of t is varied from 0 to 1.

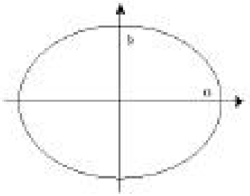
CONICSECTIONS

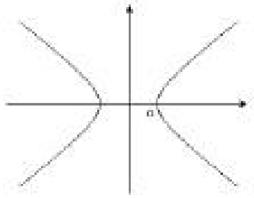
Thegeneralsecond-degreeequationfor conicsectionsis

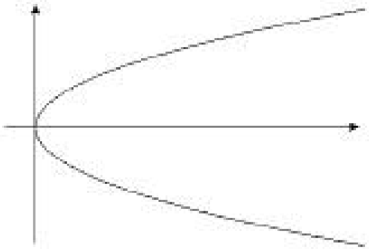
$ax^2+2bxy+cy^2+2dx+2ey+f=0$

- Bydefiningcoefficientsa,b,c, d,eandfwegetvarietyofconic sections.









ParametricEquationsofAnalyticalCurves

2. EquationofaCircle:

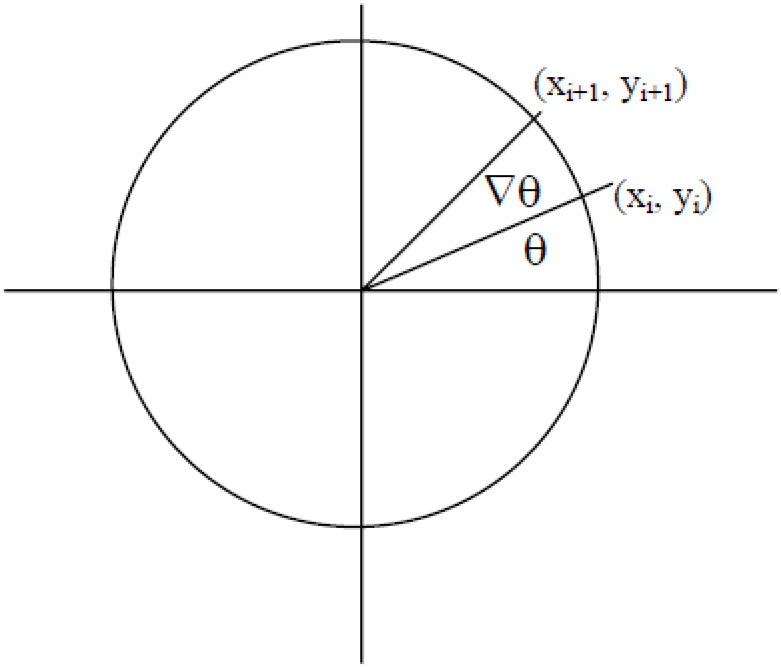
x_i=x_c

+rcosθy_i=y_c

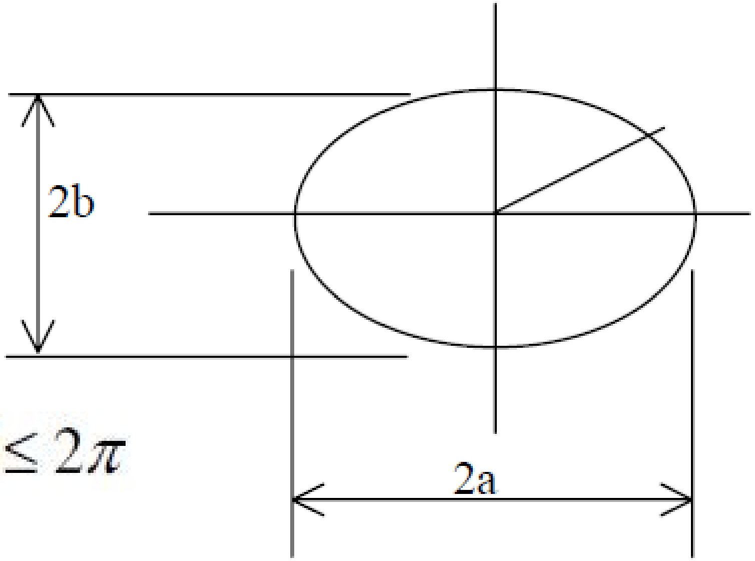
+rsinθ

$$0 \le \theta \le 2\pi$$

$$x = \frac{1-t^2}{1+t^2};\ y = \frac{2t}{1+t^2} \quad 0 \le t \le 1$$



3. EquationofanEllipse:



$$x = a\cos\theta; \qquad y = b\sin\theta \qquad 0 \le \theta \le 2\pi$$

4. EquationofParabola:

$$x = a\theta^2, \quad y = 2a\theta \qquad 0 \le \theta \le \infty$$

$$x = \tan^2\phi; \qquad y = \pm 2\sqrt{a\tan\phi} \qquad 0 \le \phi \le \tfrac{\pi}{2}$$

5. EquationofHyperbola:

$$x = \pm a\sec\theta, \quad y = \pm b\tan\theta \qquad 0 \le \theta \le \tfrac{\pi}{2}$$

$$x = a\cosh\theta, = (e^{\theta} + e^{-\theta}) \quad y = b\sinh\theta = (e^{\theta} - e^{-\theta}) \qquad 0 \le \theta \le \infty$$

ParametricEquationsofSyntheticCurves

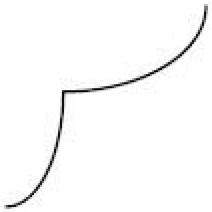
- Typesofsyntheticcurves

1. Cubiccurve(Cubicspline)
2. Beziercurve
3. B-splineCurve

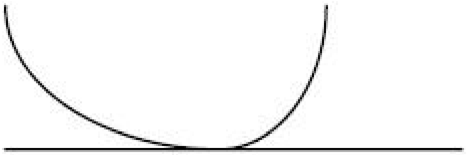
- Typesofcontinuity:

1. Zeroorderparametriccontinuity
2. Firstorderparametriccontinuity
3. Secondorderparametriccontinuity

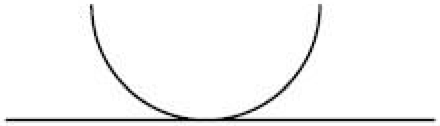
- Variouscontinuityrequirementsatthedatapointscanbespecifiedto imposevariousdegreesofsmoothnessofthecurve.
- Acomplexcurvemayconsistofseveralcurvesegmentsjoined together.
- Smoothnessoftheresultingcurveisassuredbyimposingoneofthe continuityrequirements.
- Azeroordercontinuity(C^0)assuresacontinuouscurve.
- Firstorder continuity(C^1)assuresacontinuousslope.
- Secondordercontinuity(C^2)assuresacontinuouscurvature.



C^0 Continuity – The curve is Continous everywhere



C^1 Continuity- Slope Continuity at the common point



C^2 Continuity - Curvature continuity at the common point

CURVEREPRESENTATION

- Curverepresentationmustbemathematically tractableandcomputationallyconvenient.
- Importantpropertiesforcurvedesigningand representation:
  - Controlpoints
  - Axisindependence
  - Localcontrol&Globalcontrol
  - Variationdiminishingproperty
  - Versatility
  - Orderofcontinuity

SYNTHETICCURVES

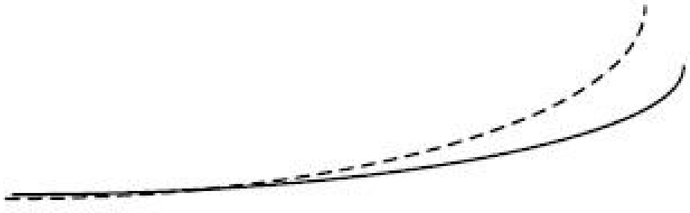
1. Cubiccurve(Cubicspline)
2. Beziercurve
3. B-splineCurve

HermiteCubicSpline

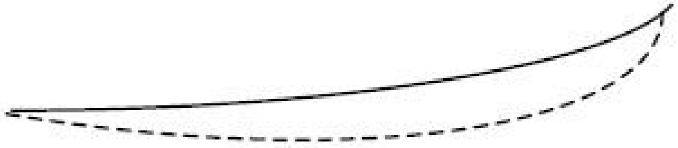
- Hermitecubiccurveisalsoknownasparametric cubiccurve,andcubicspline.
- Thiscurveisusedtointerpolategivendatapoints thatresultinasyntheticcurve,butnotafreeform,unliketheBezierandB-splinecurves.
- Thecurveisdefinedbytwodatapointsthatlieat thebeginningandattheendofthecurve,alongwiththeslopesatthesepoints.
- Itisrepresentedbyacubicpolynomial.
- Whentwoendpointsandtheirslopesdefinea curve,thecurveiscalledaHermitecubiccurve.
- Severalcubicsplinescanbejoinedtogetherbyimposingtheslopecontinuityatthecommon points.

Indesignapplications,cubicsplinesarenotaspopularasthe BezierandB-splinecurves.Therearetworeasonsforthis:

- Thecurvecannotbemodifiedlocally,i.e.,whenadatapointismoved,theentirecurveisaffected,resultinginaglobalcontrol.
- Theorderofthecurveisalwaysconstant(cubic),regardlessofthenumberofdatapoints.Increaseinthenumberofdatapointsincreasesshapeflexibility,However,this requiresmoredatapoints,creatingmoresplines,thatarejoinedtogether(onlytwodatapointsandslopesareutilizedforeachspline).



Effect of Moving the Data Point



Effect of Change in slope

EquationofHermite CubicSpline

Acubicsplineisathird-degreepolyno1nial,definedas

P(t)= aie where, $0 \leq t \leq 1$

where, $0<t\leq 1$,andP(t)isapointonthecurve.
Expandingtheaboveequation,weget

$P(t) = a_3 t^3 + a_2 t^2 + a_1 t + a_0$where, 0<t<I

If(x,y,z)arethecoordinatesofpointP

$$x(t) = a_{3x} t^3 + a_{2x} t^2 + a_{1x} t + a_{0x}$$

$$y(t) = a_{3y} t^3 + a_{2y} t^2 + a_{1y} t + a_{0y}$$

$$z(t) = a_{3z} t^3 + a_{2z} t^2 + a_{1z} t + a_{0z}$$

Thereare12unknowncoefficients ,ai;, knownasthealgebraiccoefficients.Thesecoefficientscanbe evaluatedbyapplyingthebounda1yconditionsattheendpoints.Fron1thecoordinatesoftheendpointsofeachsegn1ent,sixofthetwelveneededequationsareobtained.Theothersixequationsarefoundbyusingthetangentvectorsatthetvvoends ofeachsegrnent.Substitutingthebounda1yconditionsatt=0,andt=1,weget,

$P(O)=ao$,and

$$P(t) = a_3 t^3 + a_2 t^2 + a_1 t + a_0$$

$P(l)=a3+a2+a1+ao$

Tofir1dthetangentvectors,wedifferentiateequation

$P'(O)=a1$

$$P'(t)=3a_3t^2+2a_2t+a_i$$

$$P'(1) = 3 a_3 + 2 a_2 + a_1$$

Solvingforthecoefficientsinte1msoftheP(t)ai1dP'(t)valuesiI1eql1ations

$ao=P(O)$

$a1=P'(O)$

$a2=-3P(O)+3P(l)-2P'(O)-P'(l)$

$a3=2P(O)-2P(l)+P'(O)+P'(l)$

In111atrixf01n1 tl1eeq11ationcanbevvrittenas,

$$P(t) \;=\; [t^3 \quad t^2 \quad t \quad I] \begin{vmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & I & 0 \\ I & 0 & 0 & 0 \end{vmatrix} \begin{vmatrix} P(O) \\ P(I) \\ P'(O) \\ P'(l) \end{vmatrix}$$

Tl1eeq11ationi11short fo1111canbewrittenas: $P(t)=[t][M]H[G]$

[M]HiscalledHe11nitematrixofaCl1bicspline

BezierCurve

- In1960s,theFrenchengineerP.Bezier,whileworking fortheRenaultautomobilemanufacturer,developeda systemofcurvesthatcombinethefeaturesofboth interpolatingandapproximatingpolynomials.
- EquationoftheBeziercurveprovidesanapproximate polynomialthatpassesnearthegivencontrolpoints andinterpolatethefirstandlastpoints.
- Severalcurvescanbecombinedandblendedtogether.
- AdvantageofBeziercurveovercubicsplinecurveis thatthedirectionofthecurveatthejointscanbe defined&changedsimplyby specifyingthepositionof controlpoints.

Propertiesof BezierCurve

- Passesthroughfirst&lastcontrolpoints.Iftheycoincide,curveisclosedcurve.
- Thecurveistangenttothe correspondingedgeofcontrol pointsattheendpoints.
- Convexhullproperty.
- Donotoscillate.Variationdiminishingproperty.
- Comparedtocubicsplinecurves,itrequireslesscalculations &lessmemory.
- ABeziercurveisindependentofthecoordinatesystemused tomeasurelocationofthe controlpoints.Axis independence property.
- Globalcontrolproperty.
- Canusezero,firstandhigherordercontinuity.
- Bezier’sblendingfunctionproducesannthdegreepolynomial forn+1 controlpoints

MathematicallyBeziercurveisrepresentedas

$$P(t) = \sum_{i=0}^{n} B_i \, J_n \, I(t)$$

WhereJ_n, $I(t)$isa blendingfunction

$$J_n, I(t) = {}^{n}C_i t^i (1-t)^{n-i}$$

And${}^{n}C_i$is thebinomialcoefficient

$${}^{n}C_i = \frac{n!}{i!\,(n-i)!}$$

If3-Dlocationof controlpointB_i is(x_i, y_i, z_i)then

$$x(t) = \sum_{i=0}^{n} x_i \, J_n \, I(t)$$

$$y(t) = \sum_{i=0}^{n} y_i \, J_n \, I(t)$$

$$z(t) = \sum_{i=0}^{n} z_i \, J_n \, I(t)$$

- **ThirdOrderBezierPolynomial**

WewillsimplifytheBezier’sequationforn=3(acubiccurve). Theproceduredevelopedherecanbeextendedtotheother valuesofn.

Forn=3,wewillhavefourcontrolpoints,namely,B_{0}, B_{1},B_{2},B_{3}. iwillvaryfrom0to3.TheBezier’sequation,

$$P(t) = \sum_{i=0}^{3} B_i \, J_n \, I(t)$$

Afterexpansionitwill become

$P(t)=J_{0,3}B_0+J_{1,3}B_1+J_{2,3}$ $B_2+J_{3,3}B_3$

where

$$J_{0,3} = \frac{3!}{0!\ 3!} \ t^0 (1-t)^3 = (1-t)^3$$

$$J_{1,3} = \frac{3!}{1!\ 2!} \ t^1 (1-t)^2 = 3t\ (1-t)^2$$

$$J_{2,3} = \frac{3!}{2!\ 1!} \ t^2 (1-t)^1 = 3t^2 (1-t)$$

$$J_{3,3} = \frac{3!}{3!\ 0!} \ t^3 (1-t)^0 = t^3$$

SubstitutingvaluesofJ_n, *I(t)inequationof P(t)*
Weget,

$$P(t)=(1-t)^3B_0+3t(1-t)^2B_1+3t^2(1-t)B_2+t^3B_3$$

Inmatrixformthisequationis writtenas

$$P(t) = [t^3 \quad t^2 \quad t \quad 1] \begin{pmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} B_0 \\ B_1 \\ B_2 \\ B_3 \end{pmatrix}$$

B-SplineCurve

- B-splinecurvesuseablendingfunction,which generatesasmooth,singleparametric polynomialcurvethroughanynumberofpoints.
- TogenerateaBeziercurveofthesamequalityof smoothness,we willhavetouseseveralpiecesof Beziercurves.
- UnliketheBeziercurve,thedegreeofthepolynomialcanbe selectedindependentlyofthe numberofcontrolpoints.
- Thedegreeof theblendingfunctioncontrolsthedegreeoftheresultingB-splinecurve.

- Thecurvehasgoodlocalcontrol.

- Themathematicalderivationofthe B-splinecurveis complex.The equationis of theform:

$$P(t) = \Sigma N_{i,k}(t) V_i$$

$$\boxed{\mathbf{C}(u) = \sum_{i=0}^{n} N_{i,p}(u)\mathbf{P}_i}$$

Where, P(t) is a point on the curve.
i indicates the position of control point i
k is order of curve
$N_{i,k}$ (t) are blending functions
V_i are control points

The matrix form of the uniform cubic B-spline curve is:

$$P_i(t) = 1/6[t^3 \quad t^2 \quad t \quad 1] \begin{pmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 0 & 3 & 0 \\ 1 & 4 & 1 & 0 \end{pmatrix} \begin{pmatrix} V_{i-1} \\ V_i \\ V_{i+1} \\ V_{i+2} \end{pmatrix}$$

$$C(it) == L_{i=0}^{n} Nilp(v_{1})Pi$$

$$;..o(u) \begin{cases} 1 & \text{if } u_i \leq u < u_{i+1} \\ 0 & \text{othcr\textbackslash visc} \end{cases}$$

$$N_{i,p}(u) \quad \frac{'It-Ui}{Ui+t>-Ui} \text{---JM.p-1(tl)} + \frac{tL,:+p+1-tL}{Ui\text{-}v+l-Ui+l} \text{i-Lp-1(u)}$$

Surfaces

AnalyticSurfaces

PlaneSurface

- Planeisdefinedby3points

RuledSurface

- Ruledsurfaceis generatedby joining correspondingpointsontwospacecurvesbystraightlines.
- MaincharacteristicofaruledsurfaceisthatthereisatleastonestraightlinepassingthroughthepointP(u,v)andlyingentirelyinthesurface.
- Eg.Cones,cylinders

SurfaceofRevolution

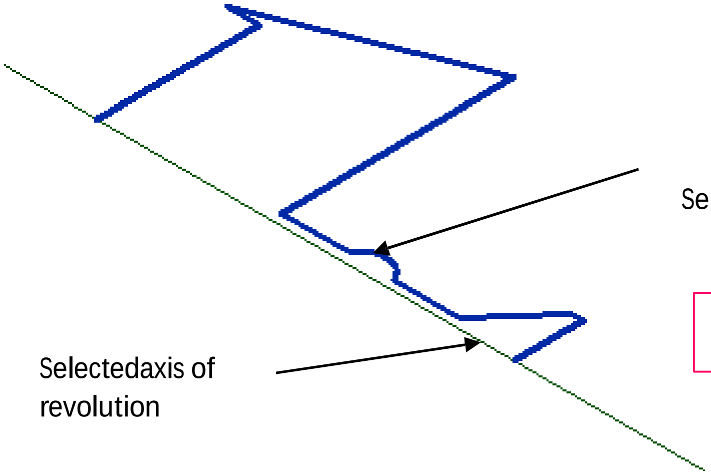
- Databaserequiresprofilecurve,axisofrotationand angleofrotationasstartingandendingangles.
- Planercurveiscalledprofile.
- Circlesinperpendicularplanearecalledparallels.
- Variouspositionsoftheprofilearoundtheaxisare calledmeridians,

TabulatedSurface/Cylinder

- Thisis defined as the surfacethat resultsfrom translatingaspace planercurve alongagiven direction.
- i.e.surfacegeneratedbymovingastraightline (calledgeneratrix)alongagivenplanercurve (called directrix).

REVSURFCommand

Itcreatesarevolvedsurfaceaboutaselected axis.
SURFTAB1specifiesthenumberoftabulationlines thataredrawninthe directionofrevolution.
SURFTAB2specifiesthenumberoftabulationlines thataredrawntodivide itintoequal-sizedintervals

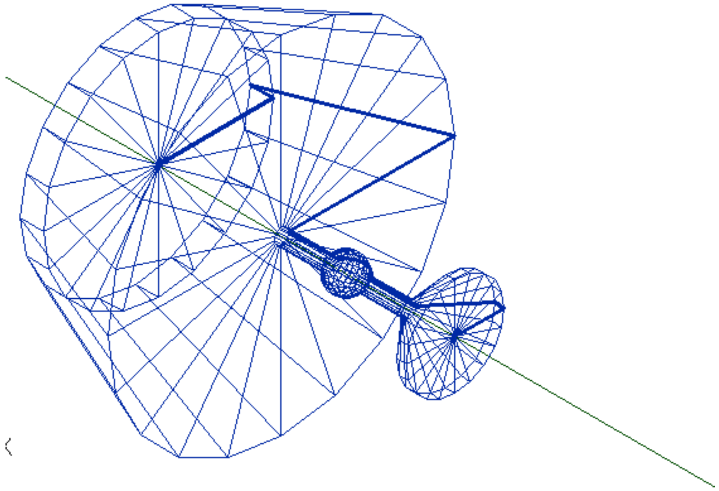


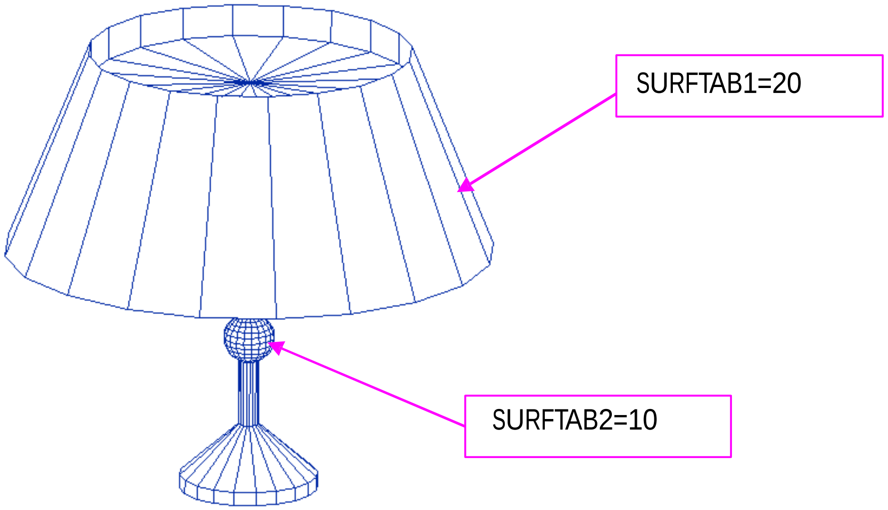
Selectedobject to revolve

Itmustbe a polyline!

Selectedaxis of revolution

NOTE!2D objectsaredrownonlyinthex-yplane.Itis importanttochoose properlocationandorientationofthecoordinatesystem.



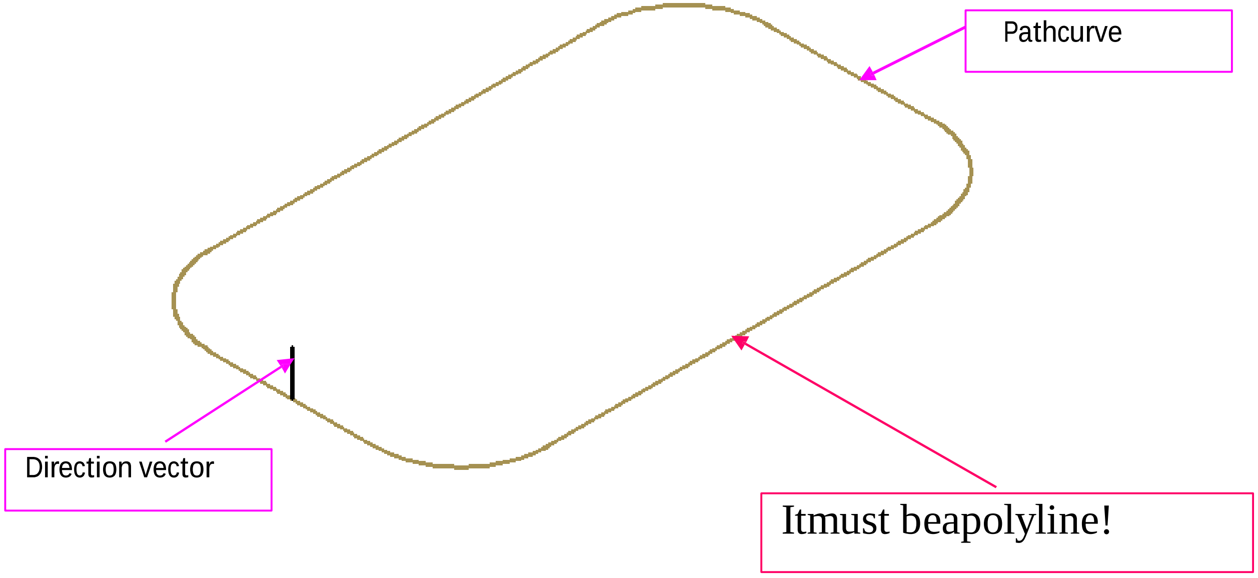


SURFTAB1=20

SURFTAB2=10

TABSURFCommand

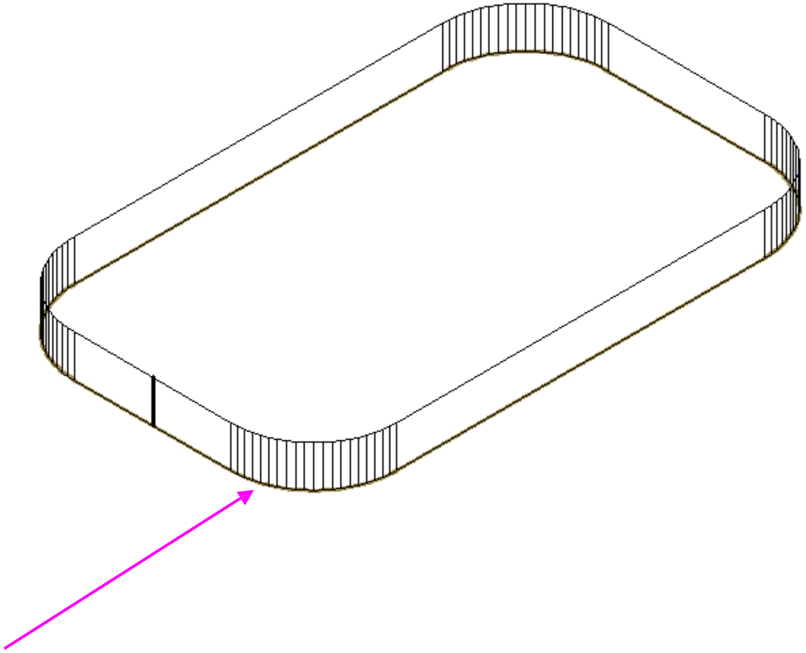
Itcreatesatabulatedsurfacefromapathcurveandadirectionvector.



Pathcurve

Direction vector

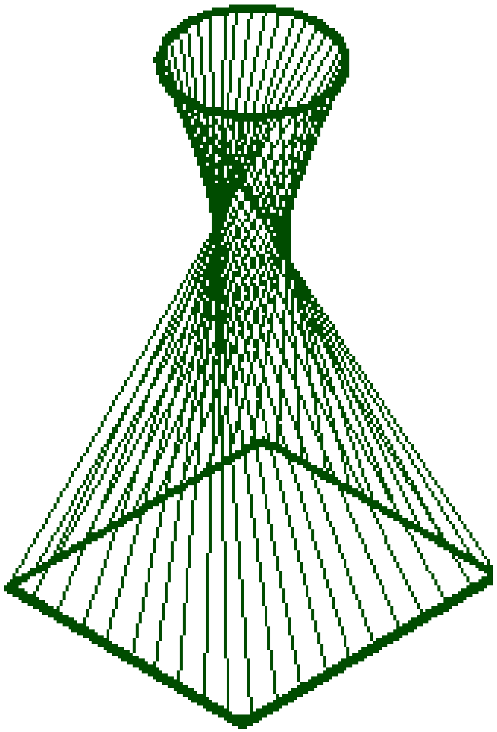
Itmust beapolyline!

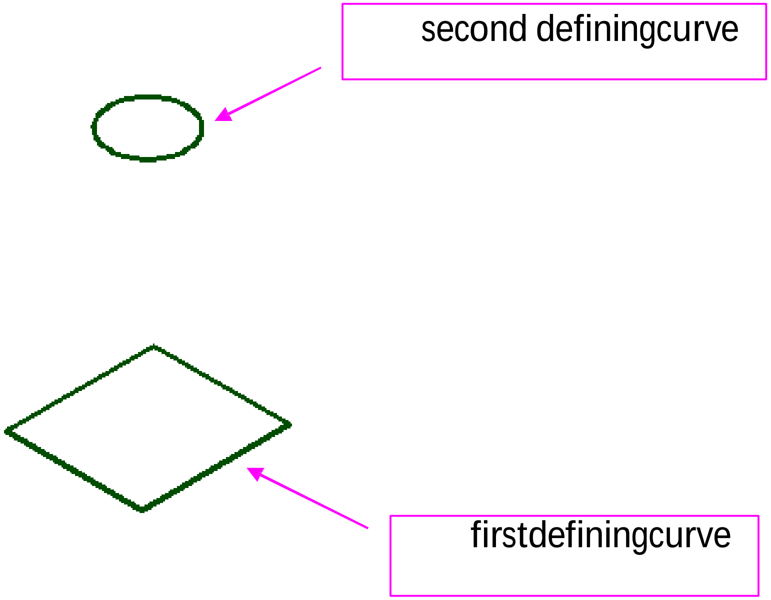


AutoCAD draws
tabulationlineswhich
dividethepathcurve
intointervalsofequal
sizesetbySURFTAB1

RULESURFCommand

Itcreatesaruledsurfacebetweentwocurves.RULESURFconstructsapolygon mesh representingtheruledsurfacebetweentwocurves.



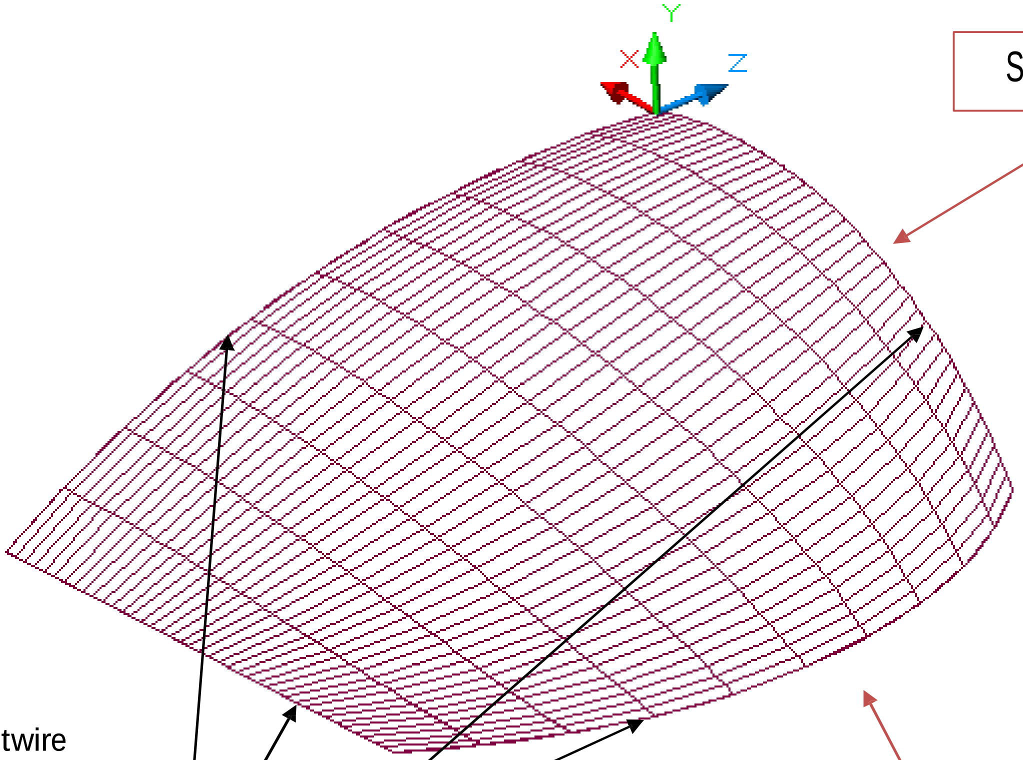


second definingcurve

firstdefiningcurve

EDGESURFCommand

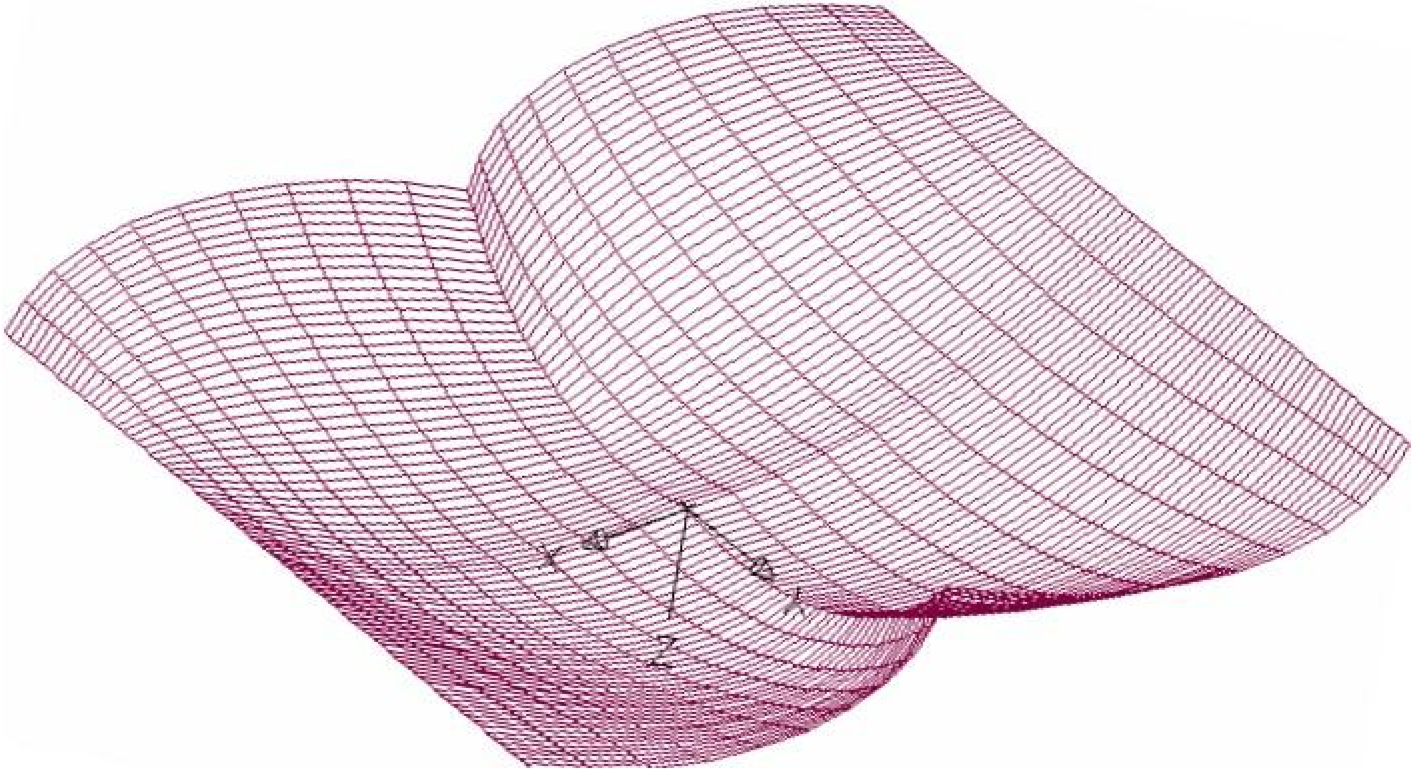
EDGESURFconstructsathree-dimensional(3D)polygonmeshapproximatingaCoons surfacepatch meshfromfour adjoiningedges.ACoonssurfacepatchmeshis a bicubic surfaceinterpolatedbetweenfouradjoiningedges(whichcan begeneralspacecurves). TheCoonssurfacepatch meshnotonlymeetsthecorners ofthedefiningedges,butalso toucheseachedge,providingcontrolovertheboundariesofthegeneratedsurfacepatch.



SURFTAB1

SURFTAB2

```
Command:EDGESURFCurrentwire
framedensity:SURFTAB1=40SURF
TAB2=10
Selectobject1forsurfaceedge:Selecto
bject2forsurfaceedge:Selectobject3f
orsurfaceedge:Selectobject4forsurfa
ceedge:
```


Y
X
Z

SYNTHETICSURFACES

- Thesurfaceentitieswhicharedefinedbythesetofdata pointsareknownassyntheticsurfaces.
- Theseareneededwhenasurfaceisrepresentedbya collectionofdatapoints.
- Representedbythepolynomials.
- Usedforrepresentingprofilesof: car bodies,shiphulls,air planewings,propellerblades,etc.
- Typesofsyntheticsurfaces:

1. Hermitebicubicsurface 2.Beziersurface

3.B-Splinesurface 4.Coonssurface

5. Blendingsurface 6.Offsetsurface

7.FilletSurface

HermiteBicubicSurface

- Theparametricbicubicsurfacepatchconnectsfourcorner datapointsandutilizesabicubicequation.

$$P(u,v)= \sum_{i=0}^{3}\sum_{j=0}^{3} \mathbf{C}_{ij}\mathbf{u}_{i}\mathbf{v}_{j} \qquad 0\leq u\leq 1, 0\leq v\leq 1$$

Inmatrixform

$$P(u,v)=U^{T}*C+V \qquad 0\leq u\leq 1, 0\leq v\leq 1$$

where $U=[u^3 u^2 u 1]^T, V=[v^3 v^2 v 1]^T$

andCoefficientMatrix[C]isgivenby

$$\begin{bmatrix} C_{33} & C_{32} & C_{31} & C_{30} \\ C_{23} & C_{22} & C_{21} & C_{20} \\ C_{13} & C_{12} & C_{11} & C_{10} \\ C_{03} & C_{02} & C_{01} & C_{00} \end{bmatrix}$$

BezierSurface

- ABéziersurfaceisdefinedbyatwo-dimensionalsetofcontrol points**p**$_{i,j}$, where*i*isintherangeof0and*m*,and*j*isintherangeof0 and*n*.Thus,inthiscase,wehave*m*+1rowsand*n*+1columnsof controlpointsandthecontrolpointonthe*i*-throwand*j*-th columnis denotedby**p**$_{i,j}$.Notethatwehave(*m*+1)(*n*+1)controlpointsintotal.
- ThefollowingistheequationofaBéziersurfacedefinedby*m*+1rows and*n*+1columnsofcontrolpoints:

$$\mathbf{p}(u,v) = \sum_{i=0}^{m} \sum_{j=0}^{n} B_{m,i}(u) B_{n,j}(v) \mathbf{p}_{ij}$$

- where$B_{m,i}$(*u*)and$B_{n,j}$(*v*)arethe*i*-thand*j*-thBézierbasisfunctionsin the*u*-and *v*-directions,respectively.

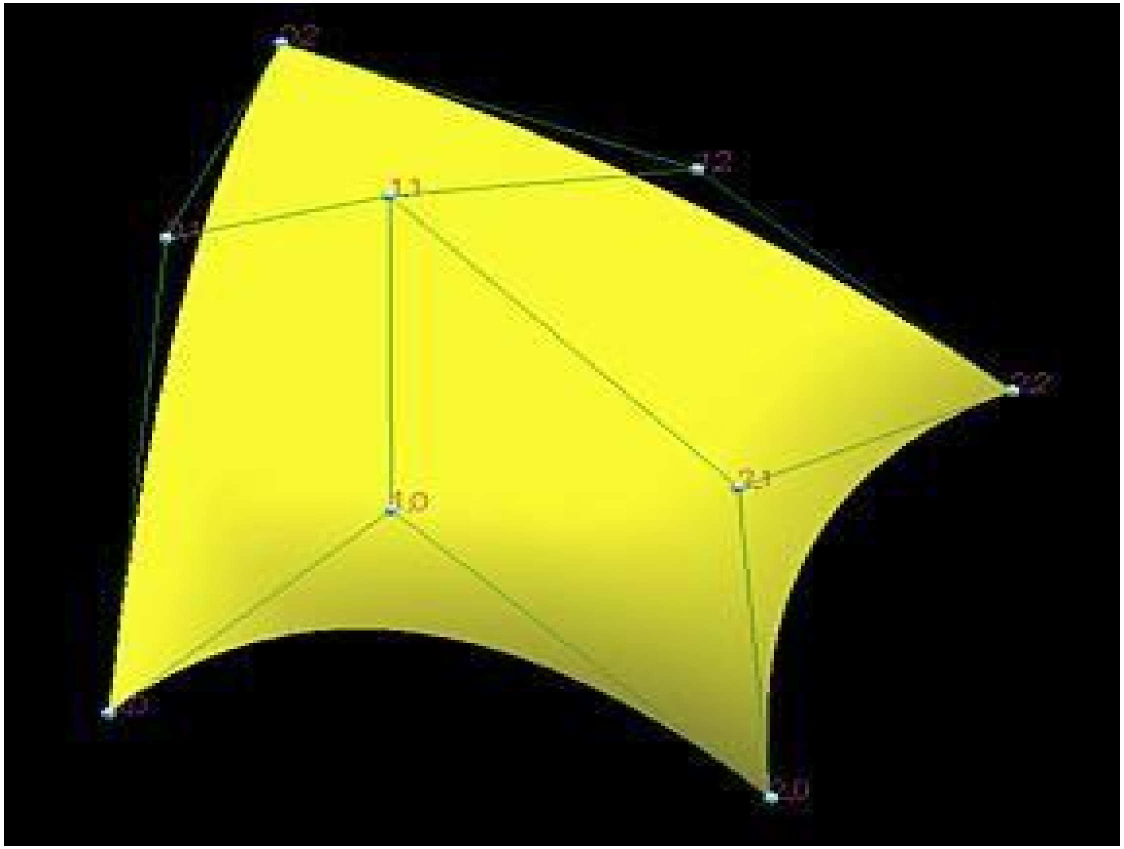
$$B_{m,i}(u) = \frac{m!}{i!(m-i)!} u^{i}(1-u)^{m-i}$$

$$B_{n,j}(v) = \frac{n!}{j!(n-j)!} v^{j}(1-v)^{n-j}$$

Since $B_{m,i}(u)$ and $B_{n,j}(v)$ are degree *m* anddegree*n*functions,weshallsaythisisa***Béziersurfaceof degree(m,n)***.Thesetofcontrolpointsisusuallyreferredtoasthe***Béziernet***or***controlnet***.Notethatparameters*u*and*v*areintherangeof0and1andhence a

Bézier surface maps the unit square to a rectangular Surface patch.

ThefollowingfigureshowsaBéziersurfacedefinedby3rowsand3columns(*i.e.*,9)controlpointsandhenceisaBéziersurfaceofdegree(2, 2).



PropertiesofBezierSurface

- **p(*u*,*v*)passesthroughthecontrolpointsatthefour cornersofthecontrolnet:$\mathbf{p}_{0,0}$,$\mathbf{p}_{m,0}$,$\mathbf{p}_{m,n}$and$\mathbf{p}_{0,n}$.**
  Infact,wehave **p**(0,0)=$\mathbf{p}_{0,0}$,**p**(1,0)=$\mathbf{p}_{m,0}$,**p**(0,1) =$\mathbf{p}_{0,n}$and**p**(1,1)=$\mathbf{p}_{m,n}$.

- **Nonnegativity:$B_{m,i}(u)B_{n,j}(v)$isnonnegativefor all*m*,*n*,*i*,*j*and *u*and*v*intherangeof0and1.**

- **PartitionofUnity:Thesumofall$B_{m,i}(u)B_{n,j}(v)$is1for all*u*and*v*intherangeof0and1.**
  Moreprecisely,thismeansforany pairof*u*and*v*inthe rangeof0and1,thefollowingholds:

$$\sum_{i=0}^{m}\sum_{j=0}^{n} B_{m,i}(u)B_{n,j}(v) = 1$$

- **ConvexHullProperty:aBéziersurfacep(*u*,*v*)liesinthe convexhulldefined byitscontrolnet.**
  Since**p**(*u*,*v*)isthelinearcombinationofallitscontrol pointswithpositive coefficientswhosesumis 1(partition ofunity),thesurfacelies intheconvexhullofitscontrol points.

- **VariationDiminishingProperty:**
  Nosuchthing exists forsurfaces.

B-Splinesurface

Iffollowinginformationisgiven:

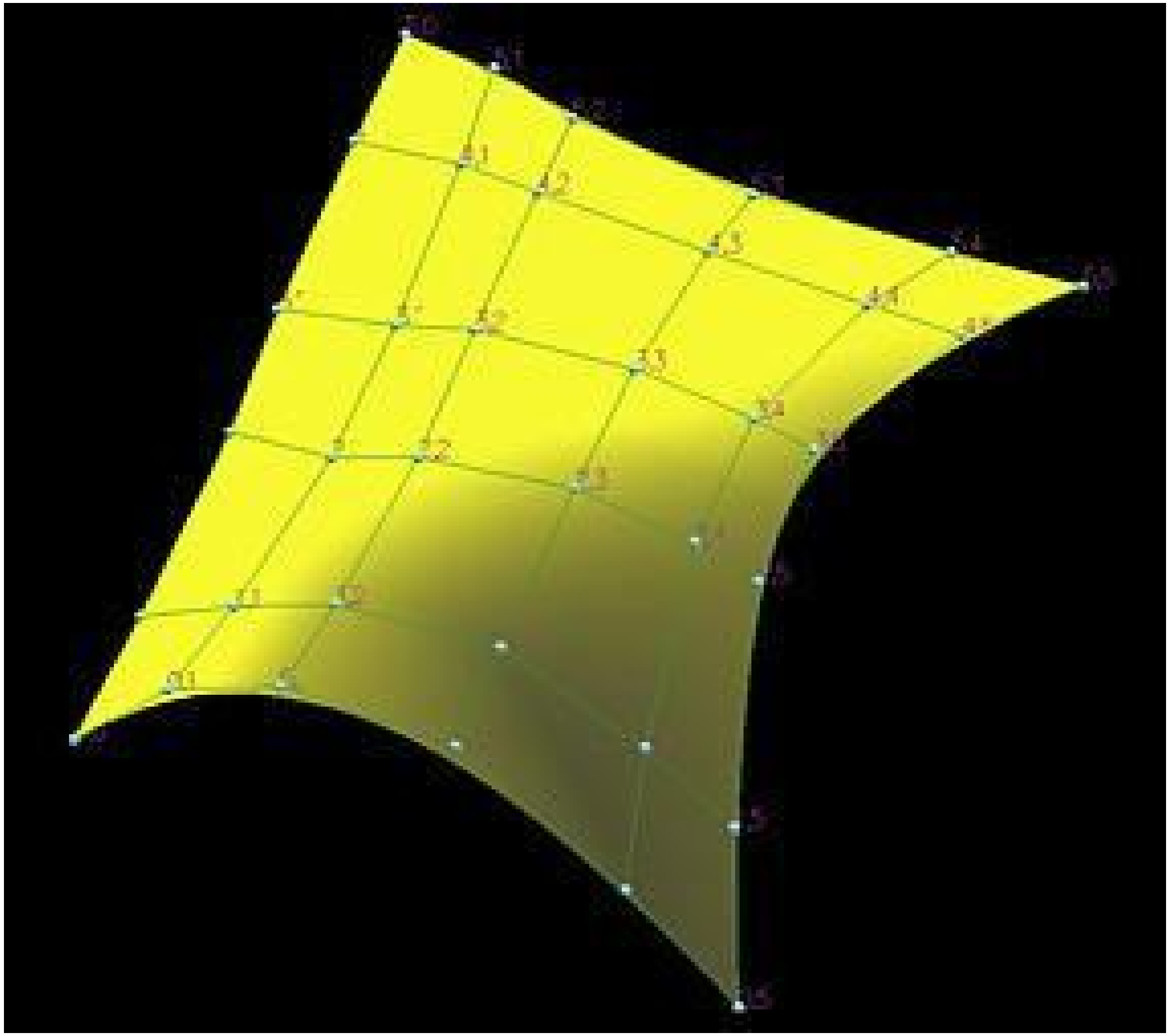
- asetof*m*+1rowsand*n*+1columnarray ofcontrolpoints$\mathbf{p}_{i,j}$,where0 <=*i*<= *m*and0<=*j*<= *n*;
- aknot vectorof*h*+1knotsinthe*u*-direction,$U=\{u_0,u_1,....,u_h\}$;
- aknotvectorof*k*+1knotsinthe*v*-direction,$V=\{v_0,v_1,....,v_k\}$;
- thedegree*p*inthe*u*-direction;and
- thedegree*q*inthe*v*-direction;

TheB-splinesurfacedefined bytheseinformationis:

$$\mathbf{p}(u,v)=\sum_{i=0}^{m}\sum_{j=0}^{n}N_{i,p}(u)N_{j,q}(v)\mathbf{p}_{i,j}$$

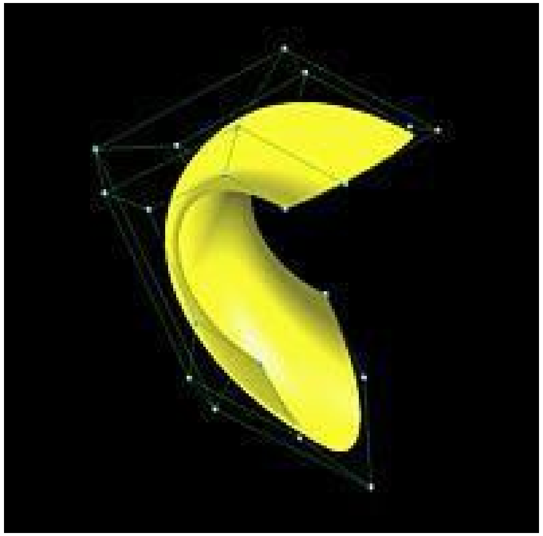
where$N_{i,p}(u)$and$N_{j,q}(v)$areB-spline basis functionsofdegree*p*and*q*, respectively.

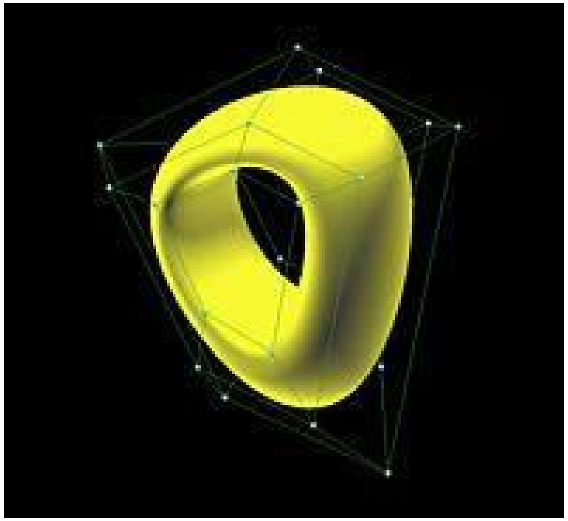
ThefollowingfigureshowsaB-splinesurfacedefinedby6 rowsand6columnsofcontrolpoints.

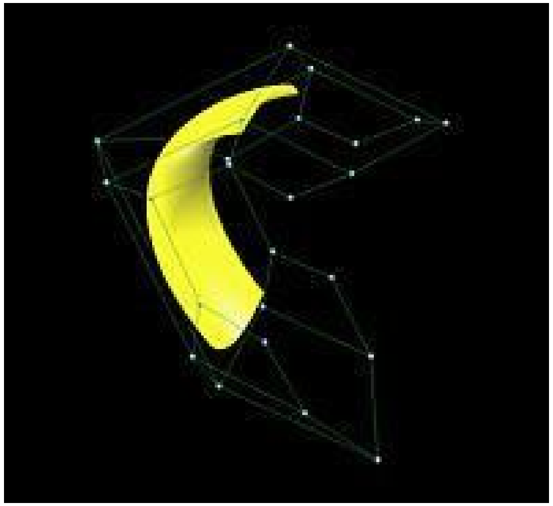


Clamped,Closedand OpenB-splineSurfaces

- SinceaB-splinecurvecanbeclamped,closedoropen,a B-splinesurfacecanalsohavethreetypes*ineachdirection*
- The followingfiguresshowthreeB-splinesurfaces clamped,closedandopeninbothdirections.Allthreesurfacesaredefinedonthesamesetofcontrolpoints; but,asinB-splinecurves,theirknotvectorsaredifferent.







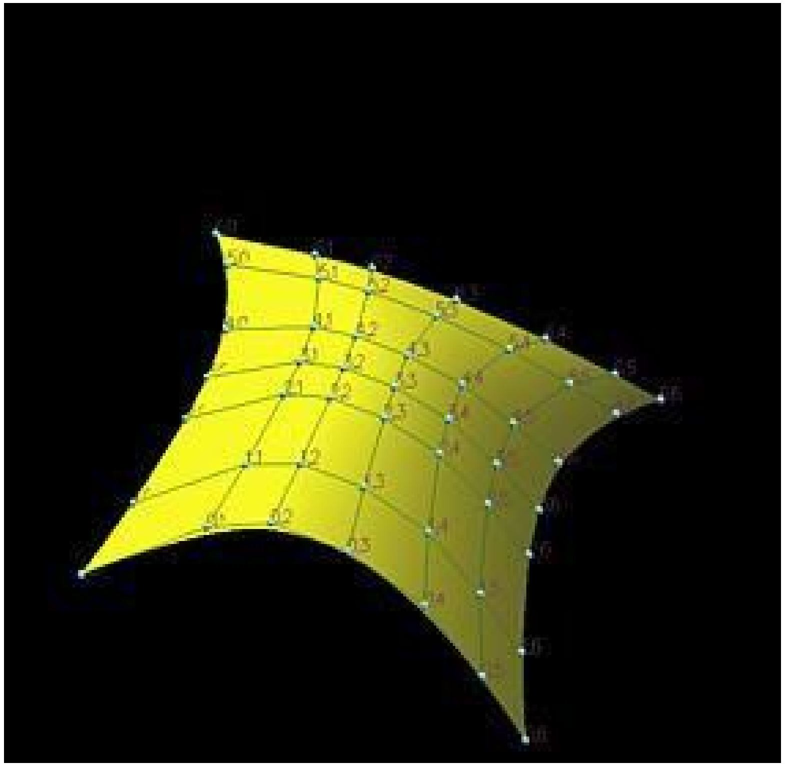
PropertiesofB-SplineSurface

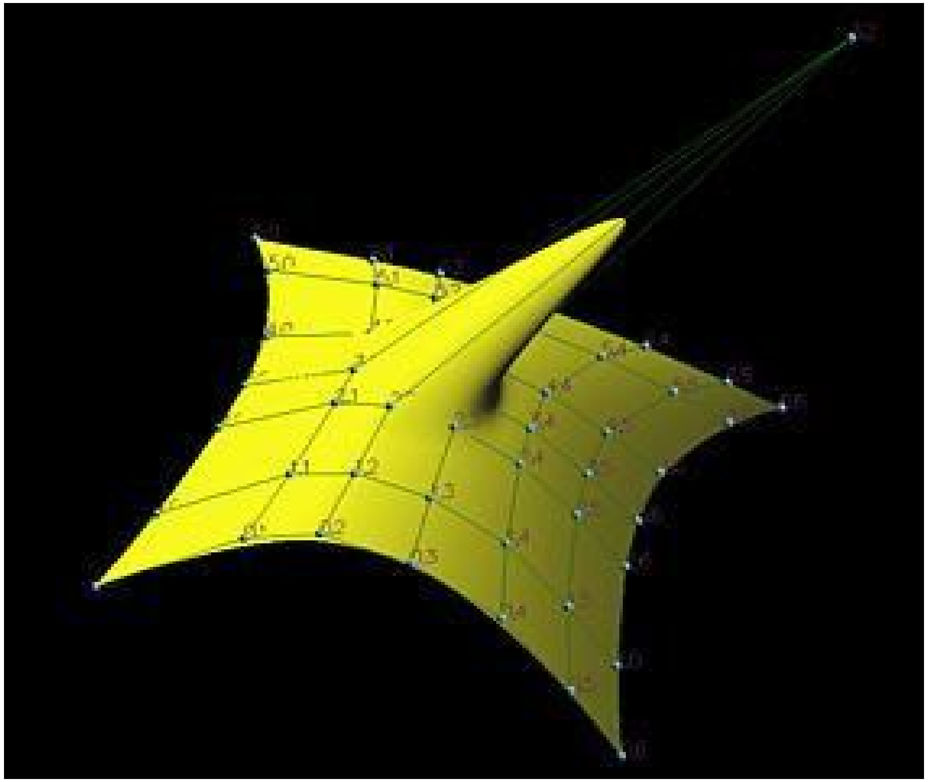
- Nonnegativity:$N_{i,p}(u)N_{j,q}(v)$isnonnegativeforall$p$,q,i ,$j$and$u$and$v$intherangeof0and1.
- PartitionofUnity:Thesumofall$N_{i,p}(u)N_{j,q}(v)$is1forall$u$and vi

$$\sum_{i=0}^{m}\sum_{j=0}^{n} N_{i,p}(u)N_{j,q}(v) = 1$$

- StrongConvexHullProperty:if(u,v)isin[u_i,u_{i+1})x[v_j,v_{j+1}),thenp(u,v) liesintheconvexhulldefinedbycontrol pointsp$_{h,k}$,where$i$-p<=h<=$i$and$j$-q<=k<=j.
- **VariationDiminishingProperty:** Nosuchthingexistsforsurfaces.

- If$m = p$,n=q,and$U$={0,0,..., 0,1,1,....,1 },thena B-splinesurfacebecomesaBéziersurface.
- LocalModificationScheme:$N_{i,p}(u)N_{j,q}(v)$iszeroif(u,v) is outsideoftherectangle[u_i,u_{i+p+1})x[v_j,v_{j+q+1})





CoonsSurface

- Coonspatch interpolatesto aninfinite numberof datapoints,thatis, to allpointsof acurve segment,to generatethe surface.

- Coonspatch is particularly usefulinblendingfour prescribedintersecting curves{ P(u,0),P(1,v), P(u,1),P(0,v)}which forma closedboundary.

- Itis assumedthatu &vrangefrom0to 1 along theseboundariesand that each pairofopposite boundarycurvesareidenticallyparameterized.

BlendingSurface

- Thisisasurfacethatconnectstwononadjacentsurfaces orpatches.
- Theblendingsurfaceisusuallycreatedtomanifest $C^0$and C^1continuitywiththetwogivenpatches.

FilletSurface

- ThisisaB-splinesurfacethatblendstwosurfacestogether.

OffsetSurface

- Existingsurfacescanbeoffsettocreatenewonesidentical inshapebutmayhavedifferentdimensions.

Modeling

- Modelingis theartofabstractingorrepresentingthe object,systemorphenomenon.

1. GeometricModeling:Itis definedas the complete representationofanobject(orasystem)withthegraphicalandnon-graphicalinformation.Itgeneratesmathematicaldescriptionoftheobjectincomputerdatabaseandimageoftheobjectonthegraphicsscreen.
2. Non-geometricModeling:Itisusuallyappliedto phenomenaorphysicalprocesses

*Methodsofgeometricmodeling:

1. Wire-framemodeling
2. Surfacemodeling
3. Solidmodeling

SalientFeaturesofGeometricModeling

- Geometricmodelisstoredinamathematicalform,soanytypeofdatarelatedwiththeobjectcanbestoredin model.
- Modelmodificationcanbecarriedoutbytheoperations like:move,rotate,scale,mirror,union,etc.
- Canbeusedtoevaluatethevariouspropertiesofan actualobjectsuchas:mass,volume,momentofinertia, etc.
- Providesasophisticatedtoolfor3-Dvisualizationofthe object.Canusedifferentcoloursandlighteffects.
- G.M.canbeautomaticallyconvertedtothe2-Dviews.
- Canbeusedbythefiniteelementanalysissoftwareto performthedifferenttypesofanalysissuchas:stress-strainanalysis,kinematicanalysis,dynamicanalysis, thermalanalysis,etc.
- CanbeusedbytheCAMsoftwaretogeneratea completetoolpathrequiredforautomaticmanufacturing.

Wire-frameModeling

- Oldest& simplest method of G.M.
- Useof 2-Dgeometricentitiessuchas: points, straightlines,curves, polygons,circles,etc.
- Themodelappears likeaframe constructedout of wire,and hence itiscalledas ‘wire-frame’model.
- Classifiedas –2D, 2½D,3D wire-framemodeling.
- 2Dwire-framemodelingis suitableforflatobjects.
- 2½DWFmodelingrepresents3dimensionalobject aslongas itdoes nothaveside walldetails.
- 3DWF modelingrepresents3 dimensionalobject withside walldetails. -useofdashedlines for hiddenedgesofthe object. -removalof hidden linesautomatically.

AdvantagesofWire-frameModeling

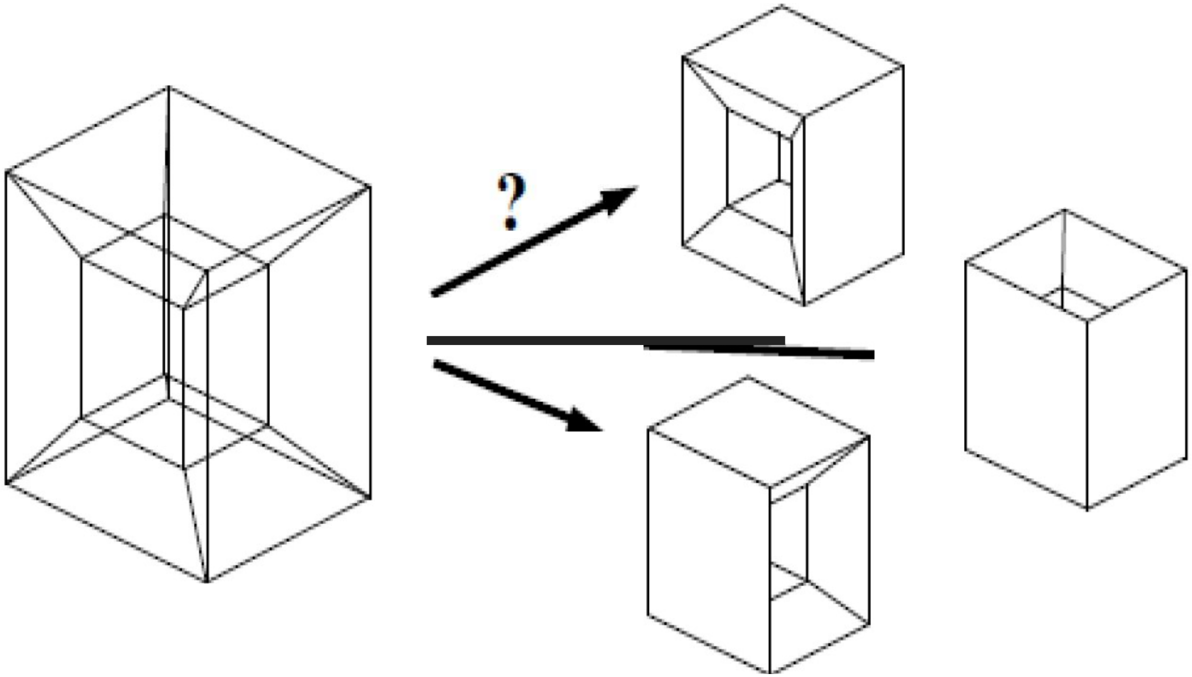
1. Simpleto construct
2. Requiresless computermemoryforstorage compared to surfaceand solidmodels.
3. Wire-framemodelsformthe basisforsurface models.
4. TheCPUtimerequiredto retrieve,editor updatethe wire-framemodelis lesscompared tosurfaceand solidmodels.

LimitationsofWire-frameModeling

1. Verydifficultandtimeconsumingtogeneratethewire-framemodelfor complicatedobjects.
2. Requiresmoreinputdatacomparedtothatofsolid models.
3. Wire-framemodelsofthecomplicatedobjectsareconfusingtotheviewerforinterpretation,especiallyif thereisnoautomatichiddenlineremoval facility.
4. Itisnotpossibletocalculatethepropertiessuchasmass, volume,momentofinertia,etc.withwireframemodels.
5. Notsuitableforapplicationslike:generatingcross-sections,checkinginterferencebetweenmatingparts,NCtool pathgeneration,andProcessplanning.
6. Wireframemodelofanobjectismoreambiguous representationthanitssurfaceandsolidmodels.

Wireframe Modeling

- Storespositionsoflines(in20or30)
- Helpfulfordrafting(easymultipleviewsandeasyediting)
- Ambiguoussurfaceslimittheautomationpossibilities
  (e.g.novolumecalculation,noNCtoolpathgeneration)



SurfaceModeling

- Asurfacemodelis generatedbyusing wire-frameentities orcurves(analyticorsynthetic)
- Inordertoassistthevisualizationofasurfaceona graphicsdisplay,artificialfairinglines,calledmeshareaddedonthe surface.Meshsizeiscontrolledbytheuser.
- Finermeshsizeonlyimprovesvisualization.
- Mostofthesurfacemodelingsoftwareareequippedwith renderingfeatures.Renderingprovidessurfaceproperties, coloureffects,lighteffectsetctoasurfacemodel.

AdvantagesofSurfaceModeling

- Complexjobscan be effectivelymodeled.
- Bettervisualizationthanwire-framemodeling.
- Completeand less ambiguousthanWFmodels.
- Suitableforapplicationslike: generatingcross-sections,checkinginterferencebetweenmating parts,NC toolpathgeneration,finiteelement modelingand Processplanning.
- Shadingofanobject is possible.
- Awireframemodelcan be extractedfroma surfacemodelbydeletingall surfaceentities.

LimitationsofSurfaceModeling

- Surfacemodelsare morecomplex,so require moreCPUtimeandcomputer memoryforstorage compared to WFmodels.
- Surfacemodelingrequire moretrainingand mathematicalbackgroundonthe part ofthe user.
- Sometimes,surfacemodelsare awkwardto createand require manipulationsof wireframe entities.eg.A surfacewithholeinitmayhaveto be createdwiththe help of wire-frameentities.

SOLIDMODELLING

Whysolidmodeling?

Recallweaknessofwireframeand surfacemodeling

- Ambiguousgeometricdescription
- incompletegeometricdescription
- lacktopologicalinformation
- Tediousmodelingprocess
- Awkwarduser interface

SolidModel

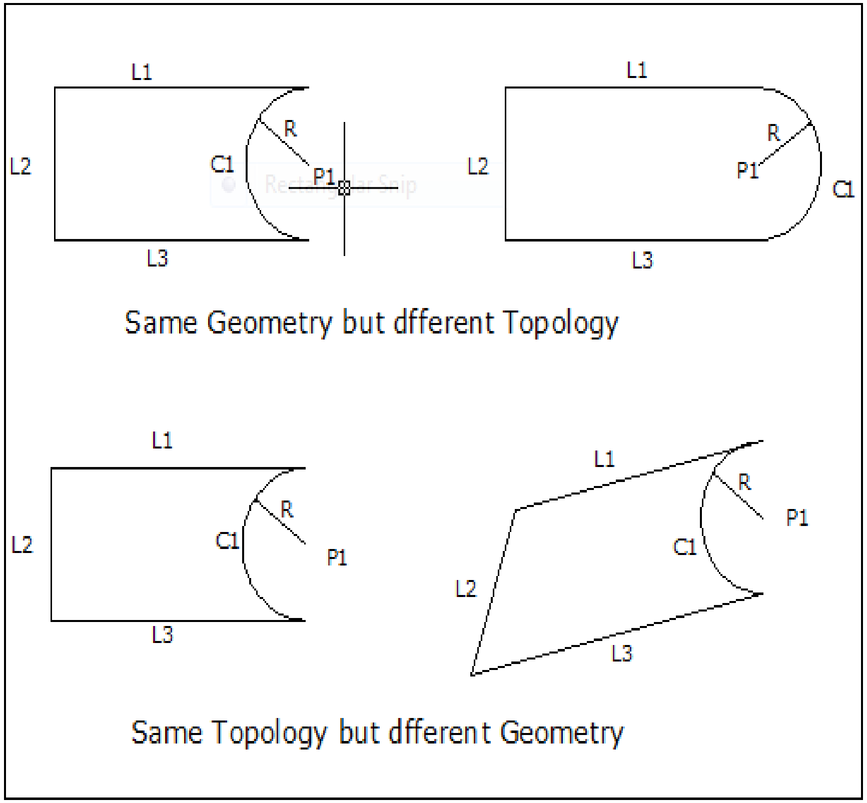
- Itistheeasiestandmostadvancedmethodofgeometric modeling.
- Solidmodelingisbasedon*complete,validand unambiguous*geometricrepresentationofphysicalobject.
    - Complete→ pointsinspacecanbeclassified.(inside/ outside)
    - Valid→vertices,edges,facesareconnectedproperly.
  - Unambiguous→ therecanonlybeoneinterpretation ofobject
- Solidmodelscanbeconvertedintowire-framemodels. Thistypeofconversionisusedtogenerateautomatically theorthographicviews.
- Analysisautomationandintegrationispossibleonlywith solidmodels→ haspropertiessuchas weight,momentof inertia,mass.

SolidModel

- Solidmodelconsistsof geometric andtopologicaldata

- Geometry→ shape,size,locationof geometricelements
- Topology→connectivityand associativityofgeometricelements

  →nongraphical,relational information



Same Geometry but dfferent Topology

Same Topology but dfferent Geometry

- **Geometry:**

Thegeometrythatdefinestheobjectis

1. ThelengthsoflinesL1,L2,L3.
2. Theanglesbetweenthelines
3. TheradiusRofhalfcircleand
4. ThecenterP1ofhalfcircle.

- **Topology:**

TheTopologythatdefinestheobjectis

1. ThelineL1sharesavertexwithlineL2andcircleC1.
2. ThelineL2sharesavertexwithlinesL1& L3.
3. ThelineL3sharesavertexwithlineL2&circleC1.
4. ThelineL1&L3donotoverlap
5. ThepointP1liesoutsidetheobject.

Neithergeometrynortopologyalonecancompletelydefine thesolidmodel.

Advantagesofsolidmodeling

- Easiestandmostadvancedmethodofgeometricmodeling.
- Containsbothgeometricandtopologicaldata.
- Providebettervisualizationas comparedtowireframe& surfacemodeling.
- Canbeconvertedintowireframemodels.
- Itis possibletocalculateautomaticallythepropertiessuchas mass,volume,momentofinertia,etc
- Producesaccuratedesigns,improvesqualityof design,and providescompletethree dimensionaldefinitionof theobjects.
- Solidmodelingis thetechnologicalsolutiontofullyintegrate andautomatedesignandmanufacturing

Limitationsofsolidmodeling

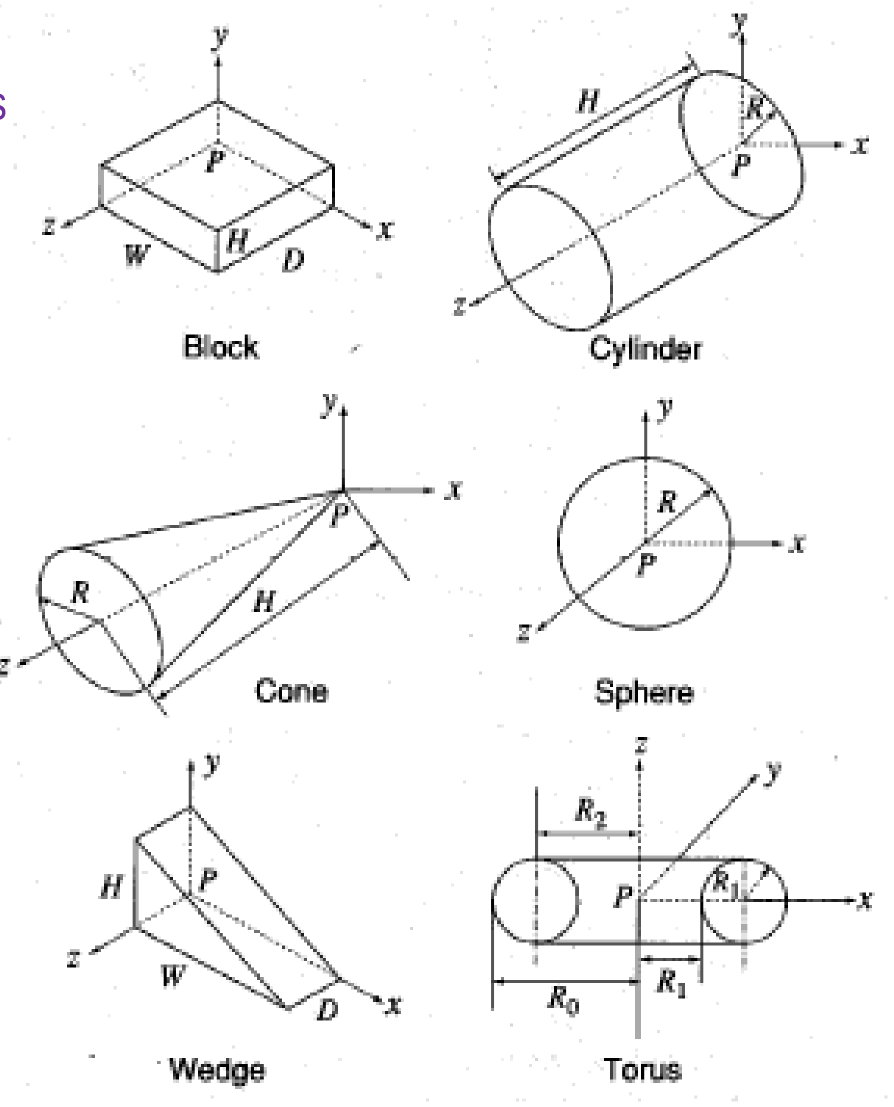
- Notpossibletocreatesolidmodelautomaticallyfromwire-frameorsurfacemodeling.
- RequiremoreCPUtimeretrieve,edit orupdatethemodel.

SolidEntities

- Solidmodelofanobjectiscreatedbyusingthethreedimensionalgeometricentities,knownasprimitives.

- Primitivesaresimplesolidshapeswithsimple mathematicalsurfaces

- Canbecontrolledbyasmallnumberofparametersandpositionedusingatransformationmatrix



Solidmodelrepresentationschemes

1. Constructivesolidgeometry(CSGorC-rep)
2. Boundaryrepresentation(B-rep)
3. Sweeping
4. Parametric(Analytical)solidmodeling
5. PrimitiveInstancing
6. FeatureBasedModeling
7. CellDecomposition
8. Spatialenumeration
9. OctreeEncoding
10. QuadtreeEncoding

Constructivesolidgeometry(CSG)

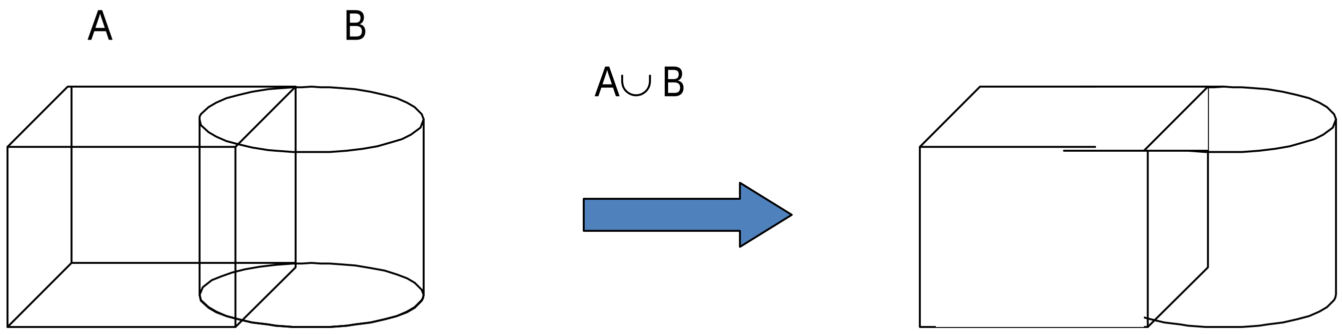
- Objectsare representedas acombinationof simplersolidobjects(*primitives*).
- The primitivesare such as cube,cylinder, cone, torus,sphere etc.
- Copiesor“instances”of these primitiveshapes arecreatedand positioned.
- Acomplete solidmodelis constructedby combiningthese “instances”usingsetofspecific, logicoperations(Boolean)

Constructivesolidgeometry(CSG)

- Booleanoperation
  - eachprimitivesolidis assumedto be a setof points,abooleanoperationis performedon pointsetsand the resultis asolidmodel.
  - Booleanoperation→ union, intersectionand difference
  - Therelativelocationandorientationofthe two primitiveshaveto be definedbeforethe booleanoperationcan be performed.
  - Booleanoperationcanbeappliedto twosolids otherthanthe primitives.

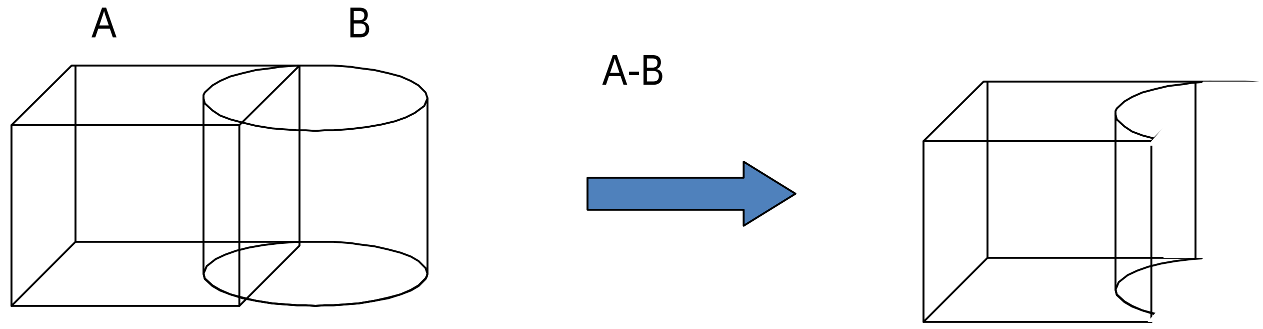
Constructivesolidgeometry(CSG)-Boolean operation

- Union
  - Thesumofall pointsineach of twodefined sets.(logical“OR”)
  - Alsoreferred to as Add, Combine, Join,Merge



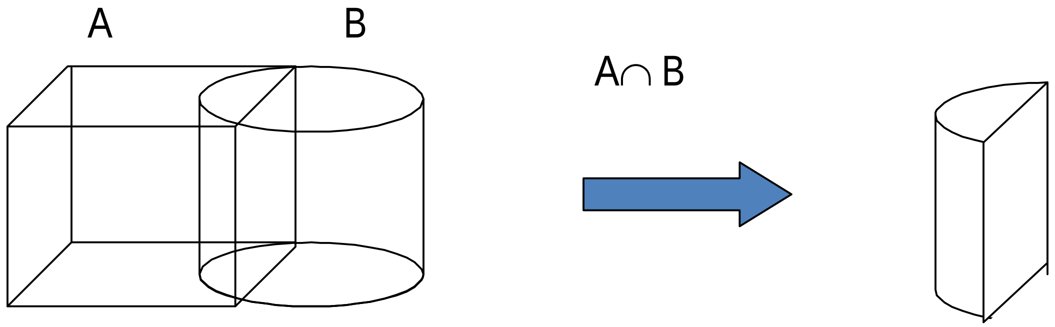
Constructivesolidgeometry(CSG)- Booleanoperation

- Difference
  - Thepointsin asourceset minusthe pointscommonto asecondset.(logical“NOT”)
  - Setmustshare commonvolume
  - Alsoreferred to assubtraction, remove,cut



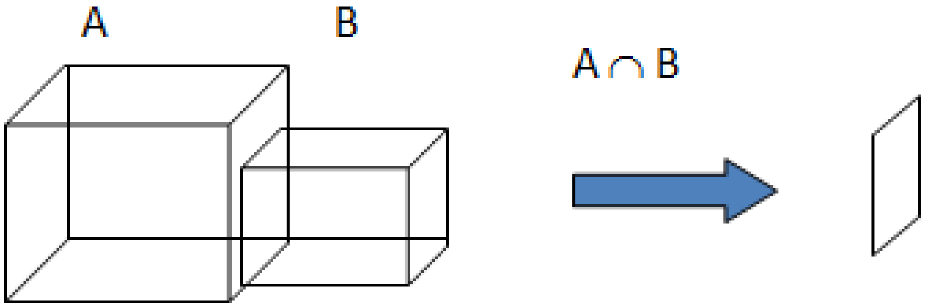
Constructivesolidgeometry(CSG)-
Booleanoperation

- Intersection
  - Thosepointscommon to each of twodefined sets(logical“AND”)
  - Setmustshare commonvolume
  - Alsoreferred to ascommon, conjoin



Constructivesolidgeometry(CSG)-Booleanoperation

- WhenusingBooleanoperation,becareful toavoidsituationthatdonotresultina validsolid



Constructivesolidgeometry(CSG)-Booleanoperation

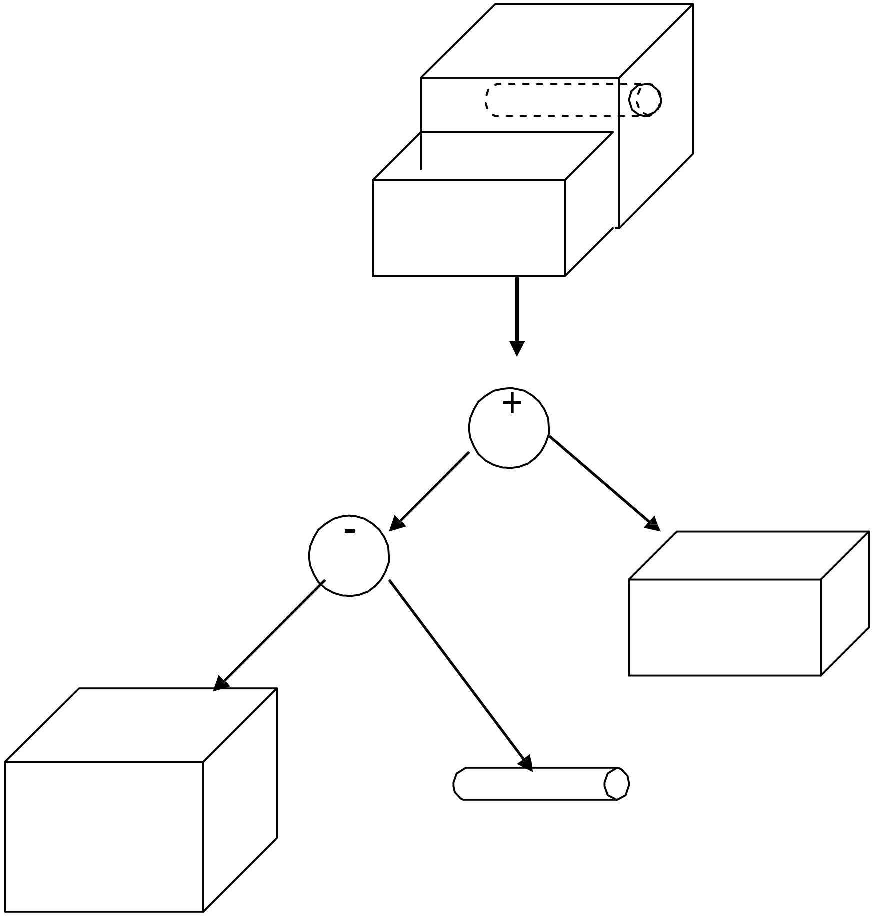
- Booleanoperation
  - Areintuitiveto user
  - Areeasyto useand understand
  - Provideforthe rapidmanipulationoflarge amountsofdata.
- Becauseofthis,manynon-CSGsystemsalsouseBooleanoperations

Constructivesolidgeometry(CSG)-datastructure

- Datastructuredoes notdefinemodelshape explicitlybutratherimpliesthegeometricshape throughaproceduraldescription
    - E.g:objectisnotdefinedas asetofedges&facesbut bytheinstruction:*unionprimitive1withprimitive2*
- Thisproceduraldatais storedin a datastructure referredto asa CSGtree
- The datastructure is simpleand storescompact data→ easyto manage

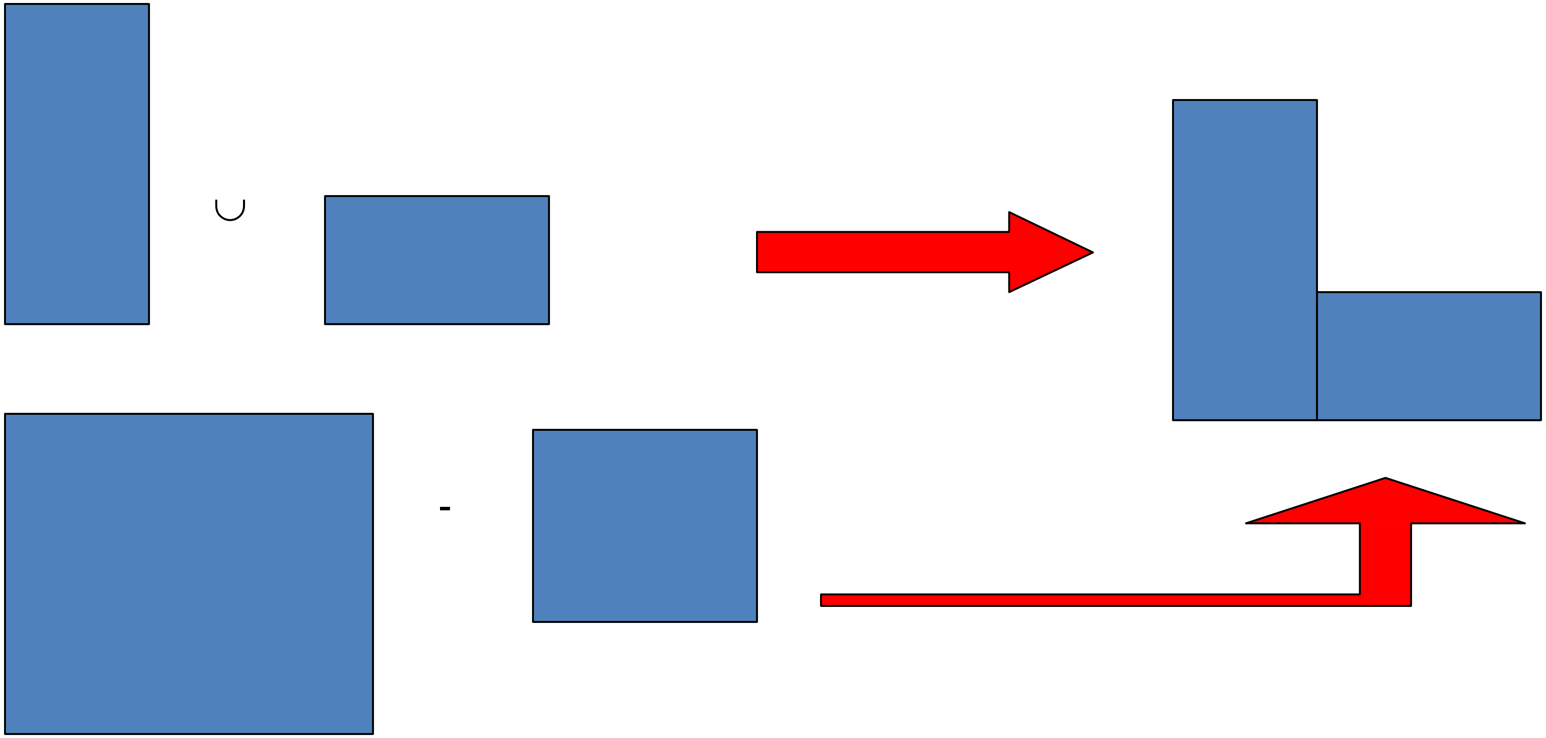
Constructivesolidgeometry(CSG)-CSGtree

- CSGtree→ storesthehistoryof applyingbooleanoperationson theprimitives.
  - Storesin abinarytree format
  - Theouterleafnodesoftree representthe primitives
  - Theinteriornodesrepresent theBooleanoperations performed.



Constructivesolidgeometry(CSG)-notunique

- Morethanoneprocedure(and hencedatabase) canbe usedto arriveatthe samegeometry.



(CSG)representation

- CSGrepresentationisunevaluated
  - Faces,edges,verticesnotdefinedinexplicit
- CSGmodelarealwaysvalid
  - Sincebuiltfromsolidelements.
- CSGmodelsarecompleteandunambiguous

(CSG)-advantage

- CSGispowerfulwithhighlevelcommand.
- Easytoconstructasolid model–minimumstep.
- CSGmodelingtechniquesleadtoa concisedatabase→ lessstorage.
  - Completehistoryofmodelisretainedandcanbe alteredatanypoint.
- Canbeconvertedtothecorrespondingboundary representation.

(CSG)-disadvantage

- Onlybooleanoperationsareallowedinthemodelingprocess → withbooleanoperationalone,therangeof shapestobemodeledisseverelyrestricted→ notpossibletoconstructunusualshape.
- Requiresagreatdealofcomputationtoderivethe informationontheboundary,facesandedgeswhichis importantfortheinteractivedisplay/manipulationof solid.

Solution

- CSGrepresentationtendstoaccompanythecorrespondingboundary representation → *hybridrepresentation*
- Maintainingconsistencybetweenthetworepresentationsisveryimportant.

Boundaryrepresentation(B-Rep)

- Solidmodelisdefinedbytheirenclosingsurfacesorboundaries.Thistechniqueconsistsofthegeometric informationaboutthefaces,edgesandverticesofan objectwiththetopologicaldataonhowtheseare connected.

- WhyB-Repincludessuchtopological information?
  - Asolidisrepresentedasa closedspacein3Dspace(surfaceconnectwithout gaps)
  - The boundaryof a solidseparatespointsinsidefrompointsoutsidesolid.

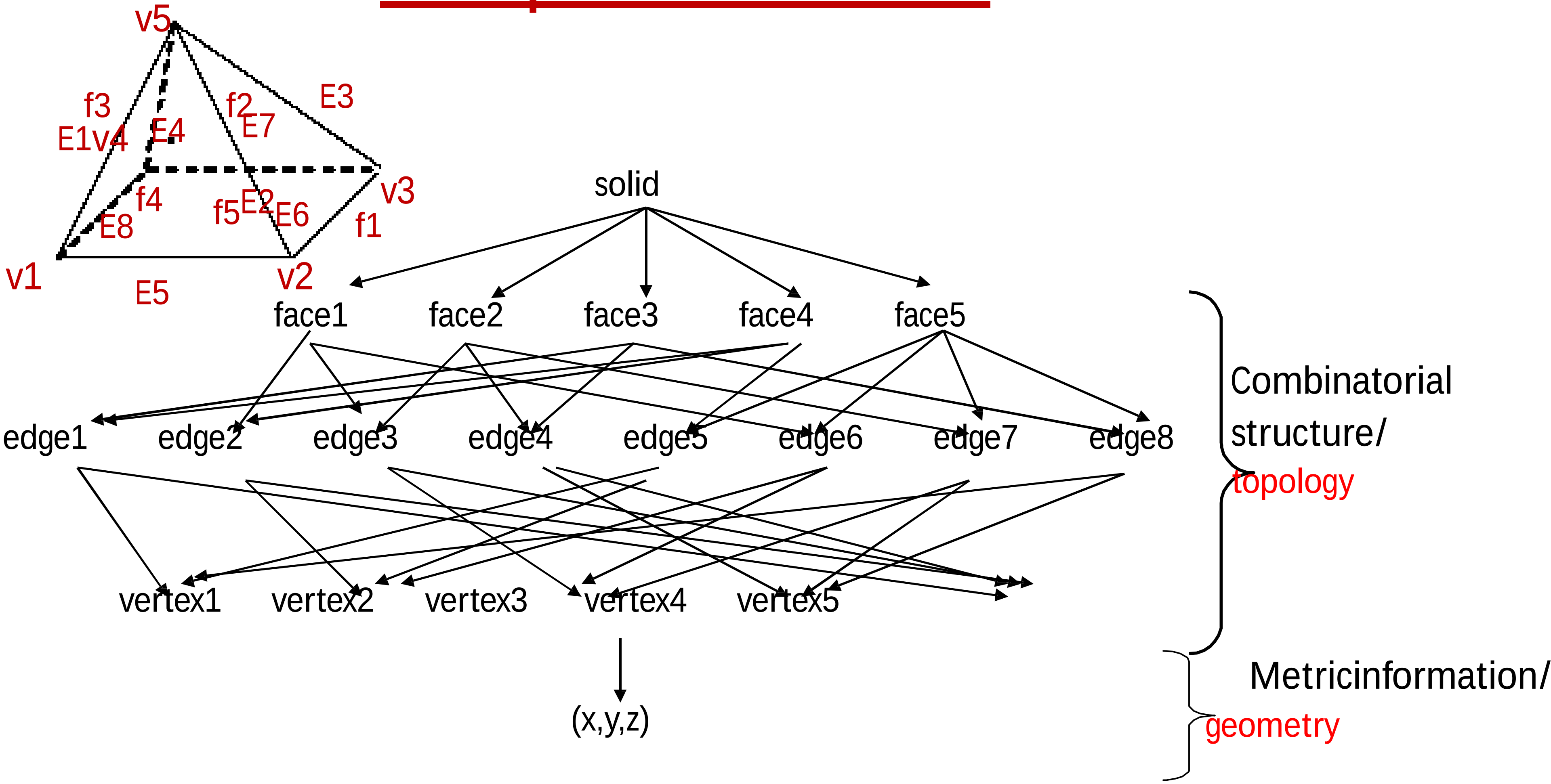
B-Repv/sSurfacemodeling

- Surfacemodel
  - Acollectionofsurfaceentitieswhichsimply encloseavolumelacks the connective datato defineasolid(i.etopology).
- B-Repmodel
  - Techniqueguarantees thatsurfacesdefinitively dividemodelspace into solidandvoid,evenafter modelmodificationcommands.

B-Repdatastructure

- B-Repgraphstoreface,edgeandverticesasnodes,with pointers,orbranchesbetweenthenodestoindicateconnectivity.

B-Repdatastructure



solid

face1 face2 face3 face4 face5

edge1 edge2 edge3 edge4 edge5 edge6 edge7 edge8

vertex1 vertex2 vertex3 vertex4 vertex5

(x,y,z)

Combinatorial structure/ topology

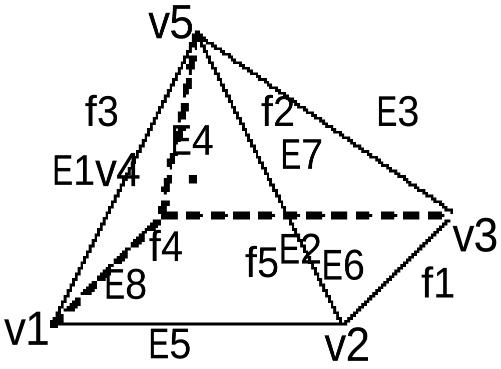
Metricinformation/ geometry

Boundaryrepresentation-validity

- Systemmustvalidatetopologyofcreatedsolid.
- B-Rephastofulfillcertain conditionstodisallowself-intersectingandopenobjects
- Thisconditioninclude
  - Eachedgeshouldadjoinexactly twofacesandhavea vertexateachend.
  - Verticesaregeometricallydescribedbypoint coordinates
  - Atleastthreeedgesmustmeetateachvertex.
  - Facesaredescribedbysurfaceequations
  - Thesetoffacesformsacompleteskinofthesolidwith nomissingparts.
  - Each face isborderedbyanorderedsetofedges formingaclosedloop.
  - Facesmustonlyintersectatcommonedgesorvertices.
  - Theboundariesoffacesdonotintersectthemselves

Boundaryrepresentation-validity

- Validityalsocheckedthroughmathematicalevaluati on
  - Evaluationis based uponEuler’s Law(validfor simplepolyhedra–nohole)
  - V–E+ F = 2 V-verticesE-edges F-faceloops



V=5, E=8, F=5

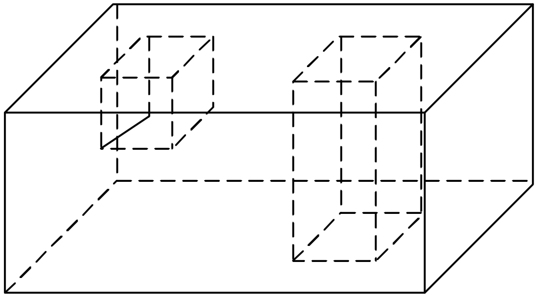
5–8+5=2

Boundaryrepresentation-validity

- ExpandedEuler's lawforcomplexpolyhedrons (with holes)
- Euler-PoincareLaw:

V-E+F-H=2(B-P)

H–numberofholesinface, P-numberofpassagesor throughholes, B-numberofseparatebodies.



V=24,E=36, F=15,H=3,P=1,B=1

Boundaryrepresentation-ambiguityand uniqueness

- ValidB-Repsareunambiguos
- Notfullyunique,butmuchmoresothanCSG
- Potentialdifferenceexistsindivisionof
  - Surfacesinto faces.
  - Curvesinto edges

Boundaryrepresentation-advantages

- Capabilityto constructunusualshapesthatwould notbepossiblewiththe availableCSG→ aircraft fuselages,swingshapes
- Lesscomputationaltimeto reconstructthe image

Boundaryrepresentation-disadvantages

- Requiresmore storage
- Morepronetο validityfailurethanCSG
- Modeldisplaylimited to planarfacesand linear edges

  -complexcurveandsurfacesonlyapproximated

CSGorC-RepApproach

- Itisveryeasytocreatea precisesolidmodeloutof theprimitives.
- ThedatabaseofCSGmodel containsconfiguration parametersofthe primitivesandboolean model.
- Requireslessstoragespace. Thusresultsinmore compactfileofthemodelin thedatabase.
- Requiresmorecomputationstoreproducethemodelanditsimage.

B-RepApproach

- Itisusefultomodeltheobjectsofunusualshapeswhicharedifficulttomodel byCSGapproach.
- ThedatabaseofB-Repmodelcontainsexplicit definitionofthemodel boundaries.
- Requiremorestoragespace.Thusresultsinlargerfileofmodelin thedatabase.
- Requiresless computationstoreproducethe modeland itsimage.

CSGorC-RepApproach

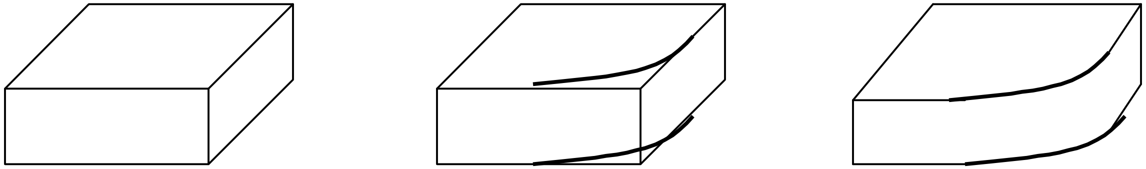
- Itisdifficulttoconvert back andforthbetweena constructivesolid geometry modelandacorrespondingwire framemodel.Itis totallylikeacreationofthenew model.

B-RepApproach

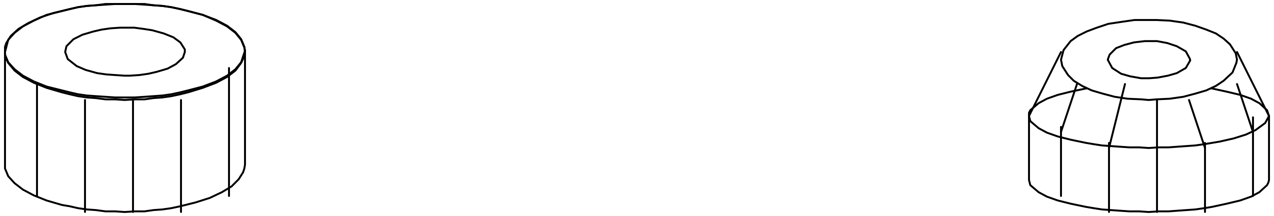
- Itisrelativelyeasyto convertbackandforth betweenaboundaryrepresentationmodelanda correspondingwireframemodel.Thisisduetothe factthatboundarydefinitionissimilar tothe wire-framedefinition.This resultsin compatibility betweenB-repand wireframemodeling

Solidobjectconstructionmethod

- Sweeping
- Boolean
- Automatedfilletingandchambering

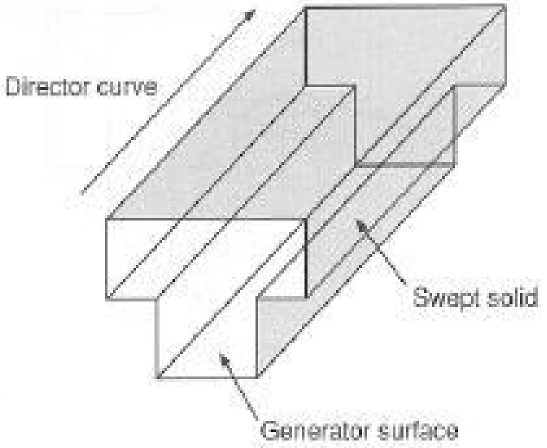


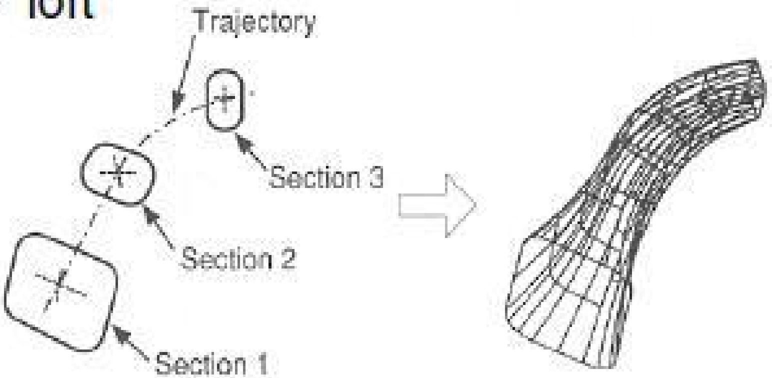
- Tweaking
  - –Faceof anobjectis movedinsomeway

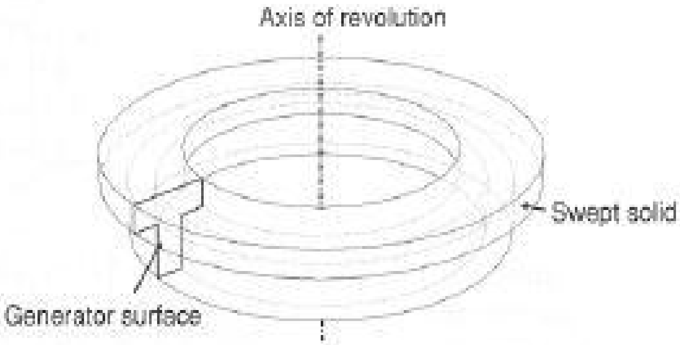


SweepingOperations

- Use 2D wireframe section(s) to generate a 3D solid.
- This includes operations such as:
  - extrude
  - revolve
  - sweep
  - loft

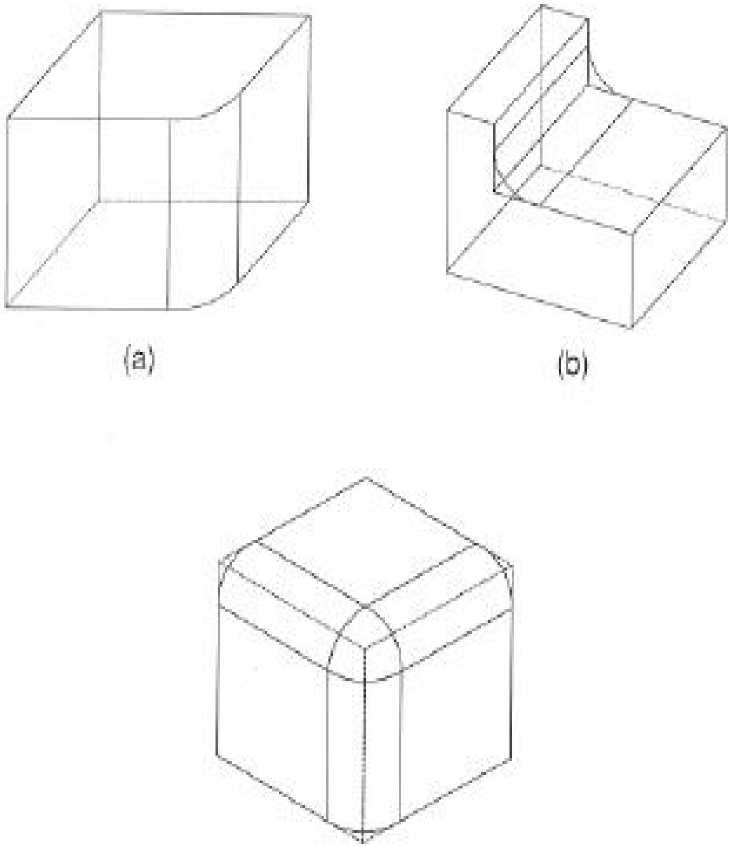






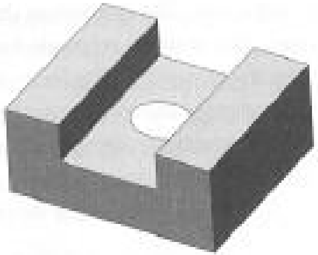
SurfaceOperations

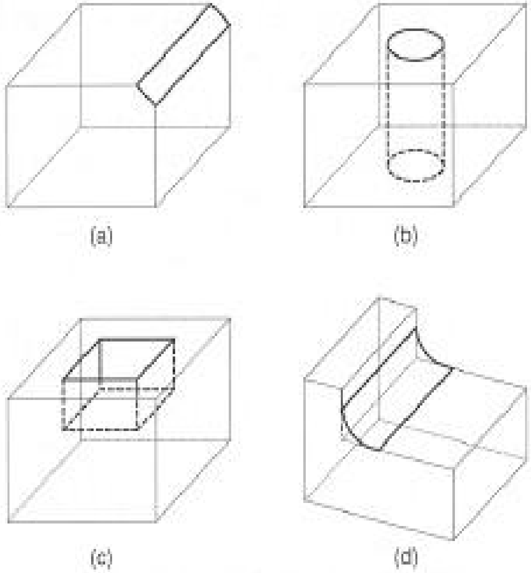
- These operate directly on the solid model surfaces, edges and vertices to create a desired modification.
- Examples:
  - chamfering
  - rounding/filleting
  - drafting
  - shelling



Feature-BasedModeling

- Features are shapes having **engineering significance.** They usually are the geometric embodiment of **machining operations** or the **function** of a component.
- Examples:
  - hole - pocket
  - slot - boss
- Many people use the term "Feature" to refer to any kind of solid modeling operation.
- Many systems provide for user-defined features.





SpatialPartitioningRepresentation

- Inthistechniquesolidis subdivided into number ofcloselyspaced,non-intersectingsmallersolids orcells.
- Cellsmayormaynotbe of sametypeas original solid.
- Cellsmayvaryin size,typeandorientation.
- Twomethods–cell decomposition

  -spatialoccupancyenumeration

CellDecomposition

- Decomposesthesolidintoasetofprimitivecellsthatareparameterized(varyinginfew parameters)
- Ithaspotentialuseinfiniteelementanalysisin which,systemtobeanalyzedisdividedinto smallerelementscalledfiniteelements.
- Buttheconditionisthatelementsshouldnot overlapeachother.

SpatialoccupancyEnumeration

- Solidissubdividedintoexactlyidentical cellsarrangedinfixed regulargrid.
- Thesecellsarecalledvoxels.
- Voxelmayhaveshapesascube,pyramid,prism,etc .
- Representationofsolidasregulararrayof cubesisknownascuberllie.
- Thisisanapproximationtechnique.