

COMPUTER AIDED DESIGN

Syllabus

- UNIT – I Introduction: Introduction to CAD/CAM, Historical developments, Industrial look at CAD/CAM, Introduction to CIM; Basics of geometric and solid modeling, explicit, implicit, intrinsic and parametric equations, coordinate systems.
- UNIT – II Transformations: Introduction, transformation of points and line, 2-D rotation, reflection, scaling and combined transformation, homogeneous coordinates, 3-D scaling, shearing, rotation, reflection and translation, combined transformations, orthographic and perspective projections, reconstruction of 3-D objects.
- UNIT – III Curves: Algebraic and geometric forms, tangents and normal, blending functions reparametrization, straight lines, conics, cubic splines, Bezier curves and B-spline curves.
- UNIT – IV Surfaces: Algebraic and geometric forms, tangents and normal, blending functions, reparametrization, sixteen point form, four curve form, plane surface, ruled surface, surface of revolution, tabulated cylinder, bi-cubic surface, bezier surface, B-spline surface.

Syllabus

- UNIT – V Solids: Solid models and representation scheme, boundary representation, constructive solid geometry, sweep representation, cell decomposition, spatial occupancy enumeration.
- UNIT – VI Automation and Numerical Control: Introduction, fixed, programmable and flexible automation, types of NC systems, MCU and other components, NC manual part programming, coordinate systems, G & M codes, Part program for simple parts, computer assisted part programming.
- UNIT – VII Group Technology: Part families, part classification and coding, production flow analysis, Machine cell design, Advantages of GT
- UNIT – VIII Flexible Manufacturing Systems & Computer aided process planning: Introduction, FMS components, types of FMS, FMS layouts, planning for FMS, advantages and applications
Conventional process planning, types of CAPP, Steps in variant process planning, planning for CAPP.

Introduction

- In general, a Computer Aided Design (CAD) package has three components: a) Design, b) Analysis, and c) Visualization. A brief description of these components follows.
 - a) **Design:** Design refers to geometric modeling, i.e., 2-D and 3-D modeling, including, drafting, part creation, creation of drawings with various views of the part, assemblies of the parts, etc.
 - b) **Analysis:** Analysis refers to finite element analysis, optimization, and other number crunching engineering analyses. In general, a geometric model is first created and then the model is analyzed for loads, stresses, moment of inertia, and volume, etc.
 - c) **Visualization:** Visualization refers to computer graphics, which includes: rendering a model, creation of pie charts, contour plots, shading a model, sizing, animation, etc.

CAD/CAM History

1960's

- Development in Interactive computer graphics research
- Sketchpad system developed by Ivan Sutherland in 1962
- **First major commercial CAD/CAM software available: CADAM by Lockheed, in 1965**
- Bell Telephone's - Graphics 1 remote display system developed

1970's

- Application of CAM in government, industry and academia
- National organization formed
- Beginning of usage of computer graphics
- Turnkey system available for drafting
- **Wireframe and surface modeling software became available**
- **Mass property calculation and FEA software became available**
- **NC tape generating, verification, and integrated circuit software became available**

CAD/CAM History

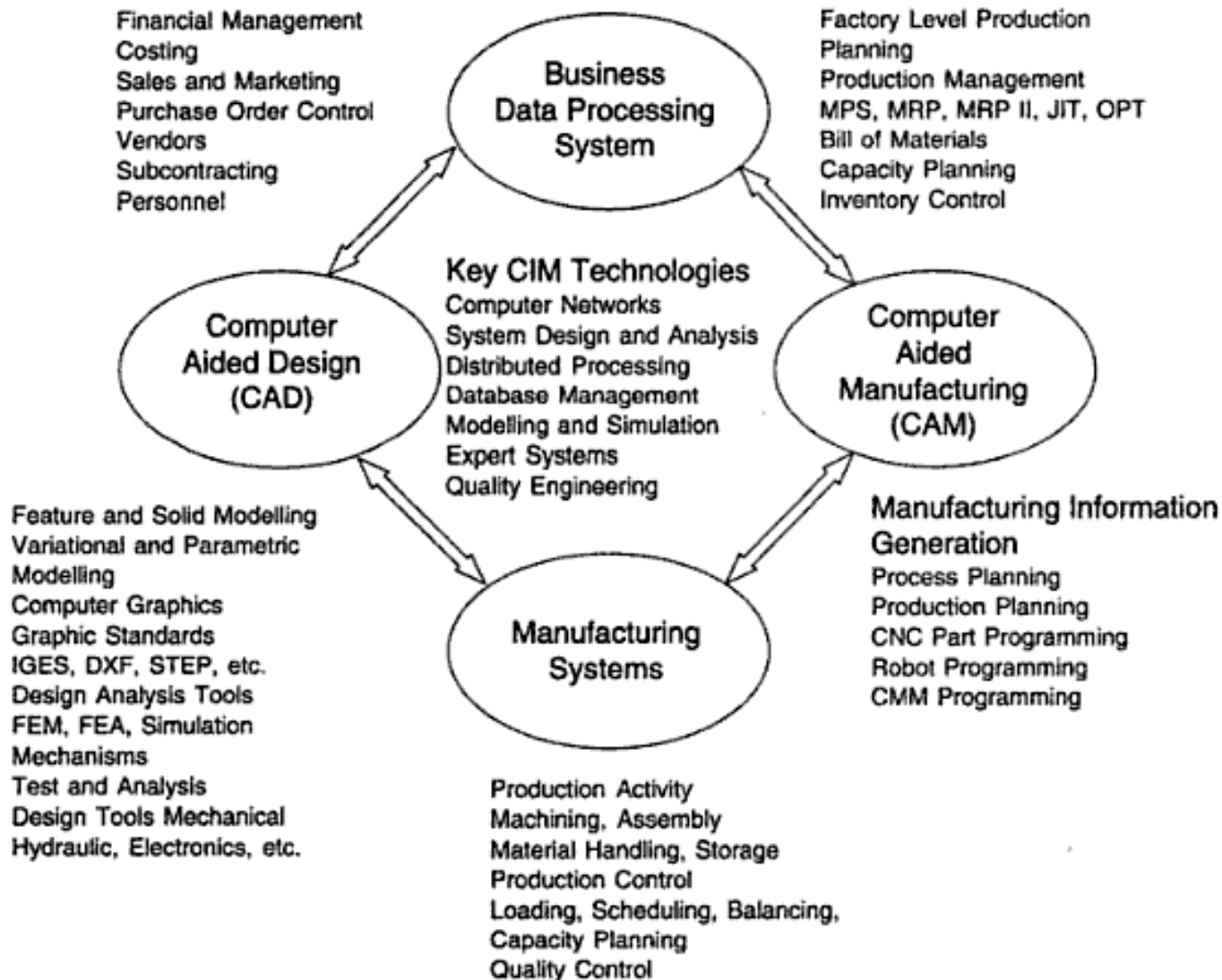
1980's

- CAD/CAM used for engineering research and development
- New CAD/CAM theories and algorithms developed
- Integration of CAD/CAM
- **Solid modeling software became available**
- Use of PCs and workstation began

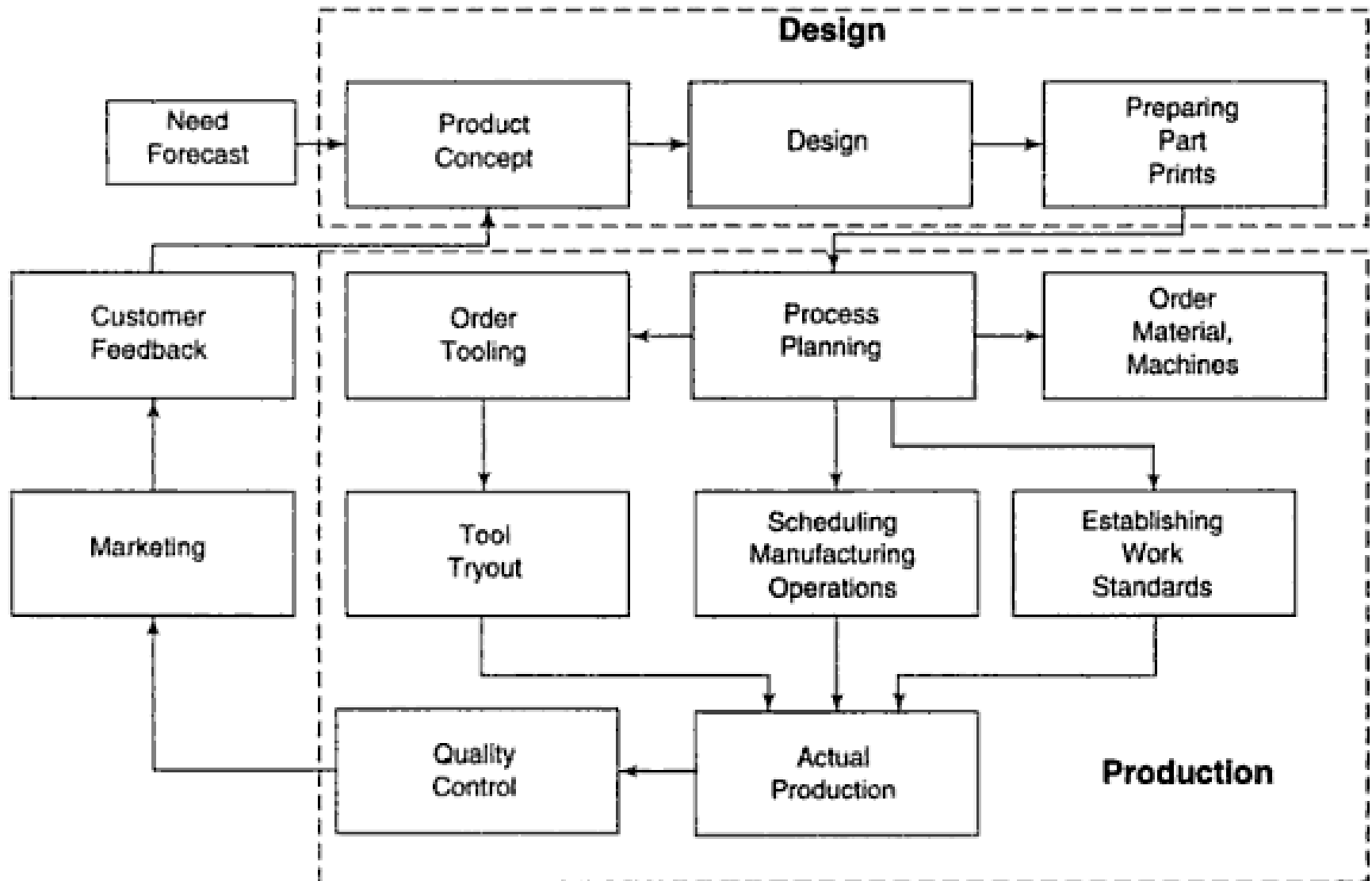
1990's

- Concept of concurrent engineering developed
- Increased use of CAD/CAM on PCs and workstations
- Improvements in hardware and software

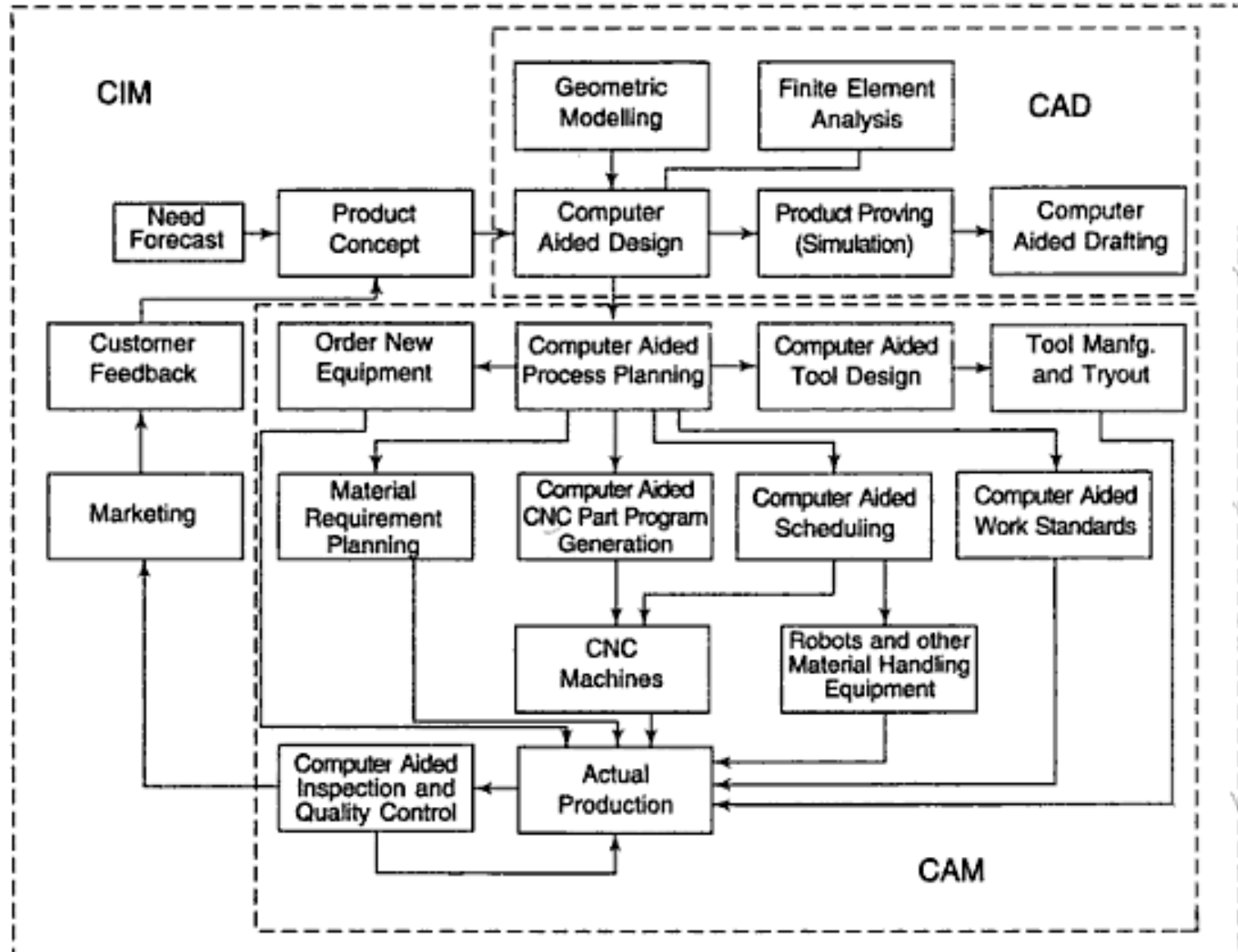
The Influence of Computers Used in Manufacturing Environment



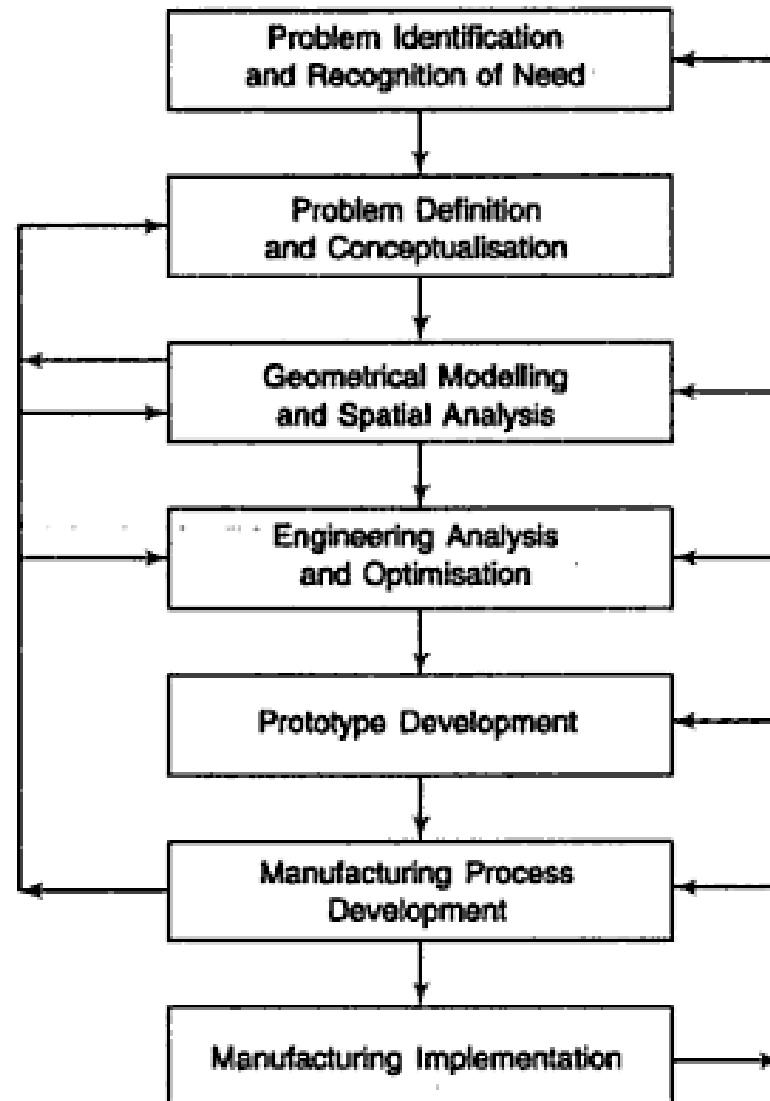
Conventional Product Cycle



Product Cycle in Computerized Manufacturing Environment



Stages in Design Process



Coordinate Systems

1. World Coordinate system
2. User Coordinate system
3. Display Coordinates
4. View Generation

Input & output Devices

- Input devices

1. Keyboard
2. Mouse
3. Light Pen
4. Joystick
5. Digitizer
6. Tablet

- Output devices

1. Cathode Ray Tube (CRT) display
2. Plasma Panel Display
3. Liquid crystal display (LCD)
4. Graphical printers
5. Plotters
6. Photographic devices

Geometric Transformation

- Computer graphics provides visual displays and manipulations of objects, e.g., transformation, editing, printing, etc.
- Fortran and visual C languages are used to effect these operations.
- Transformation is the backbone of computer graphics, enabling us to manipulate the shape, size, and location of the object.
- It can be used to effect the following changes in a geometric object:
 - Change the location
 - Change the Shape
 - Change the size
 - Rotate
 - Copy
 - Generate a surface from a line
 - Generate a solid from a surface
 - Animate the object

There are two types of transformations:

- **Modeling Transformation:** this transformation alters the coordinate values of the object. Basic operations are scaling, translation, rotation and combination of one or more of these basic transformations. Examples of these transformations can be easily found in any commercial CAD software. For instance, AutoCAD uses SCALE, MOVE, and ROTATE commands for scaling, translation, and rotation transformations, respectively.
- **Visual Transformation:** In this transformation there is no change in either the geometry or the coordinates of the object. A copy of the object is placed at the desired sight, without changing the coordinate values of the object. In AutoCAD, the ZOOM and PAN commands are good examples of visual transformation.

2D Transformations

- Translation
- Scaling
- Rotation
- Reflection
- Shear

In Homogeneous Representation, an n -dimensional space is mapped into $(n+1)$ dimensional space.

2D Transformations

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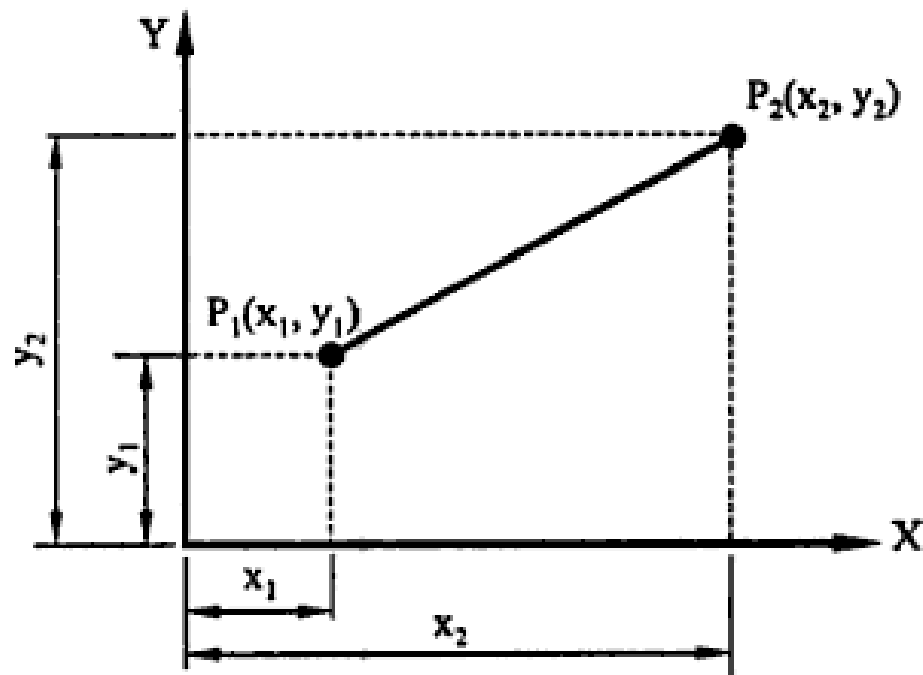
In Homogeneous Representation, an n -dimensional space is mapped into $(n+1)$ dimensional space.

A point can be represented by a 1 - row \times 2 - column matrix such that

$$\mathbf{P} = [x, y]$$

Similarly, a line can be represented (refer Fig. 3.1.1) as :

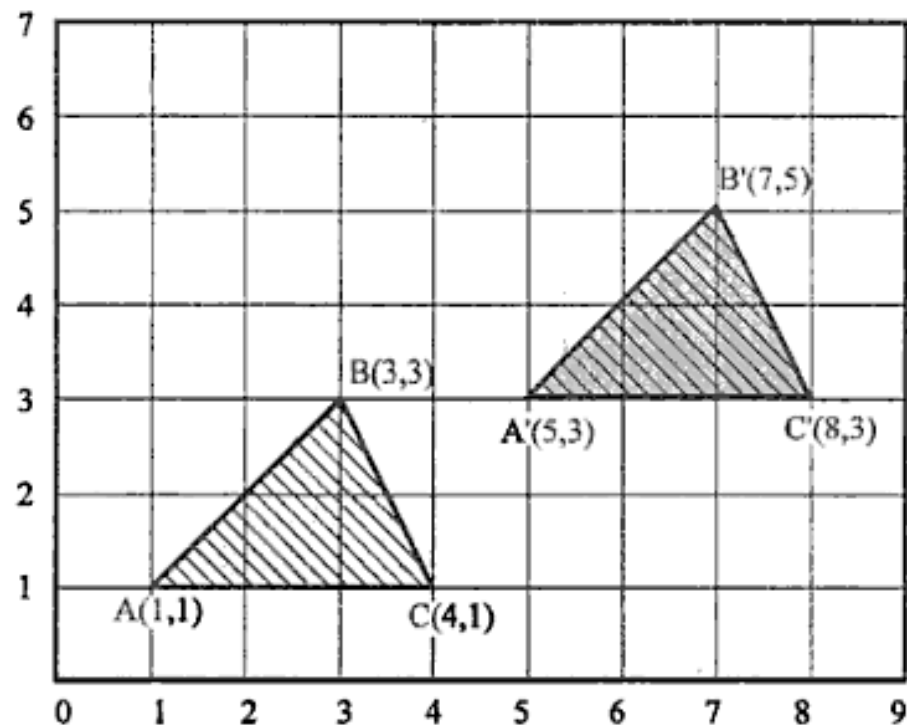
$$\mathbf{L} = \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \end{bmatrix}$$



Point and Line representation

a). Translation :

When every entity of a geometric model remains parallel to its initial position, the transformation is called as *translation*. Translating a model therefore implies that every point on it moves by an equal given distance in a given direction. A translation involves moving of an element from one location to another.

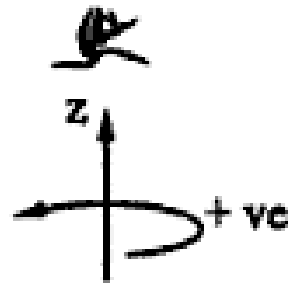


$t_x = 4$ and $t_y = 2$
Translation of a Triangle

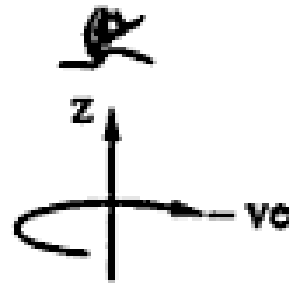
b) Rotation –

It can be used to create entities arranged in circular arrays, by creating the entity once and then rotating/copying it to the desired positions on the circumference.

Sign convention for angular movement



+ Counterclockwise



- Clockwise

Rotation of a point :

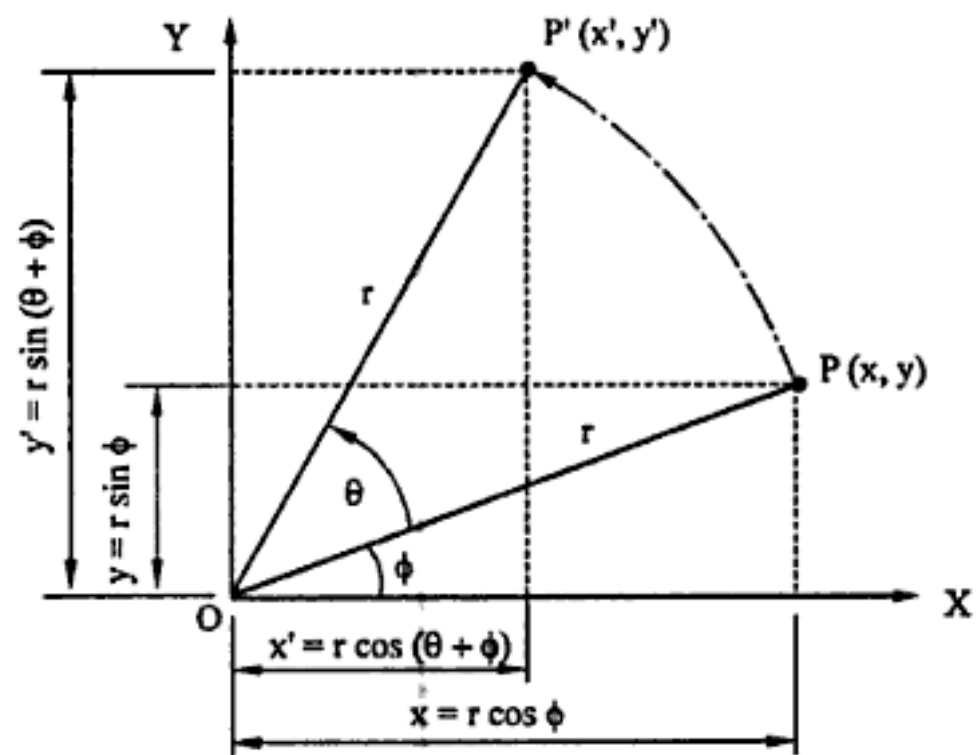
$$\begin{aligned} \mathbf{P} &= [x \ y] \\ &= [r \cos \phi \ r \sin \phi] \end{aligned}$$

$$\begin{aligned} \mathbf{P}' &= [x' \ y'] \\ &= [r \cos (\theta + \phi) \ r \sin (\theta + \phi)] \\ &= [r (\cos \theta \cos \phi - \sin \theta \sin \phi) \ r (\sin \theta \cos \phi + \cos \theta \sin \phi)] \\ &= [(x \cos \theta - y \sin \theta) \ (x \sin \theta + y \cos \theta)] \end{aligned}$$

$$= [x \ y] \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$$

$$= \mathbf{P} \cdot \mathbf{R}$$

$$\mathbf{R} = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$$

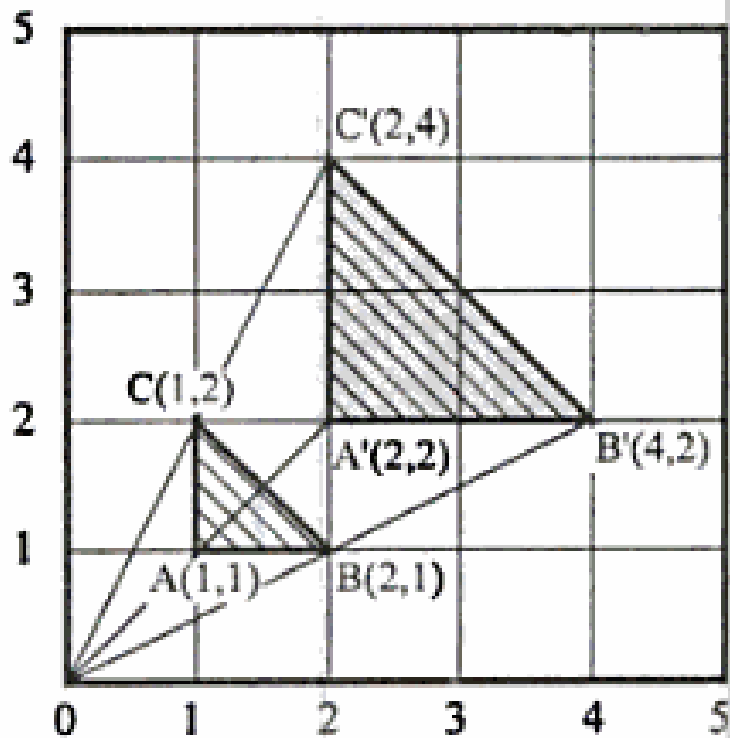


c). **Scaling**

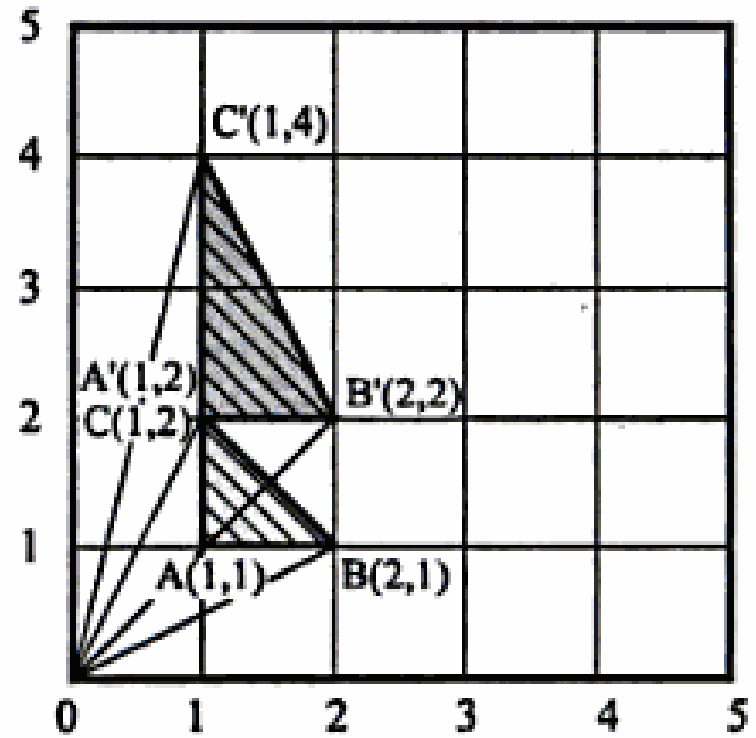
Scaling transformation alters the size of an object. Scaling can be *uniform* (i.e. equal in both X and Y directions) or *non-uniform* (i.e. different in X and Y directions). This scaling is occurs about the origin. Scaling can be achieved by the matrix transformation of the form :

$$\mathbf{P}' = \mathbf{P} \cdot \mathbf{S}$$
$$[x' \ y'] = [x \ y] \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix}$$

where S_x = Scale factor in X direction and
 S_y = Scale factor in y direction.



(a) Uniform Scaling by Factor 2

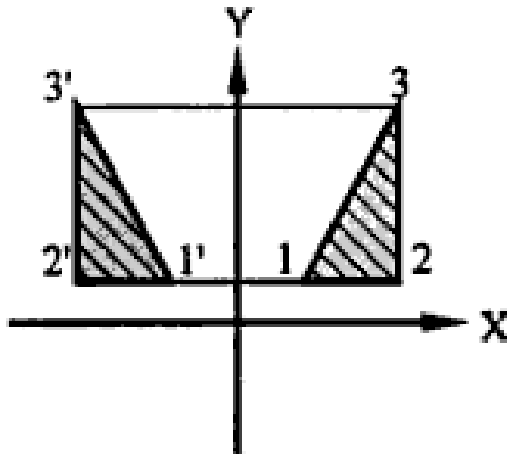


(b) Non-uniform Scaling by $S_x = 1$ and $S_y = 2$

d) Reflection (Mirror):

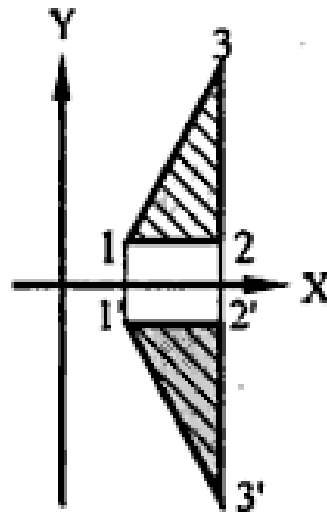
Reflection is the same as obtaining the mirror of the original object.

$$P' = P * M$$



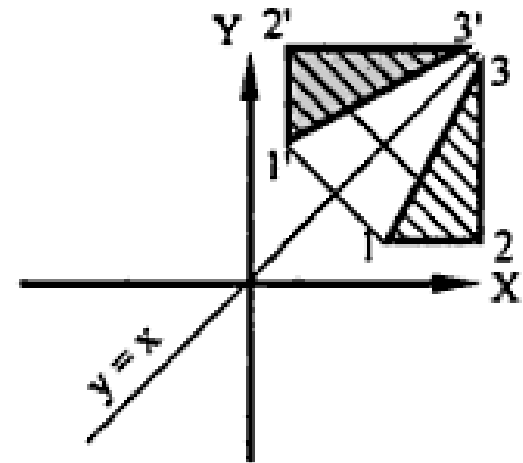
$$M = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

(a) Reflection
about Y axis



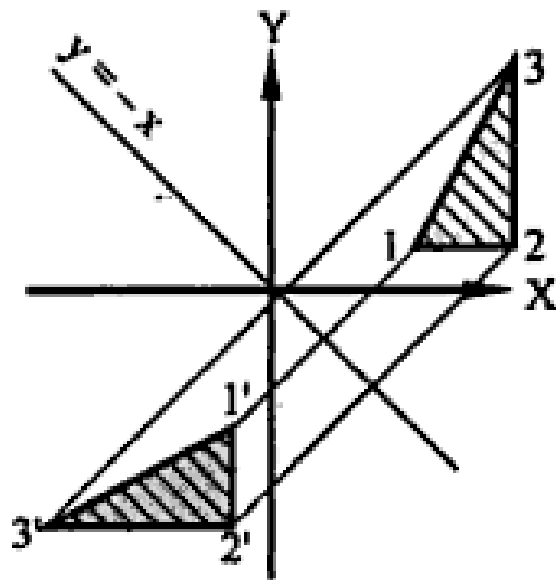
$$M = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

(b) Reflection
about X axis



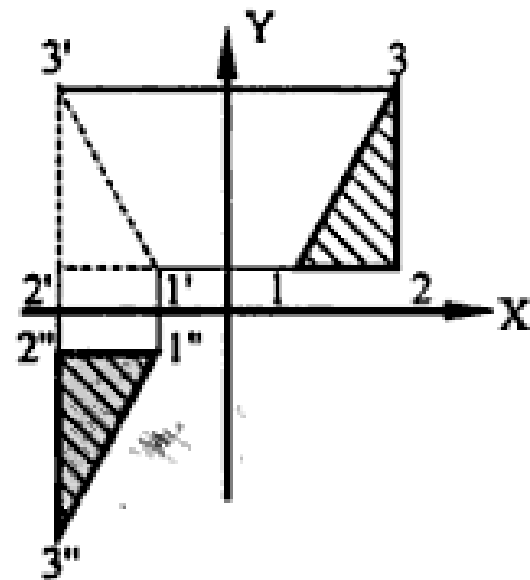
$$M = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

(c) Reflection
about Line $y = x$



$$M = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

**(d) Reflection
about Line $y = -x$**



$$M = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

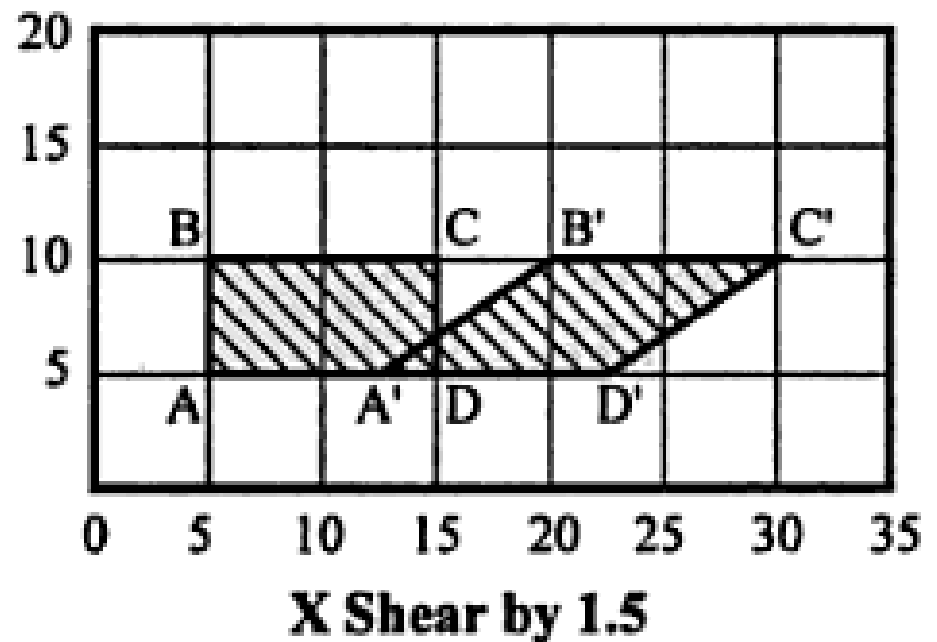
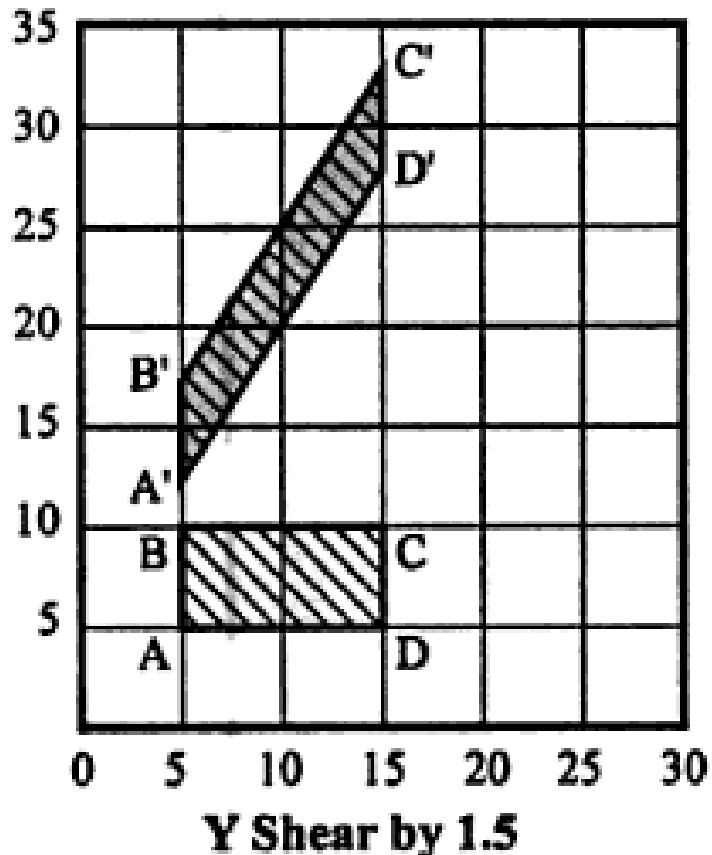
**(e) Reflection
about both X and Y axis**

e) Shear :

This transformation causes the image to slant.

$$\mathbf{P}' = \mathbf{P} \cdot \mathbf{SH}$$

$$\mathbf{SH} = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$$



Matrix Transformation in Homogeneous Coordinate System

a). Translation :

$$\begin{aligned} [x' \ y' \ 1] &= [x \ y \ 1] \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ t_x & t_y & 1 \end{bmatrix} \\ &= [x + t_x \ y + t_y \ 1] \end{aligned}$$

b). Rotation :

$$[x' \ y' \ 1] = [x \ y \ 1] \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

c). Scaling :

$$[x' \ y' \ 1] = [x \ y \ 1] \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Matrix Transformation in Homogeneous Coordinate System

d). Reflection / Mirror :

$$[x' \ y' \ 1] = [x \ y \ 1] \begin{bmatrix} a & c & 0 \\ b & d & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

e). Shear :

For y shear

$$[x' \ y' \ 1] = [x \ y \ 1] \begin{bmatrix} 1 & a & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

For x shear

$$[x' \ y' \ 1] = [x \ y \ 1] \begin{bmatrix} 1 & 0 & 0 \\ b & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Rotation of an Object about an Arbitrary Axis

Step 1: Translate the fixed axis so that it coincides with the z-axis

Step 2: Rotate the object about the axis

Step 3: Translate the fixed axis back to the original position.

Example 3: Rotate the rectangle $(0,0)$, $(2,0)$, $(2, 2)$, $(0, 2)$ shown below, 30° ccw about its centroid and find the new coordinates of the rectangle.



Solution: Centroid of the rectangle is at point (1, 1). We will first translate the centroid to the origin, then rotate the rectangle, and finally, translate the rectangle so that the centroid is restored to its original position.

1. *Translate the centroid to the origin:* The matrix equation for this step is

$$[P^*]_1 = [P] [T_t], \text{ where } [P] = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 2 & 0 & 0 & 1 \\ 2 & 2 & 0 & 1 \\ 0 & 2 & 0 & 1 \end{pmatrix}$$

$$\text{and } [T_t] = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & -1 & 0 & 1 \end{pmatrix}$$

2. **Rotate the Rectangle 30° ccw About the z-axis:** The matrix equation for this step is given as

$[P^*]_2 = [P^*]_1 [T_r]$, where, $[P^*]_1$ is the resultant points matrix obtained in step 1, and $[T_r]$ is the rotation transformation, where $\theta = 30^\circ$ ccw. The transformation matrix is,

$$[T_r]_\theta = \begin{pmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} .866 & .5 & 0 & 0 \\ -.5 & .866 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

3. *Translate the Rectangle so that the Centroid Lies at its Original Position:* The matrix equation for this step is

$[P^*]_3 = [P^*]_2 [T_{-t}]$, where $[T_{-t}]$ is the reverse translation matrix, given as

$$[T_{-t}] = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{pmatrix}$$

Now we can write the entire matrix equation that combines all the three steps outlined above. The equation is,

$$[P^*] = [P] [T_t] [T_r] [T_{-t}]$$

$$[P^*] = [P] [T_t] [T_r] [T_{-t}]$$

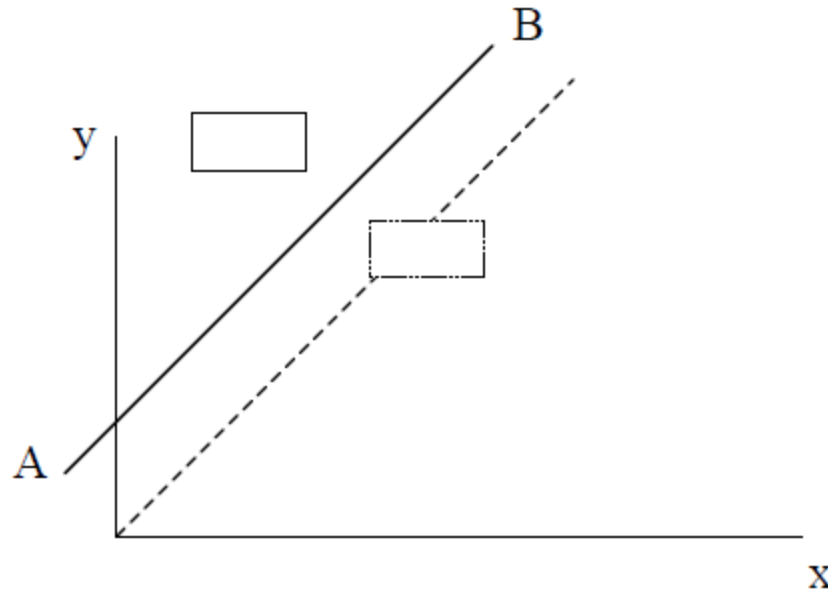
$$[P^*] = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 2 & 0 & 0 & 1 \\ 2 & 2 & 0 & 1 \\ 0 & 2 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos 30^\circ & \sin 30^\circ & 0 & 0 \\ -\sin 30^\circ & \cos 30^\circ & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{pmatrix} \mathbf{x}$$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0.6340 & -0.3660 & 0 & 1 \\ -0.3660 & 1.3660 & 0 & 1 \\ 1.3660 & 2.3660 & 0 & 1 \\ -0.3660 & 1.3660 & 0 & 1 \end{pmatrix}$$

The first two columns represent the new coordinates of the rotated rectangle.

Reflection about an arbitrary plane/line



Step 1: Translate the line AB (i.e., the plane) such that it passes through the origin, as shown by the dashed line.

Step 2: Next, rotate the line about the origin (or the z-axis) such that it coincides with x or y axes (we will use the x-axis).

Step 3: Mirror the rectangle about the x-axis.

Step 4: Rotate the line back to its original orientation.

Step 5: Translate the line back to its original position.

The new points matrix, in terms of the original points matrix and the five transformation matrices is given as,

$$[P^*] = [P] [T_t] [T_r] [T_m] [T_{-r}] [T_{-t}] \quad (\text{Note: A negative sign is used in the subscripts to indicate a reverse transformation}).$$

Where, the subscripts t, r, and m represent the translation, rotation, and mirror operations, respectively.

Combined Transformation

- Most applications require the use of more than one basic transformation to achieve desired results. As, scaling with an arbitrary fixed point involves both scaling and translation. And rotation around a given point, other than the origin involves rotation and translation. Thus these types of transformations are called as combined transformations.
- Example of Scaling with an arbitrary point.
- The transformation sequence is-

$$[P^*] = [P] [T_t] [T_s] [T_{-t}]$$

Where, $[T_t]$ is the translation transformation matrix, for translation of the fixed point to the origin,

$[T_s]$ is the scaling transformation matrix, and

$[T_{-t}]$ is the reverse translation matrix, to restore the fixed point to its original position.

The three transformation matrices $[T_t] [T_s] [T_{-t}]$ can be concatenated to produce a single transformation matrix, which uniformly scales an object while keeping the pivot point fixed.

Thus, the resultant, concatenated transformation matrix for scaling is,

$$[T_s]_R = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -x & -y & 0 & 1 \end{pmatrix} \begin{pmatrix} s & 0 & 0 & 0 \\ 0 & s & 0 & 0 \\ 0 & 0 & s & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ x & y & 0 & 1 \end{pmatrix}$$
$$= \begin{pmatrix} s & 0 & 0 & 0 \\ 0 & s & 0 & 0 \\ 0 & 0 & s & 0 \\ x-sx & y-sy & 0 & 1 \end{pmatrix}$$

The concatenated equation can be used directly instead of the step-by-step matrix solution. This form is preferable when writing a CAD program.

THREE-DIMENSIONAL TRANSFORMATION

A three-dimensional object has a three-dimensional geometry, and therefore, it requires a three-dimensional coordinate transformation. A right-handed coordinate system is used to carry out a 3-D transformation.

A general scaling transformation matrix is given as:

$$[T_s] = \begin{pmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Where, s_x , s_y , s_z are scale factors along x, y, and z-axes, respectively.

Translation Transformation matrix:

$$[T_t] = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ x & y & z & 1 \end{pmatrix}$$

Reflection of any object in 3D is obtained w.r.t. any plane like XY, YZ, ZX etc.

If the reflection is with respect to XY plane, the z-coordinate of the point flips. Thus reflection matrix will be.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotation Transformation

$$[T_{rz}] = \begin{pmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$[T_{rx}] = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$[T_{ry}] = \begin{pmatrix} \cos\theta & 0 & -\sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ \sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$